Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

PROPERTY OF
The University of Michigan Libraries
1817
ARTES SCIENTIA VERITAS

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

THE RHIND MATHEMATICAL PAPYRUS

BRITISH MUSEUM 10057 AND 10058

PHOTOGRAPHIC FACSIMILE, HIEROGLYPHIC TRANSCRIPTION, TRANSLITERATION, LITERAL TRANSLATION, FREE TRANSLATION, MATHEMATICAL COMMENTARY, AND BIBLIOGRAPHY

IN TWO VOLUMES

MATHEMATICAL ASSOCIATION OF AMERICA
OBERLIN, OHIO, U. S. A.
1927

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

THE
RHIND MATHEMATICAL PAPYRUS

BRITISH MUSEUM 10057 AND 10058

VOLUME I

FREE TRANSLATION AND COMMENTARY

BY

ARNOLD BUFFUM CHACE
Chancellor, Brown University

WITH THE ASSISTANCE OF

HENRY PARKER MANNING
Associate Professor of Mathematics, Brown University, Retired

---

BIBLIOGRAPHY OF EGYPTIAN MATHEMATICS

BY

RAYMOND CLARE ARCHIBALD
Professor of Mathematics, Brown University

MATHEMATICAL ASSOCIATION OF AMERICA
OBERLIN, OHIO, U. S. A.
1927

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Mathematics
QA
.P361

WHEN the manuscript of this treatise approached completion, Doctor Chace requested the Mathematical Association of America to undertake its publication, generously providing the means necessary to assure its appearance in a form commensurate with its importance. Upon learning of the desire of the Trustees to contribute to the success of the undertaking in every way possible, Doctor Chace made the further offer to donate to the Association a certain number of copies, adding the suggestion that these be then sold at a price considerably below the cost of publication. This was done with the desire not merely to assist financially an organization of which he was a charter member and to which he has always contributed with great liberality, but to make it possible for the individual and institutional members to purchase the work much more reasonably than would have been possible had it been simply a commercial product. The proceeds from its sale are to constitute an endowment of the Association known as the ARNOLD BUFFUM CHACE FUND.

The Association takes pleasure in making these facts known, and at the same time in expressing its high appreciation of this notable work of a well-trained scholar in mathematics, of the interest which he has always shown in the cultivation of the science in this country, and of his generosity in connection with this publication.

RAYMOND CLARE ARCHIBALD
DAVID EUGENE SMITH
Publication Committee
on behalf of
THE MATHEMATICAL ASSOCIATION OF AMERICA

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Math
Wahr
8-15-57
75772
add. cop.
2v.

127
3654

To the
Memory of my Wife
ELIZA GREENE CHACE
WITHOUT WHOSE ENTHUSIASTIC ENCOURAGEMENT
AND ASSISTANCE I SHOULD NEVER HAVE
COMPLETED THIS WORK

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

PREFACE

DURING the past fifteen years the preparation of the work now presented to the public has occupied my leisure hours, in an otherwise active business life. In 1910 my wife and I made a trip to Egypt and became much interested in its monuments and literature. The British Museum copies of the Book of the Dead and of the Rhind Mathematical Papyrus were acquired two years later and I soon commenced an intensive study of the latter. This was inspired by my interest in the history of mathematics and in the ancient Egyptian civilization.

The pleasure derived from such study has been great and especially have I enjoyed the intimate knowledge which I thereby gained regarding the reasoning power of those ancient people. I venture to suggest that if one were to ask for that single attribute of the human intellect which would most clearly indicate the degree of civilization of a race, the answer would be, the power of close reasoning, and that this power could be best determined in a general way by the mathematical skill which members of the race displayed. Judged by this standard the Egyptians of the nineteenth century before Christ had a high degree of civilization.

The great advance in our present knowledge of the Egyptian language would of itself have justified the publication in 1923 of Professor Peet's scholarly work on the Rhind papyrus, marking such a notable advance over the remarkable translation and commentary by Eisenlohr forty-six years earlier. It seemed to me, however, that there was room for yet another work on this subject intended for mathematicians and the general public, rather than for Egyptologists who will find philological matters fully discussed by Professor Peet. In my volumes the new contributions may be summarized as follows:

1. A mathematical commentary, in the preparation of which I have been greatly assisted by Doctor Manning who, while associated with me in this work during the past seven years, has displayed scholarly insight and exactness;

2. The very elaborate and notable critical Bibliography of Egyptian Mathematics contributed by Professor Archibald and including refer-

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

ence to the literature of over fifty documents dating from 3500 B.C. to about 1000 A.D.;

3. The photographic reproduction of the Rhind mathematical papyrus;

4. The presentation of the original hieratic text, the hieroglyphic transcription, the transliteration, and the literal translation in such a manner that even the reader untutored in Egyptian may pick out in the original those signs corresponding to the phrases in the free translation. Particularly in this part of the work, during the past two years, have I regarded myself as fortunate in securing the coöperation of Doctor Bull. Without such aid I could scarcely have hoped to escape severer strictures than will now be meted out to me by Egyptologists.

My grateful thanks are due to the Trustees of the British Museum for their courtesy in supplying me with photographs of the Rhind Papyrus, and in granting me permission to reproduce them. In connection with this arrangement, the kindness displayed and information furnished by Mr. S. R. K. Glanville, of the staff in the Department of Egyptian and Assyrian Antiquities of the British Museum, were highly appreciated.

The New York Historical Society has been most generous in placing its fragments of the Rhind papyrus at our disposal for study, arrangement, and photographing, and in permitting the publication of the results.

To Professor D. E. Smith, of Columbia University, who, with Professor Archibald, was appointed by the Mathematical Association of America to assist in seeing my work through the press, I am greatly indebted for encouragement and valuable suggestions.

I wish also to express thanks for assistance to Professor T. Eric Peet, of the University of Liverpool, to Dr. G. A. Reisner, of the Boston Museum of Fine Arts, and to Doctor Caroline Ransom Williams of Toledo, Ohio.

I hope that the distribution of my work by the Mathematical Association of America may lead to a deeper interest in the contributions of the golden period of Egyptian science.

A. B. C.

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

CONTENTS

PREFACE

INTRODUCTION

EGYPTIAN ARITHMETIC 3–30
Numbers and operations; Special processes emphasized in the Rhind papyrus; Table of the division of 2 by odd numbers; Division of the numbers 1–9 by 10; Multiplication by certain fractional expressions; Divisions by a fractional number; Separation in given proportions; Other arithmetical problems; Arithmetical and geometrical progressions.

EGYPTIAN MEASURES 31–34
Measures of capacity; Measures of area and length.

EGYPTIAN GEOMETRY 35–38
Determination of volumes; Determination of areas; Pyramids, *seked*.

THE METHODS AND AIMS OF THE EGYPTIAN MATHEMATICIAN . . 39–43
His methods; His mistakes; His theoretical interest.

NOTE ON THE EGYPTIAN CALENDAR AND EGYPTIAN CHRONOLOGY 43–46

NOTE ABOUT WORDS FOR GRAIN AND OTHER KINDS OF FOOD . . 46–47

FREE TRANSLATION AND COMMENTARY

TITLE, DATE AND SCRIBE 49

CHAPTER I, EGYPTIAN ARITHMETIC 50–85
I. Table of the division of 2 by odd numbers; II. Problems 1–6: Table of the division of the numbers 1–9 by 10; III. 7–20: Multiplication by certain fractional expressions; IV. 21–23: Problems in completion; V. 24–29: '*Aha*' or quantity problems; VI. 30–34: Division by a fractional expression; VII. 35–38: Division of a *hekat;* VIII. 39–40: Division of loaves. Arithmetical progression.

CHAPTER II, EGYPTIAN GEOMETRY 86–99
I. Problems 41–46: Volumes; II. 47: Division of 100 *hekat;* III. 48–55: Areas; IV. 56–60: Pyramids, *seked*.

CHAPTER III, MISCELLANEOUS PROBLEMS 100–119
Problems 61–84; Numbers 85–87.

BIBLIOGRAPHY

PREFATORY NOTE, SURVEY OF LITERATURE, CHRONOLGICAL LIST OF DOCUMENTS DISCUSSED, INDEX OF AUTHORS, SUBJECT INDEX 121–206

GENERAL INDEX TO VOLUME I . . 207–210

Digitized by Google

Original from UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

INTRODUCTION[1]

The Rhind Mathematical Papyrus was found at Thebes in the ruins of a small building near the Ramesseum. It was purchased in 1858 by A. Henry Rhind and after his death came into the possession of the British Museum. It is a copy that was made by a scribe named A'h-mosè and begins with a rather ambitious statement of what the author proposes to do. The date at which this copy was made is indicated in the old Egyptian method by the year of a certain pharaoh named 'A-user-Rê',[2] who has been identified as one of the Hyksos dynasty, living approximately 1650 B.C. The scribe further says that it is in likeness to older writings, of the time of king Ne-ma'et-Rê' (Amen-em-hêt III) who reigned from 1849 to 1801 B.C.[3] The papyrus is written in hieratic[4] and originally it was a single roll nearly 18 feet long and about 13 inches high, but it came to the British Museum broken apart, and with a number of fragments missing, the most important of which have been found in the possession of the New York Historical Society. A lithographic facsimile of the papyrus was published by the British Museum in 1898, but the photographs which I have reproduced in the second volume of this work represent it as it appears to-day. The fragments found in New York were brought over

[1] In referring to a publication listed in the Bibliography, I shall generally give with the name the year of publication, and if there is more than one publication listed under the same name in a given year, a second number will indicate the one referred to. But all references to Peet will be to his edition of the Rhind Papyrus and all references to Gunn to his review of Peet, both listed under Peet, 1923, 2, and only the number of the page or plate will be given. Nearly all references to Griffith will be to the series of articles listed under 1891 and 1892, and in these references only the numbers of the volume and page will be given.

[2] For the pronunciation of Egyptian words see the note at the beginning of the Literal Translation. Only consonants were written during most of Egyptian history, but the vowels supplied by us are sometimes inferred from those of a late period. Each vowel indicates a syllable. Hyphens are used in compound words. The sign ' represents a consonant which we need not pronounce. Thus the god's name "Rê'" at the end of a royal name may be pronounced like the English word "ray."

[3] For chronology see J. H. Breasted, *A History of Egypt*, second edition, New York, 1911, page 597.

[4] Egyptian writings that have come down to us are in four forms: (1) hieroglyphic, used in formal writings such as temple and tomb inscriptions; (2) hieratic, a more cursive form, both of these forms appearing at the beginning of Egyptian history; (3) demotic, which appeared about 800 B.C., and replaced the hieratic for everyday use; and (4) Coptic, in which the Egyptian language was written with the Greek alphabet. This last form arose in the third century A.D. and survives to-day in the liturgy of the Coptic church.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

by Edwin Smith with his papyrus (see J. H. Breasted, *The Edwin Smith Papyrus*, Oxford, in press). This was discovered in 1922 when Professor Newberry, the English Egyptologist, suspected that they were some of the missing pieces and brought tracings of them to Professor Peet (see Bibliography under Ahmes, 1898). Peet publishes a plate (Plate E) on which with much skill he has arranged nearly all of the writing on them, and I have copied this writing on my plates, placing it with the problems to which it belongs, and including its translation with the translation of those problems. I have also put along with the photographs of the papyrus a photograph of the fragments placed in their proper positions, except for a few small pieces whose positions have not yet been determined.

The rest after the introductory paragraph may be divided into three parts,—an arithmetical part, a geometrical part, and a part containing a collection of miscellaneous arithmetical problems. In my judgment the arithmetical part is more exact than the geometrical part, and exhibits more reasoning power. The geometrical part reveals considerable knowledge of geometrical facts useful for the determination of volumes, areas, and line relations. The section of miscellaneous problems is interesting because it gives us some knowledge of Egyptian customs, their method of trading, of raising taxes, of feeding animals, and of fixing the comparative values of different foods and drinks by the amount that can be made from a unit of material. It contains also various methods of solution, some of which have come down to the present day and are found in our arithmetics.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

EGYPTIAN ARITHMETIC

Numbers and Operations

We have very few traces of Egyptian arithmetic from a date earlier than the original sources of the Rhind papyrus. But before this date there was a long period of Egyptian civilization, and we may well believe that for more than a thousand years there had been a slow development of the elaborate system before us.

The Egyptians apparently conceived of two kinds of numbers, an ascending series from 1 to 1,000,000 of numbers that we call integers, and a corresponding descending series consisting of ⅔ and reciprocal numbers or unit fractions.[1]

For integers they had:

1. A well-defined decimal notation, without, however, the positional device that renders our modern notation so simple;

2. A thorough understanding of the four arithmetical operations, —addition, subtraction, multiplication, and division.

Addition and subtraction were easily accomplished. Direct multiplication by integers was generally confined to the multipliers 2 and 10; that is, in cases where the process was explicit, and it may be presumed that in all cases[2] they multiplied by repeated doubling, or occasionally multiplying by 10, and adding the products formed from multipliers that would make up the given multiplier. This is not very different from our modern way, where we multiply by the unit figure of our multiplier, and then by the ten figure, and so on, and then add the re-

[1] These are the same as our fractions with numerator 1, and it will be convenient, for the most part, to speak of them as fractions. They are sometimes called fundamental fractions (Stammbrüchen). When the Egyptians wish to express a number that is not a single unit fraction, they use a combination of such fractions. Thus for our 1¾, they say 1 ½ ¼; for 2 times ⅛, ⅓ ⅛, and so on. Such an expression we may speak of as a fractional number or fractional expression; it was understood that the fractions, or the whole number and fractions, were to be added, and I shall always write them as do the Egyptians without any sign of addition, just as we write 1½. A given number could be expressed as a sum of unit fractions, or as the sum of a whole number and such fractions, in an infinite number of ways, and sometimes the expression for a given number is varied, but the fractions of an expression must all be different fractions, and very rarely were they allowed to have a sum equal to or greater than 1.

[2] Many times the details of the multiplication are omitted in the papyrus and we cannot say with certainty what was done in every case, but it is fair to make inferences if we take into account all of the cases in which the details are given.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

sults. In most of the multiplications all through the papyrus the author checks those multipliers that he is to use.[1] Division was performed by successive multiplication of the divisor until the dividend was obtained.[2]

For the numbers of the descending series they had:

1. A notation which for reciprocal numbers was nearly like their notation for integers, these numbers being distinguished from integers by having a dot in hieratic and the sign in hieroglyphic written over them, except that the first three had special signs in hieratic, and the first, $\frac{1}{2}$, in hieroglyphic also;

2. Special devices for addition and subtraction, because it was necessary to express the result using only integers and different unit fractions (see page 3, footnote 1); and for multiplication, because, apparently the only fractions that they could use as direct multipliers were $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{10}$.

Addition and subtraction will be explained below. In multiplication they generally took $\frac{2}{3}$ and then halved to get $\frac{1}{3}$,[3] and they took $\frac{1}{10}$

[1] For the multiplication of 19 by 6 the Egyptians would say,

	1	19
	\2	38
	\4	76
Total,	6	114,

the two multipliers which make up 6 being checked; and for 12 times 23,

	1	23
	\10	230
	\2	46
Total	12	276.

See page 52.

In Problem 79, 2801 is multiplied by 7 in the same way,

1	2801
2	5602
4	11204
Total	19607.

[2] Dr. Reisner tells me that when the modern Egyptian peasant wishes to divide things of some sort among a number of persons he first distributes a certain number, say 5, to each, and then perhaps 2, and so on.

[3] Peet, page 20. Examples are in Problems 25, 29, 32, 38, 42, 43, and 67; besides 8 and 16–20, and many times in the table at the beginning. $\frac{1}{3}$ is given without any $\frac{2}{3}$ in Problems 28, 29, 32, 39, 42 (line 8 of the multiplication), and several times in 35–37. We may suppose that in these cases the author did not intend to put in the work, but only the result. Gunn says (page 125) that "The Egyptian, like everyone else, had ultimately no way of arriving at $\frac{2}{3}$ but via $\frac{1}{3}$." This is not quite true, for, whatever $\frac{2}{3}$ meant to him, he knew for a fact that it is the reciprocal of $1\frac{1}{2}$ (see, for example, page 75, footnote, also Gunn, page 129), and it was possible for him to make a table for $1\frac{1}{2}$ and read it backwards, just as he sometimes reads backwards the table for the division of 2 by odd numbers (page 20, footnote 2). A number not found in his table (for example, a number that is not a multiple of 3) could be obtained by putting other numbers together, using $\frac{2}{3}$ for $\frac{2}{3}$ of 1, and $1\frac{1}{3}$ for $\frac{2}{3}$ of 2. This is explained by Gunn himself on page 125.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

and then doubled to get $\frac{1}{5}$.[1] The reciprocals of other numbers were sometimes used as multipliers, when the numbers themselves had appeared in previous multiplications that could be transformed into the multiplications desired.[2] In particular, they often used the fact that the reciprocal of a number multiplying the number itself gives 1.

Egyptian division might be described as a second kind of multiplication, where the multiplicand and product were given to find the multiplier.[3] In the first kind of multiplication, the multiplier, being given, can be made up as a combination of the multipliers that were generally used, and the corresponding combination of products would be the required product. When it was the product that was given along with the multiplicand, various multipliers would be tried, 2, 10, and combinations of these numbers, or combinations of the fractions $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{10}$, and from the products thus obtained the Egyptians would endeavor to make up the entire given product.[4] When they succeeded in doing this the corresponding combination of multipliers would be the required multiplier. But they were not always able to get the given product at once in this way, and in such cases the complete solution of the problem involved three steps: (a) multiplications from which selected products would make a sum less than the required product but nearly equal to it; (b) determination of the remainder that must be added to this sum to make the complete product; and (c) determination of the multiplier or multipliers necessary to produce this remainder. The multipliers used in the first and third steps made up the required multiplier. The second step was called completion and will be explained below. For the third step they had

[1] Peet, page 19, footnote 1. Examples occur in Problems 21, 22, and 35.

[2] Thus in Problem 34, 4 times $1\frac{1}{2}\frac{1}{4}$ is equal to 7, and therefore $\frac{1}{7}$ of $1\frac{1}{2}\frac{1}{4}$ is equal to $\frac{1}{4}$. There are some striking illustrations in Problem 38. See also Problems 33 and 70.

[3] The usual form of expression was "Operate on (Make, Make the operation on) . . . for the finding of" . . . (Problems 21, 22, 26, 30, 56, etc., see Literal Translation). In the table at the beginning of the papyrus the Egyptian says several times, "Call 2 out of" some number, and this form of expression is used also in Problems 35, 37, 38, 63, 66, and 67. But the point of view is the same, the idea being to get 2 as a result of operating on the number (see Gunn, page 124), and the process is the same as when the former expression is used. In Problems 1–6 and in 65 the author speaks of "making" a certain number of loaves "for" a certain number of men, and in 54 and 55 he uses the expression "Take away a certain area from a certain number of fields," that is, by taking an equal amount from each field (see page 96), but these are forms of expression used for the statement of the problems and not technical expressions for any kind of division. The expression "Call 2 out of" . . . should be particularly noticed, for we should naturally take the numbers the other way. We say that "3 will go into 6 twice" and we can imagine some one saying "Call 3 out of 6," but the Egyptian says "Call 6 out of 3."

[4] This happens in Problems 34–38, also in 69. See notes to 34 and 37.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

a definite process which they generally used. The remainder, being a small number, would consist of one or more reciprocal numbers. For one of these numbers the third step may be expressed by the rule: To get the multiplier that will produce the reciprocal of a given whole number as a product multiply the multiplicand by the number itself and take the reciprocal of the result of this multiplication.[1] If, for example, we wish to multiply 17 so as to get $\frac{1}{3}$, we take 3 times 17, which is 51, and then we can say that $\frac{1}{51}$ of 17 equals $\frac{1}{3}$.[2] See page 17. In both kinds of multiplication the product is formed by adding certain multiples of the multiplicand, and so the multiplicand and product, instead of being mere numbers, may be things of some kind, while the multiplier must always be a mere number.

On the other hand, the Egyptians could not solve directly the problem of finding the multiplicand when the multiplier and product were given. At the present time we scarcely think of this as a new problem; in fact, we never notice the distinction between multiplier and multiplicand, often saying that we multiply two numbers together. Even when some of our numbers represent things, we always think of them as mere numbers while we are multiplying or dividing. The Egyptian method emphasized this distinction, even when all the quantities were mere numbers. However, it was known that the product then would be the same if the multiplier and the multiplicand were interchanged, and sometimes, even in the first kind of multiplication, they were interchanged, simply because this would make the multiplication easier.[3] When the multiplier and product were given the Egyptians usually made this interchange, thinking of all their quantities as mere numbers;[4] that is, they changed their problem into one in which the given multiplier became the multiplicand and the required multiplicand was to be obtained as a multiplier, only at the end to be interpreted as the problem required. Problems 30–34 are particularly good illustrations of this method; also some of the *pefsu* problems (69–78), where the notation sometimes

[1] To use Gunn's expression (pages 129 and 134) we might say, "Multiply by the number itself and dot the result."

[2] This rule is illustrated in Problems 34 and 70, and all through the table for the division of 2 by odd numbers. The formation of this table is for each number a multiplication problem of the second kind, and, except for the cases of 3, 35, and 91, the three steps of the process are always present.

[3] This is done in the third step in the solutions of Problems 24–27.

[4] In Problem 55, instead of regarding the number of the given product as a mere number, the author regards the given multiplier, now taken as a multiplicand, as representing things of the same kind as the product. See notes to that problem.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

tells us which numbers are to be regarded as mere numbers, and which represent things of some kind.[1]

But sometimes they wished to keep in mind the nature of the quantities in the problem, and then, instead of solving it as a problem in multiplication, they used a process of trial that has been called "false position" and will be explained below.

The Egyptians commenced their descending series with the quantity ⅔, and it is interesting to note that in the papyrus of Akhmîm (Baillet, 1892), written in Greek about 600 A.D., the Egyptian system of unit fractions is still used, and with the same apparent exception of ⅔. Griffith (volume 16, page 168), regarding the short line in the hieroglyphic sign as a half of the longer line, saw in it a symbolic expression for 1 divided by 1½, but it is known now that at first the two strokes were of the same length, so that this idea applies, if at all, only to later Egyptian times.[2]

Special Processes Emphasized in the Rhind Papyrus

I will mention three special processes:

1. A method in which a fractional expression is applied to some particular number;
2. The solution of problems by false position;
3. A process of completion used for determining the amount to be added to an approximation to a given number in order to get the number.

It may be well at this point to explain these processes somewhat carefully.

1. If we wish to add two groups of fractions, say ⅓ ⅕ and ⅕ ⅐, we reduce them to the common denominator 105. Then as many times as each denominator has to be multiplied to produce the common denominator, so many times must the numerator of that fraction be multiplied to produce the new numerator, and these new numerators,

[1] In Problem 64 also this is shown by the notation. See page 30.

[2] Sethe (1916, pages 91 ff.) finds some traces among the Egyptians, as well as with other ancient peoples, of a system of complementary fractions ⅔, ¾, ⅘, . . . They called these expressions "the 2 parts," "the 3 parts," etc., the 2 parts being what is left after taking away ⅓, and so with the others. Thus in the Hebrew Scriptures we read (Genesis 47, 24) that Joseph said to the Egyptians when they sold themselves and their land for food, "Ye shall give the fifth part unto Pharaoh and four parts shall be your own." ⅘ is the only fraction of this system to be found in the Rhind papyrus. In the papyrus of Akhmîm the sign for ⅔ is always preceded by the singular article, τὸ, as if it were one part (Baillet, page 19).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

added together, will give the numerator of the required result, in this case 82, so that the sum of the fractions will be $82\frac{1}{105}$.

Suppose that an Egyptian had the same problem. Since a fractional expression with the Egyptians was a sum of fractions, or of a whole number and fractions, we might think that he could add the two groups by writing down the four fractions together, as $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{7}$, but one law of the Egyptian fraction-system required that the fractions in an expression should all be different. To add the fractions, he would think of a number or of a group of things of some kind, to which he could suppose them to refer, a number or group such that each fraction of it will be a whole number or at least one that he can add to other numbers of the same kind. He might take 105, say 105 loaves, and suppose that he is to receive $\frac{1}{3}$ $\frac{1}{5}$ of them and also $\frac{1}{5}$ $\frac{1}{7}$ of them. He would say that $\frac{1}{3}$ of 105 loaves is 35 loaves, and $\frac{1}{5}$ is 21, making 56 loaves in the first group, and that $\frac{1}{5}$ is 21 loaves and $\frac{1}{7}$, 15, making 36 loaves in the second group, and that he will receive in all 82 loaves as his part of 105. But he could not say that he will have $82\frac{1}{105}$ of them all. All that he can do is to seek to express the 82 loaves as an aggregate of parts—different parts—of 105 loaves.

To find what parts of 105 loaves will make 82, he would take fractional multipliers and seek to multiply 105 so as to get 82. This would be a multiplication of the second kind (see page 5). He might say[1]

1	105
\ $\frac{2}{3}$	70
$\frac{1}{3}$	35
$\frac{1}{30}$	3 $\frac{1}{2}$
\ $\frac{1}{15}$	7
$\frac{1}{10}$	10 $\frac{1}{2}$
$\frac{1}{5}$	21
\ $\frac{1}{21}$	5
Total $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$	

But he might be able without going through a formal multiplication to separate 82 loaves into groups which he recognizes as parts of 105. Thus he might take 70, which is $\frac{2}{3}$ of 105, leaving 12, and these he could separate into 7 and 5, which are $\frac{1}{15}$ and $\frac{1}{21}$ of 105.

If now he finds in some way that 82 of the loaves will be $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$ of the 105 loaves, then he will know that $\frac{1}{3}$ $\frac{1}{5}$ plus $\frac{1}{5}$ $\frac{1}{7}$ make $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$ when applied to 105 loaves, and he will conclude that they always make $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$.

[1] The Rhind Papyrus contains multiplications for this purpose in Problems 21 and 22.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

This example illustrates what I mean by "applying a fractional expression to some particular number."

Sometimes the Egyptian wishes to use this method with an expression involving a whole number as well as fractions. Thus in the first multiplication of Problem 32 of the Rhind Papyrus the author desires to select partial products that will add up to 2. Regarding all of them as referring to a group of 144 things of some kind, assuming perhaps that he has several such groups, he finds the values of these partial products as applied to 144 and seeks to make up in this way the number of things in two groups, that is, 288. The multiplicand, 1 ⅓ ¼, makes one whole group and ⅓ and ¼ of another, or 228 things in all. The next expression, 1 $\frac{1}{18}$, makes 152 things, and so for the others.

The examples in the papyrus seem to indicate that there was no definite rule for determining what number would be most convenient to take. Naturally it would be desirable to take a number for which it was easy to find the parts indicated by the given fractions. The number taken is often the largest number whose reciprocal is among them. Thus in many cases the parts are not all whole numbers, but are whole numbers and simple fractions. In Problem 33 one of the parts is [1] 13 ½ ¼ $\frac{1}{14}$ $\frac{1}{28}$.

This method of applying an expression to a particular number was continued for many generations and is found in the papyrus of Akhmîm already mentioned. Rodet (1882, page 37) calls the number taken a *bloc extractif* out of which these fractions are drawn. His idea may be somewhat like that which I have expressed, but his explanation seems a little abstract, and the use of a technical term, while very convenient, makes the process seem more improbable for so ancient a people. Hultsch (1895) has a theory still more formidable. He supposes that the Egyptian introduces an auxiliary unit. In the example above he would say that the Egyptian, for purposes of addition, makes $\frac{1}{105}$ a new unit, in terms of which the given fractions become whole numbers that can be readily added. Peet (page 18) considers that the question is merely one of notation, that the Egyptian really did have the conception of a fraction with numerator greater than 1 but no notation for

[1] The problems in which there is evidence of the use of this method are 7–20, 21–23, 31–34, 36–38, and 76, and the case of 2 divided by 35 in the first table. In Problem 36 this method is used for a division process, and in the case of 2 divided by 35 for a multiplication, but generally it is used for the addition or subtraction of quantities. In all cases in the papyrus the only fractions that appear as parts of the number taken are ⅔, ½, and fractions whose denominators are powers of 2, except in Problem 33.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

such fractions.[1] In fact, he claims that the theories of Rodet and Hultsch are essentially the same as the modern theory, "bloc extractif" and "auxiliary unit" being only other names for common denominator. Perhaps he would say that my "particular number" is only another name for common denominator, but it does not seem so to me. The idea of taking a number, solving the problem for this number, and assuming that the result so obtained holds true for any number, is exactly what the boy in school is inclined to do for all sorts of problems, and what the author of our handbook does in much of his work. In fact, he always takes a particular number, and when he has solved a problem he does not hesitate to take a more complicated one of the same kind and use the same method. I do not think that the idea of a common denominator or of a fraction with numerator greater than 1 is involved in the theory as I have explained it, even though the number used is the same number as our common denominator, and some of the numerical work is the same as when a common denominator is used.[2] When I say that ⅓ of 105 is 35 and ⅕ is 21, together making 56 things, there is no suggestion that ⅓ is equal to $^{35}/_{105}$ and ⅕ to $^{21}/_{105}$.

2. The process of false position (*positio falsa*) consists in assuming a numerical answer and then by performing the operations of the problem getting a number which can be compared with a given number, the true answer having the same relation to the assumed answer that the given number has to the number thus obtained.[3] In this method we see one point of distinction between arithmetic and algebra. In algebra a letter x represents exactly the answer, and its value is obtained by solving an equation. In this method we take a number

[1] Sethe, in his review of Peet (See Bibliography under Peet, 1923, 2) expresses emphatic dissent from this view. He says "Wherever there was the notion there would have been already established the word and the sign. Indeed, it would have been entirely incomprehensible that the Egyptians should have written ½ ¼ or ½ $^1/_{12}$ if they did not also read these words, and that the Greeks, Romans, and Arabs should have held on so long to the really circuitous reckoning with fundamental fractions if the so much simpler method of mixed fractions [fractions with numerator greater than 1] had already so early been known." See also Sethe, 1916, pages 60 and 62.

[2] It is just as the modern schoolboy in solving simultaneous equations by different methods will have the same numbers to multiply or divide although the theories of the different methods are different.

[3] We might say that this is proportion, but the words "false position," with their suggestion of the days when we had only arithmetic in our schools, give us a better realization of the Egyptian point of view. The rule was called by the Arabs, *ḥisab al-Khataayn*, and hence it appears in medieval Europe as *Elchataym*. It was also known by such names as *Regula falsi*, *Regula positionum*, *False positie*. See D. E. Smith, 1923, volume 2, pages 437–438.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

which (usually) is not the answer, and get the answer by finding its relation to this number.[1]

False position[2] is used in a number of multiplication problems in which the multiplier and product are given and it is required to find the multiplicand, because by this method it is possible to keep in mind the nature of the quantities involved. Problems 24–27 exhibit the method very clearly although the required multiplicand in these problems is nothing more definite than "a quantity," and the given product, which must be a quantity of the same kind, is given simply as a number. It is instructive to compare this group of problems with Problems 30–34, which interchange multiplicand and multiplier. The problems of the two groups are problems of the same kind and all of them involve only "a quantity."

I think, however, that the *hekat* problems, 35, 37, and 38, show more clearly that this process enables us to keep in mind the nature of the quantities involved, and I will give in detail the reasoning of one of these problems as I understand it.

In Problem 35 a vessel filled $3\frac{1}{3}$ times with grain will make 1 *hekat* (about half a peck). The vessel contains so much grain and the *hekat* is so much grain. $3\frac{1}{3}$ times the former makes the latter. Here we have given the multiplier, $3\frac{1}{3}$, and the product, 1 *hekat* of grain, to find the multiplicand, which will be a certain portion of a *hekat* of grain. Assume a vessel that itself contains 1 *hekat;* this filled $3\frac{1}{3}$ times will give $3\frac{1}{3}$ *hekat*. Then the one that we have must bear the same relation to the one assumed, that the amount of grain that fills the former $3\frac{1}{3}$ times bears to the amount of grain that fills the latter $3\frac{1}{3}$ times; that is, that one *hekat* bears to $3\frac{1}{3}$ *hekat*. Thus we have to find the relation of one *hekat* to $3\frac{1}{3}$ *hekat*. Making $3\frac{1}{3}$ *hekat* the multiplicand and one *hekat* the product we get for the multiplier the number $\frac{1}{3}$ $\frac{1}{10}$. The amount of grain that the given vessel holds is then $\frac{1}{3}$ $\frac{1}{10}$ times the amount of grain that the one assumed holds; that is, it is $\frac{1}{3}$ $\frac{1}{10}$ times 1 *hekat*.[3]

[1] We might notice a point of similarity in this method and the method of applying fractions to a particular number, that in both methods we first assume a particular number or quantity. In Problem 76 the same assumed quantity, namely, 30 loaves, is used for both purposes. First the writer takes $\frac{1}{20}$ and $\frac{1}{30}$ as parts of 30 to show that the sum of these fractions is $\frac{1}{12}$, and then he takes 30 loaves as the basis of a false position process for determining the answer to the problem.

[2] False position is used in Problems 24–27, 40 and 76. It is used also in Problems 28, 29, and 35–38 as I have explained them.

[3] It is a little confusing to have 1 *hekat* come into the solution in two ways.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The process of false position can be applied to other problems besides those of division. This is well illustrated in Problem 40, and I will give an explanation of it at this point.

This problem is to divide 100 loaves among five men in such a way that the shares received shall be in arithmetical progression and that ⅐ of the sum of the largest three shares shall be equal to the sum of the smallest two. The papyrus does not say that the shares shall be in arithmetical progression, but the solution shows that this is the intention. After stating the problem it simply says that the difference is 5½ without explaining the method of obtaining this number, nor does it say that the smallest share shall be 1 nor even mention the smallest share. We may suppose that it is so natural to think of 1 as the smallest number that it does not occur to the author to mention it, even when he finally gets an answer in which the smallest number is not 1. Now to get the common difference he may have assumed first a common difference 1. The terms of the progression would be 1, 2, 3, 4, 5, the sum of the smallest two would be 3, and ⅐ of the sum of the largest three would be 1 ½ ⅐ ¼, a difference between the two sides of 1 ¼ ½₈. If he then assumed a common difference of 2, and hence the progression 1, 3, 5, 7, 9, he would find the sum of the smallest two terms to be 4, and ⅐ of the sum of the largest three, 3, making a difference between the two sides of 1. In other words, for each increase of 1 in the assumed common difference he would find the inequality between the two sides reduced by ¼ ½₈. To make the two sides equal he must multiply his increase 1 by as many times as ¼ ½₈ is contained in 1 ¼ ½₈, which is 4½, and this added to the first assumed difference 1 makes 5½ as the true common difference. This process of reasoning is exactly in accordance with Egyptian methods.[1]

To determine if the progression which he has obtained fulfills the second requirement of the problem, namely, that the number of loaves shall be 100, he proceeds as follows: Having the progression 1, 6½, 12, 17½, 23, he finds that the sum is 60 instead of 100. Therefore it is necessary to multiply by the factor that will produce 100 from 60,

[1] Peet (page 78) seems to think that this problem is like some of the inverse problems of the papyrus (see below, page 35); that the author, having a series with a common difference of 5½, noticed in this series the relation between the sum of the smallest two terms and the sum of the largest three, and made up an inverse problem with 100 for sum instead of 60. But it is as difficult to see how he should discover this relation in a series that he has before him, as it is to see how he should find the common difference, 5½, when he has given this relation.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

namely, by $1\frac{2}{3}$. The true smallest term will then be $1\frac{2}{3}$ and we shall have the true division of loaves

$$1\ \tfrac{2}{3}\quad 10\ \tfrac{2}{3}\ \tfrac{1}{6}\quad 20\quad 29\ \tfrac{1}{6}\quad 38\ \tfrac{1}{3}.$$

This problem is notable because, while the Egyptian mathematician did not have such a thing as simultaneous equations, yet by methods which were within his knowledge he could sometimes obtain the result when there were two unknown quantities, as illustrated here.

The process of false position was employed by Diophantus and by Arabic writers, and has continued in use even down to our own day, being found in older arithmetics;[1] it was probably dropped from use about the time that algebra began to be generally taught in our schools.

3. The third of the special processes was a process of completion, used for determining the amount to be added when we have very nearly a given number. It was used especially in the second kind of multiplication as explained on page 5. Problems 21–23 are given as problems in completion and show the method of solving such problems. Thus in Problem 21 we have to complete $\frac{2}{3}\ \frac{1}{15}$ to 1. To determine the answer, these fractions are applied to 15. $\frac{2}{3}\ \frac{1}{15}$ of 15 make 11 and require 4 more to make the whole of 15. 4 is the same as $\frac{1}{3}\ \frac{1}{15}$ of 15, and therefore $\frac{1}{3}\ \frac{1}{15}$ is what is required to complete the given fractions to 1. The three problems are all solved in this way.[2]

I may add that there is another group of problems, 7–20, before which the author puts the words, "Example of making complete," but probably by mistake as these problems are all simple multiplications. See page 23, footnote 2.

Table of the Division of 2 by Odd Numbers

Inasmuch as the Egyptian mathematician performed his multiplications mostly by doubling or halving it was necessary that he should be able to double any numerical quantity, a reciprocal as well as a whole number. This could easily be done with the reciprocal of an even number, but for odd numbers it was convenient to have a special table. To determine the double of a reciprocal number was the same as

[1] See, for example, Benjamin Greenleaf, *The National Arithmetic*, revised, Boston, 1853, page 286. He calls it, "Single Position."

[2] In the formation of the table for the division of 2 by odd numbers completion is one of the processes for all but three of the numbers. In simple cases the completion may not have required elaborate calculations, but whenever the author did not know at once what were the fractions required, the procedure illustrated in Problems 21–23 would be used. In six cases the word "remainder" is put in, indicating the result of a completion process. These are for 2 divided by 17, 19, 23, 37, 41, and 53. Completion is also used in Problems 30–34.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

to find by what the number itself must be multiplied to get 2, and so the problem becomes a multiplication problem of the second kind, a problem in which the given odd number is the multiplicand and 2 is the product. In the first eight pages of the papyrus these relations are obtained for all the odd numbers from 3 to 101. Theoretically the result can be put in an infinite number of forms; the forms given are generally the simplest. In the first part of the table the relations are worked out in detail in a sufficient number of examples to show the method.

In the papyrus the scribe places his answer to the left of the given number and the "reckoning" below. Hultsch (1895, page 4) and Peet (page 34) both regard the latter as the proof and not the solution. It is true that the interpretation of this as the solution leaves the table without proofs, but the proof would be somewhat like the solution and perhaps the author thought that one process would answer both purposes. The process is introduced by the word *seshemet*, put in with each number that comes at the top of a page. This word is used elsewhere in the papyrus only to introduce a solution, or the numerical work of a solution after a summary has been given in words.[1] Even if we are to regard the answer as coming before this numerical work, which is placed on lower lines, the arrangement would be like the arrangement in many other places where the numerical work is placed after the solution in words. In these problems the solution is, as I have said, a multiplication of the second kind, while the proof would be a multiplication of the first kind. To carry through the proof the Egyptian would have to multiply the given odd number by the different fractions of his answer and get the product as the sum of the partial products of these multiplications.[2] Take the case of 2 divided by 7. To prove that $\frac{1}{4}\ \frac{1}{28}$ times 7 equals 2 he would say:

1	7
$\frac{1}{2}$	$3\ \frac{1}{2}$
$\diagdown\frac{1}{4}$	$1\ \frac{1}{2}\ \frac{1}{4}$
[3] $\frac{1}{7}$	1
$\frac{1}{14}$	$\frac{1}{2}$
$\diagdown\frac{1}{28}$	$\frac{1}{4}$
Total	2.

[1] Outside of the table for the division of 2 by odd numbers, the word *seshemet* occurs nine times in the papyrus: in Problems 41, 42, 43, 44, 46, 58, 60, 65, 66. Peet himself always translates it "working out," and says that that is what it means in these nine problems (pages 22–23).

[2] See proofs of Problems 24–27, 30, 32–38. [3] See the solutions in Problems 24 and 34.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Here the sixth line is obtained by halving the preceding line in order to get the $\frac{1}{28}$ of 7 called for by the answer that we are trying to prove, but in the solution (and in the papyrus) we have $\frac{1}{4}$ as the fraction required to complete to 2 the $1\ \frac{1}{2}\ \frac{1}{4}$ in line 3, and we get $\frac{1}{28}$ as the multiplier necessary to produce it. That is, in the solution [1] we get the $\frac{1}{28}$ from the $\frac{1}{4}$, while in the proof we get the $\frac{1}{4}$ from the $\frac{1}{28}$. The only other way of carrying through the proof would be to interchange multiplier and multiplicand (see page 6). With this interchange the Egyptian would have said:

\1	$\frac{1}{4}\ \frac{1}{28}$
\2	$\frac{1}{2}\ \frac{1}{14}$
\4	$1\ \frac{1}{7}$
Total	2,

getting the total, if necessary, by applying his fractions to some number, say 28. This would be like the third step in the solutions of Problems 24–27.

In the latter part of the table the second step is omitted and only the result of the first step is given, but the third step is given for all of the numbers except 3, 35, and 91, and indicates clearly that we have, at least in each case except these, the solution and not the proof.

In the first step [2] of these solutions the author takes small multipliers

[1] In the cases of 2 divided by 17 and 19 the papyrus shows even more clearly than in the case 2 divided by 7 that the calculations given represent the solution and not the proof. See page 16.

[2] The problem worked out in this table has usually been described as the decomposition of a fraction with numerator 2 into a sum of fractions with numerator 1. Several eminent mathematicians have discussed it, notably Sylvester (1880) and Loria (1892). Others are listed in the Bibliography under Collignon, 1881. Of the discussions which I have seen the clearest is that by Loria. All of these writers, however, discuss the problem from the modern point of view. For example, a formula given as followed in some cases is $2/n = 1/a + 1/an$, where $a = (n + 1)/2$ (see Eisenlohr, 1877, pages 30–35). But no formula or rule has been discovered that will give all the results of the table, and Loria expressly says that he does not attempt to indicate how the old Egyptians obtained them.

Peet supposes that the Egyptian began by separating 2 into two parts, the first of which was greater than 1 and exactly contained in the given odd number. How to find this first part, or to know that it was contained in the given odd number, is really the thing that Peet claims that the papyrus does not tell us. However, he sees in the "reckoning" which he calls proof a clue to the method (page 36), his explanation being somewhat like mine. See also Bibliography under Neugebauer, 1926.

Hultsch declares positively that there is "no indication of the method," that the Egyptian reckoning "is indeed a study of mystery," and that "attempts to explain it . . . in spite of the darkness in which the ancient priestcraft has enveloped it, have hitherto not succeeded" (1895, page 5).

The way in which they worked out their problem is what has especially interested me.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

until he gets a result a little less than 2. Most frequently he begins with $\frac{2}{3}$ and then halves a sufficient number of times. Sometimes he halves without taking $\frac{2}{3}$, and sometimes he takes $\frac{1}{10}$ or $\frac{1}{7}$. In some cases with these fractional multipliers he gets a whole number and can use the reciprocal of this for a multiplier and get the result more quickly. He always does this with a multiple of 3, except for 3 itself, 9, and 15, because $\frac{2}{3}$ of it is a number whose reciprocal gives him at once $1\frac{1}{2}$. When the number is a multiple of 5 or 7, the fraction $\frac{1}{5}$ or $\frac{1}{7}$ gives him a whole number. In the Egyptian mind the case of $\frac{2}{3}$ is similar to $\frac{1}{5}$ and $\frac{1}{7}$. Finally, a special method seems to have been employed for 35 and 91, and still another method for 101. These cases will be explained below.

In reproducing the table, I have marked the different cases A, B, AD, BD, C, and E; that is, I have used:

A when the author first takes $\frac{2}{3}$;

B when he simply halves;

D along with A or B when he also uses $\frac{1}{10}$ or $\frac{1}{7}$;

C when at some step he gets a whole number and uses its reciprocal as a multiplier;

E for the three special cases of 35, 91, and 101.

In accordance with the foregoing rules I have marked each number with the letter indicating the kind of multiplier used, and it seems desirable to give the details of one problem of each case.

For A, I will take the case of 17, which is the first number for which the method is fully given. Beginning with the number itself the Egyptian writes:

1	17
$\frac{2}{3}$	11 $\frac{1}{3}$
$\frac{1}{3}$	5 $\frac{2}{3}$
$\frac{1}{6}$	2 $\frac{1}{2}$ $\frac{1}{3}$
$\backslash\frac{1}{12}$	1 $\frac{1}{4}$ $\frac{1}{6}$.

What remains to make 2 is the problem that constitutes the second step of this solution. This step is not explained in any of the examples that make up the table, but the method, as we have seen, is explained in Problems 21–23. In this case the amount needed is $\frac{1}{3}$ $\frac{1}{4}$.

There was, I think, no question of reducing or decomposing a fraction with numerator 2, for the Egyptians never had such a fraction to reduce. The author wishes to express, in the only way he knew, the quantities that, used as multipliers with these odd numbers, will produce 2. Nor do I believe that there was any mystery, or that the Egyptian mathematician tried to cover up his methods of reasoning. The reckoning given explains sufficiently his methods, and the methods indicated are the easiest for him to use.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

To make up $\frac{1}{3}$ the author multiplies 17 by 3, getting 51, and, because 3 times 17 is 51, it follows that $\frac{1}{51}$ of 17 is $\frac{1}{3}$. In the same way, getting 68 as 4 times 17, he knows that $\frac{1}{68}$ of 17 is $\frac{1}{4}$. Thus his answer is $\frac{1}{12}$ $\frac{1}{51}$ $\frac{1}{68}$.

He has two ways of writing down this third step. There are two multiplications, the multiplier of the second being the reciprocal of the product of the first, and differing from it only in the dot used in writing the reciprocal of a whole number. Thus in getting the $\frac{1}{3}$ these multiplications are

3 times 17 is 51 and $\frac{1}{51}$ times 17 is $\frac{1}{3}$.

But he always omits one of these two numbers, either the multiplier of the second multiplication or the product of the first. Thus in this example he might write:

	3	51	$\frac{1}{3}$,
or else	3	$\frac{1}{51}$	$\frac{1}{3}$.

As it happens, the author uses the second form in this case, but in some cases he uses the first. It may be, however, that he always has in mind the first multiplication, even when he puts a dot over the product, and that the fraction written after this multiplication is put in simply to indicate its purpose.[1]

To illustrate B, I will take the case of 13. The papyrus says:

1	13
$\frac{1}{2}$	6 $\frac{1}{2}$
$\frac{1}{4}$	3 $\frac{1}{4}$
$\diagdown\frac{1}{8}$	1 $\frac{1}{2}$ $\frac{1}{8}$.

There remains $\frac{1}{4}$ $\frac{1}{8}$ for which the multipliers are $\frac{1}{52}$ and $\frac{1}{104}$ and the answer is $\frac{1}{8}$ $\frac{1}{52}$ $\frac{1}{104}$.

An example of AD is the number 25. Here we have simply:

1	25	
$\diagdown\frac{1}{15}$	1 $\frac{2}{3}$	
$\diagdown$3	[2]75	$\frac{1}{3}$.

This would seem to indicate that the author first took $\frac{1}{10}$, getting $2\frac{1}{2}$, and

[1] Gunn thinks (pages 127–128) that the second of these two forms should always have been used, and he maintains that many of the multiplications are illogically set out in the papyrus. He would always write the middle number in the first column as a fractional multiplier and the third number in the second column as product. It seems to me that the first of the two multiplications is the more important as the one that has to be worked out, while the second is written in the line at the top anyway.

[2] The papyrus says $\frac{1}{75}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

then ⅔, ⅔ of 1/10 being 1/15. ⅔ of 2 would be 1⅓ and ⅔ of ½ would be ⅓, making 1⅔. Having obtained 1⅔ he requires ⅓ more to make 2 and the multiplier for this is 1/75. His answer therefore is 1/15 1/75.

An example of BD is the number 31. Here, supplying the first two lines, which are not given, we have:

1	31
1/10	3 1/10
\1/20	1 ½ 1/20.

There remains ¼ 1/5, for which the multipliers are obtained by the usual method, and his answer is 1/20 1/124 1/155.

For C, I will take first the number 21, which is a multiple of 3. ⅔ of 21 is 14; therefore 1/14 of 21 is 1½. There remains ½; 2 times 21 is 42, so that 1/42 of 21 is ½, and the answer is 1/14 1/42.

For a multiple of 5 take 65. We may suppose that the author proceeded in this way:

	1	65
	1/10	6 ½
	1/5	13
therefore	1/13	5
	1/26	2 ½
	\1/39	1 ⅔.

There remains ⅓; 3 times 65 is 195, and his answer is 1/39 1/195. The last line of this multiplication is obtained by taking ⅔ of the numbers in the preceding line, ⅔ of 2½ being the same as in the case of 25.

I have marked 35, 91, and 101 with E. These cases may be explained in the following way:

We have seen that the relations in this table may be regarded in two ways, either as expressing twice the reciprocal of an odd number, or as showing by what the odd number must be multiplied to make 2. In obtaining them the Egyptian nearly always takes the latter point of view, but in the case of 35 he seems to have considered that he was doubling the reciprocal of 35. To find the double of 1/35 he applies this fraction to the number 210, which is 6 times 35. This is indicated by a 6 written in red under 35. The double of 6 is 12, equal to 7 and 5, and these numbers are 1/30 and 1/42 of 210. Therefore his answer is 1/30 1/42. He writes out his answer in the usual form, but puts under each fraction the amount that it makes when applied to 210. Finally, in the lines below, where he usually puts the multiplications of his solution, he writes the two fractions of his answer with the results of multiplying

Digitized by Google Original from UNIVERSITY OF MICHIGAN

35 by them, namely, 1⅙ and ⅔ ⅙. All multiplication work is omitted and what is written could be taken as indicating a proof, especially as the second product is not in a form that could have been obtained by the last step of the process employed in the other cases.[1]

For 91 the same process that he has used for 35 will lead to the form of result that he gives. By this process he would take $\frac{1}{91}$ as $\frac{1}{91}$ of 910, which is 10. Twice this is 20, equal to 13 and 7, and the fractions that give 13 and 7 are $\frac{1}{70}$ and $\frac{1}{130}$. Multiplying 91 by these two fractions gives him 1 ⅕ $\frac{1}{10}$ and ⅔ $\frac{1}{30}$, together making 2.

In the case of 101 he takes $\frac{1}{101}$ itself for his first fraction. This makes 1 and he has to find fractions to make up another 1, and so 2 in all. He thinks of the familiar combination ½ ⅓ ⅙ and takes for the other fractions of his answer $\frac{1}{202}$, $\frac{1}{303}$, and $\frac{1}{606}$. This example is found entirely in the New York fragments. Eisenlohr and other writers before 1922 supposed that the table went only to 99.

In nearly all cases after 23, except multiples of 3, the author omits the successive multiplications by which he obtains the first fraction of his answer, and though we can tell what these multiplications were from the denominator of this fraction, it is not always easy to say in what order they were taken or just how they were carried through. We have to be guided largely by the form in which he gives the product produced by them. It may be worth while to make one or two suggestions as to the way in which he did this.

With the number 15 he begins "$\frac{1}{10}$ 1½." He may have obtained this 10 by taking ⅔ of 15 just as he takes ⅔ of 21 and of each multiple of 3 after 21, but it is more probable that he notices that he can easily divide by 10, and that this division will give him a number between 1 and 2, and so he does not take ⅔ of 15 at all. In each case of a multiple of 3 after 15, except 93, he mentions the fact that he takes ⅔.

With 25 I have supposed that the author took first $\frac{1}{10}$ and then ⅔. He may have taken ⅔ first and then $\frac{1}{10}$. ⅔ of 25 is 16⅔. ⅔ of 10 is 6⅔, and therefore $\frac{1}{10}$ of 6⅔ is ⅔, and $\frac{1}{10}$ of 16⅔ is 1⅔. Similarly with 53, if he takes ⅔ and then halves, getting 17⅓, he will then say that $\frac{1}{10}$ of 16⅔ is 1⅔ and $\frac{1}{10}$ of 1 is $\frac{1}{10}$, making 1 ⅔ $\frac{1}{10}$. That seems to be the easiest way to explain the result that he gives.

In the case of 43, if he takes ⅔ and then proceeds to halve, he will get 28⅔, 14⅓, and 7⅙, and the whole numbers suggest taking $\frac{1}{7}$ for a multiplier. There seems no other reason for his using 7 and taking $\frac{1}{42}$

[1] It may be, however, that the author was not thinking of a proof, but only attempted to put in something that would correspond to the usual form of solution.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

as his first fraction. $\frac{1}{42}$ is so small that he has to get $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{7}$ to make 2, and in his answer he has four fractions. If he had halved twice more instead of multiplying by $\frac{1}{7}$ he would have had $\frac{1}{24}$ for his first fraction and one form for his final answer would have been $\frac{1}{24}$ $\frac{1}{344}$ $\frac{1}{516}$.

When the author takes $\frac{2}{3}$ and $\frac{1}{10}$ in the same example his result in some cases comes most simply by taking $\frac{2}{3}$ first and $\frac{1}{10}$ at the end. Examples are 47, 53, 79, and 89. Yet in other cases, as of 55, 73, 83, and 95, he apparently multiplied by $\frac{1}{10}$ first. In the case of 59 he takes $\frac{2}{3}$ twice. The most natural process of obtaining the form given seems to be:

1	59
$\frac{2}{3}$	39 $\frac{1}{3}$
$\frac{1}{3}$	19 $\frac{2}{3}$
$\frac{1}{6}$	9 $\frac{1}{2}$ $\frac{1}{3}$
$\frac{1}{12}$	4 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$
$\frac{1}{18}$	3 $\frac{1}{6}$ $\frac{1}{9}$
$\frac{1}{36}$	1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$,

writing for $\frac{1}{6}$, 9 $\frac{1}{2}$ $\frac{1}{3}$ instead of 9 $\frac{2}{3}$ $\frac{1}{6}$, and halving again, so that his fractions will have even denominators, before he takes $\frac{2}{3}$ the second time.

The author uses the word *gem* meaning "find"[1] before the first denominator in the case of every number after 41 which is not a multiple of 3, except perhaps 101,[2] and also in the cases of 93 and 99, and he uses it before both denominators for 91. All of these cases, except perhaps 93 and 99, are the more difficult ones, and he does not put in the details but leaves them to the reader.

All of these various cases seem to indicate that there was no definite rule for determining the multipliers to be used, but probably the slow experience of different writers suggested different multipliers for different examples, as they seemed to them the easiest or gave results in the most satisfactory form.

In the table as here reproduced I have put: first, the letter or letters indicating the kind of multipliers employed; second, the number; third, the first fraction of the answer, this being the multiplier that produces a number a little less than 2; then the number a little less than 2 that is produced by this multiplier; the remainder necessary to make 2; and, finally, the answer.[3]

[1] Peet translates this, "found."

[2] The piece of the papyrus on which it would be written is still missing.

[3] This table is used in the following problems: 1–6, 30, 31, 33, 34, 36, 38, 42, 43, 67, 69, and 70, in some of them more than once; it is used backwards in 10, 17, 61, and 67.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

	Number	First Multiplier	Corresponding Product	Remainder	Answer
A	3	2/3	2		2/3
A	5	1/3	1 2/3	1/3	1/3 1/15
B	7	1/4	1 1/2 1/4	1/4	1/4 1/28
A	9	1/6	1 1/2	1/2	1/6 1/18
A	11	1/6	1 2/3 1/6	1/6	1/6 1/66
B	13	1/8	1 1/2 1/8	1/4 1/8	1/8 1/52 1/104
BD	15	1/10	1 1/2	1/2	1/10 1/30
A	17	1/12	1 1/4 1/6 or 1 1/3 1/12	1/3 1/4	1/12 1/51 1/68
A	19	1/12	1 1/2 1/12	1/4 1/6	1/12 1/76 1/114
C	21	1/14	1 1/2	1/2	1/14 1/42
A	23	1/12	1 1/2 1/4 1/6 or 1 2/3 1/4	1/12	1/12 1/276
AD	25	1/15	1 2/3	1/3	1/15 1/75
C	27	1/18	1 1/2	1/2	1/18 1/54
A	29	1/24	1 1/6 1/24	1/2 1/6 1/8	1/24 1/58 1/174 1/232
BD	31	1/20	1 1/2 1/20	1/4 1/5	1/20 1/124 1/155
C	33	1/22	1 1/2	1/2	1/22 1/66
E	35	1/30 1/42	1 1/6 2/3 1/6		1/30 1/42
A	37	1/24	1 1/2 1/24	1/3 1/8	1/24 1/111 1/296
C	39	1/26	1 1/2	1/2	1/26 1/78
A	41	1/24	1 2/3 1/24	1/6 1/8	1/24 1/246 1/328
AD	43	1/42	1 1/42	1/2 1/3 1/7	1/42 1/86 1/129 1/301
C	45	1/30	1 1/2	1/2	1/30 1/90
AD	47	1/30	1 1/2 1/15	1/3 1/10	1/30 1/141 1/470
BD	49	1/28	1 1/2 1/4	1/4	1/28 1/196
C	51	1/34	1 1/2	1/2	1/34 1/102
AD	53	1/30	1 2/3 1/10	1/6 1/15	1/30 1/318 1/795
AD	55	1/30	1 2/3 1/6	1/6	1/30 1/330
C	57	1/38	1 1/2	1/2	1/38 1/114
AD	59	1/36	1 1/2 1/12 1/18	1/4 1/9	1/36 1/236 1/531
BD	61	1/40	1 1/2 1/40	1/4 1/8 1/10	1/40 1/244 1/488 1/610
C	63	1/42	1 1/2	1/2	1/42 1/126
C	65	1/39	1 2/3	1/3	1/39 1/195
BD	67	1/40	1 1/2 1/8 1/20	1/5 1/8	1/40 1/335 1/536
C	69	1/46	1 1/2	1/2	1/46 1/138
BD	71	1/40	1 1/2 1/4 1/40	1/8 1/10	1/40 1/568 1/710
AD	73	1/60	1 1/6 1/20	1/3 1/4 1/8	1/60 1/219 1/292 1/365

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

	Number	First Multiplier	Corresponding Product	Remainder	Answer
C	75	$\frac{1}{50}$	1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{50}$ $\frac{1}{150}$
C	77	$\frac{1}{44}$	1 $\frac{1}{2}$ $\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{44}$ $\frac{1}{308}$
AD	79	$\frac{1}{60}$	1 $\frac{1}{4}$ $\frac{1}{15}$	$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{10}$	$\frac{1}{60}$ $\frac{1}{237}$ $\frac{1}{316}$ $\frac{1}{790}$
C	81	$\frac{1}{54}$	1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{54}$ $\frac{1}{162}$
AD	83	$\frac{1}{60}$	1 $\frac{1}{3}$ $\frac{1}{20}$	$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$	$\frac{1}{60}$ $\frac{1}{332}$ $\frac{1}{415}$ $\frac{1}{498}$
C	85	$\frac{1}{51}$	1 $\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{51}$ $\frac{1}{255}$
C	87	$\frac{1}{58}$	1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{58}$ $\frac{1}{174}$
AD	89	$\frac{1}{60}$	1 $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{20}$	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{10}$	$\frac{1}{60}$ $\frac{1}{356}$ $\frac{1}{534}$ $\frac{1}{890}$
E	91	$\frac{1}{70}$ $\frac{1}{130}$	1 $\frac{1}{5}$ $\frac{1}{10}$ $\frac{2}{3}$ $\frac{1}{30}$		$\frac{1}{70}$ $\frac{1}{130}$
C	93	$\frac{1}{62}$	1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{62}$ $\frac{1}{186}$
AD	95	$\frac{1}{60}$	1 $\frac{1}{2}$ $\frac{1}{12}$	$\frac{1}{4}$ $\frac{1}{6}$	$\frac{1}{60}$ $\frac{1}{380}$ $\frac{1}{570}$
BD	97	$\frac{1}{56}$	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$	$\frac{1}{7}$ $\frac{1}{8}$	$\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$
C	99	$\frac{1}{66}$	1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{66}$ $\frac{1}{198}$
E	101	$\frac{1}{101}$	1	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$	$\frac{1}{101}$ $\frac{1}{202}$ $\frac{1}{303}$ $\frac{1}{606}$

Division of the Numbers 1–9 by 10

Table:

Number divided by 10	Result
1	$\frac{1}{10}$
2	$\frac{1}{5}$
3	$\frac{1}{5}$ $\frac{1}{10}$
4	$\frac{1}{3}$ $\frac{1}{15}$
5	$\frac{1}{2}$
6	$\frac{1}{2}$ $\frac{1}{10}$
7	$\frac{2}{3}$ $\frac{1}{30}$
8	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
9	$\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$.

This table can be used by the author when he wishes to take $\frac{1}{10}$ of a number that is not a multiple of 10, just as the preceding table is used when he wishes to double the reciprocal of an odd number.[1]

The Rhind papyrus gives the table in full as above and then proves that it is correct for the numbers 1, 2, 6, 7, 8, and 9. Apparently the author intended to put in only the more difficult proofs. Thus he omits the proof for 5 divided by 10 because this is the same as $\frac{1}{2}$. He also omits the proof for 4 because 4 divided by 10 is the same as 2 divided by

[1] See, for example, Problem 30, proof.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

5, which we have already in the first table. But it seems strange that he should omit the proof for 3 and put in those for 1 and 2. All that remains of the second proof in the Rhind papyrus would fit 2 or 3 equally well, and Eisenlohr assumes that it is for 3, but the New York fragments show that really it is for 2.

It is not difficult to show how the Egyptians would derive the expressions in this table. Take, for example, the case of 3. To divide 3 by 10 we have to multiply 10 so as to get 3. We have:

1	10
\ 1/10	1
\ 1/5	2
Total 1/5 1/10;	

or, taking 7,

1	10
\ 2/3	6 2/3
[1] 1/10	1
1/15	2/3
\ 1/30	1/3
Total 2/3 1/30.	

In order to make these problems seem practical the author supposes that we have a certain number of loaves of bread to divide among 10 men.

Multiplication by Certain Fractional Expressions

Problems 7–20 are problems in multiplication[2] that consist of two groups. In the first the multiplier is 1 1/2 1/4 and in the second, 1 2/3 1/3. Problems 7 and 9–15 belong to the first group and problems 8 and 16–20 to the second. One problem of each group is given and then the rest of those of the first followed by the rest of those of the second. In Problem 7, 1/4 1/28 is multiplied by 1 1/2 1/4 and in each of the problems of this group

[1] Instead of this he may have said, as in the first table, the remainder is 1/2; 3 times 10 is 30 and therefore 1/30 of 10 is 1/2.

[2] Before Problem 7, the author put in the papyrus the words "Example of making complete" and some writers have explained this by saying that the problem is to add to a given number certain fractional parts of it. Neugebauer (See Bibliography, 1926) regards these as completion problems for the division of 2 by 7 and 9 in the table at the beginning of the papyrus, because they include certain multiplications that arise in those two divisions. It seems to me more natural to regard them as simple multiplications and to suppose that the heading came to be put here in some way by mistake. Perhaps the author wrote down the heading for Problems 21–23 and then discovered that he had these multiplications to put in first.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

the multiplicand is one of the numbers that can be obtained from $\frac{1}{4}$ $\frac{1}{28}$ by doubling or halving one or more times. In Problem 7, the multiplication is as follows:

1	$\frac{1}{4}$ $\frac{1}{28}$
$\frac{1}{2}$	$\frac{1}{8}$ $\frac{1}{56}$
$\frac{1}{4}$	$\frac{1}{16}$ $\frac{1}{112}$
Total	$\frac{1}{2}$.

$\frac{1}{4}$ $\frac{1}{28}$ is the number which we have found as the value of 2 divided by 7; and in some of the problems $\frac{1}{7}$ is given as $\frac{1}{2}$ of this number, $\frac{1}{14}$ as $\frac{1}{2}$ of $\frac{1}{7}$, and so on. In fact, some of these multiplications are carried through in both ways in different problems.[1]

In adding his partial products the Egyptian applies these fractional numbers to the number 28, using the method that I have already explained (page 7), and in Problems 7, 13, 14, and 15 he puts under each fraction the number that it makes when taken as a part of 28. Thus in Problem 7, $\frac{1}{4}$ and $\frac{1}{28}$ of 28 are 7 and 1, $\frac{1}{8}$ and $\frac{1}{56}$ are $3\frac{1}{2}$ and $\frac{1}{2}$, and $\frac{1}{16}$ and $\frac{1}{112}$ are 1 $\frac{1}{2}$ $\frac{1}{4}$ and $\frac{1}{4}$. These numbers added together give 14, which is $\frac{1}{2}$ of 28. Therefore the answer is $\frac{1}{2}$.

In Problem 15, $\frac{1}{128}$ and $\frac{1}{896}$ of 28 are $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ and $\frac{1}{32}$, and we have what may seem rather complicated quantities to add, even when we use 28, but they are simpler than the given quantities, being the reciprocals of powers of 2.

In the problems of the other group, 8 and 16–20, the author starts with a single fraction and multiplies by 1 $\frac{2}{3}$ $\frac{1}{3}$, so that the result is simply to double the fraction. The numbers taken are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{1}{24}$. In this group the fractional numbers are applied to 18, and in the first and last two problems we find placed under the fractions their values as parts of this number. Nearly all of these parts are themselves fractions, but they are the reciprocals of powers of 2.

There are some interesting multiplications in these solutions. In Problem 17 the author has $\frac{2}{3}$ of $\frac{1}{3}$ equal to $\frac{1}{6}$ $\frac{1}{18}$. He may have put this down from memory or have copied it from some table; or he may have applied the rule given in Problem 61 (see below). Then for a half of $\frac{1}{6}$ $\frac{1}{18}$ he writes $\frac{1}{9}$ because his table for the division of 2 by odd numbers tells him that 2 times $\frac{1}{9}$ is $\frac{1}{6}$ $\frac{1}{18}$.

Problem 61 may also be considered in this connection as it consists of a multiplication table of various fractions. With 61 is associated

[1] It is interesting to notice that the multiplicands and partial products in these multiplications can be arranged in a table formed by taking $\frac{1}{2}$ $\frac{1}{14}$ as the first number and multiplying several times by $\frac{1}{2}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

61B, which gives a rule for finding $\frac{2}{3}$ of a fraction.[1] The rule says, "To find $\frac{2}{3}$ of $\frac{1}{5}$ take thou its double and its six-fold, and do thou likewise for any fraction that may occur." In the table at the beginning of the papyrus for the division of 2 by odd numbers we notice that the quotient for 3 is simply $\frac{2}{3}$, but if the author had used the method employed for all multiples of 3 after 15 he would have obtained $\frac{1}{2}$ $\frac{1}{6}$, and he sometimes finds it convenient as here to use this expression in place of $\frac{2}{3}$. This rule is used a number of times in the arithmetical portions of the papyrus.[2] In Problem 33 it is employed with the fraction $\frac{1}{679}$.

It is to be noticed that in dealing with numbers which the modern mathematician calls fractions, the Egyptian regards the denominator as the important element, and therefore, although he wants to multiply the 5 by 2 and 6, he says, "Multiply $\frac{1}{5}$."

Division by a Fractional Number

Problems 24–38 are all essentially problems in division by a fractional expression. Problem 67 may also be included in this list. In other words, these are multiplication problems in which the product is one of the given numbers. But in all of them it is the multiplier and not the multiplicand that is the other given number. And therefore, as they stand, they cannot be solved directly by the Egyptian process of multiplication. We have seen (pages 6 and 7) that there are two methods of solving such problems. One method was to regard all of the numbers as mere numbers and change the problem so as to make the given multiplier a multiplicand and obtain the answer first as a multiplier. The other method was that of false position. The former method is used in Problems 30–34. The latter seems to be the method in all except these.

The first eleven of these problems (24–34) are sometimes called *'aha'*[3] or quantity problems because they nearly all begin with the word *'aha'* and use this word to denote the result of their calculations. In Problems 35, 37, and 38 the quantity is a measure of grain. In 36 the word for the measure is omitted and the problem by itself is a purely

[1] The word for fraction used here means literally "weak sign." Gunn (page 134) thinks that it is used for "uneven fraction." When the denominator of a fraction is an even number the rule is unnecessary; for all that is necessary is "to take $1\frac{1}{2}$ times this number and dot it."

[2] Namely, in Problems 17, 30, 32, 33, and for $\frac{2}{3}$ of $\frac{1}{3}$ in Problem 61.

[3] '*ḥ*'; this was formerly written *hau*.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

numerical one, but it is written in the same phraseology save for the omission of this word and is solved by the same method, so that we can consider the four problems together.

Problem 24 is to find a quantity that with its $\frac{1}{7}$ will make 19. The author assumes 7, which with its $\frac{1}{7}$ makes 8, and then, to find the answer, he multiplies 7 by the number that multiplying 8 will give 19; 7, 8, and 19 as well as the answer all represent quantities of the same kind.

In each of the problems 24–27 the multiplier is 1 and a single fraction and he assumes as his answer the number that is the denominator of the fraction. In 28 and 29 he has two or three fractions and assumes the product of their denominators as his answer.

Problems 30–34 are far more complicated; nowhere, indeed, in the entire papyrus is more skill shown in dealing with long fractional expressions. In these problems it is required to determine what quantity a certain fractional number must multiply in order to produce a given product. But in solving them the Egyptian changes his point of view and determines what must multiply this fractional number.[1] In the proofs, however, he goes back to his original point of view and multiplies the answer by the given fractional number to get the number given as product.

In Problem 30 the author wishes to find what quantity $\frac{2}{3}\ \frac{1}{10}$ must multiply in order to produce 10, and so changing his point of view as I have said, he proceeds to multiply $\frac{2}{3}\ \frac{1}{10}$. He finds that multipliers amounting to 13 will give 9 and a series of fractions. Then we may suppose that he takes these fractions and the given expression as parts of 30. At any rate he finds that $\frac{1}{30}$ will just complete the fractions to 1, and that $\frac{1}{23}$ of the given expression will make this $\frac{1}{30}$. Thus his answer is $13\frac{1}{23}$.

In Problems 31 and 33 he has the same fractional expression, namely, $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$,[2] the products to be obtained being 33 and 37. In both cases he multiplies this fractional expression in such a way as to get very nearly the product desired.

Thus to get 33 he doubles three times and halves twice, checking those multipliers that will give him very nearly this number. The numbers that he checks are equal to $14\frac{1}{4}$, and make up most of his answer. He adds the whole numbers and larger fractions of the corresponding

[1] See page 6.

[2] This is the same as $2\ \frac{1}{6}\ \frac{1}{7}$. It is very unusual for him to write a fractional expression in which the fractional part is itself more than 1.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

products, leaving for the present the smaller fractions, and he gets 32½. The remaining ½ required to make up all of 33 must then be equal to the smaller fractions of these products plus a further product by a multiplier yet to be determined. It remains only to get this multiplier. Choosing 42 as a convenient number, he applies his expressions to 42. The required ½, taken as ½ of 42, is equal to 21; the previously discarded fractions make 17¼ of it, and leave 3 ½ ¼ to be obtained by the multiplier that we are seeking. The original expression, 1 ⅔ ½ ¼, applied to 42, is equal to 97, and $\frac{1}{97}$ of it is 1; that is, $\frac{1}{97}$ as multiplier gives a product that, taken as a part of 42, equals 1. To get the product that equals 3 ½ ¼ he must take as multiplier[1] 3 ½ ¼ times $\frac{1}{97}$, and this multiplier, together with the 14¼ checked in the first multiplication, makes up his answer, which is

$$14\ \tfrac{1}{4}\ \tfrac{1}{56}\ \tfrac{1}{97}\ \tfrac{1}{194}\ \tfrac{1}{388}\ \tfrac{1}{679}\ \tfrac{1}{776}.$$

In Problem 33 to get nearly 37 he has only to carry his doubling one step further, 16 times 1 ⅔ ½ ¼ being equal to 36 ⅔ ¼ $\frac{1}{28}$, or only $\frac{1}{21}$ less than 37. He applies his expressions to 42 to determine that this remainder is $\frac{1}{21}$ as well as to determine the multiplier that will produce it. To do the latter, since $\frac{1}{21}$ of 42 is 2, he has to use as multiplier 2 times $\frac{1}{97}$, and his answer is

$$16\ \tfrac{1}{56}\ \tfrac{1}{679}\ \tfrac{1}{776}.$$

In Problem 32 the multiplier is 1 ⅓ ¼ and the product to be obtained is 2, and in Problem 34 the multiplier is 1 ½ ¼ and the product to be obtained is 10. The latter may be associated with Problems 7 and 9–15.

In Problem 31 the answer, as we have seen, is particularly long, and the author makes no attempt at a proof, but in the other four problems he goes through the proof very carefully, so that it becomes more prominent than the solution.

In the proof of his answer to Problem 32 he multiplies 1 ⅙ $\frac{1}{12}$ $\frac{1}{114}$ $\frac{1}{228}$ by 1 ⅓ ¼. From the partial products he selects first the larger numbers, namely, 1, ⅙, ⅓, and ¼, or 1 ½ ¼, leaving ¼ to be made up from the remaining fractions, and by applying these to 912, the largest number whose reciprocal is found among them, he finds that their sum is exactly ¼, and that his answer is correct. In the same way he carries through the somewhat easier proof of Problem 34.

In the case of Problem 33 he multiplies his answer by 1 ⅔ ½ ¼. Adding the whole numbers and larger fractions of this multiplication,

[1] To find this multiplier he takes $\frac{1}{97}$ and its double and halves twice, getting the double from the table at the beginning of the papyrus.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

that is, the parts that came from the whole number 16 of his answer, he has 36 ⅔ ¼ $\frac{1}{28}$, which is the same number that he got at the beginning as 16 times the given fractional number. The remaining fractions of his multiplication are so complicated that he seems to have some difficulty in carrying through his proof. In the first place, he applies all of his fractions to 5432, here again taking the largest number whose reciprocal is found among them, and he places under each of the three fractions of the sum already obtained, as well as under each of the fractions that he has not yet added, the number that it makes as a part of 5432. The numbers placed under ⅔, ¼, and $\frac{1}{28}$ make 5173⅓. This subtracted from 5432 leaves 258⅔, and the proof would be completed if he should show that this is the sum of the numbers placed under the fractions not yet added. Instead, he writes after ⅔ ¼ $\frac{1}{28}$ that there remains $\frac{1}{28}$ $\frac{1}{84}$ (although in the first part of the solution he had the simpler form $\frac{1}{21}$ for this remainder), and that the two fractions of this remainder make 194 and 64⅔ when taken as parts of 5432.

Problems 35–38 are numerically the same kind as the preceding. I have explained 35 on page 11 and the other three are solved in the same way.

In Problem 67 it is required to find how many cattle there are in a herd when ⅙ $\frac{1}{18}$ of them makes 70, the number due as tribute to the owner. The numerical work of the solution is exactly like that for 35–38. We can suppose that the Egyptian, without going through all of the reasoning, recognized that it was a problem of the same kind, to be solved by the same process.

Separation in Given Proportions.[1]

Four of the miscellaneous problems, 62, 63, 65, and 68, are problems of separation in given proportions.

Problem 62 is about a bag with pieces of gold, silver, and lead in it. The meaning is not very clear, but the numerical problem is simply to divide 84 into three parts proportional to 12, 6, and 3. These numbers added together make 21, 84 is 4 times 21, and therefore the parts into which 84 is to be divided are 4 times the numbers 12, 6, and 3.

Problem 63 is exactly the same kind of problem and is solved in the same way. 700 loaves are to be divided among four men in the proportion of the numbers ⅔, ½, ⅓, and ¼. The sum of these fractions is 1 ½ ¼. Instead of dividing 700 by 1 ½ ¼ the author divides 1 by this

[1] This is an important class of problems always considered in our arithmetics and algebras.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

number and multiplies 700 by the result. 1 divided by 1 ½ ¼ gives ½ ¼₁₄, 700 times ½ ¼₁₄ is 400, and 400 multiplied by the four given fractions gives the four parts into which 700 is to be divided, namely, 266⅔, 200, 133⅓, and 100.

In Problem 65, 100 loaves are to be divided among 10 men with three getting double portions, and in Problem 68, 100 *hekat* of grain are to be distributed to four overseers, the numbers of men in their gangs being 12, 8, 6, and 4, respectively. These are problems of the same kind and solved in the same way.

Other Arithmetical Problems

Problem 39 is also a division problem, but it is simply to divide 100 loaves among 10 men, 50 among 6, and 50 among 4.

In Problem 66 an amount of fat is given as required for a year, to find how much that is for a day. It is a simple division problem, solved by the multiplication of the divisor, but it seems to be regarded by the author as typical, so that at the end he tells us to do like this for any other problem of this kind.

Problems 69–78 have to do with the relative values of certain amounts of food or drink, the amount of grain in a unit of food or drink, and the amount of food or drink that can be made from a unit measure of grain. One of them, Problem 76, is particularly interesting as belonging to a class of problems that have been common in all of our arithmetics and algebras: problems in regard to the time that it takes to do a piece of work when men work together, or to fill a cistern when two or more pipes are running at the same time, or to row a certain distance up a stream and back—problems solved by first adding the reciprocals of the given numbers.[1]

Arithmetical and Geometrical Progressions

There are two problems, 40 and 64, that involve arithmetical progression, and one, 79, that involves geometrical progression. These are very interesting and seem to mark the acme of the Egyptians' skill in arithmetic.

Problem 40 is more interesting as illustrating false position, and I have explained it in connection with my discussion of that process (page 12).

[1] See D. E. Smith, 1923, volume 2, pages 536–541; also "On the Origin of Certain Typical Problems," *American Mathematical Monthly*, volume 24, pages 64–71. Problem 76 does not involve time, but it is of the same type.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In Problem 64, 10 *hekat* of barley are supposed to be divided among 10 men in such a way that the amounts received by the different men form an arithmetical progression with a common difference of $\frac{1}{8}$[1] of a *hekat*. Thus the terms, their sum, and the common difference are all concrete quantities, namely, certain amounts of barley, and the author is careful to use the special notation required for these quantities (the special forms of numbers of *hekat* and the "Horus eye" forms; see page 31), but the ordinary notation for numbers which represent the number of men, or differences, and for numbers used as multipliers. Thus there are given the number of terms, 10; the sum of the terms, 10 *hekat* of barley; and the common difference, $\frac{1}{8}$ of a *hekat*. First he gets the mean or average, which is 1 *hekat*. Then to get the largest term he would add to this average the common difference one-half as many times as there are differences in all, but as the number of differences is an odd number he cleverly adds instead one-half of the common difference as many times as there are differences, that is, 9 times. Having obtained the amount of the largest portion, he gets the others by subtracting the common difference a sufficient number of times, and writes down the entire progression in descending order, each term as a certain amount of barley.

Problem 79 is a problem in which is calculated in two ways the sum of a geometrical progression. There are two columns. The first column indicates, it seems to me, a numerical method for determining the sum when the first term is equal to the ratio. This method may be stated in the following rule: In any geometrical progression whose first term is equal to the ratio, the sum of any number of terms is equal to the sum of one less number of terms plus 1 multiplied by the ratio. This rule the author follows in the first column, and in the second he performs the ordinary process of multiplication by the ratio and adds the terms together, getting the same result and thinking perhaps that he was proving the rule.[2]

[1] I am using black-faced type to represent the "Horus eye" forms. See page 31.

[2] Neugebauer (1926) constructs a table in which these two columns appear as the last column and row, and the calculations are all reduced to doublings and additions. See Bibliography.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

EGYPTIAN MEASURES[1]

Measures of Capacity

The unit of volume or capacity, used especially in measuring grain, was the *hekat*, which can be determined as 292.24 cubic inches, or a little more than half a peck.[2] This was divided into 320 parts called *ro*, but the Egyptians also used as fractions of a *hekat* the fractions whose denominators are powers of 2 down to $\frac{1}{64}$, $\frac{1}{64}$ of a *hekat* being 5 *ro*. This series of fractions was peculiarly adapted to multiplication by doubling or halving. They were written in a special notation and have been called "Horus eye" fractions (see Bibliography under Möller, 1911).[3]

Besides using the "Horus eye" notation for parts of a *hekat*, the Egyptians had special hieratic signs for the numbers from 5 to 10 when used to express *hekat*. These signs seem to be ligatures of dots,[4] the sign for 10 being a long vertical stroke, representing perhaps ten dots one above another. The Egyptians had also a peculiar way of writing an expression for a large quantity of grain (but, so far as I know,

[1] This discussion is confined entirely to the subject as it is presented in the Rhind papyrus. A very elaborate study of Egyptian weights and measures was made by Griffith in 1892 (see Bibliography) and I have taken some statements from his article and checked many others by it, but for the most part my account is based on a study of the papyrus itself.

[2] See page 32. For some reason Peet calls the *hekat* a bushel, although he says that a *hînu* ($\frac{1}{10}$ of a *hekat*) was about 29.2 cubic inches (page 25). It is possible that he gets this word from Sethe, who uses the word *Scheffel* several times for *hekat* (1916, pages 74 and 80). Eisenlohr also uses the word *Scheffel*, but uses it for 10 *hekat*. Gunn calls the *hekat* a gallon (page 126), but the gallon is a liquid measure, at least in America, and for the same amount in dry measure it is better to say "half a peck," unless we use the word *hekat* itself. A note by G. P. G. Sobhy in *The Journal of Egyptian Archaeology*, volume 10, pages 283–284, gives the *hînu* in litres as determined by several vases which are inscribed with their values, but there is a considerable variation in the results from these different vases, the value of a *hînu* varying from .4028 to .544, making the value of a *hekat* vary from 4.028 to 5.44 litres. As the litre is 61.023 cubic inches, 292.24 cubic inches would be 4.789 litres.

[3] In the Rhind papyrus when some calculation gives a portion of a *hekat* expressed in ordinary fractions, these are reduced first to *ro*, and then to the "Horus eye" fractions. See Problems 35, 37, 38, 69, and 70. In 69 and 70, "Horus eye" fractions are changed to ordinary form before multiplication, as if for purposes of multiplication the author thought of them as mere numbers. Generally the multiplications are carried through with the "Horus eye" forms.

[4] In hieroglyphic these dots may have been written as little circles (see Möller, 1909, volume 1, page 66).

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

only in hieratic). The sign which represents a container of grain on its side, and forms a part of many words for different kinds of grain (see page 47) was also used in these expressions. When the amount was equal to or more than 100 *hekat*, this sign was written with the number of hundreds before it, and the signs for any smaller number of *hekat* next after it. Also 50 *hekat* and 25 *hekat* were put down as ½ and ¼.[1] The number of whole *hekat* was followed by "Horus eye" fractions and by *ro* and fractions of a *ro*. In the case of 2, 3, or 4 *ro* the sign for the word *ro* was written under the number, while this sign without a number stood for 1 *ro*, and the fractions of a *ro* came after the sign.

Furthermore the Egyptians had not only the system of a simple *hekat* and its parts and multiples, but also systems of a double *hekat* and a quadruple *hekat* with their parts and multiples, each part or multiple of a double *hekat* being twice the corresponding part or multiple of a simple *hekat*, and each part or multiple of a quadruple *hekat* four times the corresponding part or multiple of a simple *hekat*; and they had the peculiar way of writing an expression for a large quantity of grain in the double and quadruple systems that they used for simple *hekat*. The double *hekat* was indicated by doubling the vertical sign in the word for *hekat* and the quadruple *hekat* by introducing an additional grain sign with four grains over it , but when these signs are not given we cannot always be sure that the system indicated is the simple system.[2] The quadruple *hekat* seems to have been called sometimes "a great *hekat*" (Griffith, volume 14, page 432), or "a great quadruple *hekat*" (Problem 68[3]).

We get information as to the size of these various measures, and, in particular, of the *hekat*, from Problems 41–46, where the capacities of certain granaries are computed from their dimensions. Here the unit of length is the cubit (*meh*), supposed to be the royal cubit, equal to 20.62 inches. The author states that ⅔ of a cubed cubit is the *khar*, and then

[1] It looks as if the Egyptians thought of a hundred-*hekat* as a unit.

[2] Double *hekat* are indicated only in Problems 82 and 84. The quadruple system is used in the granary problems, Problems 41–46, in Problems 47 and 68, and in Number 86. When there is no indication of either the double or quadruple system, it may be possible sometimes to determine the system by the nature of the problem. In the *pefsu* problems (69–78) the number of loaves made from a *hekat* of material varies from 5 to 45. I should be inclined to think that even a simple *hekat* would make 5 rather large loaves. But in some of the problems the numbers may have been taken at random without thought of any particular application, and in some cases it may not have been the intention to restrict the problem to a particular system, it being equally good whether the *hekat* are simple, double or quadruple *hekat*.

[3] But not in Problems 41–47 as Peet implies (page 26).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

that it takes 20 *khar* to make 100 quadruple *hekat* or 400 simple *hekat*. This would make the *hekat* 292.24 cubic inches, as stated above.

In addition to the *hekat* and all of its parts and multiples there was the *khar* just mentioned, and the *hînu*, equal to $\frac{1}{10}$ of a *hekat*, although its origin was independent of the *hekat* system.

Measures of Area and Length

The only problems dealing with area are 48–55. The units of measure in these problems besides the cubit (page 32) are, first, the linear unit called *khet*[1] which is 100 royal cubits, and, second, the square *khet* called *setat*, which is 10,000 square cubits. The area of a field is expressed in terms of the *setat* and fractions of a *setat* in much the same way as the measure of a quantity of grain is expressed in terms of the *hekat* and fractions of a *hekat*. In the first place, the Egyptians used the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of a *setat* in the same way as they used the "Horus eye" fractions, and for these fractions also they had special forms.[2] Then smaller portions of a *setat* were expressed in terms of a unit that they called "cubit"[3] and seem to have thought of as a strip 1 *khet* or 100 cubits long and 1 cubit wide. I shall call this unit a "cubit-strip." 100 cubit-strips make a *setat*, and as $\frac{1}{8}$ of a *setat* is equal to $12\frac{1}{2}$ cubit-strips, calculations with this system of units are not quite as simple as with the *hekat* system, where the smallest "Horus eye" fraction is equal to a whole number of *ro*. Thus in Problem 54 we have $\frac{1}{5}$ of a *setat*, which is 20 cubit-strips, expressed as $\frac{1}{8}$ *setat* $7\frac{1}{2}$ cubit-strips. The double of this is $\frac{1}{4}$ $\frac{1}{8}$ *setat* $2\frac{1}{2}$ cubit-strips, and so on. Cubit-strips are denoted by placing an arm, the determinative and word-sign of the word *meh*, over the number, and *setat* by placing a little rectangle over the number, or, in hieratic, a heavy stroke or arc. In two cases (in Problem 48) this is written as a heavy dot. The special hieratic forms of 7 and 9 used for 7 and 9 *hekat*, are used also when these numbers express so many *setat*, and 10 *setat* are expressed by a heavy vertical stroke just as 10 *hekat* are expressed. (See Problems 48 and 53.) When there are more than 10 *setat* the number of 10's is expressed as an ordinary number as if a ten-*setat* was thought of as a unit. In fact, the

[1] Gardiner (1927, page 199) calls it a rod although it is more than 10 rods. Peet (page 24) compares it to our chain, which is 4 rods.

[2] I shall put these fractions, like the "Horus eye" fractions, in black-faced type.

[3] Or "cubit-of-land." This use of the word cubit in expressions of area is like our method of measuring cloth and other articles that are sold by the yard, or lots in a city that are sold by the front foot. See Griffith, volume 14, pages 410 ff., volume 15, page 306.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

line used for 10 may be only the ordinary sign for 1 although in the case of *hekat* it is regarded as a ligature for ten dots or small circles. When there are ten-*setat* and unit *setat* the *setat* sign is placed over the number denoting units. Thus 36 *setat* in Problem 48 is written as an ordinary 3 and then a 6 with the *setat* sign over it, and 72 *setat* as an ordinary 7 and a 2 with the *setat* sign over it. Besides the peculiar signs for $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ *setat* we have a special sign for $\frac{1}{2}$ of a cubit-strip (Problems 54 and 55). Sometimes this is like the ordinary sign for $\frac{1}{3}$ and sometimes the same without the dot at the end.

In some of the area problems the scribe handles his units in a way that seems a little confusing. Thus to get the area of a rectangle he sometimes multiplies its length in cubits by its width in *khet*. This of course will give him the correct answer in cubit-strips.

The pyramid problems (56–60), which follow the problems on area, introduce a new unit of length called the palm or hand-breadth (*shesep*), which is $\frac{1}{7}$ of a cubit, and which is denoted sometimes by an arc over the number; and $\frac{1}{4}$ of a palm called a finger-breadth or finger.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

EGYPTIAN GEOMETRY

The author of this papyrus was able to determine the areas of rectangles, triangles, and circles, and the volumes of cylinders and prisms, and he knew that in a right triangle the relation of the lengths of two sides determines one of the angles. In the papyrus he takes up problems of volume, then problems of area, and, finally, problems involving the relative lengths of the sides of a triangle.

Determination of Volumes

In the problems of volume, 41–46, the author calculates the amount of grain that can be stored in certain spaces or bins of given dimensions, and the dimensions of bins that will contain given amounts of grain. Rather strangely he takes up the case of cylinders first and then that of rectangular parallelepipeds. As to the latter, all that I need to say is that he multiplies the three dimensions together. Problem 44 is an example of the direct calculation of such a volume, the three dimensions being all equal to 10. Problem 45 is the inverse of 44, and 46 is a problem of the same kind as 45. It is to be noticed that the author takes first the inverse of a numerical problem that he has worked out directly, and therefore he knows the answer.[1] Problem 45 as stated has only the volume given. Taken as the inverse of 44 we should consider it as involving the taking of a cube root. What he does is to take two of the dimensions as each equal to 10, and to find the third. Problem 46 is not the inverse of a problem already solved, but here also he assumes two of the dimensions as each equal to 10 and gets the third dimension, which in this case is not the same number, but is one-third of it, or $3\frac{1}{3}$, so that the bin is a rectangular parallelepiped but not a cube.

In Problems 41–43 the author has given the dimensions of a cylindrical body to find the volume. Thus in 41 the diameter of the cylinder is given as 9 and the altitude as 10. In order to obtain the area of the base the author subtracts from the diameter its $\frac{1}{9}$, squares the remainder, and obtains 64 as the area of the circle. Multiplying this by 10 he gets 640 as the contents of the granary in cubed cubits.

Thus we see that in this problem he gives the relation between the

[1] In the same way 58 is the inverse of 57 and 59B is the inverse of 59.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

area of a circle and its diameter. He does not explain how he discovered this relation, but from the fact that this problem in volume precedes 48, which states the relation for areas, it may be supposed that he determines the volume of a cylinder by some such process as the following:

He would quite possibly start with a cylinder of 9 units diameter because 9 with the Egyptians was a very important number and represented a group of the principal divinities.[1] He would then construct square prisms of the same height but of different bases, and he would find that the cylinder filled with water would almost exactly fill the prism whose side was 8 units. He was then able to judge that the base of the prism would be determined by subtracting from the diameter its $\frac{1}{9}$, and this gives the value of π as 3.1605. The fact that in this case a whole number for the side of the prism gives such a close approximation was probably a happy accident. In the next problem, having determined this relation, he takes for diameter 10, which does not give him a whole number for the side of the prism. This method, which we have already noted (page 8), of taking a simple numerical example and generalizing from that, is not from the mathematical point of view strictly legitimate and is liable to error unless the generalization is afterwards proved.

Determination of Areas

In Problem 48 is indicated the relation between a circle and its circumscribing square.[2]

In Problems 51–53 the Egyptian determines the area of a triangle by multiplying $\frac{1}{2}$ of its base, and the area of a trapezoid by multiplying $\frac{1}{2}$ of the sum of its bases, by the length of a line (*meret*) which, so far as our present knowledge goes, might be either the side or a line representing the altitude. In the latter case he would be correct. In case the triangle is isosceles with a narrow base as compared with its height, he would be nearly correct, even if *meret* means side. Personally I am inclined to think that this word does mean side in geometry, and that the author intended to consider only isosceles triangles with narrow bases. In Problem 51 the base is comparatively narrow, 4, with *meret* equal to 10.[3] In 53 it seems to be $4\frac{1}{2}$ with *meret* 14, while in 52 the

[1] Sethe (1916) page 38. There were a greater and a lesser "ennead," the personnel of which varied in different ages and localities.

[2] Such a figure occurs on a Babylonian tablet of 2000 B.C.; compare Gadd, 1922.

[3] The error is but little over 2%. In the other cases it is still smaller.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

complete triangle has 6 for base and 60 for *meret*. This word in other connections means bank or wharf, which would indicate a side and not the altitude. It does not seem probable that the author had much conception of different kinds of triangles. We may suppose that he has in mind a piece of land, of a certain width at one end and coming to a point, or at least narrower, at the other end. Thus to get the area he thinks of a rectangle with the average width of the piece of land.[1]

Pyramids. The Relation of the Lengths of Two Sides of a Triangle

The relation of the lengths of two sides of a right triangle is illustrated in Problems 56–60, which deal with the distinguishing lines of a pyramid. In these problems the scribe uses certain special terms. In 56–59 he uses the words *ukha-thebet* and *per-em-us* for two lines, and "pyramid" for the structure. In Problem 60 he calls the structure *iwn*[2] and the two lines *sentet* and *kay-en-heru*, and the height is much greater in proportion to the base. In both cases he uses the word *seked*[3] for the relation of the lengths of the two lines, but he thinks of the *seked*, not as a ratio, but as so many palms per cubit.

The diagrams themselves do not show definitely what these lines are, and there are two opinions respecting them. Eisenlohr in his translation takes the *ukha-thebet* as the diagonal of the base and the *per-em-us* as the lateral edge, while he takes *sentet* and *kay-en-heru* as the side of the base and the altitude. Borchardt (1893) contends that *ukha-thebet* and *sentet* both mean the side of the base, and that *per-em-us* and *kay-en-heru* both mean altitude.[4]

[1] Sethe in his review of Peet (see Bibliography under Peet, 1923, 2) regards *meret* as "clearly" meaning height, because the 7 in 53 is written at the vertex of the triangle (but in 51 the 10 is written along the middle of the side), and because mention is made of only one *meret*. Peet suggests, "half doubtfully" as Sethe says, that *meret* probably means height, but his reasoning is rather inconclusive and not very clear.

Gunn suggests (page 133) that *meret* is a pair of lines forming a right angle with each other, one of them perpendicular to the base at one end, and the other passing through the vertex, or, with a truncated triangle, lying on the line opposite the base. This idea was suggested to him by a (vertical) cross-section of a wharf and the sloping bank under it, although he himself says that to the Egyptian mind triangles mostly lie flat on the ground. To me a bank or wharf suggests the dividing line between the water and the land and would be applied to the side of a triangle of land as separating the ground within from the ground outside. The idea of a cross-section of a wharf is one of four arguments that Gunn offers. The other three are not so definite and can be modified, if necessary, so as to apply equally well to the interpretation of *meret* as the side of the triangle.

[2] *yûn*, a word generally meaning "pillar."

[3] Peet translates this *batter*.

[4] This interpretation had been suggested by E. and V. Revillout (1881).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In the first interpretation the ratio involved in the *seked* in Problems 56–59B is the cosine of the angle which the lateral edge makes with the diagonal of the base, and in Problem 60 the tangent of the angle which the lateral face makes with the base. In the second interpretation the *seked* means the cotangent of the latter angle in all of the problems.[1]

Borchardt argues with considerable force from practical considerations and most Egyptologists have now accepted his interpretation, although it necessitates the assumption that the Egyptian called the same line by different names in successive problems.[2] In my Free Translation it will be convenient to translate these terms in accordance with Borchardt's theory.The whole matter is not very important from the point of view of Egyptian mathematics. The important point is that at the beginning of the 18th century B.C., and probably a thousand years earlier, when the greater pyramids were built, the Egyptian mathematician had some notion of referring a right triangle to a similar triangle, one of whose sides was a unit of measure, as a standard. Nor do the two interpretations make much difference in the angles of the structure. In fact, the actual measurements of the pyramids themselves vary so much that we cannot tell absolutely from measurement which is the more probable interpretation.

In 56, 58, 59, and 60, the lengths of the two lines are given to determine the *seked*; in 57 and 59B the base line and the *seked* are given to determine the other line. In all cases we have an isosceles triangle, a half of whose base is the base of the right triangle used.

[1] Assuming that in 60 the author divides the wrong way. See page 99.

[2] Borchardt suggests in another paper (1922, page 12) that Problem 60 may have been taken from a different source.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

THE METHODS AND AIMS OF THE EGYPTIAN MATHEMATICIAN

His Methods

The Rhind mathematical papyrus is our chief authority for the state of mathematical knowledge in Egypt about the year 1650 B.C., and while some scholars such as E. and V. Revillout (1881) see in it only the work of a school-boy, most writers have recognized the scientific character of its procedure. The Egyptians of this date did not have the mental development reached by the Greeks a thousand years later. They had a smaller store of mathematical facts and less skill in mathematical operations. Yet their skill was remarkable and there was a scientific quality in their mathematics. The author of this papyrus had an idea of general methods applicable to groups of problems, and within the groups simple problems are followed by more complicated ones that are of the same kind and are solved by the same methods.

The methods of the Egyptian were largely those of trial and what we might call approximation. That is, if he could not get the answer at once he would try to get nearly the answer first and then make up what was lacking. This appears especially in what we have called the second kind of multiplication, where he has the multiplicand and product given to find the multiplier (see page 5), while the method of false position is also a method of trial.

Yet he was quick to generalize and when he had found a solution for a simple problem he did not hesitate to solve in the same way more difficult problems of the same kind, and occasionally to state the solution as a rule. He expresses this idea at the end of Problem 66, where he says, "Do the same thing in any example like this." Some processes are repeated again and again, showing that he had a method clearly in mind, even if he did not express it as a rule. One rule is put in words in Problem 61B, and is employed in several places (see pages 24–25) as if it were well understood. There are other operations and methods of solution that are taken for granted as being familiar at least to the author. His method of dividing a number in given proportions follows a definite rule which is employed without explanation, although not formulated (see page 28). Problem 79 illustrates, as I believe, a rule for finding the sum of a geometrical progression (see page 30).

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

In the geometrical problems he has several rules that he uses without comment, rules for getting areas and volumes and for changing from one unit of measure to another. Perhaps the most striking single operation that is often used is that by which in the third step of a multiplication of the second kind he determines the multiplier that will produce as product the reciprocal of a given number (see page 6).

A striking, though natural, characteristic of the Egyptian's work is his tendency noted above to take particular cases or particular numbers and generalize from them. This is seen especially in his method of adding fractions by taking them as parts of some number and adding their values when applied to this number (see pages 7–10). See also a remark on page 36.

It is quite possible that the writer of the papyrus, and the Egyptian mathematicians who preceded him, kept the results of their multiplications and other calculations in the form of tables, and often, when they had a multiplication or division to perform, took the result from these tables instead of working it out in full.[1] This would explain the fact that details are often omitted that are more difficult than other details that are put in. Some one must have worked out these details in some way, but the result of a multiplication once worked out could be used in two or three ways (see page 83). We have called the first part of the papyrus a table and next after this there is a table given in full just before Problem 1. There are also tables for fractions of a *hekat* in Problems 47, 80, and 81.

His Mistakes

Much has been written about the mistakes that we find in the Rhind papyrus. There are occasional mistakes, mostly numerical, that are merely accidental. Some of them are found in the group of Problems 7–20 (see page 64); five times in the first table 60 is written for 80; in two problems (43 and 59) the two given numbers are interchanged in the statement as compared with the solution; in Problem 49 the dimensions given in the statement are 10 and 2, but in the solution 10 and 1; in Problem 64 the mean share is given as ½ *hekat* when it is 1 *hekat*; in Problem 43 a second method of solution is preceded by one step of the first; in the last part of Problem 82 there is a numerical mistake in the division of a quantity of grain by 2. These are examples. At the beginning it is stated that this work is a copy of writings of an earlier period, and some of the mistakes seem to be mistakes of copying. Peet has

[1] We have many elaborate tables of the Babylonians. See, for example, Hilprecht, 1906.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

explained very skillfully how they may have arisen (see pages 63, 119, and 124). There are indications in the papyrus of corrections and alterations made after it was completed.[1] We may think of an earlier author, or earlier authors, who wrote the original from which our copy was made; of the scribe A'h-mosè who may have been an ignorant copyist, but who understood something at least of the problems and may have added details of his own; corrections or changes in the copy that is now in the British Museum may have been made by a still later hand. It is not likely that A'h-mosè made all the mistakes and the earlier writers none.[2] In two or three problems there is some confusion as to the meaning of the problem or of portions of the solution (53, 84, and certain lines of 82), that may be due to the fact that some parts are missing or that the writer or some copyist attempted to put down a problem that was but imperfectly known to him. It is interesting to speculate on the relation of these different writers to the book, but for the most part we shall have to take it as we have it now, the product of Egyptian mathematics, and Egyptian mathematics at its highest development. As to the mistakes, if a problem is obscure it may be necessary to notice them, but if it is clear we are not interested in them. We wish to know what the Egyptians understood, what they could do, and the methods that they employed. Everyone makes mistakes, and almost any modern mathematical book, in spite of the careful proof-reading that it receives, contains mistakes just like those in the papyrus, and some of them contain many more than the papyrus. The remarkable thing is not that there are mistakes, but that most of the calculations are carried through without them.

There are some cases where the method employed does not give a result that is exactly correct. This indicates the limits of the Egyptian's knowledge and is a matter of interest. But we should hardly call the em-

[1] Thus in Problem 28, line 2, the number which should be 10 was originally written 20 (and it is so written in the British Museum Facsimile) but the little mark that would make the 20 appears somewhat vaguely, and, according to Peet, this is because it was imperfectly erased, so that the papyrus may now be regarded as saying 10. The reader will find that corrections were made in Problems 11 and 12 and that the form of expression was changed in certain lines of Problem 61 (see notes on Problem 61, and, in the Literal Translation, on Problems 11 and 12). Sometimes the correction was made in lighter ink. There are also many patches covering portions of the original writing that are copied on the patches. Some of these can be recognized in the photographs. An interesting example is the sign for *d* near the end of line 2 in Problem 67. This sign is made on a patch and a little bit of the end of the original sign can be seen projecting from under an edge of the patch.

[2] We do not know anything about A'h-mosè, but the mistakes are not all mistakes of copying and some of them show some understanding of the processes on the part of whoever made them.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

ployment of such a method a careless mistake. He takes for the area of a circle the square of $\frac{8}{9}$ of the diameter, and according to one interpretation he seems to take for the area of a narrow isosceles triangle the product of the length of the side and the half of the base. In Problem 53 (according to my interpretation) he tries to allow for the deficiency of his figure (ABED, see page 94) from a rectangle by taking away its $\frac{1}{10}$ from the product of its base and side. We have some noteworthy indications of the limitations of the Egyptians, but we have also remarkable examples of what they could do, and in comparison the mere mistakes that we may find are of no importance or interest.

His Theoretical Interest

A careful study of the Rhind papyrus convinced me several years ago that this work is not a mere selection of practical problems especially useful to determine land values, and that the Egyptians were not a nation of shopkeepers, interested only in that which they could use. Rather I believe that they studied mathematics and other subjects for their own sakes. In the Rhind papyrus there are problems of area and problems of volume that might be of use to the farmer who owns land and raises grain. There are pyramid problems that might furnish specifications to the builders, or enable an interested observer to determine the dimensions of a pyramid before him. Many of the arithmetical problems concern a division of loaves or of a quantity of grain among a certain number of men, or the relative values of different amounts of food or drink. But when we come to examine the conditions laid down and the numbers involved in these various problems as well as the purely numerical ones, we see that they are more like theoretical problems put in concrete form. In one (Problem 63) 700 loaves are divided among four men in shares that are proportional to the four fractions $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, the first four terms of their fraction-series. In two (Problems 40 and 64) there is a dividing into shares that form an arithmetical progression, in Problem 67 the tribute for cattle is determined as $\frac{1}{6}$ $\frac{1}{18}$ of the herd and the problem asks for the number of the herd when the number of tribute cattle is given, and Problem 31 is a problem whose answer is

$$14\ \tfrac{1}{4}\ \tfrac{1}{56}\ \tfrac{1}{97}\ \tfrac{1}{194}\ \tfrac{1}{388}\ \tfrac{1}{679}\ \tfrac{1}{776}.$$

Such problems and such quantities were not likely to occur in the daily life of the Egyptians. Thus we can say that the Rhind papyrus, while

Digitized by Google Original from UNIVERSITY OF MICHIGAN

very useful to the Egyptian, was also "an example of the cultivation of mathematics as a pure science, even in its first beginnings."[1]

NOTE ON THE EGYPTIAN CALENDAR AND EGYPTIAN CHRONOLOGY—[2] There are three references in the Rhind papyrus to the Egyptian calendar. We find in the beginning the date when it was written, in Problem 66 the fact that 365 days make a year, and in Number 87 three dates and two of the days that were called epagomenal days.

The Egyptians counted their years from the beginning of the king's reign, starting anew with each new king, and, with the many breaks in their history and the uncertainty regarding many of their kings, it is difficult now to determine the dates of events, or the lengths of intervals between them.[3] In the Egyptian calendar year there was a certain shifting of the seasons and of the dates of events depending on them, that sometimes furnishes a clue for these determinations.

The Egyptian year consisted of 12 months of 30 days each and 5 days at the end that were regarded as not belonging to any month. The five days were called epagomenal days and were supposed to be the birthdays of five of the gods.[4] There were three seasons of four months each.

[1] It has been pointed out (Archibald, review of Peet, 1923, 2) that problems involving arithmetical and geometrical progressions seem to imply considerations not strictly practical and that the frustum of a pyramid problem of the Golenishchev papyrus (Turaev, 1917) is further evidence of the Egyptians' theoretical interest in science. Wieleitner (second review by him listed in the Bibliography under Peet, 1923, 2), after referring to this, says, "But is it not sufficient simply to refer to the *Hau*-reckoning, which, even if it refers to a measure (as do 35, 37, and 38, which belong with the '*hau*-reckoning' problems), is purely theoretical and in part complicated," and he ends with the words that I have quoted. See also Karpinski, 1917, page 258.

The scientific interest of the Egyptians is especially shown in the Edwin Smith papyrus, which contains careful and systematically arranged observations on the human body, and adds many interesting details, even in cases that are classified as hopeless, with no treatment recommended. See J. H. Breasted, "The Edwin Smith Papyrus," *The New York Historical Society, Quarterly Bulletin*, volume 6, 1922, pages 5–31.

[2] References: J. H. Breasted, *Ancient Records of Egypt*, Chicago, volume 1, 1906, pages 25–48; Eduard Meyer, "Ægyptische Chronologie," 1904, "Nachträge zur ægyptische Chronologie," 1907, *Abhandl. d. Berlin. Akad. d. Wiss.*, Berlin; F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie*, Leipzig, 1906, Band I, Einleitung und Kapitel II. The last mentioned book is very complete and thorough. A view of Egyptian chronology not at present generally accepted has been advocated by F. Petrie in "The length of Egyptian history," *British School of Archaeology in Egypt. Studies*, London, 1911, volume 2, pages 10–12; and another by A. Weigall in *A History of the Pharaohs*, New York, 1925.

[3] One very uncertain period lies between the twelfth and eighteenth dynasties, which included the time of the Hyksos kings and of the writing of our papyrus.

[4] Osiris, Horus, Set, Isis, and Nephthys. In Number 87 two of these birthdays are mentioned as belonging to the first month. This, however, is certainly a mistake, although Egyptologists have found it difficult to explain how the writer came to make it. See Peet page 130.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Names were given to the months, but each month was also known simply as the first, second, third, or fourth of the season to which it belonged. The most striking event of the Egyptian year is the rising of the Nile, which begins in July, and the seasons were named with reference to this event and its consequences. Thus the first season was called the season of the inundation (*akhet*, mentioned at the beginning of the papyrus and in Number 87), the second season was the season of the "going forth" (*prôt*) when the vegetation burst forth in the fields as the waters of the river were subsiding, and the third season was the season of summer (*shômu*, mentioned also in Number 87), when the earth became dry and parched before the coming of the next inundation.

These names show that at first the calendar year was made to begin about the time that the Nile began to rise. But the rising of the Nile depends on the solar year, which is about a quarter of a day longer than this Egyptian calendar year,[1] and so the Nile began to rise a day later after four years and a month later after 120 years, and after a time the entire season of the inundation came before the river began to rise, and was the dryest season of the three. They soon found that their year was a "wandering year," but continued to use it.

The beginning of the rise of the Nile, or any other event that depends on the seasons, is not as regular, nor as easy to determine exactly as some things that may be observed of the stars. The Egyptians early discovered that in the daily apparent revolution of the sun and stars around the earth the stars were continually gaining on the sun. If they watched a star, setting perhaps not long after sunset, they would notice that it set earlier each night than the night before, until at last it would disappear almost before there was darkness enough to make it visible. Then, after a certain number of days, they would see it in the east rising before the sun and appearing in the morning sky as a harbinger of the coming day. They noticed this, in particular, of Sirius, the brightest of all the fixed stars. The day when Sirius first appeared as a morning star had a religious significance. We can imagine the priests in the early morning, gazing out over the desert, waiting for the rising of the sun, and then one morning to the south of the place where the sun would appear some one would be the first to catch a glimpse of the rising star just as its light was being put out by the advancing sunlight. This was

[1] 365¼ days is called a Julian year. The solar year is a very little shorter. Newcomb gives its length in 1900 as 365.2422 days, with a yearly shortening of .0053 seconds, which would make the solar year about 365.2425 days in early Egyptian history (Ginzel, page 32). This is the same as $365^{97}/_{400}$, the number of days in our calendar year, there being 97 leap years in 400 years.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

called the heliacal rising of Sirius, its rising with the sun, and this day marked the beginning of a kind of year that would end when the day came for the same event to occur again. This Sirius year, like the solar year, was a quarter of a day longer than the Egyptian calendar year, and its beginning, like the rise of the Nile, occurred one day later every four years.

The siderial year is longer than the solar year. The vernal equinox moves along the ecliptic at the rate of about 50″ a year, and when the sun reaches the vernal equinox it has still 50″ to go to reach the point where it passed the vernal equinox the year before.[1] If a star is on the ecliptic the time from the moment when the sun passes it to the moment when this event occurs again will be exactly a siderial year. We might suppose that the same would be true for any fixed star. If we think of the great circle of the ecliptic with the earth as a point at its center, we should say that the sun passes any fixed star when they have the same longitude, longitude being measured along the ecliptic. But the apparent daily revolution of the sun and stars around the earth is a motion parallel to the equator, and when the observer thinks of a star as overtaking and passing the sun, the passing that he thinks of is when they have the same right ascension, for right ascension is measured along the equator.

The heliacal rising of Sirius as observed by the Egyptians was not, indeed, the moment when Sirius passed the sun, even in this sense, for the star would have to rise before the upper edge of the sun in order to be visible. The refraction of the atmosphere and other conditions must be taken account of in determining this moment, and especially the fact that Sirius in the south reaches the horizon farther along on its circle of rotation than the sun. These conditions, however, for the most part, are the same from year to year and would not affect the length of time between two such events. But the rate of increase in the right ascension of the star varies with the position of the equator, and the star year is not usually the same as the siderial year. The difference is, however, a periodic one, with a period of over 25,000 years. During one part of this period the star year is longer than the siderial year, and during the other part it is shorter. It simply happens that during practically all of Egyptian history the Sirius year was almost exactly 365¼ days.[2] The

[1] The exact distance is 50.2564″ and it takes the sun 20 min. 23.8 sec., making the siderial year equal to 365.25636 days (Ginzel, pages 28 and 32).

[2] Let λ and β be the longitude and latitude of a star, ω the angle which the ecliptic makes with the equator, and γ the longitude of the point on the ecliptic which has the same right

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Egyptians have left two or three records, each giving the day and month when Sirius rose with the sun in a certain year of some king's reign. The number of days difference in the calendar dates of two such records, multiplied by 4, will enable us to determine the number of years between the two events,[1] but as the shifting of a day in four years amounts to a year in 1460 years, it may be necessary to add 1460 or a multiple of this number to get the length of the entire interval.

NOTE ABOUT WORDS FOR GRAIN AND OTHER KINDS OF FOOD—There are ten or twelve words used in the papyrus to denote some kind of grain or food. As I have translated them they are:

1. Specific grains,

yôt	barley	used in Problem	64
bôdet	spelt	used in Problems	79, 82, and 86
sûet	wheat	used in Problem	82

also

yôt mehy	Lower Egyptian barley	used in Problem	83

ascension as the star. Then from the right triangles of the figure we can derive the equation

$$\tan \gamma = \tan \lambda - \frac{\tan \omega \tan \beta}{\cos \lambda}.$$

For a fixed star β is constant but λ increases at the rate of 50″ in a calendar year. Then when $d\lambda$ is taken equal to 50″, $d\gamma$ will be the increase of γ in a year and will measure the amount by which the star year exceeds the solar year.

A study of the above equation and of the equation for $d\gamma/d\lambda$ shows that λ and γ are together at $-\pi/2$ and $\pi/2$, and, if β is positive, then as they increase from one to the other of these values, γ increases more slowly than λ until it comes to the value 0, and then more rapidly. Beyond $\pi/2$ conditions are reversed, γ increasing at first more rapidly and afterwards more slowly.

If β is negative, that is, if the star is south of the ecliptic, as is the case with Sirius, conditions between $-\pi/2$ and $\pi/2$ are the same as the conditions above between $\pi/2$ and $3\pi/2$: as λ and γ increase, γ increases more rapidly than λ until it comes to the value 0, and then more slowly. During the period of Egyptian history γ was positive and increasing more slowly than λ, $d\gamma < d\lambda$, and the Sirius year was shorter than the siderial year. Ginzel gives a table (page 185) for the length of the Sirius year at different periods, showing that it was equal to the Julian year about 3231 B.C., and that by 139 A.D. it was 1 min. 29 sec. longer.

[1] At least to within less than 4 years. The following illustration shows how this fact is used (see Ginzel pages 194 and 200):

A record of the heliacal rising of Sirius on the 28th day of the month Epiphi (the third month of the season *shômu*) under Tehût-mosè III has been fixed as occurring in 1470 B.C. Now the Ebers papyrus records a rising as on the 9th of Epiphi in the 9th year of Amen-hotpe I. From the 9th to the 28th is 19 days which would give us an interval of 76 years. Therefore the Ebers rising occurred about 76 years earlier than that recorded under Tehût-mosè, that is, about 1546 B.C., which puts the beginning of Amen-hotpe's reign at about 1555 B.C. According to Breasted this date is 1557 B.C.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

yôt shema'	Upper Egyptian barley	used in Problem	74

2. General term for grain,

shes [1]	grain	used in Problems	35, etc.

3. Meal or flour,

wedyet	*wedyet*-flour	used in Problems	72, etc.
noid	meal	used in Problems	69 and 70

4. Other foods,

'ôd	fat	used in Problem	66
besha	*besha* [2]	used in Problem	71
heket [1]	beer	used in Problems	71, 77, and 78.

The grain and flour words as generally written contain the grain sign (see page 32).

The word *shes* occurs about 16 times and is a general term for any kind of grain. Thus in Problem 82 it appears to refer to both spelt and wheat. In many places it is used to introduce the standard notation, explained on pages 31–32, with the "Horus eye" fractions, "It makes of grain" being almost equivalent to "Express in standard form."

Wedyet-flour is used in making bread and beer, and in Problem 82 in making feed for geese. Peet suggests that this may be a general term for flour or meal, just as *shes* is a general term for grain.

Peet gives a full and clear discussion of the uses of these words (pages 113–114).

[1] See notes to Problems 41 and 71 in the Literal Translation in regard to the readings of these words.

[2] A kind of grain or fruit, Erman and Grapow, *Wörterbuch der Aegyptischen Sprache*, Volume 1, Leipzig, 1926, page 478.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

FREE TRANSLATION AND COMMENTARY[1]

Title, Date, and Scribe

Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets. This book was copied in the year 33, in the fourth month of the inundation season, under the majesty of the king of Upper and Lower Egypt, 'A-user-Rê', endowed with life, in likeness to writings of old made in the time of the king of Upper and Lower Egypt, Ne-ma'et-Rê'. It is the scribe A'h-mosè who copies this writing.

[1] In the following pages I have endeavored to give in clear smooth English a free translation of the Rhind papyrus with some notes explaining details. In some places I have put in words or clauses that are omitted from the original, but found in other problems of the same group, or are required to express what I believe is the true meaning of the Egyptian. In some places where parts of a solution are misplaced I have arranged them in their proper order (see, for example, Problems 32 and 69). More particularly, I have corrected numerical mistakes and careless slips in the way of writing numbers. The reader will find the exact words of the original in the Literal Translation, and all corrections that have been made in the Free Translation may be found by a comparison of the two.

Notes giving explanations or interpretations of the text, except such as have been included in the Introduction, are placed in the following pages with the problems to which they refer. Notes relating to the Egyptian language or writing are placed with the Literal Translation.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

CHAPTER I. EGYPTIAN ARITHMETIC

SECTION I

TABLE OF THE DIVISION OF 2 BY ODD NUMBERS

2 divided by 3

Get 2 by operating on 3. ⅔ of 3 is 2.

2 divided by 5

⅓ of 5 is 1 ⅔, $\frac{1}{15}$ of 5 is ⅓.

Working out:

1	5
⅔	3 ⅓
[1]\⅓	1 ⅔
\$\frac{1}{15}$	⅓.

2 divided by 7

¼ of 7 is 1 ½ ¼, $\frac{1}{28}$ of 7 is ¼.

1	7		1	7
½	3 ½		2	14
\¼	1 ½ ¼	¼	4	28.
[2]\4	28			

2 divided by 9

⅙ of 9 is 1 ½, $\frac{1}{18}$ of 9 is ½.

1	9	
⅔	6	
⅓	3	
\⅙	1 ½	
\2	18	½.

[1] See Introduction, page 4. The use of these checks is somewhat arbitrary. They are sometimes put with the wrong numbers. Often they are used only in some of the multiplications of a problem. In Problems 7–20 there are no checks at all and in certain other problems also (45, 46, 49, 51, 56, 66, and 79) they are absent. In this Free Translation I have tried to be a little more uniform, but to follow the original approximately.

[2] See Introduction, page 17.

Digitized by Google

Original from UNIVERSITY OF MICHIGAN

2 divided by 11

⅙ of 11 is 1 ⅔ ⅙, $\frac{1}{66}$ of 11 is ⅙.

1	11		1	11	
⅔	7 ⅓		\2	22	
⅓	3 ⅔		\4	44	
\⅙	1 ⅔ ⅙	Total	6	66	⅙.

2 divided by 13

⅛ of 13 is 1 ½ ⅛, $\frac{1}{52}$ of 13 is ¼, $\frac{1}{104}$ of 13 is ⅛.

1	13		
½	6 ½		
¼	3 ¼		
\⅛	1 ½ ⅛		
\4	52	¼	
\8	104	⅛.	

2 divided by 15

$\frac{1}{10}$ of 15 is 1 ½, $\frac{1}{30}$ of 15 is ½.

1	15	
\$\frac{1}{10}$	1 ½	
\$\frac{1}{30}$	½.	

2 divided by 17

Get 2 by operating on 17. $\frac{1}{12}$ of 17 is 1 ⅓ $\frac{1}{12}$, $\frac{1}{51}$ of 17 is ⅓, $\frac{1}{68}$ of 17 is ¼.

Working out:

1	17					
⅔	11 ⅓					
⅓	5 ⅔		\1	17		
⅙	2 ½ ⅓		\2	34		
\$\frac{1}{12}$	1 ¼ ⅙	Total	3	51		⅓
Remainder	⅓ ¼		4	68		¼.

In the fourth line of the multiplication the author might have said 2 ⅔ ⅙, and then the last line would have been 1 ⅓ $\frac{1}{12}$, and he uses this form, in giving his answer above, for the part that $\frac{1}{12}$ of 17 makes towards 2. Somewhat similarly in the case of 23 he changes 1 ½ ¼ ⅙ to 1 ⅔ ¼.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 19

$\frac{1}{12}$ of 19 is 1 $\frac{1}{2}$ $\frac{1}{12}$, $\frac{1}{76}$ of 19 is $\frac{1}{4}$, $\frac{1}{114}$ of 19 is $\frac{1}{6}$.

1	19		1	19	
$\frac{2}{3}$	12 $\frac{2}{3}$		2	38	
$\frac{1}{3}$	6 $\frac{1}{3}$		4	76	$\frac{1}{4}$
$\frac{1}{6}$	3 $\frac{1}{6}$		Remainder		$\frac{1}{6}$
$\frac{1}{12}$	1 $\frac{1}{2}$ $\frac{1}{12}$				
Remainder	$\frac{1}{4}$ $\frac{1}{6}$				
			1	19	
			2	38	
			4	76	
		Total	6	114	$\frac{1}{6}$.

2 divided by 21

$\frac{1}{14}$ of 21 is 1 $\frac{1}{2}$, $\frac{1}{42}$ of 21 is $\frac{1}{2}$.

1	21	
$\frac{2}{3}$	14	1 $\frac{1}{2}$
2	42	$\frac{1}{2}$.

2 divided by 23

$\frac{1}{12}$ of 23 is 1 $\frac{2}{3}$ $\frac{1}{4}$, $\frac{1}{276}$ of 23 is $\frac{1}{12}$.

1	23		1	23	
$\frac{2}{3}$	15 $\frac{1}{3}$		10	230	
$\frac{1}{3}$	7 $\frac{2}{3}$		2	46	
$\frac{1}{6}$	3 $\frac{1}{2}$ $\frac{1}{3}$	Total	12	276	$\frac{1}{12}$.
$\frac{1}{12}$	1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$				
Remainder	$\frac{1}{12}$				

2 divided by 25

$\frac{1}{15}$ of 25 is 1 $\frac{2}{3}$, $\frac{1}{75}$ of 25 is $\frac{1}{3}$.

1	25	
$\frac{1}{15}$	1 $\frac{2}{3}$	
3	75	$\frac{1}{2}$.

2 divided by 27

$\frac{1}{18}$ of 27 is 1 $\frac{1}{2}$, $\frac{1}{54}$ of 27 is $\frac{1}{2}$.

1	27	
$\frac{2}{3}$	18	1 $\frac{1}{2}$
2	54	$\frac{1}{2}$.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

2 divided by 29

Get 2 by operating on 29. $\frac{1}{24}$ of 29 is 1 $\frac{1}{6}$ $\frac{1}{24}$, $\frac{1}{58}$ of 29 is $\frac{1}{2}$, $\frac{1}{174}$ of 29 is $\frac{1}{6}$, $\frac{1}{232}$ of 29 is $\frac{1}{8}$.

Working out:

1	29	
\ $\frac{1}{24}$	1 $\frac{1}{6}$ $\frac{1}{24}$	
\ 2	58	$\frac{1}{2}$
\ 6	174	$\frac{1}{6}$
\ 8	232	$\frac{1}{8}$.

2 divided by 31

$\frac{1}{20}$ of 31 is 1 $\frac{1}{2}$ $\frac{1}{20}$, $\frac{1}{124}$ of 31 is $\frac{1}{4}$, $\frac{1}{155}$ of 31 is $\frac{1}{5}$.

1	31	
\ $\frac{1}{20}$	1 $\frac{1}{2}$ $\frac{1}{20}$	
\ 4	124	$\frac{1}{4}$
\ 5	155	$\frac{1}{5}$.

2 divided by 33

$\frac{1}{22}$ of 33 is 1 $\frac{1}{2}$, $\frac{1}{66}$ of 33 is $\frac{1}{2}$.

1	33	
\ $\frac{2}{3}$	22	1 $\frac{1}{2}$
\ 2	66	$\frac{1}{2}$.

2 divided by 35

35. $\frac{1}{30}$ of 35 is 1 $\frac{1}{6}$, $\frac{1}{42}$ of 35 is $\frac{2}{3}$ $\frac{1}{6}$.

6 7 5

2 times $\frac{1}{35}$ is $\frac{1}{30}$ $\frac{1}{42}$. For $\frac{1}{35}$ applied to 210 gives 6; and 2 times 6 is 12, or 7 and 5, which are $\frac{1}{30}$ and $\frac{1}{42}$ of 210.

1	35
\ $\frac{1}{30}$	1 $\frac{1}{6}$
\ $\frac{1}{42}$	$\frac{2}{3}$ $\frac{1}{6}$.

This example is explained in the Introduction, page 18. In the papyrus, following his usual practice when applying fractions to a particular number, the author writes 6, 7, and 5 under the 35, $\frac{1}{30}$, and $\frac{1}{42}$ of the first line.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 37

$\frac{1}{24}$ of 37 is 1 $\frac{1}{2}$ $\frac{1}{24}$, $\frac{1}{111}$ of 37 is $\frac{1}{3}$, $\frac{1}{296}$ of 37 is $\frac{1}{8}$.

1	37		\1	37	
$\frac{2}{3}$	24 $\frac{2}{3}$		\2	74	
$\frac{1}{3}$	12 $\frac{1}{3}$	Total	3	111	$\frac{1}{3}$
$\frac{1}{6}$	6 $\frac{1}{6}$		Remainder	$\frac{1}{8}$	
$\frac{1}{12}$	3 $\frac{1}{12}$		1	37	
\$\frac{1}{24}$	1 $\frac{1}{2}$ $\frac{1}{24}$		2	74	
Remainder	$\frac{1}{3}$ $\frac{1}{8}$		4	148	
			8	296	$\frac{1}{8}$.

2 divided by 39

$\frac{1}{26}$ of 39 is 1 $\frac{1}{2}$, $\frac{1}{78}$ of 39 is $\frac{1}{2}$.

1	39	
\$\frac{2}{3}$	26	1 $\frac{1}{2}$
\2	78	$\frac{1}{2}$.

2 divided by 41

Get 2 by operating on 41. $\frac{1}{24}$ of 41 is 1 $\frac{2}{3}$ $\frac{1}{24}$, $\frac{1}{246}$ of 41 is $\frac{1}{6}$, $\frac{1}{328}$ of 41 is $\frac{1}{8}$.

Working out:

1	41		1	41	
$\frac{2}{3}$	27 $\frac{1}{3}$		\2	82	
$\frac{1}{3}$	13 $\frac{2}{3}$		\4	164	
$\frac{1}{6}$	6 $\frac{2}{3}$ $\frac{1}{6}$	Total	6	246	$\frac{1}{6}$
$\frac{1}{12}$	3 $\frac{1}{3}$ $\frac{1}{12}$		8	328	$\frac{1}{8}$.
\$\frac{1}{24}$	1 $\frac{2}{3}$ $\frac{1}{24}$				
Remainder	$\frac{1}{6}$ $\frac{1}{8}$				

2 divided by 43

$\frac{1}{42}$ of 43 is 1 $\frac{1}{42}$, $\frac{1}{86}$ of 43 is $\frac{1}{2}$, $\frac{1}{129}$ of 43 is $\frac{1}{3}$, $\frac{1}{301}$ of 43 is $\frac{1}{7}$.

	1	43	
Find	\$\frac{1}{42}$	1 $\frac{1}{42}$	
	\2	86	$\frac{1}{2}$
	\3	129	$\frac{1}{3}$
	\7	301	$\frac{1}{7}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 45

$\frac{1}{30}$ of 45 is 1 $\frac{1}{2}$, $\frac{1}{90}$ of 45 is $\frac{1}{2}$.

	1	45	
\	$\frac{2}{3}$	30	1 $\frac{1}{2}$
\	2	90	$\frac{1}{2}$.

2 divided by 47

$\frac{1}{30}$ of 47 is 1 $\frac{1}{2}$ $\frac{1}{15}$, $\frac{1}{141}$ of 47 is $\frac{1}{3}$, $\frac{1}{470}$ of 47 is $\frac{1}{10}$.

	1	47		
Find	$\frac{1}{30}$		1 $\frac{1}{2}$ $\frac{1}{15}$	
\	3	141		$\frac{1}{3}$
\	10	470		$\frac{1}{10}$.

2 divided by 49

$\frac{1}{28}$ of 49 is 1 $\frac{1}{2}$ $\frac{1}{4}$, $\frac{1}{196}$ of 49 is $\frac{1}{4}$.

	1	49		
Find	\ $\frac{1}{28}$		1 $\frac{1}{2}$ $\frac{1}{4}$	
\	4	196		$\frac{1}{4}$.

2 divided by 51

$\frac{1}{34}$ of 51 is 1 $\frac{1}{2}$, $\frac{1}{102}$ of 51 is $\frac{1}{2}$.

	1	51	
\	$\frac{2}{3}$	34	1 $\frac{1}{2}$
\	2	102	$\frac{1}{2}$.

2 divided by 53

Get 2 by operating on 53. $\frac{1}{30}$ of 53 is 1 $\frac{2}{3}$ $\frac{1}{10}$, $\frac{1}{318}$ of 53 is $\frac{1}{6}$, $\frac{1}{795}$ of 53 is $\frac{1}{15}$.

Working out:

	1	53				1	53	
Find	\ $\frac{1}{30}$		1 $\frac{2}{3}$ $\frac{1}{10}$			\ 10	530	
\	6	318		$\frac{1}{6}$		\ 5	265	
	Remainder		$\frac{1}{15}$		Total	15	795	$\frac{1}{15}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 55

$\frac{1}{30}$ of 55 is 1 $\frac{2}{3}$ $\frac{1}{6}$, $\frac{1}{330}$ of 55 is $\frac{1}{6}$.

	1	55	
Find	\ $\frac{1}{30}$	1 $\frac{2}{3}$ $\frac{1}{6}$	
	\ 6	330	$\frac{1}{6}$.

2 divided by 57

$\frac{1}{38}$ of 57 is 1 $\frac{1}{2}$, $\frac{1}{114}$ of 57 is $\frac{1}{2}$.

	1	57	
	\ $\frac{2}{3}$	38	1 $\frac{1}{2}$
	\ 2	114	$\frac{1}{2}$.

2 divided by 59

$\frac{1}{36}$ of 59 is 1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$, $\frac{1}{236}$ of 59 is $\frac{1}{4}$, $\frac{1}{531}$ of 59 is $\frac{1}{9}$.

	1	59	
Find	\ $\frac{1}{36}$	1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$	
	\ 4	236	$\frac{1}{4}$
	\ 9	531	$\frac{1}{9}$.

2 divided by 61

$\frac{1}{40}$ of 61 is 1 $\frac{1}{2}$ $\frac{1}{40}$, $\frac{1}{244}$ of 61 is $\frac{1}{4}$, $\frac{1}{488}$ of 61 is $\frac{1}{8}$, $\frac{1}{610}$ of 61 is $\frac{1}{10}$.

	1	61	
Find	\ $\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{40}$	
	\ 4	244	$\frac{1}{4}$
	\ 8	488	$\frac{1}{8}$
	\ 10	610	$\frac{1}{10}$.

2 divided by 63

$\frac{1}{42}$ of 63 is 1 $\frac{1}{2}$, $\frac{1}{126}$ of 63 is $\frac{1}{2}$.

	1	63	
	\ $\frac{2}{3}$	42	1 $\frac{1}{2}$
	\ 2	126	$\frac{1}{2}$.

2 divided by 65

Get 2 by operating on 65. $\frac{1}{39}$ of 65 is 1 $\frac{2}{3}$, $\frac{1}{195}$ of 65 is $\frac{1}{3}$.

Working out:

	1	65	
Find	\ $\frac{1}{39}$	1 $\frac{2}{3}$	
	\ 3	195	$\frac{1}{3}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 67

$\frac{1}{40}$ of 67 is 1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{20}$, $\frac{1}{335}$ of 67 is $\frac{1}{5}$, $\frac{1}{536}$ of 67 is $\frac{1}{8}$.

	1	67	
Find	\ $\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{20}$	
	\ 5	335	$\frac{1}{5}$
	\ 8	536	$\frac{1}{8}$.

2 divided by 69

$\frac{1}{46}$ of 69 is 1 $\frac{1}{2}$, $\frac{1}{138}$ of 69 is $\frac{1}{2}$.

	1	69	
	\ $\frac{2}{3}$	46	1 $\frac{1}{2}$
	\ 2	138	$\frac{1}{2}$.

2 divided by 71

$\frac{1}{40}$ of 71 is 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{40}$, $\frac{1}{568}$ of 71 is $\frac{1}{8}$, $\frac{1}{710}$ of 71 is $\frac{1}{10}$.

	1	71	
Find	\ $\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{40}$	
	\ 8	568	$\frac{1}{8}$
	\ 10	710	$\frac{1}{10}$.

2 divided by 73

$\frac{1}{60}$ of 73 is 1 $\frac{1}{6}$ $\frac{1}{20}$, $\frac{1}{219}$ of 73 is $\frac{1}{3}$, $\frac{1}{292}$ of 73 is $\frac{1}{4}$, $\frac{1}{365}$ of 73 is $\frac{1}{5}$.

	1	73	
Find	\ $\frac{1}{60}$	1 $\frac{1}{6}$ $\frac{1}{20}$	
	\ 3	219	$\frac{1}{3}$
	\ 4	292	$\frac{1}{4}$
	\ 5	365	$\frac{1}{5}$.

2 divided by 75

$\frac{1}{50}$ of 75 is 1 $\frac{1}{2}$, $\frac{1}{150}$ of 75 is $\frac{1}{2}$.

	1	75	
	\ $\frac{2}{3}$	50	1 $\frac{1}{2}$
	\ 2	150	$\frac{1}{2}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 77

Get 2 by operating on 77. $\frac{1}{44}$ of 77 is 1 $\frac{1}{2}$ $\frac{1}{4}$, $\frac{1}{308}$ of 77 is $\frac{1}{4}$.

Working out:

	1	77	
Find	\ $\frac{1}{44}$	1 $\frac{1}{2}$ $\frac{1}{4}$	
	\ 4	308	$\frac{1}{4}$.

2 divided by 79

$\frac{1}{60}$ of 79 is 1 $\frac{1}{4}$ $\frac{1}{15}$, $\frac{1}{237}$ of 79 is $\frac{1}{3}$, $\frac{1}{316}$ of 79 is $\frac{1}{4}$, $\frac{1}{790}$ of 79 is $\frac{1}{10}$.

	1	79	
Find	\ $\frac{1}{60}$	1 $\frac{1}{4}$ $\frac{1}{15}$	
	\ 3	237	$\frac{1}{3}$
	\ 4	316	$\frac{1}{4}$
	\ 10	790	$\frac{1}{10}$.

2 divided by 81

$\frac{1}{54}$ of 81 is 1 $\frac{1}{2}$, $\frac{1}{162}$ of 81 is $\frac{1}{2}$.

	1	81	
	\ $\frac{2}{3}$	54	1 $\frac{1}{2}$
	\ 2	162	$\frac{1}{2}$.

2 divided by 83

$\frac{1}{60}$ of 83 is 1 $\frac{1}{3}$ $\frac{1}{20}$, $\frac{1}{332}$ of 83 is $\frac{1}{4}$, $\frac{1}{415}$ of 83 is $\frac{1}{5}$, $\frac{1}{498}$ of 83 is $\frac{1}{6}$.

	1	83	
Find	\ $\frac{1}{60}$	1 $\frac{1}{3}$ $\frac{1}{20}$	
	\ 4	332	$\frac{1}{4}$
	\ 5	415	$\frac{1}{5}$
	\ 6	498	$\frac{1}{6}$.

2 divided by 85

$\frac{1}{51}$ of 85 is 1 $\frac{2}{3}$, $\frac{1}{255}$ of 85 is $\frac{1}{3}$.

	1	85	
Find	\ $\frac{1}{51}$	1 $\frac{2}{3}$	
	\ 3	255	$\frac{1}{3}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 87

$\frac{1}{58}$ of 87 is 1 $\frac{1}{2}$, $\frac{1}{174}$ of 87 is $\frac{1}{2}$.

	1	87	
\	$\frac{3}{3}$	58	1 $\frac{1}{2}$
\	2	174	$\frac{1}{2}$.

2 divided by 89

Get 2 by operating on 89. $\frac{1}{60}$ of 89 is 1 $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{20}$, $\frac{1}{356}$ of 89 is $\frac{1}{4}$, $\frac{1}{534}$ of 89 is $\frac{1}{6}$, $\frac{1}{890}$ of 89 is $\frac{1}{10}$.

Working out:

	1	89	
Find \	$\frac{1}{60}$	1 $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{20}$	
\	4	356	$\frac{1}{4}$
\	6	534	$\frac{1}{6}$
\	10	890	$\frac{1}{10}$.

2 divided by 91

$\frac{1}{70}$ of 91 is 1 $\frac{1}{3}$ $\frac{1}{10}$, $\frac{1}{130}$ of 91 is $\frac{2}{3}$ $\frac{1}{30}$.

	1	91	
Find \	$\frac{1}{70}$	1 $\frac{1}{3}$ $\frac{1}{10}$	
Find \	$\frac{1}{130}$	$\frac{2}{3}$ $\frac{1}{30}$.	

2 divided by 93

$\frac{1}{62}$ of 93 is 1 $\frac{1}{2}$, $\frac{1}{186}$ of 93 is $\frac{1}{2}$.

	1	93	
\	$\frac{3}{3}$	62	1 $\frac{1}{2}$
\	2	186	$\frac{1}{2}$.

2 divided by 95

$\frac{1}{60}$ of 95 is 1 $\frac{1}{2}$ $\frac{1}{12}$, $\frac{1}{380}$ of 95 is $\frac{1}{4}$, $\frac{1}{570}$ of 95 is $\frac{1}{6}$.

	1	95	
Find \	$\frac{1}{60}$	1 $\frac{1}{2}$ $\frac{1}{12}$	
\	4	380	$\frac{1}{4}$
\	6	570	$\frac{1}{6}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

2 divided by 97

$\frac{1}{56}$ of 97 is 1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$, $\frac{1}{679}$ of 97 is $\frac{1}{7}$, $\frac{1}{776}$ of 97 is $\frac{1}{8}$.

	1	97	
Find	\ $\frac{1}{56}$	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$	
	\ 7	679	$\frac{1}{7}$
	\ 8	776	$\frac{1}{8}$.

2 divided by 99

$\frac{1}{66}$ of 99 is 1 $\frac{1}{2}$, $\frac{1}{198}$ of 99 is $\frac{1}{2}$.

	1	99	
Find	\ $\frac{2}{3}$	66	1 $\frac{1}{2}$
	\ 2	198	$\frac{1}{2}$.

2 divided by 101

Get 2 by operating on 101. $\frac{1}{101}$ of 101 is 1, $\frac{1}{202}$ of 101 is $\frac{1}{2}$, $\frac{1}{303}$ of 101 is $\frac{1}{3}$, $\frac{1}{606}$ of 101 is $\frac{1}{6}$.

Working out:

\ 1	101	1
\ 2	202	$\frac{1}{2}$
\ 3	303	$\frac{1}{3}$
\ 6	606	$\frac{1}{6}$.

SECTION II

PROBLEMS 1–6. TABLE OF THE DIVISION OF THE NUMBERS 1–9 BY 10

Table	1	divided	by	10	gives	$\frac{1}{10}$
	2	"	"	"	"	$\frac{1}{5}$
	3	"	"	"	"	$\frac{1}{5}$ $\frac{1}{10}$
	4	"	"	"	"	$\frac{1}{3}$ $\frac{1}{15}$
	5	"	"	"	"	$\frac{1}{2}$
	6	"	"	"	"	$\frac{1}{2}$ $\frac{1}{10}$
	7	"	"	"	"	$\frac{2}{3}$ $\frac{1}{30}$
	8	"	"	"	"	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
	9	"	"	"	"	$\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 1

Example of dividing 1 *loaf among* 10 *men.*

Each man receives $\frac{1}{10}$.

Proof. Multiply $\frac{1}{10}$ by 10.

Do it thus:

1	$\frac{1}{10}$
\2	$\frac{1}{5}$
4	$\frac{1}{3}$ $\frac{1}{15}$
\8	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$

Total 1 loaf, which is correct.

Problem 2

Divide 2 *loaves among* 10 *men.*

Each man receives $\frac{1}{5}$.

Proof. Multiply $\frac{1}{5}$ by 10.

Do it thus:

1	$\frac{1}{5}$
\2	$\frac{1}{3}$ $\frac{1}{15}$
4	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
\8	1 $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{15}$

Total 2 loaves, which is correct.

Problem 3

Divide 6 *loaves among* 10 *men.*

Each man receives $\frac{1}{2}$ $\frac{1}{10}$.

Proof. Multiply $\frac{1}{2}$ $\frac{1}{10}$ by 10.

Do it thus:

1	$\frac{1}{2}$ $\frac{1}{10}$
\2	1 $\frac{1}{5}$
4	2 $\frac{1}{3}$ $\frac{1}{15}$
\8	4 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$

Total 6 loaves, which is correct.

Problem 4

Divide 7 *loaves among* 10 *men.*

Each man receives $\frac{2}{3}$ $\frac{1}{30}$.

Proof. Multiply $\frac{2}{3}$ $\frac{1}{30}$ by 10; the result is 7.

Do it thus:

1	$\frac{2}{3}$ $\frac{1}{30}$
\2	1 $\frac{1}{3}$ $\frac{1}{15}$
4	2 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
\8	5 $\frac{1}{2}$ $\frac{1}{10}$

Total 7 loaves, which is correct.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The answer in this case might have been written[1] ½ ⅛, but the Egyptian thinks of ⅔ as the largest fraction that there is and prefers to use it wherever he can.

In this problem and the next two the double of ⅔ ⅒ ⅓₀ is given as 1 ⅓ ⅒. Directly it is 1 ⅓ ⅓ ⅛. Perhaps these fractions were taken as parts of some number, say 30. They would make 18, or 15 and 3, which would be ⅓ and ⅒ of 30.

Problem 5

Divide 8 *loaves among* 10 *men.*

Each man receives ⅔ ⅒ ⅓₀.

Proof. Multiply ⅔ ⅒ ⅓₀ by 10; the result is 8.

Do it thus:	1	⅔ ⅒ ⅓₀
	\2	1 ⅓ ⅒
	4	3 ⅕
	\8	6 ⅓ ⅛

Total 8 loaves, which is correct.

Problem 6

Divide 9 *loaves among* 10 *men.*

Each man receives ⅔ ⅕ ⅓₀.

Proof. Multiply ⅔ ⅕ ⅓₀ by 10.

Do it thus:	1	⅔ ⅕ ⅓₀
	\2	1 ⅔ ⅒ ⅓₀
	4	3 ⅓ ⅒
	\8	7 ⅕

Total 9 loaves, which is correct.

In the first doubling we have 1 ⅓, ⅓ ⅛, and ⅛. The two ⅓'s make ⅔ and the two ⅛'s make ⅒ ⅓₀. Thus we get 1 ⅔ ⅒ ⅓₀. The next doubling is the same as in Problem 4.

SECTION III

Problems 7–20. Multiplication by Certain Fractional Expressions[2]

Problem 7

Multiply ¼ ½₂₈ *by* 1 ½ ¼.

| | | |

Problem 7B

Multiply ¼ ⅛ *by* 1 ½ ¼.

1	¼ ⅛	
½	⅛ 1/56	
¼	1/16 1/112	as parts of 28 these are 1 ½ ¼ and ¼
Total	1/28.	

Problem 8

Multiply ¼ *by* 1 ⅜ ⅛.

1	¼	as a part of 18 this is 4 ½
⅜	1/16	" " " " " " " 3
⅛	1/32	" " " " " " " 1 ½
Total	½ since	" " " " " " " 9.

Problem 9

Multiply ½ 1/14 *by* 1 ½ ¼.

1	½ 1/14
½	¼ 1/28
¼	⅛ 1/56
Total	1.

Problem 10

Multiply ¼ ⅛ *by* 1 ½ ¼.

1	¼ ⅛
½	1/7
¼	1/14
Total	½.

Problem 11

Multiply 1/7 *by* 1 ½ ¼.

1	1/7
½	1/14
¼	1/28
Total	¼.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 12

Multiply $\frac{1}{14}$ *by* $1\ \frac{1}{2}\ \frac{1}{4}$.

1	$\frac{1}{14}$
$\frac{1}{2}$	$\frac{1}{28}$
$\frac{1}{4}$	$\frac{1}{56}$
Total	$\frac{1}{8}$.

Problem 13

Multiply $\frac{1}{16}\ \frac{1}{112}$ *by* $1\ \frac{1}{2}\ \frac{1}{4}$.

1	$\frac{1}{16}\ \frac{1}{112}$	as parts of 28 these are $1\ \frac{1}{2}\ \frac{1}{4}$ and $\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{32}\ \frac{1}{224}$	" " " " " " $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$ " $\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{64}\ \frac{1}{448}$	" " " " " " $\frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}$ " $\frac{1}{16}$
Total	$\frac{1}{8}$	since as a part of 28 this is $3\ \frac{1}{2}$.

Problem 14

Multiply $\frac{1}{28}$ *by* $1\ \frac{1}{2}\ \frac{1}{4}$.

1	$\frac{1}{28}$	as a part of 28 this is 1
$\frac{1}{2}$	$\frac{1}{56}$	" " " " " " " " $\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{112}$	" " " " " " " " $\frac{1}{4}$
Total	$\frac{1}{16}$	since " " " " " " " "$1\ \frac{1}{2}\ \frac{1}{4}$.

Problem 15

Multiply $\frac{1}{32}\ \frac{1}{224}$ *by* $1\ \frac{1}{2}\ \frac{1}{4}$.

1	$\frac{1}{32}\ \frac{1}{224}$	as parts of 28 these are $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$ and $\frac{1}{8}$
$\frac{1}{2}$	$\frac{1}{64}\ \frac{1}{448}$	" " " " " " $\frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}$ " $\frac{1}{16}$
$\frac{1}{4}$	$\frac{1}{128}\ \frac{1}{896}$	" " " " " " $\frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}$ " $\frac{1}{32}$
Total	$\frac{1}{16}$	since as a part of 28 this is $1\ \frac{1}{2}\ \frac{1}{4}$.

There are several mistakes in these multiplications in the original, and at the end of Problem 15 the author puts a sign that stands for the word "error." See notes to the Literal Translation.

Problem 16

Multiply $\frac{1}{2}$ *by* $1\ \frac{2}{3}\ \frac{1}{3}$.

1	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{6}$
Total	1.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 17

Multiply ⅓ *by* 1 ⅗ ⅓.

1	⅓
⅗	⅙ 1/18
⅓	1/9
Total	⅗.

Problem 18

Multiply ⅙ *by* 1 ⅗ ⅓.

1	⅙
⅗	1/9
⅓	1/18
Total	⅓.

Problem 19

Multiply 1/12 *by* 1 ⅗ ⅓.

1	1/12	as a part of 18 this is 1 ½
⅗	1/18	" " " " " " " 1
⅓	1/36	" " " " " " " ½
Total	⅙ since	" " " " " " " 3.

Problem 20

Multiply 1/24 *by* 1 ⅗ ⅓.

1	1/24	as a part of 18 this is ½ ¼
⅗	1/36	" " " " " " " ½
⅓	1/72	" " " " " " " ¼
Total	1/12 since	" " " " " " " 1 ½.

SECTION IV

Problems 21–23. Problems in Completion

Problem 21

It is said to thee, Complete ⅗ 1/15 *to* 1.

Applied to 15, ⅗ is 10 and 1/15 is 1, making 11; the remainder is 4. Multiply 15 so as to get 4.

1	15
1/10	1 ½
\⅕	3
\1/15	1
Total	4.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Therefore $\frac{1}{3}$ $\frac{1}{15}$ is what is to be added to the given number.

For proof add them all together, namely,

$\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{15}$ $\frac{1}{15}$, making 1;

for, applied to 15, these fractions are equal to

10 3 1 1, making 15.

After obtaining the remainder, 4, our author has to determine what fractions, taken as parts of 15, make 4. But this is the same as to say, What should multiply 15 to make 4—and so he takes $\frac{1}{10}$, $\frac{1}{3}$, and $\frac{1}{15}$ of 15, and from the last two gets his result. The same process is used in the next problem, but in Problem 23 the fractions are given without explanation.

The solutions of this problem and the next are proved by adding together the fractions of the given expression and the fractions of the answer.

In the papyrus at the end of this solution are placed the words "Another, $\frac{1}{3}$ $\frac{1}{10}$ to be added." This has no connection with Problem 21, but the number $\frac{1}{3}$ $\frac{1}{10}$ is the answer to the problem that immediately follows.

Problem 22

Complete $\frac{2}{3}$ $\frac{1}{30}$ *to* 1.

Applied to 30, $\frac{2}{3}$ $\frac{1}{30}$ is 21. 30 exceeds 21 by 9. Multiply 30 so as to get 9.

1	30
\ $\frac{1}{10}$	3
\ $\frac{1}{3}$	6
Total	9.

Therefore $\frac{1}{3}$ $\frac{1}{10}$ is to be added to make the completion.

For proof add them all together, namely,

$\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{30}$, making 1;

for, applied to 30, these fractions are equal to

20 6 3 1, making 30.

Problem 23

Complete $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{10}$ $\frac{1}{30}$ $\frac{1}{45}$ *to* $\frac{2}{3}$.

Applied to 45 these are equal to

11 $\frac{1}{4}$ 5 $\frac{1}{2}$ $\frac{1}{8}$ 4 $\frac{1}{2}$ 1 $\frac{1}{2}$ 1,

which requires 6 $\frac{1}{8}$ more to make up $\frac{2}{3}$ of 45, or 30. 6 $\frac{1}{8}$ is equal to $\frac{1}{9}$ $\frac{1}{40}$ of 45. Therefore $\frac{1}{9}$ $\frac{1}{40}$ is to be added to the given number to make $\frac{2}{3}$.

For proof add them all together, namely,

$\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{30}$ $\frac{1}{40}$ $\frac{1}{45}$,

Digitized by Google Original from UNIVERSITY OF MICHIGAN

and these with an additional ⅛ make 1; for applied to 45 these fractions are equal to

11 ¼ 5 ½ ⅛ 5 4 ½ 1 ½ 1 ⅛ 1 and 15.

The author does not explain how he gets the fractions ⅙ ¼₀. Following the method of the first two problems of this section he would have to multiply 45 so as to get 6⅛. This he might have done as follows:

1	45
⅓₀	4 ½
⅕	9
\⅙	5
⅓₀	2 ¼
\¼₀	1 ⅛
Total ⅙ ¼₀.	

In the proof of this problem, where the number to be obtained is ⅜, he adds in also a ⅛ and so gets 1, which is itself the number to be obtained in Problems 21 and 22.

SECTION V

Problems 24–29. '*AHA*' or Quantity Problems

Problem 24

A quantity and its ⅐ *added together become* 19. *What is the quantity?*

Assume 7.

\1	7
\⅐	1
Total	8.

As many times as 8 must be multiplied to give 19, so many times 7 must be multiplied to give the required number.

1	8
\2	16
½	4
\¼	2
\⅛	1
Total 2 ¼ ⅛.	
\1	2 ¼ ⅛
\2	4 ½ ¼
\4	9 ½

Do it thus: The quantity is 16 ½ ⅛,

⅐	2 ¼ ⅛,
Total	19.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In the third multiplication, instead of multiplying 7 by 2 ¼ ⅛, the author multiplies 2 ¼ ⅛ by 7, this being easier. A similar change is made in each of the next three problems. See Introduction, page 6.

Problem 25

A quantity and its ½ added together become 16. *What is the quantity?*

Assume 2.

\1	2
\½	1
Total	3.

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

\1	3
2	6
\4	12
⅔	2
\⅓	1
Total 5 ⅓.	
1	5 ⅓
\2	10 ⅔

Do it thus:	The quantity is	10 ⅔
	½	5 ⅓
	Total	16.

Problem 26

A quantity and its ¼ added together become 15. *What is the quantity?*

Assume 4.

\1	4
\¼	1
Total	5.

As many times as 5 must be multiplied to give 15, so many times 4 must be multiplied to give the required number. Multiply 5 so as to get 15.

\1	5
\2	10
Total 3.	

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Multiply 3 by 4.

1	3
2	6
\4	12;

The quantity is	12
¼	3
Total	15.

Problem 27

A quantity and its ⅕ added together become 21. *What is the quantity?*

Assume 5.

\1	5
\⅕	1
Total	6.

As many times as 6 must be multiplied to give 21, so many times 5 must be multiplied to give the required number.

\1	6
\2	12
\½	3

Total 3 ½.

\1	3 ½
2	7
\4	14.

The quantity is	17 ½
⅕	3 ½
Total	21.

Problem 28

A quantity and its ⅔ are added together and from the sum ⅓ of the sum is subtracted, and 10 *remains. What is the quantity?*

Subtract from 10 its $\frac{1}{10}$, which is 1. The remainder is 9. This is the quantity; its ⅔, 6, added to 9, makes 15, and ⅓ of 15, taken away from 15, leaves 10. *Do it thus* . . . .

Digitized by Google Original from UNIVERSITY OF MICHIGAN

It may be supposed that our author first solved the problem as follows:

Assume 9.

\1	9
\⅔	6
Total	15
1	15
⅓	5
Remainder	10.

As many times as 10 must be multiplied to give 10, that is, once, so many times 9 must be multiplied to give the required number, and therefore the required number is 9. But now he notices that 9 is obtained by taking away its ⅒ from 10, so he puts in the solution given in the papyrus.

The solution does not seem to be complete. The words, *Do it thus* ("The doing as it occurs") are usually put at the beginning of numerical work, and in no other problem are they at the end of the solution. Peet has suggested (page 63) that, in copying, the scribe came to these words and unconsciously let his eye pass to the same words in the next problem, the statement of the next problem and the beginning of its solution being also omitted. A similar omission occurs in the solution of Problem 70.

Problem 29

A quantity and its ⅔ are added together, and ⅓ of the sum is added; then ⅓ of this sum is taken and the result is 10. *What is the quantity?*

\1	10
\¼	2 ½
\⅒	1
The quantity is	13 ½
⅔	9
Total	22 ½
⅓	7 ½
Total	30
⅔	20
⅓	10.

As in the preceding problem it may be supposed that our author first solved the problem as follows:

Assume 27.

\1	27
\⅔	18
Total	45
⅓	15
Total	60
⅔	40
⅓	20.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

As many times as 20 must be multiplied to give 10, so many times 27 must be multiplied to give the required number.

But at this point he seems to have changed the order of these numbers in his mind and to have said, As many times as 20 must be multiplied to give 27 so many times 10 must be multiplied to give the required number.

\1	20
½	10
\¼	5
\1/10	2

Total 1 ¼ 1/10.

Therefore we must multiply 10 by 1 ¼ 1/10 (see Peet, page 64).

SECTION VI

Problems 30–34. Division by a Fractional Expression [1]

Problem 30

If the scribe says, What is the quantity of which ⅔ 1/10 *will make* 10, *let him hear.*

Multiply ⅔ 1/10 so as to get 10.

\1	⅔ 1/10
2	1 ⅓ ⅕
\4	3 1/15
\8	6 1/10 1/30

Total 13. 13 times ⅔ 1/10 makes 9 and the fractions ⅔, 1/10, 1/15, 1/10, and 1/30. The remainder is 1/30. Take 30. ⅔ 1/10 of 30 is 23. Therefore 1/30 of 30, or 1, will be 1/23 of this. 13 1/23 is the required number.

For proof we multiply 13 1/23 by ⅔ 1/10.

1	13 1/23
\⅔	8 ⅔ ¼6 1/138
\1/10	1 ⅕ 1/10 1/230
Total	10.

To get the remainder 1/30 the author could apply the fractions of his partial products, ⅓, 1/10, etc., to 30, their values taken this way making 29 in all, and requiring 1 part more to make the full 30, so that in order to make a full 10 he would require, in addition to what he already has, 1/30.

In the multiplication of his proof we may notice that ⅓ of 13 is given at once as 8 ⅓ and that ⅓ of 1/23 is given by the rule in Problem 61 (see Introduction, page 25).

[1] Full explanations of these problems are given in the Introduction, pages 25–28. I may mention that the problems of this section and the next are numbered by Eisenlohr a little differently from the order in the papyrus, 33 and 34 coming, in fact, after 38, but belonging in the same group with 30, 31, and 32. As I have used his numbering, I have followed his order.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

$\frac{1}{10}$ also he can take without difficulty. For 13 he has $\frac{1}{10}$ of 10 equal to 1, and $\frac{1}{10}$ of 3, by the table that precedes Problem 1, is $\frac{1}{6}$ $\frac{1}{10}$. Finally, $\frac{1}{10}$ of $\frac{1}{28}$ is $\frac{1}{280}$.

In the original our author seems to say that we must multiply $\frac{1}{30}$ by $\frac{1}{28}$ to get $\frac{2}{3}$ $\frac{1}{10}$. It may be that he did not mean to put a dot (the sign for fraction) over 23, but meant to say, Multiply $\frac{1}{30}$ by 23 to get $\frac{2}{3}$ $\frac{1}{10}$. This is a correct statement but does not explain his solution. Following the method given in the other problems of this group, he should have said, Multiply $\frac{1}{28}$ by $\frac{2}{3}$ $\frac{1}{10}$ to get $\frac{1}{30}$.

Problem 31

A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{4}$, added together, become 33. *What is the quantity?*

Multiply 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ so as to get 33.

1	1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{4}$
\2	4 $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{28}$
\4	9 $\frac{1}{6}$ $\frac{1}{14}$
\8	18 $\frac{1}{3}$ $\frac{1}{7}$
$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{14}$
\$\frac{1}{4}$	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{28}$

Total 14 $\frac{1}{4}$. 14 $\frac{1}{4}$ times 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ makes 32 $\frac{1}{2}$ plus the small fractions $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$. 32 $\frac{1}{2}$ from 33 leaves the remainder $\frac{1}{2}$ to be made up by these fractions and a further product by a number yet to be determined.

$\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$

taken as parts of 42 are

6 5$\frac{1}{4}$ 3 1 $\frac{1}{2}$ 1 $\frac{1}{2}$,

making in all 17 $\frac{1}{4}$, and requiring 3 $\frac{1}{2}$ $\frac{1}{4}$ more to make 21, $\frac{1}{2}$ of 42.

Take 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ as applying to 42:

\1	42
\$\frac{2}{3}$	28
\$\frac{1}{2}$	21
\$\frac{1}{4}$	6
Total	97.

That is, 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ applied to 42 gives 97 in all. $\frac{1}{42}$ of 42, or 1, will be $\frac{1}{97}$ of this, and 3 $\frac{1}{2}$ $\frac{1}{4}$ will be 3 $\frac{1}{2}$ $\frac{1}{4}$ times as much. Therefore we multiply $\frac{1}{97}$ by 3 $\frac{1}{2}$ $\frac{1}{4}$.

[1]\$\frac{1}{97}$	$\frac{1}{42}$	or 1	as a part of 42
\$\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$	$\frac{1}{21}$	" 2	" " " " 42
\$\frac{1}{194}$	$\frac{1}{84}$	" $\frac{1}{2}$	" " " " 42
\$\frac{1}{388}$	$\frac{1}{168}$	" $\frac{1}{4}$	" " " " 42.

[1]These multiplications do not represent the multiplication of $\frac{1}{97}$ by 3 $\frac{1}{2}$ $\frac{1}{4}$, but form a continuation of the multiplication at the beginning of the solution. But the partial products of

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The total is 14 ¼ 1/56 1/97 1/194 1/388 1/679 1/776, which multiplied by 1 ⅜ ½ 1/7 makes 33.

Problem 32

A quantity, its ⅓, and its ¼, added together, become 2. What is the quantity?

Multiply 1 ⅓ ¼ so as to get 2.

1	1 ⅓ ¼
\⅜	1 1/18
\⅓	½ 1/36
\⅙	¼ 1/72
\1/12	⅛ 1/144.

Take 12 times 12.

1	12
2	24
\4	48
\8	96
Total	144.

We will apply our fractions to 144. For the given expression we have

\1	144
\⅓	48
\¼	36
Total	228.

The products above, taken as parts of 144, are equal to

228 152 76 38 19.

The sum of the numbers here that correspond to the multipliers checked is equal to 285 and requires 3 more to make up 288, or 2 times 144. As 1 ⅓ ¼ times 144 is 228 we shall have as a continuation of our first multiplication,

\1/228	1/144 or 1 as a part of 144
\1/114	1/72 " 2 " " " " 144.

Adding together all the multipliers checked in this multiplication, we have 1 ⅙ 1/12 1/114 1/228 as the required quantity.

---

the former multiplication are the multipliers of the latter and so are represented in the left-hand column here, while the multipliers of the former are the alternative numbers, 1, 2, ½, and ¼, given at the right.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof.

\1	1 ⅙ ⅟₁₂ ⅟₁₁₄ ⅟₂₂₈
⅔	⅔ ⅑ ⅟₁₈ ⅟₁₇₁ ⅟₃₄₂
\⅓	⅓ ⅟₁₈ ⅟₃₆ ⅟₃₄₂ ⅟₆₈₄
½	½ ⅟₁₂ ⅟₂₄ ⅟₂₂₈ ⅟₄₅₆
\¼	¼ ⅟₂₄ ⅟₄₈ ⅟₄₅₆ ⅟₉₁₂.

The total is 1 ½ ¼ and a series of smaller fractions. 1 ½ ¼ taken from 2 leaves a remainder of ¼. Apply the smaller fractions to 912. The fractions are:

⅟₁₂ ⅟₁₁₄ ⅟₂₂₈ ⅟₁₈ ⅟₃₆ ⅟₃₄₂ ⅟₆₈₄ ⅟₂₄ ⅟₄₈ ⅟₄₅₆ ⅟₉₁₂,

and as parts of 912 they are equal to

76 8 4 50 ⅔ 25 ⅓ 2 ⅔ 1 ⅓ 38 19 2 1,

which together make 228, or ¼ of 912. For

1	912
½	456
¼	228.

In this problem the fractional number is 1 ⅓ ¼ and the product to be obtained is 2. In order to add the partial products when multiplying 1 ⅓ ¼ so as to get 2 the author applies the fractions to 144, getting this number apparently as 12 times 12, although he might have taken it as the largest number whose reciprocal occurs among the fractions to be added, which is his usual procedure.

In his first multiplication he finds that the multiplier 1 ⅙ ⅟₁₂ gives him very nearly 2. It may be noticed that he checks the ⅔ and ⅓ instead of the first line. To produce exactly 2 he has to take as additional multipliers ⅟₂₂₈ and ⅟₁₁₄, getting ⅟₂₂₈ by the same process of reasoning that gave him ⅟₈₇ in the preceding problem and ⅟₂₃ in Problem 30.

He gives the answer as 1 ⅙ ⅟₁₂ ⅟₁₁₄ ⅟₂₂₈ and proceeds to prove that it is correct. Going back to his original point of view, he multiplies it by the given fractional number 1 ⅓ ¼, taking ⅔ and halving to get ⅓, and halving twice to get ¼.

Problem 33

A quantity, its ⅔, its ½, and its ⅐, added together, become 37. *What is the quantity?*

Multiply 1 ⅔ ½ ⅐ so as to get 37.

1	1 ⅔ ½ ⅐
2	4 ⅓ ¼ ⅟₂₈
4	9 ⅙ ⅟₁₄
8	18 ⅓ ⅐
\16	36 ⅔ ¼ ⅟₂₈.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Applying ⅔, ¼, and $\frac{1}{28}$ to 42 we have

1	42
\⅔	28
½	21
\¼	10 ½
\$\frac{1}{28}$	1 ½.[1]

The total is 40; there remains 2, or $\frac{1}{21}$ of 42. As 1 ⅔ ½ ⅐ applied to 42 gives 97, we shall have as a continuation of our first multiplication

$\frac{1}{97}$	$\frac{1}{42}$ or 1 as a part of 42
\$\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$ $\frac{1}{21}$	" 2 " " " " 42.

This $\frac{1}{21}$ with the product already obtained will make the total 37. Thus the required quantity is 16 $\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$.

Proof.

1	16 $\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$
⅔	10 ⅔ $\frac{1}{84}$ $\frac{1}{1358}$ $\frac{1}{4074}$ $\frac{1}{1164}$
½	8 $\frac{1}{112}$ $\frac{1}{1358}$ $\frac{1}{1552}$
⅐	2 ¼ $\frac{1}{28}$ $\frac{1}{392}$ $\frac{1}{4753}$ $\frac{1}{5432}$.

The whole numbers and larger fractions make 36 ⅔ ¼ $\frac{1}{28}$; the remainder is $\frac{1}{28}$ $\frac{1}{84}$. The smaller fractions applied to 5432 make

97	8	7	
64 ⅔	4	1 ⅓	4 ⅔
48 ½	4	3 ½	
13 ½ ¼ $\frac{1}{14}$ $\frac{1}{28}$	1 ⅐	1.	

⅔, ¼, $\frac{1}{28}$, and the fractions of the remainder, $\frac{1}{28}$ and $\frac{1}{84}$, applied to 5432 make 3621 ⅓, 1358, 194, and 194 and 64 ⅔; for we have

1	5432
⅔	3621 ⅓
½	2716
¼	1358
$\frac{1}{28}$	194
Total	5173 ⅓.

There remains 258 ⅔ which is equal to 194 plus 64 ⅔.

We may notice that in the course of his proof our author has ⅔ of $\frac{1}{679}$ equal to $\frac{1}{1358}$ $\frac{1}{4074}$ a remarkable application of the rule given in Problem 61.

For an explanation of the proof see Introduction, pages 27–28. The calculation by which the 97 is obtained is given above in the solution of Problem 31.

[1] Since ⅔ of 42 is 28, 1 ½ being the reciprocal of ⅔.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 34

A quantity, its ½, and its ¼, added together, become 10. *What is the quantity?*

Multiply 1 ½ ¼ so as to get 10.

\1	1 ½ ¼
2	3 ½
\4	7
\⅐	¼[1]
¼ 1/28	½
\½ 1/14	1.

The total is the quantity required, 5 ½ ⅐ 1/14.

Proof.

\1	5 ½ ⅐ 1/14
\½	2 ½ ¼ 1/14 1/28
\¼	1 ¼ ⅛ 1/28 1/56.

The whole numbers and simpler fractions (powers of ½) make a total of 9 ½ ⅛; the remainder is ¼ ⅛. The remaining fractions, namely,

⅐ 1/14 1/14 1/28 1/28 1/56,

applied to 56, are equal to

8 4 4 2 2 1,

making a total of 21, while ¼ and ⅛ make 14 and 7, and so also a total of 21. Therefore the result obtained is correct.

The first step is a multiplication of the second kind (Introduction, page 5) with 1 ½ ¼ taken as multiplicand and 10 as product. After doubling twice, also taking ⅐ and doubling this twice, the author finds a combination of products exactly equal to 10, and it is not necessary for him to take the second and third steps that belong to this kind of multiplication when the product is not obtained directly from the partial products. It is easy to see why this multiplication should be so simple; for we have already enough partial products to make any integer up to 14. Compare the notes to Problem 37.

SECTION VII

Problems 35–38. Division of a *Hekat*

In these problems in the papyrus the questions are put in a curious way: "I have gone a certain number of times into the *hekat*-measure, certain parts have been added to me, and I return filled. What is it that says this?" It is stated as if the vessel represented as speaking had gone

[1] See Introduction, page 5, footnote 2.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

into the *hekat*-measure and returned filled, but clearly it is the *hekat*-measure that is filled.

In Problem 36 the word *hekat* is omitted and the problem is stated and solved as a simple numerical problem. It is worded, however, in the same way as the other three except for the omission of the word *hekat* and it is solved in the same way. Also the proof is the same as the proof of the numerical result in each of the other three. In the three *hekat* problems the answer obtained as an ordinary fractional part of a *hekat* and proved as such is then reduced to *ro* and proved for the number of *ro*; and, finally, it is reduced to the "Horus eye" fractions as far as possible, and proved for this form of expression (see Introduction, page 31).

The method of solution of these problems is that of false position and the solution of Problem 35 is explained in full in the Introduction, page 11.

For an account of the *hekat* and its subdivisions see Introduction pages 31–33.

Problem 35

I have gone three times into the hekat-*measure, my* ⅓ *has been added to me, and I return having filled the* hekat-*measure. What is it that says this?*

Do it thus: Assume 1. Multiplying by $3\frac{1}{3}$ we have

\1	1
\2	2
\⅓	⅓
Total	$3\frac{1}{3}$.

Get 1 by operating on $3\frac{1}{3}$.

1	$3\frac{1}{3}$
$\frac{1}{10}$	⅓
⅕	$\frac{2}{3}$
Total	1,

The answer is $\frac{1}{3}\frac{1}{10}$.

Proof.

\1	$\frac{1}{3}\frac{1}{10}$
\2	$\frac{1}{2}\frac{1}{10}$
\⅓	$\frac{1}{10}$
Total	1.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Express the result in *ro*

1	320
⅒	32
⅕	64
Total	96.

Proof of the result as expressed in *ro*.

1	96
2	192
⅓	32
Total	320.

Expressed in "Horus eye" fractions[1] it makes of grain **¼ 1/32 1/64** *hekat* 1 *ro*.

Proof of the result expressed in this form.

1	**¼ 1/32 1/64** *hekat* 1 *ro*
2	**½ 1/16 1/32** " 2 "
⅓	**1/16 1/32** " 2 "
Total	1.

Problem 36

I have gone in three times, my ⅓ and my ⅕ have been added to me, and I return having filled the measure. What is the quantity that says this?

Assume 1. Multiplying by 3 ⅓ ⅕ we have

1	1
1	1
1	1
⅓	⅓
⅕	⅕
Total	3 ⅓ ⅕.

Get 1 by operating on 3 ⅓ ⅕. Apply this to 30; it makes 106. Multiply 106 so as to get 30.

1	106
½	53
\¼	26 ½
\1/106	1
\1/53	2
\1/212	½
Total	30,

[1] See Introduction, page 31. The "Horus eye" fractions are indicated by black-faced type throughout this work.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

that is, the whole of 30, or 1. The answer is ¼ ⅓53 ⅓106 ⅓212.

Proof.

1	¼ ⅓53 ⅓106 ⅓212
2	½ ⅓30 ⅓318 ⅓795 ⅓53 ⅓106
⅔	⅓12 ⅓159 ⅓318 ⅓636
⅓	⅓20 ⅓265 ⅓530 ⅓1060

The larger fractions are ½ and ¼. In order to get 1 we should have for the sum of the remaining fractions ¼. To get this apply these fractions to 1060.

The fractions

	⅓53	⅓106	⅓212				
as parts of 1060 make	20	10	5			or	35
	⅓30	⅓318	⅓795	⅓53	⅓106		
make	35 ⅓	3 ⅓	1 ⅓	20	10	"	70
	⅓12	⅓159	⅓318	⅓636			
"	88 ⅓	6 ⅔	3 ⅓	1 ⅔		"	100
	⅓20	⅓265	⅓530	⅓1060			
"	53	4	2	1		"	60

The total is 265, or ¼ of 1060; for

1	1060
½	530
¼	265
¼	265
Total	1060.

In multiplying 1 at the beginning of this solution by 3 ⅓ ⅓, instead of saying, once 1, twice 2, etc., our author actually writes down "once 1" three times and then the rest of the multiplication, and in getting 1 by operating on 3 ⅓ ⅓, instead of multiplying this expression directly by multipliers that will eventually give him 1, he applies it to 30, noting that 3 times 30 and ⅓ ⅓ of 30 make 106. Therefore to find how many times 3 ⅓ ⅓ will make 1 he determines how many times 106 will make 30 and the answer to this, ¼ ⅓53 ⅓106 ⅓212, is the answer to the problem.[1]

Problem 37

I have gone three times into the hekat-*measure, my ⅓ has been added to me, ⅓ of my ⅓ has been added to me, and my ⅓ has been added to me; I return having filled the* hekat-*measure. What is it that says this?*

[1] This is the only time that he applies his fractional expressions to a particular number for the purpose of dividing. Generally he uses the method for an addition or subtraction. See Introduction, page 9, footnote.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Assume 1. Multiplying by the given expression we have

1	1
2	2
⅓	⅓
⅓ of ⅓	⅑
⅑	⅑
Total	3 ½ 1/18.

Get 1 by operating on 3 ½ 1/18.

1	3 ½ 1/18
½	1 ½ ¼ 1/36
\¼	½ ¼ ⅛ 1/72
⅛	¼ ⅛ 1/16 1/144
1/16	⅛ 1/16 1/32 1/288
\1/32	1/16 1/32 1/64 1/576

The total, ¼ 1/32, times, 3 ½ 1/18 makes 1, for we have first to add ½, ¼, and ⅛, and then the smaller fractions

1/72 1/16 1/32 1/64 1/576,

which, taken as parts of 576, make

8 36 18 9 1,

a total of 72, or ⅛ of 576. Therefore the answer is ¼ 1/32.

Proof.

1	¼ 1/32
2	½ 1/16
⅛	1/12 1/96
⅓ of ⅓	1/36 1/288
⅑	1/36 1/288.

The total is 1, for we have first to add ½ and ¼, and then the smaller fractions

1/32 1/16 1/12 1/96 1/36 1/288 1/36 1/288,

which, taken as parts of 288, make

9 18 24 3 8 1 8 1,

a total of 72, or ¼ of 288.

Express the result in *ro*.

1	320
½	160
\¼	80
⅛	40
1/16	20
\1/32	10
Total	90.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof of the result as expressed in *ro*

\1	90
\2	180
\⅓	30
\⅓ of ⅓	10
\⅑	10
Total	320.

It amounts in grain to **¼ 1/32** *hekat.*

Proof of the result expressed in this form.

\1	**¼ 1/32**
\2	**½ 1/16**
\⅓	**1/16 1/32**
\⅓ of ⅓	**1/32**
\⅙	**1/32**
Total	**½ ⅛ ¼ ⅛.**

In this problem the *hekat* is to be divided by 3 ½ 1/18. The author expresses this number in the peculiar form 3 ⅓ ⅓of⅓ ⅙, but when he assumes 1 and multiplies 1 by this number he gets the simpler form, and this he uses in determining that it must be multiplied by ¼ 1/32 to get 1. In his proof, however, instead of taking 3 ½ 1/18, he very properly goes back to his original fractions as given in the statement of the problem, multiplying his answer by 3 ⅓ ⅓of⅓ ⅙.

The second operation is to multiply 3 ½ 1/18 so as to get 1. He proceeds by halving to obtain a series of fractional expressions, from which he finds at once a combination that makes exactly 1 (see Introduction, page 5). This would almost seem to indicate that the problem was made up from the answer. We find, however, that his procedure was most natural, and in no way does it indicate that he knew what would be the result. He wishes to get 1 or some number nearly equal to 1. The first two partial products are too large, but the third, ½ ¼ ⅛ 1/32 is less than 1 and can be used. Having selected this he might have proceeded at once to find the remainder by the process of completion, but he seems to have chosen to carry his multiplication further in the hope of getting still nearer 1. Since ½ ¼ ⅛, even without the 1/32, require only ⅙ to make 1, any further expression that he could use must be less than ⅙, and so he was compelled to pass over the next two partial products and the first one that he could try was the one beginning with 1/16. With this he finds that he gets exactly 1 and he does not have to proceed further. See notes to Problem 34.

After obtaining and proving the result expressed in the ordinary form as a fractional part of a *hekat* he reduces it to *ro* and then to the "Horus eye" fractions and proves for both of these forms that it is correct. As the denominators of the fractions that he first obtains are themselves powers of 2 he has the same fractions in his last expression, but he writes them in the "Horus eye" forms, and in his proof he has a slightly different combination because he takes ⅓ of ¼ 1/32 as 1/16 1/32 instead of 1/12 1/96, which the notation would not permit.

At the very end, instead of writing 1 *hekat* as the result of the multiplications of his proof, he writes its equivalent as **½ ⅛ ¼ ⅛.**

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 38

I have gone three times into the hekat-*measure, my* ¼ *has been added to me, and I return having filled the* hekat-*measure. What is it that says this?*

Assume 1. Multiplying by the given expression we have

\1	1
\2	2
\¼	¼
Total	3 ¼.

Get 1 by operating on 3 ¼.

1	3 ¼
$\frac{1}{22}$	¼,

for ¼ of 22 is 3 ¼,

$\frac{1}{11}$	¼ $\frac{1}{28}$
$\frac{1}{6}$ $\frac{1}{66}$	½ $\frac{1}{14}$
Total	1.

Therefore the answer is $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$.

Proof.

1	$\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$
2	½ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$
¼	$\frac{1}{22}$,

for $\frac{1}{22}$ of 7 is the expression that we have obtained.

Total	1.

Express the result in *ro*.

1	320
⅔	213 ⅓
⅓	106 ⅔
\⅙	53 ⅓
\$\frac{1}{11}$	29 $\frac{1}{11}$
\$\frac{1}{22}$	14 ½ $\frac{1}{22}$
\$\frac{1}{66}$	4 ⅔ ⅙ $\frac{1}{66}$
Total	101 ⅔ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$.

Proof of the result as expressed in *ro*.

\1	101 ⅔ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$
\2	203 ½ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$
\¼	14 ½ $\frac{1}{22}$
Total	320,

or 1 *hekat*.

It amounts in grain to **¼ 1/16** *hekat* 1 ⅔ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ *ro*.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof of the result expressed in this form.

1			¼	1/16	*hekat*	1	⅔	1/11	1/22	1/66 *ro*
2			½	⅛	"	3	½	1/11	1/33	1/66 "
1/7			1/32		"	4	½	1/22		"

The total of the larger portions of these products makes

½ ¼ ⅛ 1/16 1/32 1/64 *hekat* 4 ⅔ *ro*,

or 319 ⅔ *ro*.

The smaller fractions

1/11 1/11 1/22 1/22 1/33 1/66 1/66,

taken as parts of 66, are equal to

6 6 3 3 2 1 1.

The total is 22, which is ⅓ of 66, and this with 319 ⅔ makes 320 *ro* or 1 *hekat*, as it should.

There are some peculiarities here in the numerical work. Thus in multiplying 3 1/7 so as to get 1, the author starts by saying that 1/22 of 3 1/7 is 1/7, and his explanation, written on one side, is literally, "The making it is of 1/7 times 22 to find 3 1/7." Also a little further along, in proving his answer, ⅙ 1/11 1/22 1/66, multiplying it by 3 1/7, he writes as one step, 1/7 1/22, and explains it by saying in almost the same words, "The making it is of 1/22 times 7 to find the fraction above." It is as if he were familiar with the relation of the three numbers 7, 3 1/7 and 22. Possibly he had this relation in some table; or in some previous reckoning he had multiplied 7 "for the finding of 22," which would give him at once 3 1/7. Then he would know that 1/7 of 22 is 3 1/7, that 1/22 of 3 1/7 is 1/7, and that 1/22 of 7 is the same as the result of "getting 1 by operating on 3 1/7;" that is, it is the number that he gets in this solution, ⅙ 1/11 1/22 1/66, so that, finally, 1/7 of the last expression is equal to 1/22.[1]

The fractional expressions in the solution and different proofs are quite complicated and it is possible that he applied them to 66 each time that he had additions to make, but, except in the last case, there is no indication of this.

In the first proof he doubles ⅙ 1/11 1/22 1/66 and gets ½ 1/11 1/33 1/66. Here twice ⅙ is ⅓, twice 1/11 is ⅙ 1/66 (by the table at the beginning) and the ⅓ and ⅙ make ½; finally, twice 1/22 1/66 make 1/11 1/33. In a very similar way in the second and third proofs he doubles the fractional expression ⅔ 1/11 1/22 1/66.

In working out the expression of the result in *ro* he has 1/11 of 320 equal to 29 1/11, and in the proof which follows, 1/7 of 101 ⅔ 1/11 1/22 1/66 is given as 14 ½ 1/22; also in the last proof 1/7 of ¼ 1/16 *hekat* 1 ⅔ 1/11 1/22 1/66 *ro* is given as 1/32 *hekat* 4 ½ 1/22 *ro*. These are more difficult than most of the divisions in the papyrus, and the author does not state explicitly how he performed them.

[1] See Introduction, page 5, footnote 2.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

SECTION VIII

PROBLEMS 39–40. DIVISION OF LOAVES. ARITHMETICAL PROGRESSION

Problem 39

Example of finding the difference of share when 100 *loaves are divided among* 10 *men,* 50 *in equal shares among* 6 *men and* 50 *in equal shares among* 4. *What is the difference of the shares?*

Multiply 4 so as to get 50.

1	4
\10	40
\2	8
\½	2
Total 12 ½.	

Multiply 6 so as to get 50.

1	6
2	12
4	24
\8	48
\⅓	2
Total 8 ⅓.	

Therefore each of the 4 will receive 12 ½ and each of the 6 will receive 8 ⅓, and the difference of share will be 4 ⅙.

In this problem the result obtained and the methods used are so simple that the problem seems hardly worth while. It is possible that the author intended to state a problem in arithmetical progression like the next one.

We may notice that he considers the 4 first in the solution, although he puts the 6 first in the statement. In the papyrus he expressed the result by writing 12 ½ four times and 8 ⅓ six times.

Problem 40

Divide 100 *loaves among* 5 *men in such a way that the shares received shall be in arithmetical progression and that* ⅐ *of the sum of the largest three shares shall be equal to the sum of the smallest two. What is the difference of the shares?*

Do it thus: Make the difference of the shares 5 ½. Then the amounts that the 5 men receive will be

23 17 ½ 12 6 ½ 1, total 60.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

As many times as is necessary to multiply 60 to make 100, so many times must these terms be multiplied to make the true series.

\1	60
\⅔	40.

The total, 1 ⅔, times 60 makes 100.

Multiply by 1 ⅔

23	it	becomes	38 ⅓
17 ½	"	"	29 ⅙
12	"	"	20
6 ½	"	"	10 ⅔ ⅙
1	"	"	1 ⅔
Total 60	"	"	100.

This problem has been fully explained in the Introduction (page 12) and needs no further comment here.

In this problem and in Problem 64 our author writes his progression as a descending series, and so the smallest term is with him the last term.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

CHAPTER II. GEOMETRY

SECTION I

Problems 41–46. Problems of Volume

Problem 41

Find the volume of a cylindrical granary of diameter 9 *and height* 10.

Take away ⅑ of 9, namely, 1; the remainder is 8. Multiply 8 times 8; it makes 64. Multiply 64 times 10; it makes 640 cubed cubits. Add ½ of it to it; it makes 960, its contents in *khar*. Take ⅒ of 960, namely 48. 4800 *hekat* of grain will go into it.

Method of working out:

1	8
2	16
4	32
\8	64.
1	64
\10	640
\½	320
Total	960
⅒	96
\1/20	48.

As explained in the Introduction (pages 35–36), the author, in order to obtain the volume of the cylinder, subtracts from the diameter its ⅑, squares the remainder, and multiplies by the altitude. In this way he finds that 640 is the contents of the granary in cubed cubits. As a *khar* is ¾ of a cubed cubit he adds to 640 its ½, which gives him 960 as the number of *khar* in the granary. Finally, dividing by 20, he obtains 48 as the number of hundreds of quadruple *hekat*. (See Introduction, page 32.)

Problem 42

Find the volume of a cylindrical granary of diameter 10 *and height* 10.

Take away ⅑ of 10, namely 1 ⅑; the remainder is 8 ⅔ ⅙ 1/18. Multiply 8 ⅔ ⅙ 1/18 times 8 ⅔ ⅙ 1/18; it makes 79 1/108 1/324. Multiply 79 1/108 1/324

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

times 10; it makes 790 $\frac{1}{18}$ $\frac{1}{27}$ $\frac{1}{54}$ $\frac{1}{81}$ cubed cubits. Add $\frac{1}{2}$ of it to it; it makes 1185 $\frac{1}{6}$ $\frac{1}{54}$, its contents in *khar*. $\frac{1}{20}$ of this is 59 $\frac{1}{4}$ $\frac{1}{108}$. 59 $\frac{1}{4}$ $\frac{1}{108}$ times 100 *hekat* of grain will go into it.

Method of working out:

1	8 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$
2	17 $\frac{2}{3}$ $\frac{1}{9}$
4	35 $\frac{1}{2}$ $\frac{1}{18}$
\8	71 $\frac{1}{9}$
\$\frac{2}{3}$	5 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$ $\frac{1}{27}$
$\frac{1}{3}$	2 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{12}$ $\frac{1}{36}$ $\frac{1}{54}$
\$\frac{1}{6}$	1 $\frac{1}{3}$ $\frac{1}{12}$ $\frac{1}{24}$ $\frac{1}{72}$ $\frac{1}{108}$
\$\frac{1}{18}$	$\frac{1}{3}$ $\frac{1}{9}$ $\frac{1}{27}$ $\frac{1}{108}$ $\frac{1}{324}$
Total	79 $\frac{1}{108}$ $\frac{1}{324}$.

1	79 $\frac{1}{108}$ $\frac{1}{324}$
10	790 $\frac{1}{18}$ $\frac{1}{27}$ $\frac{1}{54}$ $\frac{1}{81}$
$\frac{1}{2}$	395 $\frac{1}{36}$ $\frac{1}{54}$ $\frac{1}{108}$ $\frac{1}{162}$
Total	1185 $\frac{1}{6}$ $\frac{1}{54}$
$\frac{1}{10}$	118 $\frac{1}{2}$ $\frac{1}{54}$
\$\frac{1}{20}$	59 $\frac{1}{4}$ $\frac{1}{108}$.

This problem is exactly like 41, but with 10 instead of 9 for diameter there are many fractions. The $\frac{1}{108}$ of 100 *hekat* should be reduced to "Horus eye" fractions and *ro*, but in the papyrus this fraction is omitted.

Problem 43

A cylindrical granary of diameter 9 *and height* 6. *What is the amount of grain that goes into it?*

Do it thus: Take away $\frac{1}{9}$ of it, namely, 1, from 9; the remainder is 8. Add to 8 its $\frac{1}{3}$; it makes 10 $\frac{2}{3}$. Multiply 10 $\frac{2}{3}$ times 10 $\frac{2}{3}$; it makes 113 $\frac{2}{3}$ $\frac{1}{9}$. Multiply 113 $\frac{2}{3}$ $\frac{1}{9}$ times 4, 4 cubits being $\frac{2}{3}$ of the height; it makes 455 $\frac{1}{9}$, its contents in *khar*. Find $\frac{1}{20}$ of this, namely, 22 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{180}$. The amount of grain that will go into it is 22 $\frac{1}{2}$ $\frac{1}{4}$ times 100 *hekat* $\frac{1}{2}$ $\frac{1}{32}$ $\frac{1}{64}$ *hekat* 2 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{36}$ *ro*.

Method of working out:

\1	8
$\frac{2}{3}$	5 $\frac{1}{3}$
\$\frac{1}{3}$	2 $\frac{2}{3}$
Total	10 $\frac{2}{3}$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1	10 ⅔
\10	106 ⅔
\⅔	7 ⅙
Total	113 ⅔ ⅙.

1	113 ⅔ ⅙
2	227 ½ ⅛
\4	455 ⅙.

1	455 ⅙
⅒	45 ½ ⅙
\⅕	22 ½ ¼ ⅛0.

This solution for a long time baffled the ingenuity of Egyptologists, but the correct interpretation was finally discovered by Schack-Schackenburg (1899, see Peet, page 83).

In the first place, the papyrus states that the height of the granary is 9 and the breadth (diameter) 6, and in the solution, when we find 4 as ⅔ of 6, the author again calls 6 the breadth; but the solution is for a cylinder in which 9 is the diameter and 6 the height.[1]

Then the method of solution is not that used in 41 and 42, but a second one (employed for a similar problem in the Kahun papyrus, Griffith, 1897), giving the volume directly in *khar* and not first in cubed cubits. It may be expressed in the following rule: Add to the diameter its ⅓; square, and multiply by ⅔ of the height. In Problem 43 the addition of its ⅓ to the diameter makes 12, the square is 144, and ⅔ of the height is 4. With these numbers the rule gives 576 *khar*, and this is just what the author would have obtained if he had followed the solutions of 41 and 42.[2]

But the author, before taking the steps of this rule, deducts from the diameter its ⅑, as by the other rule, and so obtains a result which is $(\frac{8}{9})^2$ of the correct result, namely, 455 ⅙ *khar*.[3]

[1] There is some confusion in Peet's explanation of this mistake (page 84). He supposes that the statement was correct in the original papyrus, but that "a later scribe, seeing in the first line of working the subtraction of a ninth of 9 from 9, . . . concluded that 9 must be the diameter and not the height, and so he transposed the two dimensions in the statement," but if the statement now in the papyrus is the result of such a transposition the transposition must have been the other way.

[2] In modern form the number of *khar* according to the two methods is expressed by the formulae $(\frac{8}{9}d)^2\ h.\frac{2}{3}$ and $(\frac{1}{3}d)^2.\frac{2}{3}\ h$, both of which reduce to $\frac{32}{27}\ hd^2$.

[3] Eisenlohr translated the word now read *khar* as "bodily content" and supposed that the addition of ⅓ to reduce cubed cubits to *khar* was a part of the calculation of the cubed cubits. This is the same as multiplying the base by ⅔ of the height instead of by the height itself, and to explain this he supposes that the given base in 41 and 42 is the upper base, and that the lower base is larger. When he comes to Problem 43 he has great difficulty. In the first place, he finds that ⅘ of the diameter of the base is multiplied by ⅓ (⅓ of it added) before squaring, as if the base were an ellipse with one axis ⅘ of the other, or as if our author intended to get the lower base or some section between the two. Then the area obtained is multiplied by ⅔ of the height instead of ½, which might be because he had taken

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In the last part of the solution the author expresses $\frac{1}{180}$ of 100 quadruple *hekat* in fractional parts of a quadruple *hekat* ("Horus eye" fractions) and quadruple *ro* and fractional parts. 100 times $\frac{1}{180}$ gives him $\frac{1}{2}$ $\frac{1}{32}$ $\frac{1}{64}$ and a remainder that must be multiplied further by 320 to reduce it to quadruple *ro*.

Problem 44

Example of reckoning the volume of a rectangular granary, its length being 10, *its width* 10, *and its height* 10. *What is the amount of grain that goes into it?*

Multiply 10 times 10; it makes 100. Multiply 100 times 10; it makes 1000. Add its ½; it makes 1500, its contents in *khar*. Take $\frac{1}{20}$ of 1500; it makes 75, its contents in quadruple *hekat*, namely, 7500 *hekat* of grain.

The working out:

1	10
10	100
1	100
10	1000
1	1000
½	500
1	1500
$\frac{1}{10}$	150
$\frac{1}{20}$	75.

Proof:

1	75
10	750
20	1500
$\frac{1}{10}$ of 1500	150
$\frac{1}{10}$ of $\frac{1}{10}$	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$	10.

This problem is similar to 41 and 42 except that the base is a square instead of a circle. At the end, after the author has completed his calculations, he goes through them in reverse order, apparently as a proof of the correctness of his result. This reverse calculation is the same as the calculation given as the solution of the next problem, the next problem being the reverse of the present one.

---

the larger base, or because he had some vague idea of the rule for the volume of a hemisphere (the area of the base times $\frac{2}{3}$ of the height), and applied it to this solid, even though it may not have been spherical. This is the way in which the question of a hemisphere came into the problem. See titles of articles by Borchardt (1897) and Schack-Schackenburg (1899).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 45

A rectangular granary into which there have gone 7500 *quadruple* hekat *of grain. What are its dimensions?*

Multiply 75 times 20; it makes 1500. Take 1/10 of 1500, namely, 150, 1/10 of its 1/10, 15, 2/3 of 1/10 of its 1/10, 10. Therefore the dimensions are 10 by 10 by 10.

The working out:

1	75
10	750
20	1500,

its contents in *khar*.

1	1500
1/10	150
1/10 of 1/10	15
2/3 of 1/10 of 1/10	10.

Instead of taking 2/3 at the beginning to reduce the contents to cubed cubits, as he would have done if he had exactly reversed the process of the preceding solution, the author takes 2/3 of the last quotient to find the third dimension.

Problem 46

A rectangular granary into which there have gone 2500 *quadruple* hekat *of grain. What are its dimensions?*

Multiply 25 times 20; it makes 500, its contents in *khar*. Take 1/10 of 500, namely, 50, its 1/20, 25, 1/10 of its 1/10, 5, 2/3 of 1/10 of its 1/10, 3 1/3. Therefore the dimensions are 10 by 10 by 3 1/3.

Its working out:

1	25
10	250
20	500,

its contents in *khar*.

1	500
1/10	50
1/10 of 1/10	5
2/3 of 1/10 of 1/10	3 1/3.

The dimensions of the granary are therefore in cubits 10 by 10 by 3 1/3.

The 1/20 that the author takes in this solution is not necessary and is not used.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

SECTION II

Problem 47. Division of 100 *Hekat*

Problem 47

Suppose the scribe says to thee, Let me know what is the result when 100 *quadruple* hekat *are divided by* 10 *and its multiples, in a rectangular or circular granary.*

1/10	becomes of grain	10	quadruple	*hekat*						
1/20	"	"	"	5			*hekat*			
1/30	"	"	"	3	1/4	1/16 1/64	"	1 2/3		*ro*
1/40	"	"	"	2	1/2		"			
1/50	"	"	"	2			"			
1/60	"	"	"	1	1/2	1/8 1/32	"	3 1/3		"
1/70	"	"	"	1	1/4	1/8 1/32 1/64	"	2 1/14 1/21 1/42		"
1/80	"	"	"	1	1/4		"			
1/90	"	"	"	1	1/16	1/32 1/64	"	1/2 1/18		"
1/100	"	"	"	1			"			

In the original the scribe is made to ask only for 1/10, but the author gives 1/10, 1/20, etc.

SECTION III

Problems 48–55. Problems of Area

Problem 48

Compare the area of a circle and of its circumscribing square.

The circle of diameter 9.		The square of side 9.	
1	8 *setat*	\1	9 *setat*
2	16 "	2	18 "
4	32 "	4	36 "
\8	64 "	\8	72 "
		Total	81 "

Problem 49

Example of reckoning area. Suppose it is said to thee, What is the area of a rectangle of land of 10 khet *by* 1 khet?

Do it thus:

1	1,000
10	10,000
100	100,000
1/10	10,000
1/10 of 1/10	1,000.

This is its area.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The papyrus states the problem for a field of 10 *khet* by 2 *khet*, and these numbers are in the figure, but the solution is for 10 *khet* by 1 *khet*, or 1,000 cubits by 100 cubits. Multiplying these numbers together gives 100,000 square cubits. Dividing this by 100 gives 1,000 cubit-strips, strips 1 cubit wide and 1 *khet* long.

Problem 50

Example of a round field of diameter 9 khet. *What is its area?*

Take away ⅑ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
⅑	1;

this taken away leaves 8

1	8
2	16
4	32
\8	64.

Its area is 64 *setat.*

Problem 51

Example of a triangle of land. Suppose it is said to thee, What is the area of a triangle of side[1] 10 khet *and of base* 4 khet?

Do it thus:

1	400
½	200
1	1,000
2	2,000.

Its area is 20 *setat.*

Take ½ of 4, in order to get its rectangle. Multiply 10 times 2; this is its area.

The author seems to put the reckoning before the explanation. In the reckoning he puts down the base as 400 and the side as 1,000; that is, he expresses these lengths in cubits. Dividing 400 by 2 he gets 200 and 1,000 as the dimensions of the equivalent rectangle. Then to obtain the area expressed as so many cubit-strips he multiplies 1,000, not by 200, but by 2, as if he thought of the rectangle as made up of 1,000 pairs of cubit-strips. Finally, he writes down 2, that is, 20 *setat* (2 ten-*setat*), as the standard form of expressing the result.

[1] For a discussion of the question whether this word should be *side* or *altitude* see Introduction, pages 36–37, and Bibliography, pages 132–134.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In his explanation he uses the number of *khet* in the base and the number of *khet* in the side, and says that 10 times 2 will give the area.

A similar problem is given on a fragment of the Golenishchev papyrus (see Bibliography under Tsinserling, 1925).

Problem 52

Example of a cut-off (truncated) triangle of land. Suppose it is said to thee, What is the area of a cut-off triangle of land of 20 khet *in its side,* 6 khet *in its base,* 4 khet *in its cut-off line?*

Add its base to its cut-off line; it makes 10. Take ½ of 10, that is 5, in order to get its rectangle. Multiply 20 times 5; it makes 10 (10 ten-*setat*). This is its area.

Do it thus:

1	1,000
½	500
\1	2,000
2	4,000
\4	8,000
Total	10,000.

Its area is 100 *setat* (10 ten-*setat*).

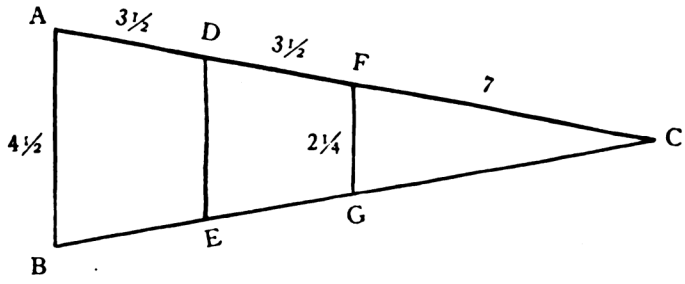
This solution is very much like the preceding. Taking ½ of the sum of the bases he gets 500 cubits or 5 *khet* as the base of the equivalent rectangle with its side 2,000 cubits. Multiplying 2,000 by 5 he gets 10,000 cubit-strips for the area. Finally he expresses this in *setat* (or ten-*setat*). In the papyrus 20 was written where 10 should have been.

Problem 53

Areas of sections of a triangle.

\1	4 ½	*setat*	
\2	9	"	
\½	2 ¼	"	
Total	15 ½ ¼	"	
1/10	1 ½	"	7 ½ cubit-strips.
Its 1/10 taken away leaves the area	14 ⅛	"	5 "
1	7	"	
\2	14	"	
½	3 ½	"	
\¼	1 ½ ¼	"	
Total	15 ½ ¼	"	
½	7 ½ ¼ ⅛	"	

Digitized by Google Original from UNIVERSITY OF MICHIGAN



This problem is difficult to explain, and the difficulty is increased by some numerical mistakes. The most probable explanation that I can give is that the author is undertaking to determine the areas of certain sections of an isosceles triangle. The drawing suggests that 14 is the common length of the two equal sides and that the sections are made by two lines parallel to the base. One of these lines seems to bisect the two equal sides and perhaps it was intended that the other should bisect the parts next to the base, so that the lengths of the sides of the three sections would be 3 ½, 3 ½, and 7. The length of the shorter dividing line is put down as 2 ¼, and this would make the base 4 ½ and the longer dividing line 3 ¼ ⅛. In his figure the author puts down 6 as the length of both of these lines, but in his calculations he seems to take 4 ½ for the base.

He first undertakes to determine the area of the largest section. Apparently he intended to multiply the base, 4 ½, by the height or side 3 ½. This would give him 15 ½ ¼ *setat.* He recognizes that the area thus determined would be too large, because the top is less than the base. He does not seem able to find the length of the upper line, but proposes arbitrarily to take away ⅒ of the area that he has obtained. In this way, if he had finished his solution, he would have had, finally, as the area, 14 ⅛ *setat* 5 cubit-strips.[1] This taking away of ⅒ reminds us of his solution of Problem 28 and of his method of determining the area of a circle by taking away ⅑ of the diameter. In Problem 82 we shall have an example where he obtains a quantity somewhat smaller than a given quantity by taking away ⅒ of ⅗ of it.

The true area, if 3 ½ were the height, would be easily found. AB and FG being 4 ½ and 2 ¼,

AB + FG will be	6 ½ ¼
Its ½, DE	3 ¼ ⅛
Adding AB we get	7 ½ ¼ ⅛
Its ½	3 ½ ¼ ⅛ 1/16,

one-half of the sum of the bases of the trapezoid ABED. Multiplying by 3½ we have

\1	3 ½ ¼ ⅛ 1/16
\2	7 ½ ¼ ⅛
\½	1 ½ ¼ ⅛ 1/16 1/32
Total	13 ½ ¼ 1/32.

[1] Actually the author multiplied 4 ½ by 1, 2, ½ and ¼, checking the 1 and 2. Then he put down as his total 5 ½ ⅛, which is the sum of the first and last of these partial products. Finally he put down as ⅒, 1 ¼ ⅛ *setat* 10 cubit-strips. This is very nearly ⅒ of 15 ½ ¼, and much too large to be ⅒ of his 5 ½ ⅛.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

This result expressed in standard form (see Introduction, page 33) is 13½ ¼ *setat* 3⅛ cubit-strips. The result obtained by the author's method exceeds this by $\frac{63}{100}$ *setat*, or a little less than 3%.

He next determines the area of the apex. To do this he multiplies 7 by 2¼ and divides by 2, getting 7½ ¼ ⅛ *setat* as this area.

Finally, to determine the area of the middle section he would simply have to subtract the areas of the other two sections from the area of the whole triangle.

Problems 54 and 55 are exactly alike, although the first is given in the papyrus more briefly and with some words missing. I will explain them together.

Problem 54

What equal areas should be taken from 10 *fields if the sum of these areas is to be* 7 setat?

Multiply 10 so as to get 7.

1	10
\½	5
\⅕	2

Total ½ ⅕.[1]

Expressed as parts of a *setat* and cubit-strips this is ½ ⅛ *setat* 7 ½ cubit-strips.

Proof.

1	½ ⅛	*setat*	7 ½	cubit-strips
\2	1 ¼ ⅛	"	2 ½	"
4	2 ½ ¼	"	5	"
\8	5 ½	"	10	"
Total	7	"		

Problem 55

What equal areas should be taken from 5 *fields if the sum of these areas is to be* 3 setat?

Multiply 5 so as to get 3.

1	5
½	2 ½
⅒	½

Total ½ ⅒.

Expressed as parts of a *setat* and cubit-strips this is ½ *setat* 10 cubit-strips.

[1] In Problem 4 this is given as ⅔ ⅓₀. See notes to that problem.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof.

\1	½	*setat* 10	cubit-strips	
2	1 ⅛	"	7 ½	"
\4	2 ¼ ⅛	"	2 ½	"
Total	3.			

Thus you find that the area is 3 *setat*.

These two problems are simple division problems—10 is multiplied so as to get 7, and 5 so as to get 3—and they have been translated both by Eisenlohr and by Peet, "Divide . . . into . . . fields." But the preposition sometimes means *from* and does not mean *into*, and the verb at the beginning, which is used several times in the papyrus, elsewhere always means *take away* or *subtract*. Gunn (page 133) has suggested that these words can be used here with their ordinary meanings in the sense of taking away an equal part from each field.

In each of these problems a product and multiplier are given to find the multiplicand. Problem 54 is, How large a field taken 10 times (once from each of the given 10 fields) will make 7 *setat*, and Problem 55, How large a field taken 5 times will make 3 *setat*? As the Egyptian cannot solve these problems directly, he forms new ones in which the multipliers in these become multiplicands and the answers are obtained first as multipliers (see Introduction, page 6). In writing down the multiplications of these new problems he writes all of his numbers as mere numbers, but in Problem 55 he writes first the statement of his new problem as a problem in *setat*.[1] The answer to this new problem is ½ ⅒, and if it is taken as a problem in *setat*, the argument for the answer to the given problem will be, ½ ⅒ times 5 *setat* makes 3 *setat*, therefore 5 times ½ ⅒ of a *setat* (or, as he has to write it, **½** *setat* 10 cubit-strips) will be 3 *setat*, and so the answer to the given problem is **½** *setat* 10 cubit-strips.

SECTION IV

Problems 56–60. Pyramids; the Relation of the Lengths of Two Sides of a Triangle

Problem 56

If a pyramid is 250 *cubits high*[2] *and the side of its base* 360 *cubits long, what is its* seked?

Take ½ of 360; it makes 180. Multiply 250 so as to get 180; it makes ½ ⅕ ⅒ of a cubit. A cubit is 7 palms. Multiply 7 by ½ ⅕ ⅒.

[1] He says, "Multiply 5 *setat* so as to get fields of 3 *setat*." See Literal Translation. Peet (page 96) says that the Egyptian "illogically" divides 3 *setat* by 5 *setat*. But it would seem to be as logical to make the new problem one of *setat* as to make it one of mere numbers. Peet explains that it is impossible for the Egyptian to obtain the quotient otherwise than as a mere number; that the Egyptian can only divide 3 by 5 (so he says), giving ½ ⅒; and then he adds, "Thus in the present case an Egyptian could not mark ½ ⅒ as *setat*, because the *setat*-notation does not recognize such a quantity." But the important question here is, How can ½ ⅒ be regarded as ½ ⅒ of a *setat*?

[2] For a discussion of the terms used in these problems see Introduction, pages 37–38.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1	7
½	3 ½
⅓	1 ⅓ ⅕
⅙₀	⅒ ⅖₅

The *seked* is 5 ⅖₅ palms.

Problem 57

If the seked *of a pyramid is* 5 *palms* 1 *finger per cubit and the side of its base* 140 *cubits, what is its altitude?*

Divide 1 cubit by the *seked* doubled, which is 10 ½. Multiply 10 ½ so as to get 7, for this is a cubit: 7 is ⅔ of 10 ½. Operate on 140, which is the side of the base: ⅔ of 140 is 93 ⅓. This is the altitude.

In this inverse problem and in 59B the author doubles the *seked* instead of taking ½ of the side of the base, and instead of dividing the *seked* doubled by 7 and dividing the side of the base by the result, he divides 7 by the *seked* doubled and multiplies the side of the base by the result, which amounts to the same thing.

Problem 58

If a pyramid is 93 ⅓ *cubits high and the side of its base* 140 *cubits long, what is its* seked?

Take ½ of 140, which is 70. Multiply 93 ⅓ so as to get 70. ½ is 46 ⅔, ¼ is 23 ⅓. Make thou ½ ¼ of a cubit. Multiply 7 by ½ ¼. ½ of 7 is 3 ½, ¼ is 1 ½ ¼, together 5 palms 1 finger. This is its *seked*.

The working out:

1	93 ⅓
\½	46 ⅔
\¼	23 ⅓
Total ½ ¼.	

Make thou ½ ¼ of a cubit; a cubit is 7 palms.

1	7
½	3 ½
¼	1 ½ ¼
Total	5 palms 1 finger.

This is its *seked*.

Problem 59

If a pyramid is 8 *cubits high and the side of its base* 12 *cubits long, what is its* seked?

Multiply 8 so as to get 6, for this is ½ of the side of the base.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1	8
\½	4
\¼	2

Total ½ ¼.

Take ½ ¼ of 7; this is a cubit.

1	7
\½	3 ½
\¼	1 ½ ¼.

The result is 5 palms 1 finger. This is its *seked.*

As in Problem 43, the two numbers given are interchanged in the statement in the papyrus, or we might say that the side of the base and the altitude are interchanged; and then once in the solution "altitude" is written for "side of the base."

The original contains two or three words at the end of the solution that are uncertain. See Literal Translation.

The next problem is the inverse of the preceding and Eisenlohr counted the two together as 59. Indeed, the author may have regarded them as one problem, since none of the second is written in red. In order to hold to the numbering of Eisenlohr I have followed Peet in calling it 59B.

Problem 59B

If the seked *of a pyramid is* 5 *palms* 1 *finger per cubit and the side of its base* 12 *cubits long, what is its altitude?*

Multiply 5 palms 1 finger doubled, which is 10 ½, so as to get 1 cubit; a cubit is 7 palms. ⅔ of 10 ½ is 7; therefore ⅔ of 12, which is 8, is the altitude.

Problem 60

If a pillar (*?*) [1] *is* 30 *cubits high and the side* (*diameter?*) *of its base* 15 *cubits, what is its* seked?

Take ½ of 15; it is 7 ½. Multiply 30 so as to get 7 ½; the result is ¼. This is the *seked.*

The working out:

1	15
\½	7 ½
1	30
½	15
\¼	7 ½.

[1] For a discussion of the meaning of this word see Peet, page 101.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In the papyrus, instead of dividing the base of the right triangle by the other given line, the author divides the other given line by the base. I follow Borchardt (1893) in treating this as a mistake.

At the end he does not multiply by 7 so as to express the *seked* in palms, as he does in the other problems.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

CHAPTER III. MISCELLANEOUS PROBLEMS

Problem 61

Table for multiplication of fractions.

$\frac{2}{3}$ of $\frac{2}{3}$ is $\frac{1}{3}$ $\frac{1}{9}$
$\frac{1}{3}$ " $\frac{2}{3}$ " $\frac{1}{6}$ $\frac{1}{18}$
$\frac{2}{3}$ " $\frac{1}{3}$ " $\frac{1}{6}$ $\frac{1}{18}$
$\frac{2}{3}$ " $\frac{1}{6}$ " $\frac{1}{12}$ $\frac{1}{36}$
$\frac{2}{3}$ " $\frac{1}{2}$ " $\frac{1}{3}$
$\frac{1}{3}$ " $\frac{1}{2}$ " $\frac{1}{6}$
$\frac{1}{6}$ " $\frac{1}{2}$ " $\frac{1}{12}$
$\frac{1}{12}$ " $\frac{1}{2}$ " $\frac{1}{24}$
$\frac{1}{6}$ of $\frac{2}{3}$ is $\frac{1}{18}$ $\frac{1}{54}$; $\frac{1}{6}$, $\frac{2}{3}$ of it is $\frac{1}{18}$ $\frac{1}{54}$
. . . . . .
$\frac{1}{5}$, $\frac{1}{4}$ of it is $\frac{1}{20}$
$\frac{1}{7}$, $\frac{2}{3}$ " " " $\frac{1}{14}$ $\frac{1}{42}$
$\frac{1}{7}$, $\frac{1}{2}$ " " " $\frac{1}{14}$
$\frac{1}{11}$, $\frac{2}{3}$ " " " $\frac{1}{22}$ $\frac{1}{66}$, $\frac{1}{3}$ of it is $\frac{1}{33}$
$\frac{1}{11}$, $\frac{1}{2}$ " " " $\frac{1}{22}$, $\frac{1}{4}$ " " " $\frac{1}{44}$.

Peet points out (page 103) that the above table contains two forms of statement which have an interesting significance. In the first four lines we find forms of the type, $\frac{2}{3}$ of $\frac{2}{3}$ is $\frac{1}{3}$ $\frac{1}{6}$, while in the last five are such forms as, $\frac{1}{5}$, $\frac{1}{4}$ of it is $\frac{1}{20}$. In the ninth line both forms are given, the statement being made twice, while the four lines that immediately precede seem to have been originally in the second form but to have been changed to the first; see Literal Translation. The reason for the second form is that of the fractions in this table the only ones that were legitimate multipliers are $\frac{2}{3}$ and $\frac{1}{2}$ and fractions obtained from them by halving. Thus the Egyptian could not directly say, "$\frac{1}{3}$ of $\frac{1}{4}$" but only "$\frac{1}{3}$, $\frac{1}{4}$ of it." He could have used the second form in all of these cases, but when he has a legitimate multiplier he prefers the first form, and so the author, perceiving after they were written that lines 5–8, like the first four, involve only these multipliers, changed them to the first form.

Problem[1] *61B*

Rule for getting $\frac{2}{3}$ of the reciprocal of an odd number.

To get $\frac{2}{3}$ of $\frac{1}{5}$ take the reciprocals of 2 times 5 and 6 times 5, and in the same way get $\frac{2}{3}$ of the reciprocal of any odd number.

[1] See Introduction, pages 24–25.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Problem 62

Example of reckoning the contents of a bag of various precious metals. Suppose it is said to thee, A bag containing equal weights of gold, silver and lead, has been bought for 84 sha'ty. *What is the amount in it of each precious metal, that which is given for a* deben *of gold being* 12 sha'ty, *for a* deben *of silver* 6 sha'ty, *and for a* deben *of lead* 3 sha'ty?

Add that which is given for a *deben* of each precious metal. The result is 21 *sha'ty*. Multiply 21 so as to get 84, the 84 *sha'ty* for which this bag was bought. The result is 4, which is the number of *deben* of each precious metal.

Do it thus:

Multiply 12 by 4 getting 48 *sha'ty* for the gold in the bag,
Multiply 6 by 4 getting 24 *sha'ty* for the silver,
Multiply 3 by 4 getting 12 *sha'ty* for the lead,
Multiply 21 by 4 getting 84 *sha'ty* altogether.

The *sha'ty* was a seal, and the word here represents a unit of value (see Weill, 1925). The *deben* was a unit of weight, equal to about 91 grammes. The papyrus does not say that the bag contains equal weights of gold, silver, and lead, but in the solution the author proceeds as if this condition was understood.

Problem 63

Example of dividing 700 *loaves among four men in the proportion of the numbers* ⅔, ½, ⅓ *and* ¼. *Let me know the share that each man receives.*

Add ⅔, ½, ⅓ and ¼; it makes 1 ½ ¼. Get 1 by operating on 1 ½ ¼; it makes ½ 1/14. Take ½ 1/14 of 700; it is 400. ⅔, ½, ⅓ and ¼ of 400 will give the shares of the four men.

Do it thus:

The quantity is	700
½ 1/14 of it is	400
⅔ of 400 to one is	266 ⅔
½ of 400 to one is	200
⅓ of 400 to one is	133 ⅓
¼ of 400 to one is	100
Total	700.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 64

Example of determining an arithmetical progression. Suppose it is said to thee, Distribute 10 hekat *of barley among* 10 *men in such a way that the shares shall be in arithmetical progression with a common difference of* ⅛ hekat. *What is the share of each?*

The average share is 1 *hekat*. The number of differences will be 9, 1 less than the number of men. Take ½ of the common difference; it makes 1/16 *hekat*. Multiply this by 9; it makes ½ 1/16 *hekat*. Add it to the average share; 1 ½ 1/16 *hekat* is the largest share. Subtract the ⅛ *hekat* for each man until you reach the last.

Do it thus:

The ten portions will be

1 ½ 1/16, 1 ¼ ⅛ 1/16, 1 ¼ 1/16, 1⅛ 1/16, 1 1/16, ½ ¼ ⅛ 1/16, ½ ¼ 1/16, ½ ⅛ 1/16, ½ 1/16, ¼ ⅛ 1/16.

The total is 10 *hekat*.

This problem is explained in the Introduction, page 30. It will be seen from the Literal Translation that the author wrote "Example of distributing the differences," but the common difference is one of the numbers given in the statement of the problem, and the problem is to get the numbers in the progression itself.

Problem 65

Example of dividing 100 *loaves among* 10 *men, including a boatman, a foreman, and a door-keeper, who receive double portions. What is the share of each?*

The working out. Add to the number of the men 3 for those with double portions; it makes 13. Multiply 13 so as to get 100; the result is 7 ⅔ 1/39. This then is the ration for seven of the men, the boatman, the foreman, and the door-keeper receiving double portions.

For proof we add 7 ⅔ 1/39 taken 7 times and 15 ⅓ 1/26 1/78 taken 3 times for the boatman, the foreman, and the door-keeper. The total is 100.

Problem 66

If 10 hekat *of fat is given out for a year, what is the amount used in a day?*

The working out. Reduce the 10 *hekat* to *ro;* it makes 3200. Reduce the year to days; it makes 365. Get 3200 by operating on 365. The

Digitized by Google Original from UNIVERSITY OF MICHIGAN

result is 8 ⅔ ⅒ ½₁₉₀. This makes for a day 1/64 *hekat* 3 ⅔ ⅒ ½₁₉₀ *ro*.

Do it thus:

1	365
2	730
4	1460
8	2920
⅔	243 ⅓
⅒	36 ½
½₁₉₀	⅙

Total 8 ⅔ ⅒ ½₁₉₀.

Do the same thing in any example like this.

Here, as in Problem 61, our author generalizes, suggesting that the method can be applied in any case like this.

Problem 67

Example of reckoning the cattle of a herd. How many cattle are there in a herd when ⅔ of ⅓ of them make 70, *the number due as tribute to the owner?*

The herdsman came to the stock-taking with 70 cattle. The accountant said to the herdsman, Very few tribute-cattle art thou bringing; pray where are all thy tribute-cattle? The herdsman replied to him, What I have brought is ⅔ of ⅓ of the cattle that thou hast committed to me. Count and thou wilt find that I have brought the full number.

Do it thus:

1	1
⅔	⅔
⅓	⅓
⅔ of ⅓	⅙ ⅑₁₈.

Get 1 by operating on ⅙ ⅑₁₈.

1	⅙ ⅑₁₈
2	⅓ ⅑
\4	⅔ ⅙ ⅑₁₈
\½	⅑

Total 4 ½.

Multiply 70 by 4 ½; it makes 315. These are those committed to him.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof. Find ⅔ of ⅓ of 315.

1	315
⅔	210
⅓	105
⅔ of ⅓	70.

These are those that he brought.

This problem indicates the method by which, when the herdsman brings to the owner or his accountant his tribute of cattle, the total count of the herd can be determined.

In the papyrus the author wrote, "Example of reckoning tribute," and though the tribute, 70 cattle, is then given, the conversation between the accountant and the herdsman implies that this is to be calculated from the number of the entire herd. But the calculations given are a solution of the inverse problem, to find the entire herd from the number of tribute-cattle, and only at the end, as a proof of these calculations, is the number of tribute-cattle determined from the number of the entire herd.

The problem is very similar to Problems 35–38, and the method of solution is the same as the method used in those problems. See Introduction, page 28.

Problem 68

Suppose a scribe says to thee, Four overseers have drawn 100 *great quadruple* hekat *of grain, their gangs consisting, respectively, of* 12, 8, 6 *and* 4 *men. How much does each overseer receive?*

There are 30 men in all. Multiply 30 so as to get 100; it makes 3 ⅓. The amount given for each man is therefore 3 ¼ 1/16 1/64 *hekat* 1 ⅔ *ro*, Take this amount 12 times for the first overseer, 8 times for the second. 6 times for the third, and 4 times for the fourth.

The multiplication.

1	3 ¼ 1/16 1/64 *hekat*	1 ⅔ *ro*
2	6 ½ ⅛ 1/32 "	3 ⅓ "
4	13 ¼ 1/16 1/64 "	1 ⅔ "
8	26 ½ ⅛ 1/32 "	3 ⅓ "

List of the amounts of grain for the four overseers:

The first with 12 workmen will have		
¼ of 100 *hekat* 15 *hekat*	or 40	*hekat*
The second with 8 workmen will have		
¼ of 100 *hekat* 1 ½ ⅛ 1/32 *hekat* 3 ⅓ *ro*	" 26 ⅔	"
The third with 6 workmen will have		
20 *hekat*	" 20	"

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The fourth with 4 workmen will have
13 ¼ 1/16 1/64 *hekat* 1⅔ *ro* or 13 ⅓ *hekat*
The four with 30 workmen will have
100 *hekat* " 100 "

This problem is merely to divide 100 into four parts proportional to the numbers 12, 8, 6, and 4. The laborious numerical calculations are due to the use of the "Horus eye" fractions. See Introduction, pages 31–32.

The solution was lengthened in the papyrus also by much repetition. In particular, the multiplication of 3 ¼ 1/16 1/64 *hekat* 1 ⅔ *ro* was carried out four times, as far as 8 twice, and as far as 4 twice, the required products being checked and the total put down for each of the four overseers.

Problem 69 introduces the word *pefsu*, and Problems 69–78 are problems about *pefsu*. This word is derived from the stem of the verb "to cook." It meant something like "cooking ratio," that is, the number of units of food or drink that could be made from a unit of material in the process of cooking, and it determined the relative value of any food or drink. To preserve the technical sense I will use the word *pefsu* in my translation. We may note that the lower the *pefsu* the more valuable the unit of food.

The *pefsu* problems are very simple examples in arithmetic, made to seem more abstruse by the use of this word *pefsu*, and, in some of them, by the use of the word "exchange." We shall see that they can be stated in a way that will make explanations almost unnecessary.

Problem 69

3 ½ hekat *of meal is made into* 80 *loaves of bread. Let me know the amount of meal in each loaf and what is the* pefsu.

Multiply 3 ½ so as to get 80.

1	3 ½
10	35
\20	70
\2	7
\⅔	2 ⅓
\1/21	1/6
\1/7	½.

The *pefsu* is 22 ⅔ 1/7 1/21.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Proof.

\1	22 ⅔ 1/7 1/21
\2	45 ⅓ ¼ 1/14 1/28 1/42
\½	11 ⅓ 1/14 1/42
Total	80.

3 ½ *hekat* makes 1120 *ro*, for

\1	320
\2	640
\½	160
Total	1120.

Therefore multiply 80 so as to get 1120.

Do it thus:

1	80
\10	800
2	160
\4	320
Total 14.	

That is, one of the loaves contains 14 *ro*, or 1/32 *hekat* 4 *ro*, of meal.

Proof.

1	1/32	*hekat* 4	*ro*
2	1/16 1/64	" 3	"
4	1/8 1/32 1/64	" 1	"
8	¼ 1/16 1/32	" 2	"
\16	½ 1/8 1/16	" 4	"
32	1 ¼ 1/8 1/64	" 3	"
\64	2 ½ ¼ 1/32 1/64	" 1	"

It makes 3 ½ *hekat* of meal for the 80 loaves.

The author asks two questions, first, what is the amount of meal in 1 loaf of bread, and second, how many loaves of bread one *hekat* of meal will make. In the solution, however, he determines first the number of loaves that one *hekat* will make, which he finds to be 22 ⅔ 1/7 1/21. This is the *pefsu*. Before determining the amount of meal in one loaf he reduces 3 ½ *hekat* to 1120 *ro*, so that he can say, If 80 loaves take 1120 *ro*, 1 loaf will take 14 *ro*, for 80 times 14, or, what is the same thing, 14 times 80 makes 1120. The 14 *ro* is written in standard from as 1/32 *hekat* 4 *ro*, and this is the answer to the first question.

In the first multiplication it would seem as if the easiest way to get 1/21 would be from 1/7 by halving and taking ⅔. Perhaps originally it was obtained in that way. In the second line of the first proof there are two applications of the table at the beginning of the papyrus, the double of 1/7 being ¼ 1/28 and the double of 1/21, 1/14 1/42.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 70

7 ½ ¼ ⅛ hekat *of meal is made into* 100 *loaves of bread. What is the amount of meal in each loaf and what is the* pefsu?

Multiply 7 ½ ¼ ⅛ so as to get 100.

1	7 ½ ¼ ⅛
2	15 ½ ¼
\4	31 ½
\8	63
\⅔	5 ¼
Total	99 ½ ¼
Remainder	¼
1/63	⅛.

Double the fraction for ¼

\1/42 1/126	¼.

The *pefsu* is 12 ⅔ 1/42 1/126.

Proof.

\1	12 ⅔ 1/42 1/126
\2	25 ⅓ 1/21 1/63
\4	50 ⅔ 1/14 1/21 1/126
\½	6 ⅓ 1/84 1/252
\¼	3 ⅙ 1/168 1/504
\⅛	1 ½ 1/12 1/336 1/1008
Total	100.

7½ ¼ ⅛ *hekat* make 2520 *ro*, for

1	320
2	640
4	1280
½	160
¼	80
⅛	40
Total	2520.

Therefore multiply 100 so as to get 2520.

1	100
10	1000
\20	2000
\5	500
\⅕	20

Total 25 ⅕.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

That is, one of the loaves contains 25 ⅕ *ro*, or $\frac{1}{16}\ \frac{1}{64}$ *hekat* ⅕ *ro*.

Proof.

1	$\frac{1}{16}\ \frac{1}{64}$	*hekat*	⅕ *ro*
10	$\frac{1}{2}\ \frac{1}{4}\ \frac{1}{32}$	"	2 "
100	$7\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$	"	

This problem is similar to the preceding. The reduction of the given number of *hekat* to *ro* is omitted in the papyrus, but the result, 2520, is given, and given as if it were the result of the multiplication used to prove the first part of the solution. Evidently, in copying this solution, the scribe let his eye drop from one "Total" to the next, and so left out the result of one multiplication and all the partial products of the next (Peet, page 116). See note to Problem 28.

There are some interesting steps in the first multiplication and in the first proof. We may notice also that in the first multiplication, as also in Problem 69, we have the "Horus eye" fractions given in the statement of the problem written as ordinary fractions. See Introduction, page 31, footnote 3.

Problem 71

From 1 des-*measure of beer* ¼ *has been poured off, and then the measure has been filled up with water. What is the* pefsu *of the diluted beer?*

Reckon the amount of *besha*[1] in 1 *des* of beer; it is $\frac{1}{2}$ *hekat*. Take away ¼ of it, namely $\frac{1}{8}$ *hekat;* the remainder is $\frac{1}{4}\ \frac{1}{8}$ *hekat*. Multiply $\frac{1}{4}\ \frac{1}{8}$ *hekat* so as to get 1 *hekat*; the result is 2 ⅔, and this is the *pefsu*.

The solution as given means that when *besha* is made into beer, $\frac{1}{2}$ *hekat* of *besha* will make 1 *des*-jug of beer. If the beer is made weaker by pouring off ¼ of it and filling with water, it will take less *besha* by ¼ to make enough of this diluted beer to fill the jug, namely, $\frac{1}{4}\ \frac{1}{8}$ *hekat*, and so 1 *hekat* of *besha* will make 2 ⅔ *des* of this weaker kind of beer. *Pefsu* then means here the number of *des*-jugs of beer that a *hekat* of *besha* will make, just as in the preceding two problems it means the number of loaves that a *hekat* of meal will make.

The first $\frac{1}{2}$ *hekat* in the solution is written by mistake in the papyrus as an ordinary ½. Elsewhere the fractions of a *hekat* are written in the "Horus eye" form.

The mention of *besha* and of other kinds of food may tend to make these solutions seem more obscure, but in no way do they have anything to do with the reasoning or numerical work, and we can leave them out of consideration in trying to understand this.

Problem 72

Example of exchanging loaves for other loaves. Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for a number of loaves of* pefsu 45. *How many of these will there be?*

[1] For a list of kinds of grain or food mentioned in the papyrus see Introduction, page 46.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Find the excess of 45 over 10; it is 35. Multiply 10 so as to get 35; it makes 3 ½. Multiply 100 by 3 ½; it makes 350. Add 100 thereto; it makes 450. Say then that there are exchanged 100 loaves of *pefsu* 10 for 450 loaves of *pefsu* 45, making in *wedyet*-flour 10 *hekat*.

In this Problem our author asks the question, If one has 100 loaves with a *pefsu* of 10, for how many loaves with a *pefsu* of 45 can he exchange them? He arrives at the result in a round-about way. He could simply have determined that 10 *hekat*, the amount of meal required to produce the given 100 loaves, would have produced 10 times 45 loaves of the second kind, and he would thus have arrived at the same result. This is the way in which he solves the next problem and Problem 75, both of which are problems of the same kind.

Problem 73

Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for loaves of* pefsu 15. *How many of these will there be?*

Reckon the amount of *wedyet*-flour in these 100 loaves; it is 10 *hekat*. Multiply 10 by 15; it makes 150. This then is the number of loaves for the exchange.

Do it thus: 100 loaves of *pefsu* 10 would be exchanged for 150 loaves of *pefsu* 15. It takes 10 *hekat*.

As in Problem 72, 100 loaves of *pefsu* 10 require 10 *hekat* of *wedyet*-flour, for "*pefsu* 10" means that 1 *hekat* makes 10 loaves. Now in this problem a *hekat* will make 15 loaves of the second kind, and so 10 *hekat* will make 150 loaves. Therefore 150 loaves of the second kind will be the equivalent of the 100 given loaves of the first kind.

Problem 74

Another problem. 1000 *loaves of* pefsu 5 *are to be exchanged, a half for loaves of* pefsu 10, *and a half for loaves of* pefsu 20. *How many of each will there be?*

1000 loaves of *pefsu* 5 will take 200 *hekat* of Upper Egyptian barley. This then is the amount of *wedyet*-flour in these loaves. ½ of the 200 *hekat* is 100 *hekat*. Multiply 100 by 10; it makes 1000, the number of loaves of *pefsu* 10. Multiply 100 by 20; it makes 2000, the number of loaves of *pefsu* 20.

Do it thus: 1000 loaves of *pefsu* 5, made from 200 *hekat* of *wedyet*-flour, can be exchanged for 1000 loaves of *pefsu* 10, taking 100 *hekat*, and 2000 loaves of *pefsu* 20, taking 100 *hekat*.

In this problem the given 1000 loaves require 200 *hekat* of *wedyet*-flour. Then, if the same amount is used to make two kinds of loaves, one-half of it at the rate of 10 loaves to a *hekat*, and the other half at the rate of 20 loaves to a *hekat*, the result will be 1000 loaves of the first kind and 2000 loaves of the second kind.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 75

Another problem. 155 *loaves of* pefsu 20 *are to be exchanged for loaves of* pefsu 30. *How many of these will there be?*

The amount of *wedyet*-flour in the 155 loaves of *pefsu* 20 is 7 **½ ¼** *hekat*. Multiply this by 30; it makes 232 ½.

Do it thus: 155 loaves of *pefsu* 20, made from 7 **½ ¼** *hekat* of *wedyet*-flour, can be exchanged for 232 ½ loaves of *pefsu* 30. It takes 7 **½ ¼** *hekat*.

This problem is like Problems 72 and 73, and the solution given is like that for Problem 73.

Problem 76

Another problem. 1000 *loaves of* pefsu 10 *are to be exchanged for a number of loaves of* pefsu 20 *and the same number of* pefsu 30. *How many of each kind will there be?*

One loaf of each kind will take

$\frac{1}{20}$ and $\frac{1}{30}$ of a *hekat*.

As parts of 30 these are

1 ½ and 1, together 2 ½.

Multiply 2 ½ so as to get 30

1	2 ½
\10	25
\2	5

Total 12.

Therefore 2 ½ is $\frac{1}{12}$ of 30, so that $\frac{1}{20}$ $\frac{1}{30}$ equals $\frac{1}{12}$. Two loaves, one of each kind, will take $\frac{1}{12}$ of a *hekat* and 1 *hekat* will make 12 loaves of each kind.

The quantity of *wedyet*-flour in the 1000 loaves is 100 *hekat*. Multiply 100 by 12; the result is 1200, which is the number of loaves of each kind for the exchange. That is

1000 loaves of *pefsu* 10, making in *wedyet*-flour 100 *hekat* can be exchanged for

1200 loaves of *pefsu* 20, " " " ½ of " 10 *hekat*
and 1200 " " " 30 " " " ¼ of " 15 "

In this problem, as in Problem 74, the author wishes to know how many loaves of two kinds can be made from a certain amount of *wedyet*-flour, but in Problem 74 he uses half of the flour for one kind of loaf and half for the other. This time he wishes to

Digitized by Google Original from UNIVERSITY OF MICHIGAN

make the same number of loaves of each kind, which makes the problem typical of a very interesting class of problems. See Introduction, page 29.

The amount given is the amount required to make 1000 loaves with a *pefsu* of 10; that is, it is 100 *hekat*. The two kinds of loaves that he wishes to get are loaves with a *pefsu* of 20 and loaves with a *pefsu* of 30. One loaf of the first kind will take $\frac{1}{20}$ of a *hekat* and one loaf of the second kind will take $\frac{1}{30}$ of a *hekat*. He applies these fractions to 30. Perhaps he thinks of determining how much flour it will take to make 30 loaves of each kind, and this he finds to be 2 $\frac{1}{2}$ *hekat*. Now as many times as 2 $\frac{1}{2}$ must be multiplied to produce 30, namely, 12 times, 1 *hekat* making 12 loaves of each kind, so many times must 100 be multiplied to produce the number of loaves of each kind that 100 *hekat* will make.

At the end in what appears to be a proof he puts down the amount of *wedyet*-flour that 1200 loaves of each kind will require, showing that it makes just 100 *hekat*. We may notice the form in which he expresses these amounts, writing the quarter and half of 100 *hekat* and the number of *hekat* in addition. This is the form used for large quantities when he would use the "Horus eye" fractions for small quantities. The same forms are used in Problem 68 and in some of the problems that are to follow. See Introduction, pages 31-32.

Problem 77

Example of exchanging beer for bread. Suppose it is said to thee, 10 des *of beer* (*of* pefsu 2) *are to be exchanged for loaves of* pefsu 5. *How many loaves will there be?*

Reckon the amount of *wedyet*-flour in 10 *des* of beer; it is 5 *hekat*. Multiply 5 by 5; it makes 25. Say then that it takes 25 loaves to make the exchange.

Do it thus:

10 *des* of beer taking 5 *hekat* of *wedyet*-flour
can be exchanged for
25 loaves of bread of *pefsu* 5; for these also take 5 *hekat* of *wedyet*-flour.

Problem 78

Example of exchanging bread for beer. Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for a quantity of beer of* pefsu 2. *How many* des *of beer will there be?*

Reckon the amount of *wedyet*-flour in 100 loaves of *pefsu* 10; it is 10 *hekat*. Multiply 10 by 2; it makes 20. Say then that it takes 20 *des* of beer to make the exchange.

These two problems are like Problems 72, 73 and 75, but instead of two kinds of loaves we have here an exchange of loaves and beer.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 79

Sum the geometrical progression of five terms, of which the first term is 7 and the multiplier 7.

The sum according to the rule. Multiply 2801 by 7.

1	2801
2	5602
4	11204
Total	19607.

The sum by addition.

houses	7
cats	49
mice	343
spelt	2401
hekat	16807
Total	19607.

I have discussed the numerical work of this problem in the Introduction, page 30.

In the second column the author places before the successive powers of 7 the words, houses, cats, mice, spelt and *hekat*. Eisenlohr regards these words as names given to the powers of 7. Another interpretation is that the problem intended is like this: In each of 7 houses are 7 cats, each cat kills 7 mice, each mouse would have eaten 7 ears of spelt, and each ear of spelt will produce 7 *hekat* of grain; how much grain is thereby saved? But the author adds all of these quantities together, showing that he is more interested in the numerical problem of the sum of these numbers.

This interpretation, coupled with the number 7, reminds us of the children's rhyme, of which one version is the following:

"As I was going to Saint Ives,
I met a man with seven wives.
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were there going to Saint Ives?" [1]

Here, again, it is suggested that the sum total of a geometrical progression be calculated but there is a "joker" in the actual wording of the first and last lines.

Rodet (1882, page 111) found in the *Liber Abaci* of Leonardo of Pisa (see Bibliography, 1857) a problem of a geometrical progression expressed in much the same way, and having the ratio 7, and he suggests that Problem 79, absurd as is its heterogeneous addition, has perpetuated itself through all the centuries from the times of the ancient Egyptians.

[1] *Every Child's Mother Goose*, with Introduction by Carolyn Wells, New York, 1918, page 111.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Problem 80

Express the "Horus eye" fractions in terms of the hînu.

The following vessels are used[1] in measuring grain by the functionaries of the granary:

1	*hekat*	makes	10	*hînu*
½	"	"	5	"
¼	"	"	2 ½	"
⅛	"	"	1 ¼	"
1/16	"	"	½ ⅛	"
1/32	"	"	¼ 1/16	"
1/64	"	"	⅛ 1/32	"

Problem 81

Another reckoning. Express fractions of a hekat *as "Horus eye" fractions and in terms of the* hînu.

Now	½	*hekat* makes	5	*hînu*
	¼	" "	2 ½	"
	⅛	" "	1 ¼	"
	1/16	" "	½ ⅛	"
	1/32	" "	¼ 1/16	"
	1/64	" "	⅛ 1/32	"

a Now														
½	¼	⅛	*hekat*			makes	8	½ ¼	*hînu*					
½	¼		"			"	7	½	"					
½	⅛	1/32	"	3 ⅓	*ro*	"	6	⅔	"	it is	⅔	of a	*hekat*	
½	⅛		"			"	6	¼	"	" "	½ ⅛	" "	"	
¼	⅛		"			"	3	½ ¼	"	" "	¼ ⅛	" "	"	
¼	1/16	1/64	"	1 ⅔	"	"	3	⅓	"	" "	⅓	" "	"	
¼			"			"	2	½	"	" "	¼	" "	"	
⅛	1/16		"	4	"	"	2		"	" "	⅕	" "	"	
⅛	1/32		"	3 ⅓	"	"	1	⅔	"	" "	1/6	" "	"	
b Now														
⅛	1/16		"	4	"	"	2		"	" "	⅕	" "	"	
1/16	1/32		"	2	"	"	1		"	" "	1/10	" "	"	
1/32	1/64		"	1	"	"		½	"	" "	1/20	" "	"	
1/64			"	3	"	"		¼	"	" "	1/40	" "	"	
1/16			"	1 ⅓	"	"		⅔	"	" "	1/15	" "	"	

[1] This interpretation is suggested by Gunn (page 136).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

c													
$\frac{1}{32}$			*hekat*	$\frac{2}{3}$	*ro*	makes	$\frac{1}{3}$	*hînu*	it is	$\frac{1}{30}$	of a	*hekat*
$\frac{1}{64}$			"	$\frac{1}{3}$	"	"	$\frac{1}{6}$	"	" "	$\frac{1}{60}$	" "	"
$\frac{1}{2}$			"			"	5	"	" "	$\frac{1}{2}$	" "	"
$\frac{1}{4}$			"			"	2 $\frac{1}{2}$	"	" "	$\frac{1}{4}$	" "	"
$\frac{1}{2}$	$\frac{1}{4}$		"			"	7 $\frac{1}{2}$	"	" "	$\frac{1}{2}$ $\frac{1}{4}$	" "	"
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	"			"	8 $\frac{1}{2}$ $\frac{1}{4}$	"	" "	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$	" "	"
d												
$\frac{1}{2}$	$\frac{1}{8}$		"			"	6 $\frac{1}{4}$	"	" "	$\frac{1}{2}$ $\frac{1}{8}$	" "	"
$\frac{1}{4}$	$\frac{1}{8}$		"			"	3 $\frac{1}{2}$ $\frac{1}{4}$	"	" "	$\frac{1}{4}$ $\frac{1}{8}$	" "	"
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$	"	3 $\frac{1}{3}$	"	"	6 $\frac{2}{3}$	"	" "	$\frac{2}{3}$	" "	"
$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	"	1 $\frac{2}{3}$	"	"	3 $\frac{1}{3}$	"	" "	$\frac{1}{3}$	" "	"
$\frac{1}{8}$			"			"	1 $\frac{1}{4}$	"	" "	$\frac{1}{8}$	" "	"
$\frac{1}{16}$			"			"	$\frac{1}{2}$ $\frac{1}{8}$	"	" "	$\frac{1}{16}$	" "	"
e												
$\frac{1}{32}$			"			"	$\frac{1}{4}$ $\frac{1}{16}$	"	" "	$\frac{1}{32}$	" "	"
$\frac{1}{64}$			"			"	$\frac{1}{8}$ $\frac{1}{32}$	"	" "	$\frac{1}{64}$	" "	"

This problem is interesting since it gives a very full table of fractional parts of a *hekat* in terms of "Horus eye" fractions and in terms of the *hînu*. There are a few errors in the papyrus and in some places it is difficult to determine just what the author intended to write. In the translation given above I have made the corrections that seem most probable.

Except for the first section, which is a repetition of Problem 80, the table is in three columns. The first column gives an expression in "Horus eye" fractions, the second column in terms of the *hînu* [1] and the third (in red in the papyrus) as a simple part of a *hekat*. In section *a* this third column was placed before the other two and written in black. Peet has suggested (page 124) that the scribe, writing the black portions first, forgot to leave room for this first red section in its proper place.

In the fourth and fifth lines of section *a* the quantities expressed are $\frac{1}{2}$ $\frac{1}{8}$ and $\frac{1}{4}$ $\frac{1}{8}$ *hekat*, equal to 5 and 3 times $\frac{1}{8}$ *hekat*. In the third column (placed in the papyrus before the other two) the author wrote $\frac{1}{5}$ and 3. If $\frac{1}{5}$ were a mistake for 5, a superfluous dot being inserted, we might suppose that he thought of $\frac{1}{8}$ *hekat* as a kind of unit of which he takes 5 and 3, respectively. The expressions in these two lines are given correctly as lines 1 and 2 of section *d*.

The sixth line of section *a*, as written in the papyrus, is the most confusing. The best explanation seems to be that the author intended to write $\frac{1}{3}$ of a *hekat*, getting this line by halving the quantities in the third line, although he says $\frac{1}{7}$. In the *hînu* column he wrote for 3 $\frac{1}{3}$, 3 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{2}{3}$, and in the third line for 6 $\frac{2}{3}$, 6 $\frac{1}{2}$ $\frac{1}{16}$. In a large part of the table the fractions of a *hînu* happen to be fractions whose denominators are powers of 2 like the "Horus eye" fractions and it may be that he attempted to write the fractions in the third and sixth lines in this way. In lines 3 and 4 of section *d* he gives correctly

[1] It may be noted that the fractions of a *hînu* are not written like the "Horus eye" fractions, even when the denominators are powers of 2.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

the expressions for ⅔ and ¼ of a *hekat*, and in the above table I have corrected lines 3 and 6 of section *a* so that they are the same as lines 3 and 4 of section *d*.

In the first line of section *c*, at the end, the papyrus says 1/60 of a *hekat* when it should be 1/30, and if the amount in the second line is intended to be a half of the amount in the first line, as is probable, then the middle number should be 1/6, and the last number 1/60, instead of ⅓ and 1/30, as in the papyrus. In the sixth line of the same section the fraction ¼ is omitted in the middle number, and a little further on, in the second line of section *d*, the 3 is omitted from the middle number. I have corrected all of these mistakes in the table above.

Problems 82–84 have to do with the feed of birds and oxen and are interesting chiefly as showing with how much care the Egyptian farmer estimated the amount of feed necessary for each bird or animal. There are many mistakes in these problems, and the last one, Problem 84, is unintelligible; at least no one has yet explained it.

Problem 82

Estimate in wedyet-*flour, made into bread, the daily portion of feed for geese.*

10 fatted geese eat daily	2½ *hekat,*
Taking for 10 days	¼ of 100 *hekat;*
Taking for 40 days	100 *hekat.*

The amount of spelt that has to be ground to produce it is
1 ½ times 100 *hekat* 16 **½ ⅛ 1/32** *hekat* 3 ⅓ *ro.*
The amount of wheat is
⅓ ¼ of 100 *hekat* 8 **¼ 1/16 1/64** *hekat* 1 ⅔ *ro.*
That which has to be taken away is 1/10, of this, namely,
6 **½ ⅛ 1/32** *hekat* 3 ⅓ *ro.*
The remainder, which is the amount of grain required, is
½ ¼ of 100 *hekat* 18 **¼ 1/16 1/64** *hekat* 1 ⅔ *ro.*
Expressed in double *hekat* this is
¼ of 100 *hekat* 21 **½ ⅛ 1/32** *hekat* 3 ⅓ *ro.*

Problem 82B

Estimate the amount of feed for other geese.

If to fatten 10 geese it takes daily	1 **¼** *hekat,*
It will take for 10 days	12 **½** "
And for 40 days	½ of 100 *hekat*

The amount of grain to be ground in double *hekat* is
23 **¼ 1/16 1/64** *hekat* 1 ⅔ *ro.*

Digitized by Google Original from UNIVERSITY OF MICHIGAN

These are problems, about the feed for fattening geese. First the author determines that it takes 100 *hekat* to feed 10 fattening geese for 40 days. Then he asks how much grain is required to produce in grinding 100 *hekat*.

He begins by mentioning two kinds of grain. The first appears to be spelt. The second is wheat. Adding to 100 *hekat* ⅔ of 100 *hekat*, which would make 166 ⅔ *hekat*, he says that this is the amount of spelt that has to be ground to produce 100 *hekat* of meal. Then he simply takes ⅔ of 100 *hekat*, namely, 66 ⅔ *hekat*, and says that this is the amount of wheat that would be required. In the next line he takes 1/10 of the latter quantity, says that this is the amount to be taken away, and subtracts it from 100 *hekat*, presenting the remainder as the solution of the problem. In other words, whatever the two lines about spelt and wheat may mean, we may suppose that grinding increases the bulk of the grain (wheat perhaps), and that the amount required to produce 100 *hekat* of meal was to be determined by taking away from 100 *hekat* of grain 1/10 of ⅔ of it. This may have been a rule that had been established by experiment and was well-known, or the author may have made up an empirical rule to determine somewhat roughly the smaller amount of grain that he knew would be sufficient to make a given amount of meal. See notes to Problem 53, page 94.

In Problem 82B, which Eisenlohr included in the previous problem, we have a similar series of calculations for geese, assuming this time that it takes just half as much to fatten them. All the numbers are half as large as in Problem 82, and so all of the steps are omitted and only the last line is given.

The quantities in these solutions are expressed in the form which uses the "Horus eye" fractions of a *hekat*, and for larger quantities writes the number of times 100 *hekat*, and writes 50 *hekat* and 25 *hekat* as ½ and ¼ of 100 *hekat*. See the notes to Problem 76, page 111, and Introduction, pages 31–32. In this notation ⅔ of 100 *hekat*, or 66 ⅔ *hekat* would be written ½ of 100 *hekat* 16 **½ ⅛ 1/32** *hekat* 3 ½ *ro*, and this is the way in which the author writes it first, in the expression for 1 ⅔ times 100 *hekat*, but in the next expression he writes ½, and then for the other third just a half of these numbers. I am inclined to think that the ½ was a little irregular, and that the scribe became confused in trying to write down ⅔. 1/10 of ⅔ of 100 *hekat*, or 6 ⅔ *hekat* is written correctly in the papyrus. 6 ⅔ from 100 leaves 93 ⅓, and 93 ⅓ *hekat* reduces without difficulty to the expression given. We can perform these various operations directly with the forms used by the Egyptian.

We may notice that we have in this problem the double *hekat* and its parts with the double *ro*. The double *hekat* is also mentioned in Problem 84. See Introduction, page 32.

Problem 83

Estimate the feed necessary for various kinds of birds.

If the feed of four geese that are cooped up is 1 *hînu* of Lower-Egyptian grain, the portion of one of the geese is **1/64** *hekat* 3 *ro*.

If the feed of a goose that goes into the pond is **1/16 1/32** *hekat* 2 *ro*, it is 1 *hînu* for 1 goose.

For 10 geese it takes 1 *hekat* of Lower-Egyptian grain.

For 10 days 10 *hekat*.

For a month ¼ of 100 *hekat* 5 *hekat*.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

The daily portion of feed to fatten

A goose	is	1/8	1/32	*hekat*	3 1/3	*ro*	for	1	bird
A *terp*-goose	"	1/8	1/32	"	3 1/3	"	"	"	"
A crane	"	1/8	1/32	"	3 1/3	"	"	"	"
A *set*-duck	"	1/32	1/64	"	1	"	"	"	"
A *ser*-goose	"	1/64		"	3	"	"	"	"
A dove	"				3	"	"	"	"
A quail	"				3	"	"	"	"

In regard to the identification of these birds see Peet, page 126.

Problem 84

Estimate the feed of a stall of oxen.

	. . . food	common . . . food
4 fine Upper Egyptian bulls eat	24 *hekat*	2 *hekat*
2 fine Upper Egyptian bulls eat	22 "	6 "
3 common . . . cattle eat	20 "	2 "
1 . . . ox	20 "	
Total of this feed	86 "	10 "
It makes in spelt	9 "	7 1/2 "
It makes for 10 days	1/2 1/4 of 100 *hekat* 15 *hekat*	1/2 1/4 of 100 *hekat*
It makes for a month	200 *hekat*	1/2 1/4 of 100 *hekat* 15 *hekat*
It makes in double *hekat*	1/2 of 100 *hekat* 11 1/2 1/8 *hekat* 3 *ro*	1/4 of 100 *hekat* 5 *hekat*

Eisenlohr gave the numbers 85, 86, and 87 to certain fragments that are not a part of the mathematical work of the papyrus, but are of interest, although incomplete and more or less unintelligible. As they cannot in any sense be called problems I will use the word "Number" when referring to them, but will give them these numbers, as does Eisenlohr, in continuation of the numbering of the problems.[1]

[1] Strictly Number 87 should be numbered before Number 86, which is at the very end of the papyrus. See Diagram in the second volume.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Number 85

This is a group of cursive hieroglyphic signs, written upside down on the back of the papyrus. It has been suggested that the scribe was merely trying his pen (see Peet, page 128). Eisenlohr attempted to give a meaning to the group, calling it a "Motto," and translating it somewhat as follows: Kill vermin, mice, fresh weeds, numerous spiders. Pray the god Rê' for warmth, wind and high water.

Gunn, however, claims (page 136) that we have here an early example of the so-called enigmatic writing, and gives as a tentative translation, "Interpret this strange matter, which the scribe wrote . . . according to what he knew."

Numbers 86 and 87 are pieces from some other writings pasted on the back of the papyrus to strengthen it or to mend places where it was torn. Number 86 is upside down. My translation of these fragments follows chiefly that of Peet.

Number 86

This seems to be from some account or memorandum. There are three pieces which appear separated in the British Museum Facsimile, but since that was made they have been placed together in their proper relative positions as may be seen in Photograph 31, volume 2. There are eighteen lines, but parts are missing from both ends of the lines. The following is a translation of the words that remain in the eighteen lines:

1. . . . living forever. List of the food in Hebenti . . .
2. . . . his brother the steward Ka-mosè . . .
3. . . . of his year, silver, 50 pieces twice in the year . . .
4. . . . cattle 2, in silver 3 pieces in the year . . .
5. . . . one twice; that is, ⅙ and ⅙. Now as for one . . .
6. . . . 12 *hînu;* that is, silver, ¼ piece; one . . .
7. . . . (gold or silver) 5 pieces, their price therefor; fish, 120, twice . . .
8. . . . year, barley, in quadruple *hekat*, ½ ¼ of 100 *hekat* 15 *hekat;* spelt, 100 *hekat* . . . *hekat* . . .
9. . . . barley, in quadruple *hekat*, ½ ¼ of 100 *hekat* 15 *hekat;* spelt, 1 ½ ¼ times 100 *hekat* 17 *hekat* . . .
10. . . . 146 ½; barley, 1 ½ ¼ times 100 *hekat* 10 *hekat*; spelt, 300 *hekat* . . . *hekat* . . .
11. . . . ½, there was brought wine, 1 ass(-load?) . . .
12. . . . silver ½ piece; . . . 4; that is, in silver . . .
13. . . . 1 ¼; fat, 36 *hînu;* that is, in silver . . .
14. . . . 1 ½ ¼ times 100 *hekat* 21 *hekat*; spelt, in quadruple *hekat*, 400 *hekat* 10 *hekat* . . .

15–18. [These lines are repetitions of line 14.]

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Number 87

This seems to be a memorandum of some incidents, not very coherent, but apparently complete.

Year 11, second month of the harvest season, Heliopolis was entered.

The first month of the inundation season, 23rd day, the commander (?) of the army (?) attacked (?) Zaru.

25th day, it was heard that Zaru was entered.

Year 11, first month of the inundation season, third day, Birth of Set; the majesty of this god caused his voice to be heard.

Birth of Isis, the heavens rained.

See the note on the Egyptian calendar and Egyptian chronology, Introduction, page 43.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

BIBLIOGRAPHY OF EGYPTIAN MATHEMATICS

with special reference to the

RHIND MATHEMATICAL PAPYRUS

AND SOURCES OF INTEREST IN ITS STUDY

BY

RAYMOND CLARE ARCHIBALD

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

PREFATORY NOTE

The hope is entertained that no item of prime importance may be found wanting in this Bibliography which, within certain limits, has been made as comprehensive as possible. Apart from publications devoted wholly to topics of the Rhind Mathematical Papyrus, numerous references are given to other mathematical, scientific, Egyptological, and popular papers and works. Consideration of these as a whole enables the student more truly to appreciate the setting, appeal, and contents of the work. With very few exceptions titles of publications dealing with Egyptian astronomy, time reckoning, contracts, and business documents, have been omitted, and only a few references have been given to the considerable literature on Egyptian metrology. Some titles in the literature of early Babylonian mathematics have been introduced. The list of publications concerning the comparatively late demotic, Greek, and Coptic papyri, and ostraca, perpetuating Egyptian arithmetic methods, has been made as complete as possible. Reviews have occasionally been listed; the value of those written by well-known scholars will be readily recognized.

Critical and explanatory notes have also been added in connection with some of the items, more particularly where the titles did not appear to be sufficiently descriptive, in the hope that they might be suggestive to the inquirer wishing to inform himself rather thoroughly on questions of the Papyrus. No inference should be drawn from the absence of a note regarding the contents of an item. Warning comments, appended to several items of a trivial nature for the purposes of this Bibliography, may save waste of time. Since everything listed has been personally inspected it is hoped that slips found may not be numerous.

Items indicated as trivial for the purposes of this Bibliography have been listed partly because references to such items without characterization are frequent in the literature, partly because such items are often by scholars whose brief though well considered remarks might be of both value and interest, and partly on the basis of considerations including the output in this connection of a given country. Unimportant

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

statements in works whose titles might suggest something different, are usually so characterized.

The chronological arrangement, according to the date of publication, followed by an alphabetical list of authors, renders the Bibliography of service for ready reference and checking.

Except in the case of those in Russian all titles of books, periodicals and articles, are given in the original language. For Russian publications it was thought best to employ a transliteration, namely that used in the British Museum; English or French translations are also given. Since the name of Bobynin is well established in mathematical literature, the form Bobuinin, which is in accordance with the above mentioned transliteration, has not been employed.

In assembling the material I have been greatly aided by the cordial assistance of scholars in different countries, but especially of certain ones in libraries of Rome, in the Preussische Staats-Bibliothek, Berlin, and in the British Museum. Among others, Professor T. Eric Peet, of the University of Liverpool, directed my attention to five titles of importance for my inquiry. Through the great courtesy of the late Doctor Georg Valentin, six titles were obtained from his extraordinary mathematical bibliography, 1462–1899. I am also indebted for four titles to the late Doctor Gustaf Eneström, of Stockholm, who was always so generous with his wonderful knowledge of mathematical history and bibliography. My friend Doctor Manning has rendered valuable help with some notes, particularly in connection with papers by Sylvester and Möller.

In conclusion, reference may be given to two unpublished documents. In the National-Bibliothek, Vienna, there is a large Greek papyrus (no. Gr. 19996) which appears to be of geometrical content. It was found at Soknopaiu Nesos (Dîmeh), in the Fayûm, and, according to the writing, dates from about the fourth century of the Christian era. This papyrus has not yet been translated.

The British Museum possesses a second mathematical work in hieratic script, which may some day still further extend our knowledge of Egyptian mathematics. It is a leather roll, numbered 10250, and, so far as the Museum records suggest, it was acquired in 1864 along with the Rhind mathematical papyrus, and hence may also have been purchased by Rhind at Luxor in 1858. It is about 10½ inches wide and probably not more than 20 inches long. On account of its brittle nature this roll has never been opened during the more than sixty years that it has been in the Museum's possession. A distinguished

Digitized by Google Original from UNIVERSITY OF MICHIGAN

chemist has recently treated a small piece of the leather roll with a celluloid solution and found that it became pliable while all previous tendency to crumbling disappeared. It is expected that the whole roll will be similarly treated within a very short time. I am indebted to Professor Griffith for the expression of an opinion that the roll is of about the same age as the Rhind papyrus. No previous description of this roll seems to have been published, although Doctor Budge reported its existence to Professor Eisenlohr who made public a note of the fact.

Brown University
Providence, Rhode Island
February, 1927

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

1706

GRAM, H., *De origine geometriæ apud Aegyptios . . . in collegio Elersiano publicè propositum à Johanne Grammio, respondente . . . Petro Nicolao Möllero, die 17. Junii, anno MDCCVI*, Copenhagen, [1706], 44 pp., small quarto.

This is the first of four publications in the series *Cogitationum ad loca scriptorum antiqviorum specimen*, I–IV, 1706–1708, and the first publication (when the author was twenty years old) of this notable Danish professor and librarian. To indicate the contents of the pamphlet perhaps I cannot do better than give a free translation of its summary made by M. C. Gertz (*Dansk Biografisk Lexikon*, Copenhagen, vol. 6, 1892, p. 179): "In this publication the author combats the then generally accepted opinion that Egyptian geometry first appeared in the time of Sesostris, and had its origin in the need of the redetermination of boundaries of fields after the overflow of the Nile had obliterated them; and he rightly shows that this statement by Herodotus is only a hypothesis, and that this opinion is not founded on any definite facts. In accordance with other places in ancient authors he asserts that Geometry is older, and indeed dates back to the time of the mythical Egyptian Hermes Trismegistus (Theuth or Thoth). In this manner he arrived at the peculiar conclusion, which he believed he had proved, with the aid of passages from much later Jewish and Christian writings of Greek authorship, that this Thoth must be identical with the Biblical Abraham."

1737

NEWTON, I., "A dissertation upon the *sacred cubit* of the *Jews* and the *cubits* of the several nations; in which, from the dimensions of the greatest Egyptian pyramid, as taken by Mr. John Greaves, the antient cubit of *Memphis* is determined. *Translated from the* Latin *of Sir* Isaac Newton, *not yet published*," J. Greaves, *Miscellaneous Works*, London, 1737, vol. 2, pp. 405–433.

Also in C. P. Smyth, *Life and Work at the Great Pyramid*, Edinburgh, 1867, vol. 2, pp. 341–366.

"Dissertatio de sacro Judæorum cubito et de cubito aliarum gentium. Edita Anglicè in Miscellaneis Operibus Johannis Gravii," in I. Newton, *Opuscula Mathematica, Philosophica et Philologica*, Lausanne and Geneva, 1744, vol. 3, pp. 491–510 + 1 plate.

Quotation from Borchardt (see later, under 1922), p. 36: "Es ist jedenfalls ein fast komisch zu nennender Zufall, dass Newton aus drei falschen Voraussetzungen die Länge der ägyptischen Elle bis auf rund 3 mm. genau bestimmte."

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

1809

GIRARD, P. S., "Mémoire sur les mesures agraires des anciens Égyptiens." *Description de l'Égypte ou Recueil des Observations et des Recherches qui ont été faites en Égypte pendant l'Expédition de l'Armée Française, . . . Antiquités, Mémoires*, Paris, vol. 1, 1809, pp. 325–356, folio.

Also in *Description de l'Égypte . . .*, "second édition," Paris, vol. 8, 1822, pp. 145–209, octavo.

JOMARD, E. F., "Mémoire sur le système métrique des anciens Égyptiens, contenant des recherches sur leurs connoissances géométriques et sur les mesures des autres peuples de l'antiquité," *Description de l'Égypte . . .*, vol. 1, 1809, pp. 495–802 + 10 plates, folio.

Also issued as a separate volume, Paris, 1817, 307 pp. + 10 plates, folio.

Also in *Description de l'Égypte . . .*, "second édition," Paris, vol. 7, 1822, 575 pp. + 10 plates, octavo; the first three words of the original title have here been changed to "Exposition du."

1818

JOMARD, E. F., "Remarques sur les signes numériques des anciens Égyptiens, fragment d'un ouvrage ayant pour titre: Observations et recherches nouvelles sur les hiéroglyphes accompagnées d'un tableau méthodique des signes," *Description de l'Égypte . . .*, vol. 2, 1818, pp. 57–70 + 1 plate, folio.

Also in *Description de l'Égypte . . .*, "second édition," Paris, vol. 9, 1829, pp. 75–102 + 1 plate, octavo.

1819

JOMARD, E. F., *Notice sur les signes numériques des anciens Égyptiens; précédée du plan d'un ouvrage ayant pour titre: Observations et recherches nouvelles sur les hiéroglyphes accompagnées d'un tableau méthodique des signes*, Paris, 1819, 31 pp. + 1 plate, duodecimo.

Devéria states (see under 1862) that Jomard here was the first to recognize the principal hieroglyphic signs for numbers. J. D. Akerblad, in his *Lettre sur l'Inscription Égyptienne de Rosette . . .*, Paris, 1802, was the first to surmise that the Egyptians like other ancient peoples used certain signs for indicating numerical magnitudes.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1825

QUINTINO, G. DI SAINT, *Saggio sopra il sistema de'Numeri presso gli antichi Egiziani*, Turin, 1825, 17 pp. + 1 plate.

Unimportant.

1829

SEYFFARTH, G., [Remarks on Egyptian number systems], *Leipziger Literatur-Zeitung*, Leipzig, no. 220, September 12, 1829, cols. 1753–1756.

Seyffarth discusses writers on demotic number systems of the ancient Egyptians, from 1802 on, and gives the signs for the numerals 1–46, and 66.

1849

BRUGSCH, H. K., *Numerorum apud veteres Aegyptios Demoticorum doctrina. Ex papyris et inscriptionibus nunc primum illustrata* auctore Henrico Brugsch, Berlin, 1849. 37 pp. + 5 plates.

JACOBI, C. G. J., "Über das Vorkommen eines ägyptischen Bruchnamens in Ptolemaeus Geographie," Akademie der Wissenschaften zu Berlin, *Berichte über die . . . Verhandlungen*, August, 1849, pp. 222–226.

Also in C. G. J. Jacobi, *Gesammelte Werke*, Berlin, vol. 7, 1891, pp. 346–350.

The fraction in question is two thirds.

1850

DAVIES, T. S., "Geometrical Notes, I. The early geometry of the Greeks and Egyptians," *The Mechanics' Magazine, Museum, Register, Journal and Gazette*, vol. 53, 1850, pp. 150–154, 169–174.

Thoughtful exposition attributing to the ancient Egyptians geometrical concepts not generally believed to have been attained to by them.

1854

RÖBER, F., *Beiträge zur Erforschung der geometrischen Grundformen in den alten Tempeln Aegyptens und deren Beziehung zur alten Naturerkenntniss*, Dresden, 1854, 2 + 58 pp. + 4 plates, quarto.

Third paragraph of the "Vorwort:" "Es gründen sich diese Untersuchungen, die ein neues noch völlig unbekanntes Gebiet der ägyptischen Denkmälerkunde aufschliessen und nachwiesen werden, dass die Aegypter schon in den ältesten Zeiten eine echt wissenschaftliche Naturforschung besessen haben, auf geo-

Digitized by Google Original from UNIVERSITY OF MICHIGAN

metrische Elementarsätze, die der allgemeinen Gestaltungslehre angehören und von meinem verstorbenen Vater endeckt worden sind."

The father, F. G. Röber, was professor of architecture in the academy at Dresden. In studying the construction of the Temple at Edfu he conceived that he could explain it as being connected with the inscription of a regular heptagon in a circle, the first account of which is given on pages 15–16 of the above mentioned *Beiträge*.[1] In this connection the following quotations may be made from a letter[2] of William Rowan Hamilton, dated September 15, 1862, and addressed to the son-in-law of the Archbishop of Dublin.

"A wish to gratify the archbishop and yourself was the first motive for my attempting to examine to some extent the Essays of Röber which you had the goodness to leave for me a few days ago, and to form some opinion of their value, unimportant as that opinion might be. But the Memoir on the ancient temples of Egypt (Röber, Dresden, 1854) has interested me profoundly. Indeed I have scarcely been able, since I opened it, to attend to anything else; and it led me into some long calculations which I have only just completed to my satisfaction. As I have paid no special attention to Egyptian Antiquities, nor meditated much on such mystical guesses as some have made at their inner meaning, the only point which I could hope to study usefully was the *geometrical discovery* announced in the first memoir, namely 'the construction of the regular heptagon', which the elder Röber appears to have *divined*, from the study of the ancient Temple Architecture.

"I entered on the subject, perhaps with prejudice; for like most (if not all) modern geometers, I have been accustomed to hold, and indeed still do hold, that it is *impossible* to construct such a heptagon with the 'right line' and 'circle' *alone*. Yet to my great surprise, I found no error in Röber's *numbers*, and on repeating the calculations on another plan, with Taylor's seven-figure logarithms, I found myself quite unable to pronounce whether Röber's *arc* erred in excess or in defect from the *exact* seventh part of the circumference; for that it *must err* I felt assured.

"It seemed, therefore, worth while to go much more closely to work; and laying tables entirely aside, to perform the *whole* of the work for myself, by *arithmetic alone*, and especially by extractions of *square roots*. And to be quite sure of a high degree of accuracy in the final result, I made it a rule to work with not fewer than *fifteen decimal places*, besides employing all verifications that I could think of in the progress of calculation, which thus laboriously conducted, has covered many sheets of paper, and cost many hours on two or three successive days.

"At last, however, it is finished; and I should have no hesitation to commit myself publicly to the result, which is, technically expressed, that the natural *cosines* of the angle assigned by Röber's construction is, to thirteen decimals,

[1] This publication of the heptagon construction was repeated in the following posthumous work (pp. 20–22), edited by the son: F. G. Röber, *Elementar-Beiträge zur Bestimmung des Naturgesetzes der Gestaltung und des Widerstandes, und Anwendung dieser Beiträge auf Natur und alte Kunstgestaltung*, Leipzig, 1861.

[2] R. P. Graves, *Life of Sir William Rowan Hamilton*, vol. 3, Dublin, 1889, pp. 143–145; see further pp. 141–148, 584–587.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

0.6234900759241, whereas the true cosine of the seventh part of four right angles deduced to a corresponding accuracy from a known cubic equation I find to be a *little less*, namely, 0.6234898018587. Admitting, though I do not believe it, that the two or three last of these decimals *may* be wrong after all the precautions taken, I am quite satisfied that the cosine of the *Egyptian Angle* —for really Röber seems to make it likely that the Egyptians did employ it— is *somewhat greater*, and that the cosine of the *true* or *geometrical* angle (for of course we can in geometrical *conception* divide the circumference into *any number* of equal parts) is somewhat less than 0.62349, and consequently that the supposed *Egyptian rule* of the *heptagon* is *not mathematically perfect*, though Röber seems to suppose it to be so. . . .

"But now let us turn to the tables and inquire *how near* does the supposed ancient rule come to the truth? *How small* in *practice* is the *error* which *theory* pronounces to *exist?* And I answer that in *practice* the *error* does not exist *at all.* I do not think that *experiments* of measurement, &c., *could* be so conducted by men, at least in the present age, as to prove to sight that there was any error. For *practical purposes*, then, the elder of the Röbers, or the old Egyptian sage whose secrets he supposed himself to have divined, has *done the impossible.* . . . Yet the *practical success* of the rule is to me absolutely *wonderful:* and it is long since any discovery in science produced in me such a sensation of *surprise.*"

In the same month that this letter was written one finds that Hamilton and De Morgan refer to the matter in correspondence. In Hamilton's paper "On Röber's construction of the heptagon," *Philosophical Magazine*, series 4, vol. 27, 1864, pp. 124–132, he describes Röber's diagram as "not very complex and may even be considered elegant;" he then proceeds to indicate the derivation of the various quadratic equations leading to the result.[1]

F. Röber is also the author of *Die aegyptischen Pyramiden in ihren ursprünglichen Bildungen, nebst einer Darstellung der Proportionalen Verhältnisse im Parthenon zu Athen*, Dresden, 1855, 6 + 28pp. + 1 plate. The discussion refers to the regular heptagon as fundamental.

Among other early publications in this connection perhaps the most notable are those of John Taylor: *The Great Pyramid, why was it built? & Who built it?* London, 1859; second edition, 1864, 327pp.; an appendix, issued also as a separate publication, was *The Battle of the Standards: the ancient, of four thousand years, against the modern, of the last fifty years—the less perfect of the two*, London, 1864, 79pp. Of the first, De Morgan, *Budget of Paradoxes*, London, 1872, p. 311, remarks that it is a "very learned" work which "may be referred to for the history of previous speculations. It professes to connect the dimensions of the Pyramid with a system of metrology which is supposed to have left strong traces in the systems of modern times; showing the Egyptians to have had good approximate knowledge of the dimensions of the earth, and of the quadrature of the circle. These are points on which coincidence is hard to distinguish from intention. Sir John Herschel noticed this work and gave several

[1] More details in this connection are given in my article on "Problems discussed by Huygens" in *American Mathematical Monthly*, vol. 28, 1921, pp. 477–479.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

coincidences in the *Athenaeum*," for 1860. There is a good deal in the work of mathematical interest.

Taylor's theories were enthusiastically supported and developed by his friend C. P. Smyth, astronomer royal for Scotland, and professor of practical astronomy in the University of Edinburgh, in his: *Our Inheritance in the Great Pyramid*, London, 1864, 400pp.; second edition, 1874; third, 1877; fourth, 1880, 16 + 677pp. + 25 plates; fifth, abridged, 1890. And in his: *Life and Work at the Great Pyramid during the months of January, . . ., and April, A. D. 1865 . . .*, 3 vols., Edinburgh, 1867. De Morgan remarks (*l. c.*, p. 292) that Smyth's "word on Egypt is paradox of a very high order, backed by a great quantity of useful labour. . . ."

In *The Builder*, vol. 24, 1866, p. 97, G. Thurnell has an article on "The geometrical formation of the great pyramid;" it is "proved" that the pyramid of Taylor, Smyth, and others is that of Herodotus (book ii, chapter 124) about 450 B. C. On pages 150–152 of this same volume of *The Builder*, E. L. Garbett contributes an excellent article on "pyramid geometry" clearly exhibiting unscientific deductions made from data of various workers. "Now, I cannot but ask, with this bird's eye view before us, where have we any evidence *yet* of the Egyptians embodying in these works any geometry, or any theoretic or liberal science at all." The article won applause from "A. X." on pages 200–201. No very different attitude is maintained by R. A. Proctor, in his *The Great Pyramid Observatory, Tomb, and Temple*, London, 1883.

1855

BRUGSCH, H. K., "Aegyptische Studien, III. Ueber die ἐπαφροδισία und den Symbolismus der Zahl 30 in den Hieroglyphen," *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, Leipzig, vol. 9, 1855, pp. 492–499 + 1 plate.

1856

LEPSIUS, K. R., "Ueber eine hieroglyphische Inschrift am Tempel von Edfu" . . . , Akademie der Wissenschaften zu Berlin, *Abhandlungen, aus dem Jahre 1855*, Berlin, 1856, pp. 69–114 + 6 plates.

This deals with a great dedicatory inscription of about 100 B. C., where reference is made to a large number of four-sided fields. For each of these the lengths of the sides (which we may call, in order as we go around, a, b, c, d) and their areas are given; these areas may be determined by the formula[1] $\frac{1}{2}(a + c) \cdot \frac{1}{2}(b + d) = \frac{1}{4}(ab + ad + bc + cd)$, while the true value is $\frac{1}{4}[ab \sin (ab) +$

[1] This formula was inferred by Lepsius as a result of the numerical cases in the inscription; for example (p. 79), 45¼, 17, 33½¼, 15 leading to the area 632. Heron of Alexandria (third century?) used the same method, *Heronis Alexandrini Opera quae supersunt omnia*, Leipzig, vol. 4, ed. by Heiberg, 1912, pp. 208–209. It is to be noted that if $d = 0$ and $c = a$ we have for the area of an isosceles triangle $\frac{1}{2}a.b$ which is exactly the formula used in the Edfu inscription; for example, pp. 82–83, where the sides of a triangle are: (i) 5, 17, 17,; (ii) 2, 3, 3. This formula persisted till about 1000 A.D.; see Gerbert, *Oeuvres*, ed. by A. Olleris, Paris,

Digitized by Google Original from UNIVERSITY OF MICHIGAN

ad sin (*ad*) + *bc* sin (*bc*) + *cd* sin (*cd*)]. Hence, except in the case of a rectangle the formula gives a result which is too *large*. Peet (1923, 2)[1] is therefore in error when he argues (p. 94) that the tenant "never lost by this rough system of measurement" in paying taxes according to the extent of his field. "This method of measuring was admittedly no more than an approximation for taxation purpose, and fractions less than $\frac{1}{64}$ of a square *Khet*, sometimes even $\frac{1}{32}$ of a square *Khet*, were omitted" (Peet); compare H. Maspero, *Les finances de l'Égypte sous les Lagides*, Paris, 1905, p. 135.

An important supplement to the memoir of Lepsius is H. K. Brugsch *Thesaurus Inscriptionum Aegyptiacarum. Astronomische und astrologische Inschriften, altaegyptischer Denkmaeler*, 3. Abteilung, *Geographische Inschriften* Leipzig, 1884, pp. 531–618. For two other references to sources using the above method for computing quadrilateral areas, between 100 B. C. and about 550 A. D., see *Greek Papyri in the British Museum. Catalogue, with Texts*, edited by F. G. Kenyon, London, vol. 2, 1898, pp. 129–141;[2] B. P. Grenfell, A. S. Hunt, J. G. Smyly, *The Tebtunis Papyri*, London, part 1, 1902, pp. 385–393;[3] and H. R. Hall, *Coptic and Greek Texts of the Christian Period from Ostraka in the British Museum*, London, 1905, p. 128, ostracon 29750, and plate 88.[4] Compare Peet (1923, 2) pp. 93–95. A reference may also be given to: H. Weissenborn, "Das Trapez bei Euklid, Heron und Brahmagupta," *Abhandlungen zur Geschichte der Mathematik*, Leipzig, Heft 2, 1879, especially

---

1867, p. 460; also Gerbert, *Opera Mathematica*, ed. by N. Bubnov, Berlin, 1899, p. 354. Compare Gunn under 1923.

The Edfu formulæ for finding the areas of both an isosceles triangle (30, 30, 18) and a quadrilateral (34, 30, 32, 32) were used in the eighth century in a work attributed to Bede the Venerable, *Venerabilis Bædæ . . . Opera*, Cologne, 1612, vol. 1, cols. 106, 109. For Roman use in a quadrilateral (30, 6, 40, 20) see F. Blume, K. Lachmann, A. Rudorff, *Die Schriften der römischen Feldmesser herausgegeben und erläutert*, Berlin, vol. 1, 1848, p. 355.

[1] The form of abbreviation, "Peet (1923)," referring to a publication of Peet in 1923 listed under this date in this Bibliography, will be constantly employed for referring to authors in the following pages. If there is more than one publication by the same author listed under a given date, a second number in the parentheses will identify the one referred to. Since, for example, under 1923 there are two titles of Peet's publications the second will be singled out by the notation: Peet (1923, 2); similarly for the first.

[2] The papyrus here discussed is no. CCLXVII of the late first, or second, century (compare p. xxvi). Although the beginning and end of the papyrus are lost, about 8½ feet, with 21 columns, are still preserved. Most of this is transcribed (without translation) and columns 17–18 are reproduced on plate 45 of the volume of Facsimiles. The papyrus is a register of land containing statements of the extents of land, calculated in arurae, assigned for sowing, woodland, uncultivated land, etc. In one of the fields (line 100) the sides are given as $1\frac{7}{16}$, $21\frac{1}{8}$, $21\frac{15}{32}$, and $21\frac{9}{32}$ and the area is found to be $41\frac{27}{64}$ arurae. The linear unit is 100 royal cubits called a schoenia, the square of which is an arura. The above result is slightly large but correct, according to the rule for areas given above, to within less than a sixty-fourth of an arura.

[3] In these pages is given the transcription of the text of a late second century B. C. village survey list (30 × 66.5 cm.) numbered 87, together with introductory comment and notes.

[4] This is a brief memorandum of land measurements on a pottery ostracon (5 × 4 in.) hardly earlier than the sixth century.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

pp. 169–172; also in *Zeitschrift für Mathematik und Physik*, vol. 24, 1879, Supplement zur historisch-literarischen Abteilung.

In problem 52 of the Rhind papyrus, we consider the quadrilateral field with two sides parallel; its area follows by substituting in the formula $\frac{1}{2}(a + c)\,h$, where h is the distance between the parallel sides.

In the review of Peet (1923, 2), Gunn (1926) states that J. H. Cole of the Egyptian Survey Department had informed him that the methods of calculating the area of four- and three-sided fields exhibited by the Edfu inscription are precisely those used by Egyptian natives to-day.

1857

LEONARDO PISANO (FIBONACCI), *Scritti* . . . pubblicati da B. Boncompagni, Rome, vol. 1, 1857.

"Liber Abaci" (written in 1202): "de disgregatione partium in singulis partibus," and "regula uniuersalis in disgregatione partium numerorum," pp. 77–83; the problem of the seven old wives,[1] pp. 311–312. These sections are discussed in Cantor, *Vorlesungen über Geschichte der Mathematik*, Leipzig, vol. 2, second ed., 1900, pp. 12–13, 26–27. See also L. Rodet (1881,1), Bobynin (1890); Bobynin (1899), and Hultsch (1895). Compare Collignon (1881).

1860

PIHAN, A. P., *Exposé des Signes de Numération usités chez les Peuples Orientaux anciens et modernes*, Paris, 1860.

"Numération Égyptienne," pp. 25–41; "numération Assyrienne," pp. 42–44.

1862

DEVÉRIA, T., "Notation des centaines de mille et des millions dans le système hiéroglyphique des anciens Égyptiens," *Revue Archéologique*, Paris, new series, vol. 6, 1862, pp. 253–263.

Reprinted in his *Mémoires et Fragments*, publiés par G. C. C. Maspero, (Bibliothèque Égyptologique, vol. 4), Paris, vol. 1, 1896, pp. 257–268.

1865

BRUGSCH, H. K., "Ein altägyptisches Rechenexempel," *Zeitschrift für Ägyptische Sprache und Alterthumskunde*, Leipzig, vol. 3, 1865, pp. 65–70, 77.

[1] "Septem uetule uadunt romam; quarum quelibet habet burdones 7; et in quolibet burdone sunt saculi 7; et in quolibet saculo panes 7; et quilibet panis habet cultellos 7; et quilibet cultellus habet uagines 7. Queritus summa omnium predictarum." This is of interest in connection with no. 79 of the Rhind papyrus.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

LEPSIUS, K. R., "Die Regel in den hieroglyphischen Bruchbezeichnungen," *Zeitschrift für Ägyptische Sprache . . .*, vol. 3, 1865, pp. 101–110.

1866

LEPSIUS, K. R., "Die altägyptische Elle und ihre Eintheilung," Akademie der Wissenschaften zu Berlin, *Philologische und historische Abhandlungen, aus dem Jahre 1865*, Berlin, 1866, pp. 1–56 + 5 plates + Nachtrag, pp. 57–63 + zweiter Nachtrag, pp. 64*–64***

1867

GOODWIN, C. W., "Notes on Egyptian numerals," *Zeitschrift für Ägyptische Sprache . . .*, vol. 5, 1867, pp. 94–5, 98–101; see also vol. 9, 1871, pp. 126–128.

With particular reference to Pleyte 1867 [1868] which[1] in turn criticised part of the contents of a letter by Goodwin published in *Zeitschrift für Ägyptische Sprache . . .*, vol. 2, 1864, p. 39, with comments by Brugsch. See also Brugsch's later article in the same *Zeitschrift*, vol. 9, 1871, p. 139 f.

LENORMANT, F., "Note relative à un papyrus égyptien contenant un fragment d'un traité de géométrie appliquée à l'arpentage," *Comptes Rendus . . . de l'Académie des Sciences*, Paris, vol. 65, Nov. 25, 1867, p. 903.

The following is the full note: "Je pense que l'Académie des sciences apprendra avec quelque intérêt l'acquisition que vient de faire le Musée Britannique[2] d'un papyrus égyptien en écriture hyératique, contenant le fragment d'un Traité de Géométrie appliquée à l'arpentage, avec figures. Ce fragment, que j'ai eu ces jours derniers l'occasion d'étudier à Londres, comprend les méthodes pour mesurer l'aire d'un carré, d'un parallélogramme, de diverses espèces de triangles, pour mesurer la superficie d'un terrain de forme irrégulière au moyen de triangles, et pour déterminer le volume d'une pyramide. Le type paléographique de l'écriture reporte ce manuscrit au temps de la XIIe dynastie, c'est-à-dire le fait environ contemporain de Salomon; de plus, une Note qu'il contient dit qu'il est la copie d'un text notablement plus ancien." The note then continues:

"Ce papyrus, si précieux pour l'histoire de la Science et qui nous fournit les premières données positives sur l'antique géométrie égyptienne, va être publié prochainement en *fac-simile* par les soins des *trustees* du Musée Britannique."

[1] For the meaning of the notation "Pleyte 1867 [1868]" see the note introducing the Index of Authors.

[2] The Rhind mathematical papyrus was purchased by the trustees of the British Museum in 1864. It was acquired by A. Henry Rhind at Luxor in 1858. Lenormant's note is, of course, very inaccurately descriptive.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1868

BIRCH, S., "Geometric Papyrus," *Zeitschrift für Ägyptische Sprache . . .* , Leipzig, vol. 6, 1868, pp. 108–110.

A discussion of the period, and an account of the contents (especially the geometric portions), of the Rhind mathematical papyrus. Of course the papyrus is incorrectly characterized as "geometric."

GOODWIN, C. W., "On the Egyptian numerals," *Zeitschrift für Ägyptische Sprache . . .* , vol. 6, 1868, pp. 106–108.

PLEYTE, W., *Les Papyrus Rollin, de la Bibliothèque Impériale de Paris, publiés et commentés* par W. Pleyte, Leyden, 1868, 43 pp. + 21 plates, large quarto.

These papyri date from the time of Seti I (about 1350 B. C.). Critical commentary was made of Pleyte's work by F. Chabas, "Sur quelques données des Papyrus Rollin," *Zeitschrift für Ägyptische Sprache . . .*, vol. 7, 1869, pp. 57–63. (Compare J. Dümichen, "Ueber einige altägyptische Rechnungen aus der Zeit des Rampsinit," *Zeitschrift für Ägyptische Sprache . . .*, vol. 8, 1870, pp. 41–46). Eisenlohr's corrections are brought together in A. Eisenlohr, "The Rollin papyri and baking calculations," *Proceedings of the Society of Biblical Archæology*, vol. 19, 1897, pp. 91–104, 115–120, 147–155, 252–265. To a certain extent Eisenlohr took account of a notable work listed later, namely Spiegelberg (1896). Stripped of footnotes the first paragraph of Spiegelberg's preface is as follows: "Ein Teil der Papyri, mit welchen ich die vorliegende Arbeit beschäftigt, ist bereits vor mehr als 25 Jahren von PLEYTE herausgegeben und erklärt worden. Wer auf dem Boden einer historischen Wissenschaft steht, wird die Bedeutung dieses ersten Entzifferungsversuchs bei dem damaligen Stande der Ägyptologie anerkennen, anderseits aber begreifen, dass jene Arbeit den heutigen Anforderungen der Wissenschaft in keiner Weise mehr genügt. Überdies war die Ausgabe von vornherein nach des Verfassers eigener Ansicht für palaeographische Zwecke völlig wertlos, und doch besteht gerade in letzterer Hinsicht ein ganz hervorragender Wert der Dokumente, welche uns ein vorzügliches Bild der Uncialschrift zur Zeit der Regierung Setis I geben."

Pages 28–37 of Pleyte's work are occupied with a "Traité sur les signes numériques;" this is followed by "Les fractions" (pp. 37–40). The first sentence on page 28 is: "Je crois nécessaire avant de terminer le sujet des Papyrus Rollin, de rectifier et d'amplifier mes notes concernant les signes numériques communiquées dans la Zeitschrift 1867, p. 1ss.," that is, in "La prononciation phonétique des noms de nombres égyptiens," *Zeitschrift für Ägyptische Sprache . . .*, 1867, pp. 1–3, 9–14, 26–31. Pleyte here sums up everything he found on the subject in hieroglyphic, hieratic, demotic, and Coptic texts, making use of the important suggestion of Goodwin (1864) concerning section numbering of the Leyden papyrus I, 350; compare Goodwin (1867). The Leyden

Digitized by Google Original from UNIVERSITY OF MICHIGAN

papyrus was published by C. Leemans in *Monumens Égyptiens du Musée d'Antiquités des Pays-Bas à Leide . . . II. Monumens Civils*, Leiden, 1846; text, pp. 74–75; plates CLIX–CLXVII. Another edition of part of the papyrus with a German translation appeared in F. J. Lauth, *Moses der Ebräer*, Munich, 1868; more recently this part of the papyrus was reedited in W. Spiegelberg, "Das Geschäftsjournal eines ägyptischen Beamten," *Recueil de Travaux relatifs à la Philologie et à l'Archéologie Égyptiennes et Assyriennes*, vol. 17, 1895, pp. 143–160. A transcription and translation of the recto is given in A. H. Gardiner, "Hymns to Amon from a Leyden Papyrus," *Zeitschrift für Ägyptische Sprache . . .*, vol. 42, 1906, pp. 12–42; (although every page of this article is dated 1905 it was not published till "22 März 1906.")

ROUGÉ, O. C. C. E. DE, *Chrestomathie Égyptienne, . . . Abrégé Grammatical*, deuxième fasicule, Paris, 1868.

"Noms de nombre," pp. 106–119 + 6 plates; "mesures diverses," pp. 119–123; "des calculs," pp. 124–128. There is a reference to Bulak papyrus no. 18 on p. 114. Compare Mariette (1872).

1870

BRETSCHNEIDER, C. A., *Die Geometrie und die Geometer vor Euklides*, Leipzig, 1870.

"Die Geometrie der Aegypter," pp. 5–22; "Der Uebergang ægyptischer Mathematik an die Griechen," pp. 22–35. There is discussion based on Birch (1868).

1872

MARIETTE, A., *Les Papyrus Égyptiens du Musée de Boulaq*, Paris, vol. 2, 1872, Papyrus no. 18, large folio.

This papyrus is reproduced on plates 14–55; there are some introductory notes, pages 6–8. The papyrus was found at Thebes in 1860. It is a sort of book containing the accounts of various supplies and payments for the house of an unknown queen and of some other members of the royal family. Attention was first drawn to this papyrus in Rougé (1868). L. Borchardt was the first to interpret the contents of the papyrus in some details in his "Ein Rechnungsbuch des königlichen Hofes aus dem Ende des mittleren Reiches," *Zeitschrift für Ägyptische Sprache . . .*, vol. 28, 1890, pp. 65–103. These details were supplemented by F. L. Griffith, "The account papyrus no. 18 of Boulaq," *Zeitschrift für Ägyptische Sprache . . .*, vol. 29, 1891, pp. 102–116. The most recent discussion is the dissertation of A. Scharff (1920) of which an extract was published as "Ein Rechnungsbuch des königlichen Hofes aus der 13. Dynastie (Papyrus Boulaq Nr. 18)," *Zeitschrift für Ägyptische Sprache . . .*, vol. 57, 1922, pp. 51–68, and plates 1**–24** which contain a hieroglyphic transcription of the papyrus made from a new copy of the original by Gardiner. This copy corrected Mariette's plates in a number of places. Bulak papy-

Digitized by Google Original from UNIVERSITY OF MICHIGAN

rus 18 and the Rollin papyri, Spiegelberg (1896), are the most important early accounts papyri.

1873

SUTER, H., *Geschichte der mathematischen Wissenschaften.* Zweite Auflage. Zürich, 1873, Theil 1, p. 13.

Unimportant; quotes Bretschneider (1870).

1874

BRUGSCH, H. K., "Ueber die Aussprache einiger Zahlwörter im Altägyptischen," *Zeitschrift für Ägyptische Sprache* . . . , vol. 12, 1874, pp. 145–147.

BRUGSCH, H. K., "Ueber den mathematischen Papyrus im britischen Museum zu London," *Zeitschrift für Ägyptische Sprache* . . . , vol. 12, 1874, pp. 147–149 + 1 plate.

Gives some of the more important technical terms used, mainly in the geometrical portion of the papyrus; compare Eisenlohr (1875, "Berichtigung").

1875

CANTOR, M., *Die römischen Agrimensoren und ihre Stellung in der Geschichte der Feldmesskunst. Eine historisch-mathematische Untersuchung*, Leipzig, 1875, pp. 32–35.

Refers to Lepsius (1855), Birch (1868), and Eisenlohr (1875), and notes that the Rhind papyrus deals with areas of isosceles triangles and trapeziums (as the term is used outside of the United States), and also with the squaring of a circle.

Review by A. Favaro, *Bullettino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche*, Rome, vol. 9, 1876; see especially pp. 172–173.

Review by G. V. Schiaparelli, "Cenno dei recenti studj del Dr Cantor sulla storia dell'agrimensura," *Reale Istituto Lombardo di Scienze e Lettere, Rendiconti*, Milan, series 2, vol. 9, 1876, pp. 25–32. Also in G. Schiaparelli, *Scritti sulla Storia della Astronomia Antica*, Bologna, part 1, vol. 2, 1926, pp. 289-301.

DÜMICHEN, J., "Über altägyptische Hohlmaasse," *Zeitschrift für Ägyptische Sprache* . . . , vol. 13, 1875, pp. 91–98.

Part of a communication addressed "An Herrn Professor Eisenlohr."

EISENLOHR, A., "Berichtigung," *Zeitschrift für Ägyptische Sprache* . . . , vol. 13, 1875, pp. 26–29.

Quotation: "In der Nov. Dec. Nummer des letzten Jahrganges der Zeitschrift p. 147ff. ist nun ein kleiner Aufsatz von Prof. Brugsch . . . erschienen,

Digitized by Google Original from UNIVERSITY OF MICHIGAN

welcher mehrfache Unrichtigkeiten enthält, welche ich im Interesse der Leser der Zeitschrift berichtigen zu müssen glaube. Damit möchte ich gleichzeitig die Behauptung Brugsch's widerlegen, 'dass dem Verständniss des Ganzen kaum nennenswerthe Schwierigkeiten entgegen treten.' "

EISENLOHR, A., "Ueber altägyptische Maasse. Nach einem auf dem internationalen Orientalisten-Congress zu London gehaltenen Vortrage," *Zeitschrift für Ägyptische Sprache . . .*, vol. 13, 1875, pp. 40–50 + 1 plate.

Dated February 9, 1875.

1876

EISENLOHR, A., "Des mesures égyptiennes résultats des études du papyrus mathématique du musée Britannique," *Transactions of the Second Session of the International Congress of Orientalists, Held in London in September, 1874*, London, 1876, pp. 282–288.

A fuller report of this address is given in Eisenlohr (1875, 2).

GÜNTHER, S., *Ziele und Resultate der neueren mathematisch-historischen Forschung*. Erlangen, 1876, pp. 98–104.

Brief critical comments especially on the geometrical part of the Rhind papyrus.

1877

CANTOR, M., "Wie man vor vierthalbtausend Jahren rechnete," *Beilage zur Allgemeinen Zeitung*, Augsburg, no. 249, September 6, 1877, pp. 3745–3747.

Popular account of Egyptian mathematics written shortly after the appearance of Eisenlohr (1877). In this same periodical, no. 263, September 20, 1877, pp. 3953–3954, "Der geometrische Papyrus," F. J. Lauth discusses the date of the reign of Amememhat III and the date of the Rhind papyrus.

EISENLOHR, A., *Ein mathematisches Handbuch der alten Ägypter (Papyrus Rhind des British Museum) übersetzt und erklärt*, Leipzig, 1877, vol. 1, *Commentar*, 4to, 4 + 296 pp. + 2 plates; vol. 2, *Tafeln*, large folio (40 × 48 cm.), 24 plates in black and red.

"Zweite Ausgabe (ohne Tafeln)," 1891 (although the date on the title page is 1877), 4 + 278 pp. + 2 plates.

In this second edition before page 9 are the same tables as in the first edition; then follow four unnumbered pages of text which were pages 9–12 of the first edition. Pages 13–26 of the first edition are lacking in the second. With page 9 of the second edition (= page 27 of the first edition) commences an un-

Digitized by Google Original from UNIVERSITY OF MICHIGAN

changed reprint of the first edition, except in connection with the long list of "Zusätzen und Berichtigungen" at the end. In the second edition cross references have not been changed, but are still to the first edition.

Until the publication of Peet (1923, 2) this was the fundamental work for the study of the Papyrus, even though it has, of late years, been considerably out of date. Quotation (p. 2): "Zu dem sachlichen Verständniss des Papyrus hat nicht wenig die Hülfe beigetragen, welche mir zwei Mathematiker der hiesigen Universität [Heidelberg], der eben erwähnte Prof. Dr. Cantor und mein Bruder Prof. Dr. Friedrich Eisenlohr gewährt haben. Ich bin denselben dafür dankbarst verflichtet."

"Though I have seen the original of the papyrus both before and after my publication was issued, I was unable, being far off, to refer to the original, but depended only on the copy, confining my alterations to correcting the evident displacement of some of the fragments and joining the text on one plate, which was in the copy divided into two."—Eisenlohr (1891).

Compare Ahmes (1898) and Eisenlohr (1899). The heiratic for nos. 49–52 is given in G. Möller, *Hieratische Lesestücke für den Akademischen Gebrauch*, Leipzig, 1909, p. 22. On p. 20 there is an extract from Papyrus Bulak 18.

Review by G. E[bers]. *Literarisches Centralblatt für Deutschland*, Leipzig, Oct. 12, 1878, cols. 1350–1353. Last paragraph: "Der Verf. hat mit der vorliegenden mühevollen und sorgsam zu Ende geführten Arbeit nicht nur seiner Specialwissenschaft, sondern auch der Kulturgeschichte ein Werthvolles Geschenck gemacht, das ihm und seinem Scharfsinn zur Ehre gereicht. Wir empfehlen es nicht nur den Fachgenossen, sondern auch den Mathematikern, die auf Grund der Eisenlohr'schen Umschriften und Uebersetzungen das Gegebene prüfen können und gegenüber mancher zweifelhaften Stelle Gelegenheit finden werden, durch neue Deutungen der rechten Meinung des ägyptischen Autors auf die Spur zu kommen."

1878

BORTOLOTTI, P., "Del primitivo cubito Egizio e de' suoi geometrici rapporti colle altre unità di misura e di peso Egiziane e straniere," *Memorie della Regia Accademia di Scienze, Lettere, ed Arti in Modena*, Sezione di Lettere, Modena, vol. 18, 1878, pp. 63–238; vol. 19, 1879, pp. 69–274; series 2, vol. 1, 1883, pp. 59–134.

Reprinted as a volume of 469 pages with the above title, but without date (Harvard University Library). The British Museum copy with title page dated 1878, has 456 pages only, that is, it lacks the "Appendice al Capo V," pages 457–469.

Refers to Lepsius (1865), Eisenlohr (1875, 1877), Favaro (1879), Aurès (1880), etc., pp. 382 ff. of the reprint.

Review by E. Babelon, *Bulletin Critique*, Paris, vol. 3, 1882, pp. 363–365.

GÜNTHER, S., "Antike Näherungsmethoden im Lichte moderner Mathematik," *Abhandlungen der königlich-böhmischen Gesellschaft der*

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Wissenschaften, mathematisch-naturwiss. Classe, Prague, series 6, vol. 9, no. 4, 1878, pp. 8–10.

π approximation; unimportant.

MATTHIESSEN, L., *Grundzüge der antiken und modernen Algebra der litteralen Gleichungen*, Leipzig, 1878.

Zweite, wohlfeile Ausgabe, Leipzig, 1896, pp. 269, 966.

Of value only for the historical setting, particularly in connection with equations of the first degree and the method of "false position." The references to the Rhind papyrus are of no importance.

RODET, L., "Sur un manuel du calculateur découvert dans un papyrus égyptien," *Bulletin de la Société Mathématique de France*, vol. 6, 1878, pp. 139–149.

1879

FAVARO, A., "Sulla interpretazione matematica del papiro Rhind pubblicato ed illustrato dal Prof. Augusto Eisenlohr," *Memorie della Regia Accademia di Scienze, Lettere, ed Arti in Modena*, Sezione di scienze, Modena, vol. 19, 1879, pp. 89–143.

Also reprinted, Modena, 1879, 55 pp.

1880

AURÈS, A., 1 "Métrologie égyptienne. Détermination géométrique des mesures de capacité dont les anciens se sont servis en Égypte, précédée d'explications relatives aux mesures de capacité grecques et romaines." Académie de Nimes, *Mémoires*, année 1879, Nimes, series 7, vol. 2, 1880, pp. 1–166 + 2 plates.

2 "Métrologie égyptienne. Appendice à la détermination géométrique des mesures de capacité dont les anciens se sont servis en Égypte." Académie de Nimes, *Mémoires*, année 1884, series 7, vol. 7, 1885, pp. 7–28.

The first of these was reprinted with two additional pages of contents (172 pp.) in 1880; the second appeared later as pp. 173–193.

CANTOR, M., *Vorlesungen über Geschichte der Mathematik*, Leipzig, vol. 1, 1880, pp. 18–41, 46–52; and numerous other references in the index under "Ahmes."

Zweite Auflage, vol. 1, 1894, pp. 20–47, 52–60; and numerous other references in the index under "Ahmes."

On page 23 occurs the following sentence: Ein weiterer mathematischer Papyrus, von dessen Inhalt leider nicht einmal Andeuten bekannt sind, gehört Herrn

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Wladimir Golenischeff an, Conservator der Sammlung in der Ermitage in Petersburg." This is the Moscow papyrus described in Turaev (1917).

Dritte Auflage, vol. 1, 1907, pp. 57–81, 85, 91–94, 96–100, 186; and numerous other references in the index under "Ahmes."

Volume 2, 1892, pp. 25, 83.

Zweite Auflage, vol. 2, 1900, pp. 27, 92.

Valuable survey.

Extract translated: F. I. Studnička, "O nejstarším Spisu mathematickém vůbec" [On the earliest mathematical writing], *Časopis pro Pěstování Matematiky a Fysiky*, Budapest, vol. 10, 1881, pp. 185–187.

Review by S. Günther, *Beilage zur Allgemeinen Zeitung*, Tübingen, no. 112, Apr. 22, 1881, pp. 1633–1634.

Review by F. Hultsch, *Jahrbücher für Classische Philologie*, Leipzig, vol. 26, 1881, pp. 569–573.

SYLVESTER, J. J., "On a point in the theory of vulgar fractions" and "Postscript to Note on a point in vulgar fractions," *American Journal of Mathematics*, vol. 3, 1880, pp. 332–335, 388–389.

Also in J. J. Sylvester, *Collected Mathematical Papers*, Cambridge, vol. 3, 1909, pp. 440–445.

Sylvester states that this paper was suggested by the "chapter in Cantor's *Geschichte der Mathematik* which gives an account of the singular method in use among the ancient Egyptians for working with fractions. It was their curious custom to resolve every fraction into a sum of simple fractions according to a certain traditional method, not leading, I need hardly say, except in a few of the simplest cases, to the expansion under the special form to which I have, in what precedes, given the name of a fractional *sorites*." Compare Sylvester (1882).

TANNERY, P., "Thalès et ses empruntes à l'Égypte," *Revue Philosophique de la France et de l'Étranger*, vol. 9, 1880, pp. 299–318.

Interesting paper and valuable at the time it was written; published simultaneously with Cantor (1880).

1881

COLLIGNON, E., ["Étant donnée une fraction p/q irreductible et moindre que l'unité, la mettre sous la forme d'une somme de parties aliquotes de l'unité"], *Bulletin de la Société Philomathique de Paris*, Paris, series 7, vol. 6, 1881, pp. 88–94.

Paper read Nov. 22, 1881, was, apparently, suggested by Favaro (1879), the only reference given by the author. Among the writers who have treated this subject are the following: Bobynin (1890, 1896, 1899), Hultsch (1895, 1901),

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Loria (1892, 1894), Mansion (1888), Simon (1907), Sylvester (1880), Tannery (1884), Vasconcellos (1923), and Vetter (1922).

EISENLOHR, A., "An historical monument," *Proceedings of the Society of Biblical Archæology*, vol. 3, 1881, pp. 97–98.

Note regarding a fragment of a wooden tablet, at the Berlin Museum, bearing the name of the Hyksos king Apophis in whose reign the Rhind papyrus was written down by the scribe Ahmes. A calculation follows suggesting that the date of the copy was approximately 2098 B. C. This date is incorrect according to the opinion of to-day, which on the basis of the same facts would make it much later. The tablet is no. 7798 and from the Fayûm. There is a brief description of it in *Königliche Museen zu Berlin, Ausführliches Verzeichnis der aegyptischen Altertümer und Gipsabgüsse*, zweite völlig umgearbeitete Auflage, Berlin, 1899, p. 217. A copy of the hieroglyphic is given in *Aegyptische Inschriften aus den königlichen Museen zu Berlin*, Leipzig, Heft 4, 1913, pp. 264–265.

REVILLOUT, E. ET V., "Note sur l'équerre égyptienne et son emploi d'après le papyrus mathématique," *Revue Égyptologique*, Paris, vol. 2, 1881, pp. 304–314.

First sentences: "Dans la *Revue bibliographique* qui précéde, nous avons longuement parlé de la partie arithmétique du papyrus mathématique, partie si heureusement étudiée par MM. Eisenlohr et Cantor. Il ne sera pas inutile de dire maintenant quelques mots sur une question qui éclaire les points les plus obscurs de la partie géométrique, jusqu' ici beaucoup moins bien comprise.

"Les savants éditeurs de notre curieux document semblent ne pas avoir saisi une vérité de la plus grande importance, c'est que ce fut primitivement un cahier d'élève[1] renfermant les devoirs donnés dans une école communale, pour ainsi dire, dans une de ces *maison d'enseignement* (*a-sbo*), dont il est fréquemment question dans les papyrus, et où l'on montrait les éléments des opérations qu'ils pouvaient avoir à exécuter dans la suite aux enfants destinés à devenir plus tard des propriétaires de campagne, des intendants, des cultivateurs, des arpenteurs, des constructeurs, des entrepreneurs, etc." . . .

The authors here suggest the important cotangent interpretation in connection with the seked of pyramids; compare Borchardt (1893), Petrie (1883), Griffith (1891–94), and Heath (1921).

RODET, L., "Les prétendus problèmes d'algèbre du manuel du calculateur égyptien (Papyrus Rhind)," *Journal Asiatique*, Paris, series 7, vol. 18, 1881, pp. 184–232, 390–459.

Also issued separately in a single volume, Paris, 1882, 122 pp.

Review by M. Cantor, *Zeitschrift für Mathematik und Physik*, vol. 27, 1882, Historisch-literarische Abtheilung, p. 117. The review is as follows: "Dfe in

[1] With this point of view compare that of Bobynin (1909); Egyptologists do not regard the Revillouts as authorities of the first rank.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

der Ueberschrift genannte Abhandlung sucht zu beweisen, dass weder der Herausgeber des mathematischen Papyrus, Prof. August Eisenlohr, noch der Referent die sogenannten Sequem- und Hau-Aufgaben der Aegypter verstanden hätten. Wir sind der entgegengesetzten Meinung, welche wir nicht verfehlt haben in unseren sogleich Mitte April an das Journal Asiatique eingesandten[1] Entgegnungen zu begründen. Da der Abdruck dieser Entgegnungen nicht allzufrühe stattfinden dürfte, so sei einstweilen auf dieselben hingewiesen, um die Meinung nicht aufkommen zu lassen, als hätten die Rodet'schen Ausführungen unsere früher ausgesprochenen Ansichten irgendwie erschüttert."

Communication by A. Eisenlohr [dated April, 1882], *Journal Asiatique*, Paris, series 7, vol. 19, 1882, pp. 515–518. Effective reply to attacks of Rodet (1881) on his *Handbuch* (1877). Quotations: "J'aurais désiré voir M. Rodet, dans un article si développé, contribuer à éclaircir quelques points restés obscur dans mon interprétation du papyrus, plutôt que de réfuter ce qui y était déjà bien expliqué." "Pour l'éclaircissement ingénieux du no. 79 du papyrus, d'après le *Liber abaci* de Léonard de Pise, j'ai déjà fait à M. Rodet mes compliments bien sincères: Sept écrivains ont chacun sept chats, chaque chat detruit sept souris, chaque souris aurait mangé sept épis, et chaque épi aurait produit sept boisseaux de blé."

In the last paragraph of the communication there is a quotation regarding problems 35–38 from Schack-Schackenburg (1882).

Review by E. et V. Revillout [in 'Revue bibliographique'], *Revue Égyptologique*, Paris, vol. 2, 1881, pp. 287–303. Last paragraph: "Tout le reste de la dernière œuvre de M. Rodet nous paraît manquer de sérieux comme d'utilité pratique. Nous avons déjà vu que tout le fonds mathématique était emprunté à MM. Eisenlohr et Cantor, et que M. Rodet n'y avait joint que des confusions et de fausses comparaisons avec certaines opérations des Indiens, des Arabes, et du moyen âge. Il en est de même pour le fonds philologique. Toutes les corrections faites par M. Rodet au texte égyptien avaient été faites par M. Eisenlohr, ses reproductions de fac-similé empruntées au fac-similé de M. Eisenlohr, et pour des points déjà expliqués de même par le savant allemand. Quant aux nouvelles déductions philologiques de M. Rodet et à ses nouvelles explications elles prouvent toutes la plus profonde ignorance de la langue et de la philologie égyptienne. M. Rodet est–personne n'en doute–un bon mathématicien; mais cela ne suffit pas pour faire de la linguistique et surtout pour donner des solutions magistrales sur une langue qu'on n'a jamais étudiée. Je ne dirai pas: 'Soyez plûtot maçon si c'est votre métier.' Mais, en vérité, l'indien (pour lequel j'avoue ma complète incompétence) offre aux études de M. Rodet un champs assez vaste. Rien n'est facile, mais rien n'est dangereux comme de vouloir paraître universel."

RODET, L., *Sur les Notations Numériques et Algébriques antérieurement au XVIe siècle*, Paris, 1881, pp. 48–51.

[1] Apparently Eisenlohr 1882 [1881], next paragraph, since nothing else of the kind seems to have been published.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

TREUTLEIN, J. P., "Ein mathematischer Beitrag zur Kulturgeschichte," *Verhandlungen des naturwissenschaftlichen Vereins in Karlsruhe*, Carlsruhe, Heft 8, 1881, pp. 283–298.

Address delivered June 20, 1879 (compare *Verhandlungen*, p. 162), based mainly on Eisenlohr (1877). In Heft 8 are reports on the activities of the Verein from May, 1875–April, 1880; but apparently no part of the volume was published till 1881.

[VERSLUYS, J., (?), the editor] (?), "Het oudste rekenkunstig geschrift," *Tijdschrift voor Vormleer, Rekenkunde en de Beginseln der Wiskunde*, Groningen, vol. 3, 1881, pp. 202–204.

An unimportant sketch based, apparently, on Eisenlohr (1877).

1882

BOBYNIN, V. V., *Matematika drevnikh Egiptyan (po papirusu Rinda)* [Mathematics of the old Egyptians according to the Rhind papyrus], Moscow, 1882, 2 + 198 pp.[1] + 2 plates.

This volume (a copy of which may be seen in Mittag-Leffler's Mathematical Institute) was published (with the approval of the dean of the Faculty of Physics and Mathematics of the University of Moscow) in a somewhat abridged form (apparently, 176 instead of 198 pages) as Supplement to volume 2 (the last) of the periodical *Matematicheskiya Listok* [Mathematical Transactions], Moscow, 1881–1882, edited by A. F. Gol'denberg with Bobynin as a collaborator. Compare Bobynin (1905).

RODET, L., [Reduction of fractions to the same common denominator, séance 1881], [Problems 28, 36–38 of the Rhind papyrus and false position, séance 1882], *Bulletin des Séances de la Société Philologique [1880–1882]*, Paris, vol. 1, 1882, pp. 132–139, 226–232.

References to Rodet (1878) and Rodet (1881).

SCHACK-SCHACKENBURG,[2] H., "Bemerkung zu Prof. Dr. Eisenlohr's Ausgabe des mathematischen Papyrus Rhind," *Recueil de Travaux relatifs à la Philologie et à l'Archéologie Égyptiennes et Assyriennes*, Paris, vol. 3, 1882, pp. 152–154.

Concerning the division of 2 by 93, 95, and 97, and nos. 35–38 and 82. The article is signed "Gr. Schack."

SYLVESTER, J. J., "6919. If $u_{x+1} - u_x^2 + u_x - 1 = 0$, prove that $\sum \frac{1}{u_x} = \frac{1}{u_0 - 1} - \frac{1}{u_{x+1} - 1}$. Thus, suppose $u_0 = 5$; then $u_1 = 21$,

[1] In Bobynin (1909), page 290, there is a misprint where "138" pp. is given.

[2] In Ibrahim-Hilmy's *The Literature of Egypt and the Soudan*, London, vol. 2, 1888, this name is given as "Shack (A. F.), *Graf von*."

Digitized by Google Original from UNIVERSITY OF MICHIGAN

$u_2 = 421$; and, if $x = 1$, we ought to find $\frac{1}{5} + \frac{1}{21} = \frac{1}{4} - \frac{1}{420}$, which is right," *Mathematical Questions with their Solutions from the 'Educational Times'*, London, vol. 37, 1882, pp. 42–43, 80.

Solutions of this problem are given by H. W. L. Tanner, R. Harley, et al. The following note is appended to the problem: "Prof. Sylvester states that this question arises naturally out of the very beautiful Ancient Egyptian method of expressing all fractions under the form of a sum of simple fractions with continually increasing denominators, that is, as the sum of the reciprocals of continually increasing integers. It is easy to prove that *in the limiting case* these integers must be subject to the law expressed by the above equation in differences.[1] The successive integers are, as in the parallel method of continued fractions, supposed to be all made maxima, which renders the process, in this as for that method, perfectly determinate; though the Egyptians did not uniformly observe this condition in their praxis. The successive fractions, it is easy to conclude from the above equation, (as an example taken at random, such as $\sqrt{2}$ for instance, will readily confirm,) decrease with enormous rapidity. The Proposer adds, that this easy but deeply interesting question—which he wishes to bear the title of *Egyptian Arithmetic*—furnishes a new instance of the wonderful wisdom of the Egyptians." Brief note by T. Muir, p. 80.

Compare Sylvester (1880).

[1] Doctor H. P. Manning has kindly supplied me with the following footnote:

This sentence, though not very clear, does not seem to represent exactly Sylvester's theory as given in Sylvester (1880), and the next sentence is incorrect since it is not the successive integers that are made maxima, but their reciprocals, the successive fractions. Even Sylvester's explanation is a little obscure, but the following will give a consistent theory that is practically the same.

A *sorites* is a series of reciprocals of positive integers $\frac{1}{u_0} + \frac{1}{u_1} + \ldots$,

where each denominator after the first satisfies the condition $u_{x+1} \geq u_x^2 - u_x + 1$. The number of terms may be finite or infinite. A finite sorites is equal to a rational proper fraction. If Q is a finite sorites and $1/u_r$ is any one of its terms, then

$$Q < \frac{1}{u_0} + \frac{1}{u_1} + \ldots + \frac{1}{u_r - 1},$$

that is, $1/u_0$ is the maximum reciprocal that does not exceed Q, $1/u_1$ the maximum reciprocal that together with $1/u_0$ does not exceed Q, and so on. Conversely, any positive rational proper fraction can be expressed, and in only one way, as a finite sorites. In this statement the reciprocal of a positive integer may be regarded as a sorites of one term. In general we take for $1/u_0$ the maximum reciprocal that does not exceed the given fraction, subtract $1/u_0$ and proceed in the same way with the remainder, and continue thus until we get a remainder that is a reciprocal. This will happen after a finite number of steps because the numerator of each remainder after the first is less than that of the preceding remainder.

When all the integers, $u_1, u_2, \ldots$, of an infinite sorites satisfy the equation $u_{x+1} = u_x^2 - u_x + 1$, the sorites is called a *limiting sorites*. It is then equal to $\frac{1}{u_0 - 1}$, and so the reciprocal of any positive integer can be expressed as a limiting sorites. When a

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1883

MEYERSON, E., "O matematyce starożytnych Egipeyan" [Ancient Egyptian mathematics], *Ateneum Pismo Naukowe I Literackie*, Warsaw, vol. 32, 1883, pp. 137–151, 268–279, 403–404.

PETRIE, W. M. F., *The Pyramids and Temples of Gizeh*, London, 1883, 16 + 250 pp. + plates.

In the chapter commencing on page 162 Petrie discusses "architectural ideas of the pyramid builders" which some seem to think of importance in properly interpreting the sections of the Rhind papyrus dealing with pyramids.

Petrie writes: "The design of the various slopes [of pyramids] that are met with, appears to be always a simple relation of the vertical and horizontal distances. It is important to settle this as it bears strongly on the whole planning of each building; and though the vertical and horizontal distances would seem to be the natural elements for setting out a slope, yet proof of this is necessary; for Ahmes, in his mathematical papyrus, defines pyramids by their sloping heights up the arris edge, and their diagonal of the base beneath that line. Such a method of measurement would naturally be adopted, when the knowledge of the design was lost, as the arris height could be easiest measured; but it is very unlikely as a specification of design."

Petrie then gives the "observed" and "theoretical angle" (that is, angle which the faces of the pyramid make with the square base) of each of "various pyramids, and so-called pyramids," and other material providing the "proof." The angles observed by Petrie are given by him with the range of error; for example: Mastaba 44, Gizeh, a tomb of the Ancient Empire, 76° 0′ with range of error ± 5′. In other words, observations at different parts of the tomb had different values of which the *mean* was 76° 0′. Then the "theoretical angle" is given as 75° 57′ 50″, that is, the nearest angle whose cotangent is expressible as the ratio of simple integers,—in this case 1 : 4. For this angle the difference is 2′ ± 5′.

Another observation of Petrie which he here records was in connection with the second pyramid of Gizeh 53° 10′ (error, ± 4′), theoretical angle 53° 7′ 48″, 3 : 4 (difference, 2′ ± 4′).

Petrie gives also the following two observations made under Vyse's direction: Mastaba Pyramid, Medum 74° 10′ (error, 1° ?), theoretical angle 75° 57′ 50″, 1 : 4 (error, 1° 47′ ± 1° ?); South stone pyramid base, Dahshur 54° 14′ 46″

---

rational fraction is expressed as a finite sorites, we can expand the last term as a limiting sorites, and thus express the fraction as an infinite sorites with this equation holding true for all the terms after a given term. Conversely, if this equation holds true for all the terms of an infinite sorites after a given term, the sorites will represent a rational fraction. When there is no term of an infinite sorites after which this equation holds true for all the terms, the sorites represents an irrational number.

For a discussion of a similar problem involving also the same equation see O. D. Kellogg, "On a Diophantine problem," *American Mathematical Monthly*, vol. 28, 1921, pp. 300–303; and D. R. Curtiss, "On Kellogg's Diophantine problem," *ibid.*, vol. 29, 1922, pp. 380–387.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

(no error given), theoretical angle 54° 9′ 46″, 13 : 18 (difference, 5′).[1] Petrie well remarked that these values "cannot be considered as very satisfactory." He found later that the first of these two measurements was entirely wrong; that the observed angle was "within a few minutes of 51° 52′," "just that of the Great pyramid of Gizeh" (W. M. F. Petrie, et al., *Medum*, London, 1892, p. 6). After ten observations in the second case he found the mean to be 55° 1′, the variations of observations being + 22′ to − 23′ (W. M. F. Petrie, *A Season in Egypt, 1887*, London, 1888, p. 30).

If we interpret the seked of the Rhind mathematical papyrus, *not* as Eisenlohr (1877), Petrie (1883), and Cantor (1885) do, but as the Revillouts (1881), followed by Borchardt (1893), and Petrie (later),[2] we find the following ratios for the sekeds in the six examples on pyramidal shapes in the Rhind papyrus: no. 56, in Eisenlohr (1877), 18 : 25, corresponding to an angle of 54° 14′ 46″; nos. 57, 58, 59, 59a, 3 : 4, corresponding to 53° 7′ 48″; and no. 60, 1 : 4 corresponding to 75° 57′ 50″.

Compare Heath (1921), where the reason for giving the details indicated above will be apparent.

VASCHCHENKO-ZAKHARCHENKO, M. E., *Istoriya matematiki. Istoricheskii ocherk razvitiya geometrii* [History of Mathematics. Historical sketch of the development of geometry], Kiev, vol. 1, 1883, 11 + 684 pp.

The chapter on Egyptian mathematics is on pages 327–350. The work is of minor importance; compare the review by Bobynin in *Bibliotheca Mathematica*, series 2, vol. 1, 1887, pp. 114–116.

1884

GOW, J., *A Short History of Greek Mathematics*, Cambridge, 1884, pp. 16–21, 126–129, 142, 285, 286.

Refers to Eisenlohr (1877) and Cantor (1880).

LEPSIUS, K. R., "Über die 6 palmige grosse Elle von 7 kleinen Palmen Länge in dem 'Mathematischen Handbuche' von Eisenlohr," *Zeitschrift für Ägyptische Sprache . . .*, vol. 22, 1884, pp. 6–11.

Especially discusses the pyramid problems, nos. 56–59a; compare Lepsius (1866). See Heath (1921).

TANNERY, P., "Questions Héroniennes," *Bulletin des Sciences Mathématiques*, Paris, series 2, vol. 8, 1884, pp. 329–344 and 359–376.

[1] Borchardt (1893), p. 16, pointed out that 54° 14′ 46″ corresponds to 18 : 25 (difference, 0″).

[2] In the new, revised, and much abridged edition of his *Pyramids . . .*, London, [1885], p. 68, Petrie states: "The design of the various slopes that are met with, appears to be always a simple relation of the vertical and horizontal distance, agreeing with the method of stating the slopes in the mathematical papyrus of Aahmes."

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Reprinted in Paul Tannery, *Mémoires Scientifiques*, Toulouse, vol. 2, 1912, pp. 137–178.

First paragraph: "Je crois avoir établi. . . que la restitution du mode d'extraction des racines carées incommensurables dans l'école héronienne est intimement liée au système de représentation des fractions par une suite de *quantièmes*, c'est-à-dire de fractions ayant pour numérateur l'unité, et, pour dénominateurs, des nombres allant en croissant. Je me propose aujourd'hui de réunir quelques observations sur l'histoire de ce système, en remontant à son origine, c'est-à-dire en commençant par l'étudier dans le papyrus égyptien, traduit et commenté par August Eisenlohr (Leipzig, 1877)."

WEYR, E., "Ueber die Geometrie der alten Ægypter. Vortrag gehalten in der feierlichen Sitzung der kaiserlichen Akademie der Wissenschaften am XXIX. Mai, MDCCCLXXXIV," *Almanach der kaiserlichen Akademie der Wissenschaften*, Vienna, vol. 34, 1884, pp. 213–247.

References to the Rhind papyrus commence on page 230.

WIEDEMANN, A., *Ägyptische Geschichte*, Gotha, 1884, pp. 63–64.

Of slight value in this connection.

WILCKEN, U., "Aus griechischen Papyruskunden, II. Ueber den angeblichen Bruchstrich," *Hermes*[;] *Zeitschrift für classische Philologie*, Berlin, vol. 19, 1884, pp. 291–292.

Quotations from page 292: "Peyron zuerst hat aufgestellt (*Papyri graeci*, Turin, 1827, Taf. VI, no. 5), dass Brüche in griechischen Documenten durch die Nebeneinanderstellung der beiden Ziffern für Zähler und Nenner mit Zuhilfenahme eines Bruchstrichs [*sic*] nach heutiger Manier ausgedrückt seien." "Die Annahme eines Bruchstriches ist daher aus der Griechischen Paläographie zu streichen." Footnote by "Th. M"[ommsen].

1885

CANTOR, M., "Über die sogennanten Seqt der ägyptischen Mathematiker," *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften*, Abtheilung II, Vienna, Jahrgang 1884, vol. 90, 1885, pp. 475–477.

"Aus einem Briefe von Prof. Moritz Cantor . . . an Prof. Emil Weyr. (Vorgelegt in der Sitzung am 9. October, 1884)." Discusses views of the Revillouts (1881) and Eisenlohr (1877), and offers a new explanation.

ERMAN, A., *Aegypten und Aegyptisches Leben im Altertum*, Tübingen [1885], pp. 487–492.

The only references are to Eisenlohr (1877).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Another edition, "neu bearbeitet," by H. Ranke, Tübingen, 1923, pp. 423–428.

References to fourteen items in this bibliography.

1886

DEMME, C., "Bemerkungen zu den Regeln des Ahmes und des Baudhâyana über die Quadratur des Kreises," *Zeitschrift für Mathematik und Physik*, vol. 31, 1886, Historisch-literarische Abtheilung, pp. 132–134 + plate with figures.

Seeks to find the method by which the author of the Rhind papyrus arrived at his formula for the area of a circle.

1887

WESSELY, K. F. J., "Die Zahl Neunundneunzig," *Mitteilungen aus der Sammlung der Papyrus Erzeherzog Rainer*, Vienna, vol. 1, 1887, pp. 113–116.

1888

MANSION, P., ["Sur une table du papyrus Rhind"], *Annales de la Société Scientifique de Bruxelles*, Louvain, vol. 12, 1888, pp. 44–46.

Reprinted in P. Mansion, "Sur une table du papyrus Rhind," *Mélanges Mathématiques (1883–1898)*, Paris, 1898, pp. 1–3.

Last sentence (p. 46): "En résumé, *les décompositions de la table du papyrus sont toujours, à un point de vue ou à un autre, plus simple que toute autre décomposition possible.*"

1889

BOBYNIN, V. V., "Quelques mots sur l'histoire des connaissances mathématiques antérieures à la science," *Bibliotheca Mathematica*, series 2, vol. 3, 1889, pp. 105–108.

1890

BOBYNIN, V. V., "Sur le procédé employé dans le papyrus de Rhind pour réduire les fractions en quantièmes," *Bibliotheca Mathematica*, series 2, vol. 4, 1890, pp. 109–112.

On reducing $2/(2n + 1)$ to fractions with numerators unity. The author seeks to show that the process of Leonardo Pisano (1857), the "regula universalis in disgregatione partium numerorum," as given in his work of 1202, is related to that of the Rhind papyrus.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

JAROLIMEK, A., "Der mathematische Schlüssel zu der Pyramide des Cheops," *Wochenschrift des österreichischen Ingineur- und Architekten-Vereines*, Vienna, 1890, pp. 187–189, 195–198, 203–206.

Compare Jarolimek (1910).

1891

BRUGSCH, H. K., *Die Aegyptologie. Abriss der Antzifferungen und Forschungen auf dem Gebiete der Aegyptischen Schrift, Sprache und Alterthumskunde*, Leipzig, 1891, 8 + 535 pp.

Neue billige Ausgabe, Leipzig, 1897, 8 + 535 pp.

"Rechenkunst und Mathematik," pp. 366–370; "Die Metrologie," pp. 370–384. Refers to Eisenlohr (1875 and 1877), Erman (1885), and Lepsius (1884).

BRUGSCH, H. K., "Die ältesten Rechentafeln der Welt," *Die Vossische Zeitung*, Berlin, Sept. 20 and 27, 1891 (nos. 439 and 451), *Sonntagsbeilage* (nos. 38 and 39).

Reprinted in H. Brugsch, *Aus dem Morgenlande. Altes und Neues*, (Reclam's Universal-Bibliothek, nos. 3151–3152), Leipzig, 1893, pp. 25–43, under the title: "Die älteste Rechenkunst."

Pages 25–31 are taken up with a survey of arithmetic and geometric knowledge of the ancient Egyptians, and pp. 32–35 with an account of the Rhind papyrus. While Brugsch sets (p. 35) the date of the Rhind papyrus between 1800 and 2000 B. C., he describes (pp. 36–42) [compare Cantor (1880), 3. Aufl., p. 74] two tablets on wood which he discovered in April, 1891, in the Museum at Gizeh (now the Egyptian Museum, Cairo), and which date back to the 11th or 12th dynasties, that is, to about 2000 B. C. It is these tablets which Brugsch calls "die ältesten Rechentafeln der Welt." These are also the tablets catalogued in Daressy (1901).

Review by F. J. Van den Berg, "De oudste rekentafels der Wereld," *Nieuw Archief voor Wiskunde*, Amsterdam, vol. 19, 1892, pp. 211–215.

ERMAN, A., "Miscellen. Ein Datum," *Zeitschrift für Ägyptische Sprache . . .*, vol. 29, 1891, pp. 59–60.

Eisenlohr (1877) remarked that no. 87 of the Rhind papyrus did not appear to belong to the papyrus but was possibly a piece of a journal in which the most important happenings were noted. In the above note Erman gave the first translation of the entries regarding the weather and showed their import. Griffith 1894 [1891], pp. 245–246 gives a complete transcription and translation of no. 87 with comment. In Möller (1909), p. 18 the opinion is expressed that nos. 86 and 87 were somewhat later in date than the earlier numbers, and possibly originated from the same hand. Petrie's "A seasonal date of the Hyksos period," a one-page leaflet in his *Historical Studies* (British School of

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Archaeology in Egypt Studies, vol. 2), London, 1911, is an argument that no. 87 furnishes the means for fixing the date of the Hyksos period. With two of the three assumptions here made by Petrie, Peet (1923, 2), p. 130, has no sympathy.

GRIFFITH, F. L., "The Rhind mathematical papyrus," *Proceedings of the Society of Biblical Archæology*, vol. 13, 1891, pp. 328–332; vol. 14, 1891, pp. 26–31; vol. 16, 1894, pp. 164–173, 201–208, 230–248 + 2 plates.

These articles are valuable, especially in the last volumes. Quotation (first paragraph vol. 16, p. 164): "Two years and a half elapsed since my last notes on the Rhind Papyrus were published: on proceeding with it it became evident that the metrology of Ancient Egypt ought to be thoroughly worked over as a preliminary study, in order to give a better mastery of the subjects dealt with in this important document. My notes were soon afterwards ready to be issued, but other matters have interfered with their publication until the present time, and in some ways they have profited by the delay. I do not pretend to have solved all the problems that were outstanding after Professor Eisenlohr's edition, but I have done what I can to supply certain new information about the papyrus itself, and to make such observations as are calculated to render the study of the document easier to those who have not hitherto paid special attention to ancient Egyptian Arithmetic and Metrology."

[Letter from A. Eisenlohr criticising interpretations of Griffith (1891)], *Proceedings of the Society of Biblical Archæology*, vol. 13, 1891, pp. 596–598.

See also Eisenlohr (1892).

PIEHL, K., "Un nouveau nom de nombre en ancien égyptien," *Proceedings of the Society of Biblical Archæology*, vol. 13, 1891, pp. 199–200.

STERNER, M., *Geschichte der Rechenkunst*, (Principielle Darstellung des Rechenunterrichtes auf historischer Grundlage, Teil 1), Munich and Leipzig, 1891, 533 pp.

"Die Rechenkunst der alten Ägypter," pp. 13–25.

1892

BAILLET, J., "Le papyrus mathématique d'Akhmîm," *Mémoires publiés par les membres de la mission archéologique française au Caire*, Paris, vol. 9, fasc. 1, 1892, pp. 2 + 1–89 + 8 plates, quarto.

A Greek papyrus not earlier than the sixth nor later than the ninth century, and hence it is almost our latest mathematical document to exhibit the methods of Egyptians 2500 years earlier. The papyrus contains a set of tables and 50 problems. The tables give (pp. 24–31): (1) the products of 2, 3, . . ., 10, 20, 30, . . ., 100, 200, . . ., 1000, 2000, . . ., 10000, by $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$; (2) the products of 1, 2, . . ., and n by $1/n$, $n = 11, 12,$

Digitized by Google Original from UNIVERSITY OF MICHIGAN

. . ., 20. The products are expressed in the Egyptian manner; for example, $\frac{1}{19}$ of 3 is given as $\frac{1}{15}$ $\frac{1}{20}$ $\frac{1}{57}$ $\frac{1}{76}$ $\frac{1}{95}$: Baillet gives a detailed discussion of the probable methods employed in preparing this table.

The problems (pp. 63–88) deal with a variety of subjects, such as volumes of containers, various operations on fractions, proportion, and interest on money. Of the three geometrical problems the first appears to be to find the contents of a cistern in the form of a frustum of a cone, given the perimeters of the bases as 20 and 12 units while the depth is $6\frac{1}{2}$. The data[1] are similar to those in a problem of Schubart (1916). Compare Grenfell (1903).

Baillet gives extended and valuable commentary, pp. 1–23, 32–62; this includes (pp. 59–62) "comparaison avec le papyrus Rhind." An important supplement to one part of his work will be found in P. Tannery, "Le Calcul des partes proportionnelles chez les Byzantines," *Revue des Études Grecques*, Paris, vol. 7, 1894, pp. 204–208; also in P. Tannery, *Mémoires Scientifiques*, Toulouse and Paris, vol. 4, 1920, pp. 283–287.

Review by V. V. Bobynin, "Greko-Egipetskii matematicheckii papirus iz Akmima" [The Greek-Egyptian papyrus of Akhmîm], *Fiziko-matematicheskiya Nauki v ikh Nastoyashchem i Proshedshem* [Physical and Mathematical Sciences in the Present and in the Past], Moscow, vol. 12, no. 4, 1895, pp. 301–340.

Review by M. Cantor, "Ein mathematischer Papyrus in griechischer Sprache," *Zeitschrift für Mathematik und Physik*, vol. 38, 1893, Historisch-literarische Abtheilung, pp. 81–87.

Review by F. Hultsch, *Berliner Philologische Wochenschrift*, vol. 14, Oct. 13, 1894, cols. 1327–1331. See also Hultsch (1894).

See also Loria (1893) and Heath (1921), vol. 2, pp. 543–545.

EISENLOHR, A., "Remarks on [Revillout's] 'Un papyrus bilingue du temps de Philipator'," *Proceedings of the Society of Biblical Archæology*, vol. 14, 1892, pp. 341–342; compare p. 102.

The demotic-Greek papyrus here referred to, British Museum 10463, was discussed by E. Revillout in this same volume of the *Proceedings*, 1891, pp. 60–97; 1892, pp. 120–132, 229–255. See British Museum, *A Guide to the Fourth, Fifth and Sixth Egyptian Rooms and the Coptic Room*, London, 1922, p. 308; an English Translation of the Greek docket is here given and indicates that it is a portion of a contract for the sale of a field of $11\frac{5}{8}$ arurae. Revillout devotes pp. 63–80 to "La question des mesures superficielles." Griffith (1892) refers to Revillout's article.

Only two references to the Rhind papyrus. The reference on page 342 is as follows: "When you say, page 102, Prof. Dr. Eisenlohr, who was the first to publish a facsimile of the Rhind Mathematical Papyrus, you restrain the importance of my work. As you know perfectly well, I have not only given the facsimile of the papyrus, but have translated and commented, verbally and scientifically, the papyrus, adding a dictionary of the whole. I have already

[1] Baillet finds this problem quite incorrect. But if instead of dividing by 36 the author had divided by 12, it would seem as if he would have arrived at the same result as did the author of Oxyrhynchus papyrus 470; compare Grenfell (1903).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

been compelled to defend my priority[1] against the pretensions of Mr. Griffith, and must very earnestly protest against the allegations that Mr. Griffith has in any way contributed to a better understanding of that papyrus." For a telling reply to this see Griffith (1892), p. 414.

GRIFFITH, F. L., "Note on Egyptian weights and measures," *Proceedings of the Society of Biblical Archæology*, vol. 14, 1892, pp. 403–450; vol. 15, 1893, pp. 301–315.

Many references to Eisenlohr. There are 44 problems in the Rhind papyrus where references are made to weights or measures. Griffith considers all of these. Quotation from Peet (1923, 2), p. 24: "Though weights and measures form in some sense an essential portion of mathematics no attempt to treat them has been made in this volume, which could be vastly increased in size by such a discussion. An even better reason for the abstention is the fact that the subject has been already admirably dealt with in an epoch-making article by Griffith [a reference is then given to the above article]. Despite the writer's modest disclaimer of finality there is practically nothing to be added to this treatment which was based on a careful reading of all previous work on the subject backed by what is more important, a first-hand acquaintance with the Egyptian texts."

LORIA, G., "Congetture e ricerche sull'aritmetica degli antichi Egiziani," *Bibliotheca Mathematica*, series 2, vol. 6, 1892, pp. 97–109.

Conjectures as to the method of resolving fractions in the Rhind papyrus.

1893

BORCHARDT, L., "Wie wurden die Böschungen der Pyramiden bestimmt?" *Zeitschrift für Ägyptische Sprache . . .*, vol. 31, 1893, pp. 9–17.

In opposition to Eisenlohr (1877), Petrie (1883), and Cantor (1885), he advocates, with new discussion, the Revillouts' cotangent interpretation (1881) regarding the seked of a pyramid. As a basis for discussion he gives a hieroglyphic transcription of nos. 56–60, checked with the original of the Rhind papyrus. Compare also Lepsius (1884), Calice (1903), and Borchardt (1922). Erroneous suggestions are dealt with under Heath (1921).

LORIA, G., "Un nuovo documento relativo alla logistica greco-egiziana," *Bibliotheca Mathematica*, series 2, vol. 7, 1893, pp. 79–89.

Report on the contents of the arithmetic in the Akhmîm papyrus [Baillet (1892)] and comparison with similar calculations found in the Rhind papyrus, and in Heron's writings. Compare Tannery (1884).

[1] As indicated in the note under Griffith (1891)?

Digitized by Google Original from UNIVERSITY OF MICHIGAN

LORIA, G., "Le scienze esatte nell' antica Grecia," *Memorie della Regia Accademia di Scienze, Lettere, ed Arti in Modena*, sezione di scienze, Modena, series 2, vol. 10, 1893–1894, 168 pp. + 2 plates.

"Cenno delle ricerche geometriche compiute nell' antichità da gli Egiziani ed i Babilonesi," pp. 155–160. This memoir was the first of five published 1893–1902 under the above mentioned general heading.

Second edition *Le Scienze Esatte nell' antica Grecia* (Manuali Hoepli) Milan, 1914, pp. 167–173.

MILHAUD, G., *Leçons sur les Origines de la Science Grecque*, Paris, 1893.

Pages 91–101 of the lectures on "L'Orient et l'Égypte." See Milhaud (1911).

SETHE, K. H., "Zum Zahlwort 'hundert'," *Zeitschrift für Ägyptische Sprache . . .*, vol. 31, 1893, pp. 112–113.

Compare Spiegelberg (1898).

ZEUTHEN, H. G., *Forelæsning over Mathematikens Historie. . . . Oldtid og Middelalder*. Copenhagen, 1893, pp. 9–11, 217, 242.

German translation: *Geschichte der Mathematik im Altertum und Mittelalter, Vorlesungen*. Copenhagen, 1896, pp. 9–12, 247, 275.

French translation: *Histoire des Mathématiques dans l'Antiquité et le moyen Âge*. Paris, 1902, pp. 8–10, 207, 232.

Of slight value in this connection.

1894

BOBYNIN, V. V., "Sur les méthodes primitives qui ont servi à résoudre des questions arithmétique," *Bibliotheca Mathematica*, series 2, vol. 8, 1894, pp. 55–60.

Some extracts from the "history of the teaching of mathematics," Bobynin (1905); the Rhind papyrus is referred to on page 58.

GEGENBAUER, L., "Das älteste Aufgabenheft," *Zeitschrift des österreichischen Ingineur- und Architekten-Vereines*, Vienna, vol. 46, 1894, pp. 441–444, 448–452, 455–459, large quarto.

Lecture delivered at the meeting of April 14, 1894, and dealing with the Rhind mathematical papyrus. References to Bobynin, Cantor, Eisenlohr, Fibonacci, the Revillouts, Rodet, and others.

HULTSCH, F., "Das elfte Problem des mathematischen Papyrus von Akhmîm. Ein Beitrag zur Verwaltungsgeschichte der Provinz Ägypten," *Historische Untersuchungen Ernst Förstemann zum*

Digitized by Google Original from UNIVERSITY OF MICHIGAN

fünfzigjährigen Doctorjubiläum gewidmet von der Historischen Gesellschaft zu Dresden, Leipzig, 1894, pp. 39–56.

Thorough study of the problem, philological and otherwise. Brief references to the Rhind papyrus, pages 44, 54, 56.

LORIA, G., "Studi intorno alla logistica greco-egiziana," *Giornale di matematiche di Battaglini*, vol. 32, 1894, pp. 28–57.

Discusses breaking up fractions of the form $2/(2n+1)$ into a sum of two to four fractions with unity in the numerator. The author believes that he has discovered new points of view more in accordance with the methods of the Rhind papyrus. See criticisms of this work by Hultsch (1895). Compare Collignon (1881).

WILSKE, K., "Über Arithmetik und rechnende Geometrie der alten Ägypter um 2000 vor Christus," *Mittelschule (und höhere Mädchenschule). Zeitschrift für die gesamten Interessen des deutschen Mittelschulwesens* . . . , Halle, vol. 8, 1894, pp. 345–356, 365–374, 385–394, 405–412.

1895

GEGENBAUER, L., "Die Mathematik der alten Ägypter," *Die Aula. Wochenblatt für die akademische Welt*, Munich, vol. 1, 1895, cols. 111–115, 142–148.

HULTSCH, F., "Die Elemente der ägyptischen Theilungsrechnung. Erste Abhandlung," *Abhandlungen der philologisch-historischen Classe der königlich-sächsischen Gesellschaft der Wissenschaft*, Leipzig, vol. 17, no. 1, 1895, 192 pp.

Valuable memoir. Contents:

Introduction, pages 3–15;

I. The Egyptian number system, etc., 16–29;
II. Half as an arithmetic notion, etc. [various fractional forms], 29–47;
III. Division in the papyrus replaced by multiplication, etc., 48–55.
IV. Survey of cases where members of the increasing series of numbers are used in the plural, etc., 55–66.
V. Trial multiplication as substitute for division, 66–90.
VI. The theory of division, 91–110.
VII. The summation of unity parts, 110–145.
VIII. The theory of partition (Zerlegung), 146–175.
IX. Survey of the "table of Ahmes" and the solution for each problem, 175–187. He concludes on page 187, with the following paragraphs:

"Diese Methoden darzustellen wird die Aufgabe einer zweiten Abhandlung[1]

[1] Never published; Hultsch died in 1906. For a biography and list of his writings see F. Rudio's "Friedrich Hultsch," *Bibliotheca Mathematica*, series 3, vol. 8, 1908, pp. 325–402.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

sein, und zwar wird Abschnitt X. der unmittelbaren Zerlegung XI der durch Erweiterung herbeigeführten Zerlegung von Brüchen mit theilbarem Nenner und im Zusammenhange damit der Zurückführung auf den günstigsten Fall gewidmet sein. In XII. Abschnitt soll die Theilung der 2 durch Primzahlen nach der Tabelle des Ahmes behandelt werden, woran sich im XIII. die übrigen bei Ahmes und im Papyrus von Akhmim überlieferten Theilungen durch eine Primzahl schliessen werden. Für den XIV. Abschnitt ist eine zusammenhängende Erklärung der in der Tabelle des Papyrus von Akhmim aufgeführten Divisionen durch theilbare Zahlen vorgesehen.

"Wie die ausserdem noch gesammelten Materielien im einzelnen auszuarbeiten sind, wird sich erst später festellen lassen."

Compare Hultsch (1901).

ISELY, L., "Les connaisances mathématiques et astronomiques des anciens Égyptiens," *Bulletin de la Société des Sciences Naturelles de Neuchâtel*, Geneva, vol. 23, 1895, pp. 23–36; synopsis of the mathematical part ("et astronomiques" not appearing in the title) in *Bibliothèque Universelle, Archives des Sciences Physiques et Naturelles*,compte rendu des séances de la Société des Sciences Naturelles de Neuchâtel, troisième période, vol. 33, 1895, pp. 587–589.

Descriptive; refers to Eisenlohr (1877), Tannery (1884), Rodet (1881), Brugsch, and Cantor; nothing of value and not always accurate; astronomical part of the article, pp. 29–35.

REVILLOUT, E., *Mélanges sur la métrologie, l'économie politique et l'histoire de l'ancienne Égypte avec de nombreux textes démotiques, hiéroglyphiques, hiératiques ou Grecs inédits ou antérieurement mal publiés*, Paris, 1895.

Autograph print. "Les mesures superficielles du Pap. mathématique," pp. IV–XLII. "Note sur les fractions égyptiennes," pp. LXIX–LXXIII; this note deals with a demotic mathematical papyrus (previously unpublished) of the Egypt Exploration Society; it is of a period about 1800 years later than The Rhind mathematical papyrus. This papyrus contains tables of fractions as follows: $\frac{1}{7}$ of 1, 2, 3, . . ., 7; $\frac{1}{8}$ of 1, 2, 3, . . ., 8; $\frac{1}{9}$ of 1, 7, 8, 9; $\frac{1}{10}$ of 1, 2; $\frac{1}{11}$ of 1, 2, . . ., 7; $\frac{1}{12}$ of 1, 2, . . ., 8; $\frac{1}{13}$ of 1, 2, . . ., 7; $\frac{1}{14}$ of 1, 2, . . ., 6; $\frac{1}{15}$ of 1, 2, . . ., 6. The extent to which these solutions agree with corresponding ones given in the Rhind papyrus, Eisenlohr (1877) and in the Akhmîm papyrus, Baillet (1892), is indicated in Hultsch (1901).

1896

BOBYNIN, V. V., "Esquisse de l'histoire du calcul fractionnaire," *Bibliotheca Mathematica*, series 2, vol. 10, 1896, especially pp. 99–101.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

BORCHARDT, L., "Altägyptische Werkzeichnungen," *Zeitschrift für Ägyptische Sprache . . .*, vol. 34, 1896, pp. 69–76.

"Construction einer Ellipse, aus Luqsor," pp. 75–76 + plate; an ellipse, with lines for its construction, in the east wall of the Temple of Luxor. Borchardt compares the analysis of this construction with apparently similar reasoning for the quadrature of the circle in the Rhind papyrus (no. 48).

EISENLOHR, A., "Altægyptische Maasse," *Recueil de Travaux Relatifs à la Philologie et à l'Archéologie Egyptiennes et Assyriennes*, Paris, vol. 18, 1896, pp. 29–46.

Numerous references to Eisenlohr (1877); he insists (pages 42-43) on his interpretation of the seked of a pyramid—rejected by other scholars.

SETHE, K. H., [Das Zahlwort Zehn], *Zeitschrift für Ägyptische Sprache . . .*, vol. 34, 1896, p. 90.

SPIEGELBERG, W., *Rechnungen aus der Zeit Setis I (circa 1350 v. Chr.) mit anderen Rechnungen des neuen Reiches* herausgegeben und erklärt. 2 vols. Strasbourg, 1896, folio. Vol. 1, Text, 8 + 100 pp.; vol. 2, 43 plates.

This work deals with the Rollin papyri, of importance for the study of accounts, and especially of weights and measures. Compare Pleyte (1868).

1897

BOBYNIN, V. V., "Egipetskaya forma tablichnavo sposova umnozheniya v russkoi narodnoi arithmetikye" [Egyptian form of an aid to multiplication in common Russian arithmetic], *Fiziko-mathematicheskiya Nauki v ikh Nastoyashchem i Proshedshem*, vol. 13, 1897, pp. 77–80.

Bobynin here refers to what is usually known in mathematical literature as the "Russian peasant method of multiplication." For various references in this connection see R. C. Archibald, "The binary scale of notation, a Russian method of multiplication, the game of nim, and Cardan's rings," *American Mathematical Monthly*, vol. 25, 1918, pp. 139–142.

BORCHARDT, L., "Der Inhalt der Halbkugel nach einen Papyrus fragment des mittleren Reiches," *Zeitschrift für Ägyptische Sprache . . .*, vol. 35, 1897, pp. 150–152.

An attempt to explain, by means of a hemisphere, problem (*c*) below, Griffith (1897). Schack-Schackenburg (1899) showed that the solid in question was probably a right circular cylinder, and this interpretation has been accepted by Peet (1923, 2) and others.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

CAJORI, F., *A History of Elementary Mathematics*, New York, 1897. Revised and enlarged edition, New York, 1917.

Pages 19–26, and other references in the index under "Ahmes" and "Egyptians." Descriptive sketch referring to writings of Baillet, Cantor, Eisenlohr, Loria, Matthiessen, and Sylvester. In one place Cajori gives in hieroglyphics what purports to be a quotation from the Rhind papyrus; this is, of course, the usual manner of quoting the hieratic of the original.

GRIFFITH, F. L., *The Petrie Papyri. Hieratic Papyri from Kahun and Gurob (principally of the Middle Kingdom)*, edited by F. L. Griffith, 2 vols. London, 1898. Vol. 1, pp. 15–18, 101, 107; vol. 2, plate VIII. In March, 1897, part 1, "containing Plates I to VIII and the text pertaining to them, were issued to subscribers" (Preface).

In this volume six bits of hieratic mathematical papyri, dating from about 1800 B. C. are reproduced in facsimile, translated, and discussed; they were found by Petrie at Kahun in 1889, and are now in University College, London. On these papyri are found: (*a*) a table for expressing fractions $2/n$, $n = 3, 5, 7, \ldots, 21$ in terms of the sum of fractions with numerators unity; the results are identical with those given in the Rhind papyrus in this same connection. (*b*) $\frac{1}{3} + \frac{1}{12}$ is multiplied by 9 giving $3 + \frac{2}{3} + \frac{1}{12}$; in the next line the number 110 having apparently been divided by 8 gives $13 + \frac{2}{3} + \frac{1}{12}$ which is 10 in excess of the above product; $1\frac{9}{12}$ is subtracted 9 times from $13 + \frac{2}{3} + \frac{1}{12}$, and its successive remainders, giving once more a series of numbers in arithmetic progression as in the Rhind papyrus, nos. 40 and 64. (*c*) It seems that the problem is to find the contents of a right circular cylindrical granary whose diameter is 12 and height is 8 cubits, the result being given in terms of the unit khar (two-thirds of a cubic cubit). Schack-Schackenburg (1899) was the first to explain this problem and the corresponding very difficult no. 43 in the Rhind papyrus; (*d*) shows the hieratic form of eight very large numbers. (*e*) Problem $\frac{1}{2}x - \frac{1}{4}x = 5$, what is x? (*f*) a vague arithmetic problem apparently requiring the base dimensions of a rectangular parallelopipedal container given the area of the base and the ratio of its sides. (The following misprint in Griffith's commentary may cause trouble for a moment: "$\frac{3}{4} = 1 \times 1\ \frac{1}{3}$"). Griffith found a new word for square root in this problem. Schack-Schackenburg (1900) observed that we here have the equivalent of the solution of a two-term quadratic equation, or of two simultaneous equations $x : y = 1 : \frac{3}{4}$, $xy = 12$, one of these equation being identical with what is called for in Berlin papyrus 6619, while both equations arise in the first problem of the Golenishchev papyrus. (*g*) Accounts of a poultry-yard.

Griffith does not here make any reference to another Kahun fragment which he described as follows on page 48 of W. M. F. Petrie, *Illahun, Kahun, and Gurob, 1889–90*, London, 1891: "one [fragment] I fear is beyond hope: it was beautifully written in columns, and still contains the most tantalising phrase 'multiply by $\frac{1}{2}$ to infinity.' Was it the famous problem that 'took in' Hercules?" Griffith probably made a slip in referring to Hercules when he had in

Digitized by Google Original from UNIVERSITY OF MICHIGAN

mind one of the paradoxes of Zeno who flourished in the fifth century before Christ. It is also noted (p. 49) that about 1890 Prof. Adolf Erman had put together some small fragments of Middle Kingdom writing [2000–1800 B. C.] from Thebes making a calculation in which expressions resemble those in the Rhind papyrus. I have verified through Professor Erman that the fragments here referred to are papyrus 6619 in Berlin; see Schack-Schackenburg (1900).

Turning to another part of Petrie's *Illahun*, written by himself we find the following (p. 15): "A most curious article among the small objects (Pl. VIII, 17) is a counting stick. It is not intended for doing operations on like an abacus but apparently for teaching children. It is made from an old piece of furniture; the holes on the left hand being for fastening pegs. It begins with *nefer* [good, or beautiful], and then dots for 1 to 9; then 10, and dots for 11 to 19; then 20, 25, and 30 to 90; and lastly the sign for 100." This is not connected in any way with what are technically known as counting rods, which are somewhat related to the abacus. Such things were wholly unknown in Egypt (oral statement to me by Sir Flinders Petrie, July, 1926).

LINDEMANN, F., "Zur Geschichte der Polyeder und der Zahlzeichen," *Sitzungsberichte der mathematisch-physikalischen Classe der königlichen bayerischen Akademie der Wissenschaften zu München*, Jahrgang 1896, 1897, vol. 26, pp. 625–758 + 9 plates.

Signs for numbers in the papyrus discussed, directly or indirectly, on pages 664–670; 698–709.

1898

AHMES, scribe, *Facsimile of the Rhind Mathematical Papyrus*, London, British Museum, Department of Oriental Antiquities, 1898. Large folio, 56 × 38 cm. 3 pp. + 21 plates.

British Museum Papyrus, nos. 10057–10058, with Introduction and Bibliography by E. A. T. W. Budge. Doctor Budge states: "The attention of Egyptologists was first called to the existence of this unique mathematical document ["unique" no longer—compare Turaev (1917)] by the late Dr. Samuel Birch who published a brief but accurate account of it in the *Zeitschrift für Ägyptische Sprache* . . . (Bd. VI, 1868);" the inaccuracy of the word "first" in this statement will be noted on referring to the title given above for 1867. The same misstatement was made by Eisenlohr (1877), p. 2.

The titles given in the Bibliography, and references, are as follows: Birch (1868), Brugsch (1874 and 1891, 1), Cantor (1880), Ebers (1878), Eisenlohr (1875, 1876, 1877, and 1881), Erman (1891), Griffith (1891–1894), Rodet (1878 and 1881), Revillout (1881), Schack-Schackenburg (1882), Suter (1873), and Wiedemann (1884).

An inspection of this "facsimile" might readily suggest that the papyrus had been beautifully reproduced in natural colors, and so that even the texture of the papyrus is clear. Such an inference would be quite erroneous. On a fake background is given a reproduction of an imperfect hand copy of the

Digitized by Google Original from UNIVERSITY OF MICHIGAN

original. Peet (1923, 2), p. 136, considered that the list of all the differences between the original and the so-called facsimile was too long for him to publish. It should be added that to the Egyptologist the word facsimile, as used above, would not convey any erroneous impression.

In preparing his hieratic text Eisenlohr (1877) confesses to have made changes in the British Museum "facsimile" loaned to him but he made some slips, as Griffith (1894) has pointed out. With these facts in mind it will be clear that a wrong inference might readily be drawn from another statement in Budge's Introduction, namely the following: "As early as 1869 the preparation of a facsimile of the papyrus and descriptive text was authorized by the Trustees of the British Museum; but the progress of the work was delayed, and in 1877 Prof. Eisenlohr published an edition entitled," . . . "the facsimile which accompanied it being produced from plates already made for the Trustee's edition, and lent to Prof. Eisenlohr."[1]

The papyri 10057, 10058 were originally one papyrus of 14 leaves, each leaf about 15 inches wide. Some 7½ inches from the middle of the sixth leaf were regarded as irretreviably lost, till 1922 when fragments belonging to this missing portion were discovered in New York City by Mr. Newberry; compare Peet (1923, 2) and Archibald 1925 [1923].

Review by F. L. Griffith, *Orientalistische Litteratur-Zeitung*, vol. 2, 1899, cols. 116–117. Quotation: "This book has evidently been printed only to make use of plates, prepared thirty years ago for Dr. Birch, whose projected publication of this famous papyrus was never issued. It was from proofs of these plates that Eisenlohr made his edition. . . . The plates of the British Museum edition are very handsomely reproduced, and the book is very cheap; beyond this there is little to be said in its favor: no more inconvenient style of publication could be imagined. The plates, representing as they do the whole of the *recto* and those portions of the *verso* on which there is writing, exactly as the papyrus is now mounted, may give a somewhat better idea of the positions of the different entries in the original; and the colour of the papyrus is imitated, as well as the inks of the writing. In these details alone is there any superiority to the plates of Dr. Eisenlohr. The few mistakes in the facsimile which the present writer (P. S. B. A. 1894) found in the Eisenlohr's plates appear here also, and in neither edition is there any clear marking of the repairs and modern ink restorations on the *verso*. On the other hand, the new publication is far less adapted for use than its predecessor, . . . The pages are mercilessly cut up (to measure!) in the plates, without the slightest reference to their meaning (!). Beyond the mere numbering of the plates no reference numbers are given to the often intricate writing. Clearly therefore, the present publication was not undertaken to benefit Egyptologists, but simply as a utilization of the old plates. Dr. Birch must have felt that these would have to be entirely redrawn, and so the publication was indefinitely shelved. It is a pity that so

[1] "Als ich im Frühjahr 1872 England besuchte, war Dr. Birch so gütig, mir ein Exemplar des lithographirten, aber noch nicht erschienen, Abdrucks jenes Papyrus zu zustellen," Eisenlohr, *op. cit.*, p. 2. I have verified that Eisenlohr's publication of these plates was not effected with the consent of the Trustees of the British Museum. Compare Quaritch's Catalogue, no. 184 and Eisenlohr (1899).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

cheap and attractive a work should not have been made of more practical value."

Anonymously reviewed in "Early Mathematics," *Nature*, vol. 59, Nov. 24, 1898, pp. 73–74; compare Heath (1921).

BOBYNIN, V. V., "Lekchii istorii matematiki" [Lectures on the history of mathematics], published as supplements to the periodical conducted by Bobynin: *Fiziko-matematicheskiya Nauki v ikh Nastoyashchem i Proshedshem*, Moscow, 1898–?.

Pages 161–243 of this history [a copy of which, so far as published, is in the library of the late Dr. G. Eneström, now the property of the Högskola, Stockholm] deal with Egyptian fractions. Pages 161–198 appeared as supplementary to volume 11 of the periodical for the year 1892, with title page dated 1898. Pages 199–243 were supplementary to a later undetermined volume.

CANTOR, M., "Die mathematischen Papyrusfragmente von Kahun," *Orientalistische Litteratur-Zeitung*, Berlin, vol. 1, 1898, cols. 306–308.

Makes note of three "remarkable similarities" between the Kahun and Rhind papyri. Compare Griffith (1897).

GOODSPEED, E. J., "The Ayer papyrus: a mathematical fragment," *The American Journal of Philology*, vol. 19, 1898, pp. 25–39 + plate (facsimile).

Abridgement in *American Mathematical Monthly*, vol. 10, 1903, pp. 133–135.

Transcription and translation, with commentary, of a Greek papyrus (size 21.30 ×40.5 cm.) which originally formed part of a papyrus roll and appears to have been found in the Fayûm, near the pyramid. It was probably written about 150 A. D. Each of the three problems in the papyrus is to find an area: (1, 2) of a quadrilateral, two of whose sides are parallel 2–6 and 16–10 units respectively, and whose other sides are 13–13 and 15–15, (3) of a rhombus one of whose sides is 10 units and one of whose diagonals is 12 units long. In the course of the discussion of the first problem part of the work corresponds to solving the simultaneous equations $x^2 - y^2 = 56$, $x + y = 14$.

Compare Schubart (1916).

HAAS, K., "Die Mathematik der alten Ägypter. Vortrag gehalten . . . 19. März, 1898," *Österreichische Mittelschule*, vol. 12, 1898, pp. 259–271.

Critically descriptive with special reference to Eisenlohr (1877), Cantor and Lepsius (1856).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

SPIEGELBERG, W., "Die Lesung des Zahlwortes 'hundert'," *Zeitschrift für Ägyptische Sprache . . .*, vol. 36, 1898, pp. 135–139.

Compare Sethe (1893).

1899

BARROS RAJA GABAGLIA, E. DE, *O Mais Antigo Documento Matematico Conhecido (Papyro Rhind)*, Rio de Janeiro, 1899, 136 pp., octavo.

Refers to Birch (1868), Brugsch (1874), Cantor (1875), Eisenlohr (1877), Favaro (1879), Rodet (1881), the Revillouts (1881), Gow (1884), Baillet (1892), Bobynin (1894), Loria (1894), Ahmes (1898).

BOBYNIN, V. V., "Développement des procédés servants à décomposer le quotient en quantièmes," *Abhandlungen zur Geschichte der Mathematik*, Leipzig, Heft 9, 1899, pp. 1–13.

This Heft was also issued as a supplement to *Zeitschrift für Mathematik und Physik*, vol. 44.

First of a series of papers in the volume prepared in honor of Moritz Cantor's seventieth birthday, August 23, 1899. Discussion with special reference to the Rhind papyrus, the Akhmîm papyrus, and the Liber Abaci of Leonardo.

EISENLOHR, A., [Letter dated November 30, 1898], *Proceedings of the Society of Biblical Archæology*, vol. 21, 1899, pp. 49–50.

With reference to an entry in Quaritch's Catalogue no. 184 casting reflections on Eisenlohr (1877). Quotation: "The same day I received the catalogue from Mr. Quaritch, I wrote to him inquiring who was responsible for the remarks, and refuting the imputation by pointing to p. 2, line 7, of my *Mathematisches Handbuch* (Leipzig, Hinrichs, 1877), where I explained that a copy of the papyrus was given to me in the spring of 1872 by Dr. Samuel Birch, the keeper of the Egyptian antiquities in the British Museum; further, that my work was not issued before I got permission of the same Dr. Birch. For the attack on my scientific fame, no man can pretend that my translation, after five years' hard work, was hastily prepared, and that numerous errors render it of little use." At the place mentioned in his *Handbuch* there is no reference to his having obtained "permission of the same Dr. Birch," but only what is quoted in the footnote under Ahmes (1898), above.

LAURITZEN, C. J. C., "Om Ægyptens Pyramider," *Nyt Tidsskrift for Matematik, A*, Copenhagen, vol. 10, 1899, pp. 51–52.

Unimportant. Suggested by J. Michelsen, "Om Ægyptens Pyramider," pages 22–23 of the same volume.

SCHACK-SCHACKENBURG, H., "Die angebliche Berechnung der Halbkugel," *Zeitschrift für Ägyptische Sprache . . .*, vol. 37, 1899, pp. 78–79.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

First sentence: "Die von Borchardt (*ÄZ* 1897, S. 150) auf die Berechnung einer Halbkugel bedeutete Stelle des mathematischen Papyrus von Kahun scheint mir eine andere Erklärung zu fordern." Most important interpretation which led to the final understanding of the difficult no. 43 of the Rhind mathematical papyrus. Compare Griffith (1897).

1900

BRAUNMÜHL, A. VON, *Vorlesungen über Geschichte der Trigonometrie*, Leipzig, Erster Teil, 1900, pp. 1–3.

Unimportant in this connection.

QUIBELL, J. E., *Hierakonopolis*, Part I (Egyptian Research Account), London, 1900.

Plate XXVIB represents a great mace of the time of King Nar-mer, who flourished before the first dynasty, about 3500 B. C. There is here "a register of captive animals, 'oxen 400,000, goats 1,422,000,' showing that the system of numeration was as fully developed before the 1st dynasty as it was in any later time" (p. 9). There is also a reference to "captives 120,000."

SCHACK-SCHACKENBURG, H., "Der Berliner Papyrus 6619," *Zeitschrift für Ägyptische Sprache . . .*, vol. 38, 1900, pp. 135–140 + plate.

First sentence: "Aus Taf. 8 der Kahuner Papyri hat Griffith [Griffith (1897)] zum ersten Male eine ägyptische Rechnung veröffentlicht, die unseren rein quadratischen Gleichungen entspricht; dem gütigen Entgegenkommen der Berliner Museum Verwaltung verdanke ich die Möglichkeit, ein zweites Beispiel aus dem Berliner Papyrus 6619 vorlegen zu können." The problem here referred to may be stated thus: Distribute 100 square ells between two squares whose sides are in the ratio 1 to $\frac{3}{4}$; whence the equations $x^2 + y^2 = 100$, $x : y = 1 : \frac{3}{4}$, corresponding to those given in Griffith (1897). The equations are solved by the method of false position and the solution of two term quadratic equations. On the back of this fragment of the papyrus is another problem somewhat similar to no. 69 of the Rhind papyrus. A translation of both sides of this fragment is given in A. Erman and F. Krebs, *Aus den Papyrus der königlichen Museen*, Berlin, 1899, pp. 81–82, "Aus einem Rechenbuch;" but the authors acknowledged their inability to give an explanation. The papyrus became the property of the Museum in 1887.

The method of false position or "false hypothesis" was a favorite one of Diophantus of Alexandria (about 250 A. D.); compare Heath (1910), pp. 175, 195; also Heath (1921), vol. 2, pp. 441, 488, 489. The following are examples of equations solved by false position in connection with problems of the Rhind papyrus: $\frac{1}{7}x + x = 19$ (no. 24); $x + \frac{2}{3}x - \frac{1}{3}(x + \frac{2}{3}x) = 10$ (no. 28); $\frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x + x = 33$ (no. 33).

See also under 1902.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

SETHE, K. H., "Eine bisher unbeachtete Bildung fur die Ordinalzahlworte in Neuägyptischen," *Zeitschrift für Ägyptische Sprache . . .*, vol. 38, 1900, pp. 144–145.

WARREN, C., "Egyptian weights and measures since the eighteenth dynasty and of the Rhind mathematical papyrus," *Palestine Exploration Fund, Quarterly Statement*, 1900, pp. 149–150.

Comment with special reference to parts of Griffith (1892).

1901

DARESSY, G., *Catalogue Général desAntiquités Egyptiennes du Musée du Caire, Nos 25001–25385, Ostraca*, Cairo, 1901.

On pages 95–96 are given brief descriptions of the two tablets 25367, 25368 to which attention was drawn by Brugsch in 1891; see Brugsch (1891). Plates 62–64 contain fine photographic reproductions of both sides of the tablets, which were acquired by the Museum in 1885. Both tablets were written by the same hand. See also Daressy (1906), Möller (1911), and Peet (1923, 1).

HULTSCH, F., "Neue Beiträge zur ägyptischen Teilungsrechnung," *Bibliotheca Mathematica*, series 3, vol. 2, 1901, pp. 177–184.

Study of Revillout (1895) and a detailed comparison of the material with Eisenlohr (1877), and Baillet (1892). An important supplement to Hultsch (1895).

HULTSCH, F., *Procli Diadochi in Platonis Rem Pvblicam Commentarii*, edidit G. Kroll, Leipzig, vol. 2, 1901.

In the third of "Drei Exkurse von F. Hultsch," "Die geometrische Zahl Platons," pp. 400-413; calculations of one Paterius connected with Egyptian "Theilungsrechnung." For Rudio's comments on this and other publications of Hultsch see his memoir, referred to in a footnote under 1895 of this Bibliography.

1902

CRUM, W. E., *Coptic Ostraca from the Collections of the Egypt Exploration Fund, the Cairo Museum and others, the texts edited with translations and commentaries*, London, 1902.

Ostracon 480, p. 46 of the translations and p. 78 of the texts. At the first reference Crum remarks: "Apparently arithmetical tables; but their system and purpose are to me unintelligible." Sethe discovered the translation and it is given in Sethe (1916), p. 71. It is a table of the values of $\frac{1}{31}$ of $\frac{1}{3}$, $\frac{1}{2}$, 1, 2, . . ., 31. For example $\frac{1}{31}$ of 25 is given as $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{31}$ $\frac{1}{62}$ $\frac{1}{124}$. This ostracon is from Denderah and is in University College, London.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

In W. E. Crum, *Short Texts from Coptic Ostraca and Papyri*, Oxford, 1921. number 442 (p. 115) is a transcription, without translation, of a 11 line pottery tablet in the Manchester University Museum. It appears to be a table of numbers, the total of which in each of the first seven lines is equal to 17; but the purpose is obscure. The number in the Museum catalogue is 6221.

SCHACK-SCHACKENBURG, H., "Das kleinere Fragment des Berliner Papyrus 6619," *Zeitschrift für Ägyptische Sprache . . .*, vol. 40, 1902, pp. 65–66.

Compare under 1900 for an account of the larger fragment. We have here a problem similar to that in the larger fragment, and leading to the equations $x^2 + y^2 = 400$, $x : y = 2 : 1\frac{1}{2}$. The facsimile of the smaller fragment is given on the same plate as that containing the larger.

TROPFKE, J., *Geschichte der Elementar-Mathematik*, Leipzig, 2 vols., 1902–1903.

Very numerous references in the index under: "Ägyptische Geometrie," "Ägyptisches Rechnen," and "Ahmes."

Zweite verbesserte und sehr vermehrte Auflage, Berlin and Leipzig, vol. 1, 1921, pp. 89–90, 94, 118–120, 171–172; vol. 2, 1921, pp. 8, 104, 134; vol. 3, 1922, pp. 20–24, 27, 111, 119; vol. 4, 1923, pp. 3, 4, 10, 14, 61, 64, 78, 79, 83, 94, 99, 127, 154–155, 181, 195, 211, 215; vol. 5, 1923, pp. 4, 11, 23; vol. 6, 1924, pp. 3, 4, 15; vol. 7, 1924, pp. 3, 4, 20.

1903

CALICE, F. v., "Zur Böschungsbestimmung im Pap. Rhind," *Zeitschrift für Ägyptische Sprache . . .*, vol. 40, 1903, p. 147.

An interpretation of problem 60, of the Rhind papyrus, without emendation of the text; refers to Borchardt (1893). This note was corrected in Schack-Schackenburg (1904, 1).

FAZZARI, G., "Breve storia dell' aritmetica e dell' algebra nei tempi antichi," *Il Pitagora*, vol. 10, Nov., 1903, pp. 14–19.

These pages in a series of articles, deal with "Aritmetica ed algebra presso gli Egiziani;" there are references to Eisenlohr (1877, 1891), Sylvester (1880), Loria (1892, 1893, 1894), and Baillet (1892). Practically the same material is given in Chapter II, pp. 17–25, of G. Fazzari, *Breve Storia della Matematica dai Tempi Antichi al Medio-evo*, Milano, [1907].

Not important in this connection.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

GRENFELL, B. P. and HUNT, A. S., *The Oxyrhynchus Papyri, Part III, edited with Translations and Notes*, (Egypt Exploration Fund, Graeco-Roman Branch), London, 1903.

Number 470 (pages 141–146) in this collection is described as a "mathematical treatise" 16.7 x 19 cm., of the third century A. D.; "a leaf, of which the top is lost, from a papyrus book . . . containing apparently descriptions of astronomical instruments." Lines 31 to the end are concerned with the construction of a water-clock in the shape of a frustum of a right circular cone of which the diameter of the upper section is 24 fingers, of the lower 12, while the distance between the sections is 18 fingers. (The cotangent of the angle which a generator of the frustum makes with its projection on the plane of the lower section (seked?) is, therefore, $\frac{1}{3}$, corresponding to an angle of 71° 34′). If the distance, h, between the sections be divided into 18 equal parts each part may be regarded as the altitude (1 finger) of a new frustum. In the papyrus the volumes of the upper six of these frusta are found by calculations equivalent to using the formula (D and d being the diameters of the circular sections):
$h\,(\pi/3)\,[(D+d)/2]\cdot(\pi/4)\,(D+d)/2 = (\frac{1}{3})\,h\,\pi\,(\pi/4)\,(\frac{1}{4})\,(D^2+2\,Dd+d^2)$
if π were taken equal to 3 this formula and that for the volume of a right circular cylinder of height h and the diameter of whose base is $\frac{1}{2}\,(D+d)$ would reduce to the same expression. The correct formula for the frustum is $(\frac{1}{3})\,h\pi\cdot(\frac{1}{4})\,(D^2+Dd+d^2)$. The introduction of "(i. e. by π)," p. 145, line 25, is a very misleading comment on the part of the editors.

It is interesting to note that Heron of Alexandria, who probably flourished about the time that this papyrus was written, carried through a numerical problem which seemed to indicate familiarity with the second of these formulæ, and with the formula $h\,(\pi/4)\,(\frac{1}{4})\,(D+d)^2$; compare *Heronis Alexandrini Opera quae supersunt omnia*, Leipzig, vol. 5, ed. by Heiberg, 1914, pp. 12–17, and vol. 3, ed. by Schöne, 1903, pp. 116–119; in both cases π is taken as $22/7$, and the numerical work of the first corresponded rather to the formula for the volume in the form: $\frac{1}{4}h\pi\,[(\frac{1}{2})^2\,(D+d)^2+(\frac{1}{3})\,(\frac{1}{2})^2\,(D-d)^2]$.

Papyrus 470, which is now in the Library of Trinity College, Dublin, is reproduced in facsimile in its natural size on plates 7–8 of L. Borchardt, *Die Altägyptische Zeitmessungen* (Die Geschichte der Zeitmessungen und der Uhren, herausgegehen von Bassermann-Jordan, vol. 1, part B), Berlin and Leipzig, 1920. The transcription and translation are practically identical with those given by Grenfell and Hunt. The oldest known specimen of a water clock of the above form, from which time was determined by the lowering of the water level, dates from about 1400 B. C.[1] It was discovered at Karnak in 1904 and is now in the Museum at Cairo; it is discussed and pictured by Borchardt, pp. 6B–7B and plates 1–3.

D. Limongelli considered, "Note sur une clepsydra antique," *Bulletin de L'Institut Égyptien*, series 5, vol. 9, pp. 51–52, what shape a clepsydra must have in order that the level of the water be proportional to the time, and found

[1] The first published description of this transparent alabaster clepsydra was in G. Daressy, "Deux clepsydra antiques," *Bulletin de L'Institut Égyptien*, Cairo, series 5, vol. 9, 1916, pp. 9–14.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

that it would be obtained by the revolution of a parabola of the fourth degree about the y-axis, $x^4 = By$, y representing the height of the water. This curve called a "Flachparabel" (A. Haas, *Lehrbuch der Differentialrechnung*, Teil 3, 1894, p. 113 and 271) was first discussed by Fagnano, *Giornale de'Letterati d'Italia*, Venice, vol. 22, 1715, p. 256, etc.

Another example of the calculation of the frustum of a cone, in connection with the seating of a theatre, may be found in the fourth (?) century Oxyrhynchus papyrus no. 186 (size 7 x 14 cm.) at Florence, published in *Papiri Greci e Latini*, (Publicazioni della Società Italiana per le Ricerca dei Papiri Greci e Latini in Egitto), Florence, vol. 3, 1914, pp. 44–45.

In his discussion of the difficult no. 43 of the Rhind papyrus Eisenlohr (1877) suggested (p. 94) that the problem was to find the volume of a frustum of a certain cone. But his interpretation was incorrect as Peet (1923, 2) summing up, see Griffith (1897), made clear.

See also Schubart (1916).

HULTSCH, F., "Beiträge zur ägyptischen Metrologie," *Archiv für Papyrusforschung und verwandte Gebiete*, Leipzig, vol. 2, 1903, pp. 87–93, 273–293, 521–528; see especially pp. 88–89.

"La quadrature du cercle dans l'ancienne Égypte," *Revue Scientifique*, Paris, series 4, vol. 20, July 18, 1903, p. 91.

Anonymous note of no importance.

1904

ERMAN, A., *Aegyptische Chrestomathie zum Gebrauch auf Universitäten und zum Selbstunterricht*, Berlin, 1904, pp. 92–93 (numbers 26, 35, 41 of the Rhind papyrus), 46*–47* (notes).

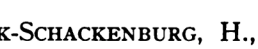
Not important in this connection.

MAHLER, E., "Az Egyiptomiak mathematikai és astronomiai ismeretei," [Mathematical and astronomical knowledge of the Egyptians], *Mathematikai és Physikai Lapok*, Budapest, vol. 13, 1904, pp. 30–53 (mathematics), 128–142 (astronomy).

SCHACK-SCHACKENBURG, H., "Nr. 60 des mathematischen Handbuchs," *Zeitschrift für Ägyptische Sprache . . .*, vol. 41, 1904, pp. 77–78.

Discussion suggested by Borchardt (1893) and Calice (1903), correcting the latter. With references to one of the Kahun papyri, to the Berlin papyrus 6619, and to the Rhind papyrus, Schack-Schackenburg suggests the omission of a certain word introduced into the text through an error, as he believed; Peet (1923, 2) approves of this. But see the important correction of this in Gunn 1926 [1923], p. 132.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

SCHACK-SCHACKENBURG, H., "  ," [I have returned complete], *Zeitschrift für Ägyptische Sprache . . .*, vol. 41, 1904, pp. 79–80.

Refers especially to words in nos. 35–38 of the Rhind papyrus, and constitutes an addition to his paper of 1882. I have followed Peet (1923, 2) in the above translation of the title. Gunn 1926 [1923] thinks that it is possible that the word "become" should be substituted for "returned."

SIMON, M., "Über die Mathematik der Ägypter," *Verhandlung des III. Internationalen Mathematiker Kongresses in Heidelberg*, Leipzig, 1904, pp. 526–535.

A review (not always convincing) of some known facts concerning the Rhind papyrus.

1905

BOBYNIN, V. V., "Iz lektsii istorii prepodavaniya matematiki. Pervichnuie metodni ryesheniya voprosov iz oblasti nauki chisel" [Some pages from the history of the teaching of mathematics. The earliest methods of solution of questions in the field of the science of numbers], *Zhurnal Ministerstva Narodnago Prosvycshcheniya* [Journal of the Ministry of Public Instruction], St. Petersburg, April, 1905, part 2, pp. 314–357.

In part supplementary to Bobynin (1882); see Bobynin (1909). Some extracts from the "history" were published in Bobynin (1894). An abstract appeared in *Dnevnik ix-go Syezda Russkikh Estestvoispuitatelei i Vrachei* [Diary of the IX. Congress of Russian Naturalists and Physicists], no. 7, pp. 20–21.

MILLER, G. A., "The mathematical handbook of Ahmes," *School Science and Mathematics*, Chicago, vol. 5, 1905, pp. 567–574.

Descriptive.

CRUM, W. E., *Catalogue of the Coptic Manuscripts in the British Museum*, London, 1905, no. 528, pp. 256–260.

Ms. Or. 5707, a palimpsest in Coptic and Greek, containing tables and problems. Red ink is used for dividing lines, red and green for ornamental headings; red, green, and yellow for the pictures. Only a small part of the text is transcribed, and very little of that is translated. The extant text is divided into two parts. The first set of tables (folios 1a–5b) contains the products of 7, 8, 9, 10, 20, 30, . . ., 90, 100, 200, . . ., 900, 1000, 2000, . . ., 9000 by 2, 20, 200, 2000, 3, 30, 300, 3000, . . ., 9, 90, 900, 9000. This statement, checked with the manuscript, corrects and slightly supplements that made by Crum. It is

Digitized by Google Original from UNIVERSITY OF MICHIGAN

supposed that the folios missing before folio 1 contained tables for the products of 1, 2, 3, 4, 5, 6, by the same series of numbers. The second set of tables (folios 6a–8b) gives the products of 1, 2, 3, . . ., 10, 20, 30, . . ., 100, 200, . . ., 1000, 2000, . . ., 10000 by $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{20}$, $\frac{1}{24}$, $\frac{1}{48}$. The second to the tenth of these latter tables are identical with the corresponding tables in Baillet (1892); the rest of these tables, as well as the fifteenth and sixteenth tables, do not occur in Baillet (1892). In addition to these tables there are 80 problems (folios 9a–13b). Of these 18 relate to a measure of land; 39 to measures of capacity, 15 to a corn measure and 8 to a liquid measure. None of these problems are translated and only some of them have been transcribed.

SIMON, M., "Zur ägyptischen Mathematik," *Archiv der Mathematik und Physik*, series 3, vol. 9, 1905, pp. 102–103.

Mostly concerning the value of π in the Rhind papyrus; references to Borchardt (1897) and Griffith (1897).

1906

BOBYNIN, V. V., "Méthode expérimentale dans la science des nombres et principaux résultats obtenus," *L'Enseignement Mathématique*, vol. 8, 1906, pp. 177–190.

The method of "false position" (regula falsi) illustrated by examples from the Rhind papyrus, nos. 24, 31–34, 40, pp. 177–179, 185–189, from Leonardo Pisano's Liber Abaci; also from Russian mss. and arithmetics of the seventeenth and eighteenth centuries.

DARESSY, G., "Calculs Égyptiens du moyen-empire," *Recueil de Travaux relatifs à l'Archéologie Égyptiennes et Assyriennes*, Paris, vol. 28, 1906, pp. 62–72.

Detailed discussion of the tablets first described in Brugsch (1891) and catalogued in Daressy (1901). The Tablets are made of wood covered on both sides with a layer of polished plaster to take the writing which is in black ink in the hieratic script. Each tablet measures about 18 by 10 inches. They are said to have been found at Akhmîm and to date from about 2000 B. C. They give lists of servants and five mathematical calculations for expressing parts of a hekat in terms of known measures. Slips made by Daressy were noted by Möller (1911); but the definitive interpretation given by Peet (1923, 1), with comment by Gunn 1926 [1923], showed how fundamentally wrong was that of Daressy. Refers to Eisenlohr (1877), Baillet (1892).

GARDINER, A. H., "Four papyri of the 18th dynasty from Kahun," *Zeitschrift für Ägyptische Sprache* . . ., vol. 43, 1906, pp. 27–47.

On pages 46–47 there is a very important section for clearing up the difficulty of no. 62, in the Rhind papyrus, for which Griffith (1892) proposed three possible

Digitized by Google Original from UNIVERSITY OF MICHIGAN

versions without explaining any one of them. Two of the "Four papyri" here considered are in the Berlin Museum numbered 9784, 9785; the other two are discussed in Griffith (1897) among the Gurob legal documents.

HILPRECHT, H. V., *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur*, (Babylonian Expedition of the University of Pennsylvania, series A: Cuneiform Texts, vol. 20, part 1), Philadelphia, 1906. 18 + 70 pp. + 45 plates.

A study of fifty mathematical tablets illustrating Babylonian mathematics. Hilprecht assigns approximate dates to all of these tablets, 1 about 2400 B. C., 1 about 2350, 25 about 2200, 1 about 2000 and 22 about 1350 B. C. Present day scholarship calls for a later dating of the first 28 of these tablets. The date "*c.* 2200" was evidently fixed by Hilprecht as "the period of the first dynasty of Isin" (p. 10). Meissner (1925), p. 445 and *British Museum. A Guide to the Babylonian and Assyrian Antiquities*, third edition, London, 1922, p. 245, agree in fixing the period of this dynasty as about 2180–1960 B. C., the mean of which 2070 is therefore to be substituted for 2200. On the basis of similar considerations I substitute 2200 for 2400, 2150 for 2350, and 1800 for 2000.

Thirty of the plates contain hand copies of 48 of the tablets; 10 more give phototype reproduction of 13 of these. Four more plates present phototype reproductions of both sides of two other tablets. The oldest of these tablets, 10201, in the Museum of the University of Pennsylvania, contains divisions of 12,960,000 which is connected (pp. 29–34) with Plato's famous "geometrical number."

Review by D. E. Smith, "The mathematical tablets of Nippur," *Bulletin of the American Mathematical Society*, vol. 13, 1907, pp. 392–398.

Other papers with features of interest to the students of Babylonian mathematics are: F. Thureau-Dangin, "L'u, le qa et la mine, leur mesure et leur rapport," *Journal Asiatique*, series 10, vol. 13, 1909, pp. 79–111 + 1 plate; F. Thureau-Dangin, "Numération et métrolologie sumériennes," *Revue d'Assyriologie et d'Archéologie Orientale*, vol. 18, 1921, pp. 123–142; H. F. Lutz, "A mathematical cuneiform tablet," *The American Journal of Semitic Languages and Literatures*, vol. 36, 1920, pp. 249–257. This last mentioned tablet is CBS8536 in the Museum of the University of Pennsylvania;[1] for an article based on that by Lutz see F. Cajori, "Sexagesimal fractions among the Babylonians," *American Mathematical Monthly*, vol. 29, 1922, pp. 8–10. Another recent article is by Q. Vetter, "Babylonské násobení a dělení," *Časopis pro Pěstování Matematiky e Fysiky*, vol. 51, 1922, pp. 271–278; French synopsis, "Multiplication et division babyloniennes," pp. 334–336; slightly revised as "La moltiplicazione e la divisione babilonese" *Archivio di Storia della Scienza*, vol. 4, 1923, pp. 233–240; based on Hilprecht (1903, 1906), Sethe (1916), and Zimmern (1916). Many earlier references are given by Hilprecht; the accounts of Babylonian mathematics in the three editions of Cantor's *Vorlesungen*,

[1] In this document dating back to about 1900 B. C. we find among many multiplications that of $44\frac{4}{9}$ by itself and the result is correctly given as $1975\frac{25}{81}$, the $\frac{25}{81}$ being expressed in terms of fractions with 60 in the denominator. Sumerians would write $\frac{20}{27}$ as $(44\frac{1}{3}\frac{1}{9})/60$.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

vol. 1, were authoritative when published. See also Fettweis (1923), Gadd (1922), Karpinski (1917, 1926), Loeffler (1912), Meissner (1925), Ungnad (1916), Vetter (1926), and Weidner (1916).

1907

RUDIO, R., editor and translator, *Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates*, Leipzig, 1907.

"Die Kreisquadratur bei den Ägyptern," pp. 85–87.

SIMON, M., "Zur altägyptischen Bruchrechnung," *Archiv der Mathematik und Physik*, series 3, vol. 12, 1907, p. 377.

1908

GÜNTHER, S., *Geschichte der Mathematik, I. Teil, Von den ältesten Zeiten bis Cartesius*, Leipzig, 1908, pp. 24–35.

Descriptive and critical.

RUSKA, J., "Das ägyptische Zeichen für die Quadratwurzel," *Mitteilungen zur Geschichte der Medizin und der Naturwissenschaften*, vol. 7, 1908, pp. 337–338.

1909

BOBYNIN, V. V., "Drevne egipetskaya matematika v epokhu vladuichestva giksov" [The old Egyptian mathematics during the reign of the Hyksos kings], *Zhurnal Ministerstva Narodnago Prosvycshcheniya*, St. Petersburg, October 1909, part 2, pp. 290–328; November, part 2, pp. 1–50.

In so far as this relates to the Rhind papyrus Bobynin makes a new study of the work [compare Bobynin (1882, 1905)] and comes to the conclusion that "it is a collection of known results in arithmetic and practical geometry, containing the germ of all mathematical literature which later sprang up. It is a book of reference of theoretical and practical mathematics for farmers, surveyors, and architects, a text-book, and finally a work for the needs of the investigator himself, dealing with questions not yet solved." (*Jahrbuch über die Fortschritte der Mathematik.*)

GRIFFITH, F. L., *Catalogue of the Demotic Papyri in the John Rylands Library*, Manchester, vol. 3, 1909, quarto.

"Numerals," pages 415–418. "The chief sources for numerals in these papyri, apart from the dates, are the dowries, collected together on pp. 269–271, and the account papyrus No. XL, transcribed on pp. 292–6." Reference may also

Digitized by Google Original from UNIVERSITY OF MICHIGAN

be given to H. K. Brugsch, *Grammaire Démotique, contenant les Principes généraux de la Langue et de l'Écriture Populaires des anciens Égyptiens*, Berlin, 1855; chapter 5, pages 58–70, is entitled: "Des véritables signes idéographiques des chiffres." From these two sources plate 1 of Sethe (1916) was made up. More recent is W. Spiegelberg, *Demotische Grammatik*, Heidelberg, 1925, "Zahlwörter," pp. 47–51; but these pages were largely based on Sethe (1916).

MÖLLER, G., *Hieratische Paläographie. Die aegyptische Buchschrift in ihrer Entwicklung von der fünften Dynaste bis zur römischen Kaiserzeit*, Leipzig, 3 vols., 1909, 1909, 1912.

This is the fundamental work of reference in the subject and was constantly employed by Chancellor Chace in preparing plates for the second volume of his edition of the Rhind papyrus. The origin of the hieratic forms of number signs from the original hieroglyphic pictures can here be readily seen. Numerals, Numerals in dates (days of the month), Fractions, and Measures, are dealt with in volumes 1–3, nos. 614–719; most of nos. 56–60 of the Rhind papyrus, in two colors, is on plate VII in volume 1. Moreover we have in volume 1 descriptions (pp. 13–18) of the Rhind and Bulak 18 papyri, and a catalogue of signs occuring in them; similarly for the Rollin papyri in volume 2.

SIMON, M., *Geschichte der Mathematik im Altertum in Verbindung mit antiker Kulturgeschichte*, Berlin, 1909, pp. 24–53.

Popular review of contents of the Rhind papyrus, and of the varied interpretations of difficult points.

VACCA, G., "Sulla quadratura del circolo secondo l'egiziano Ahmes," *Bollettino di Bibliografia e Storia delle Scienze Matematiche*, Turin, July–September, 1908, vol. 11, pp. 65–67.

Gives a figure by means of which the value $(16/9)^2$ for π, as given in the Rhind papyrus is found; also other values which may be derived in a similar manner. The article is dated: "Firenze, Gennaio 1909."

Compare a fifteenth century anonymous paper "De inquisicione capacitatis figurarum" edited with notes by M. Curtze, *Abhandlungen zur Geschichte der Mathematik*, Heft 8, 1898, pp. 45–46, 63; this Heft appeared also as a supplement to *Zeitschrift für Mathematik und Physik*, vol. 42.

1910

EVERITT, E., "Algebra-History," *Encyclopædia Britannica*, eleventh edition, Cambridge, vol. 1, 1910, p. 616.

Not important for our inquiry.

GRIFFITH, F. L., "Egypt-mathematics," *Encyclopædia Britannica*, eleventh edition, Cambridge, vol. 9, 1910, pp. 46–47.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

HEATH, T. L., *Diophantus of Alexandria, A Study in the History of Greek Algebra*, Second Edition, Cambridge, 1910.

Pages 111–113 in the chapter: "The place of Diophantus."

JAROLIMEK, A., "Die Rätsel der Cheops-Pyramide," *Prometheus. Illustrierte Wochenschrift über die Fortschritte in Gewerbe, Industrie und Wissenschaft*, Berlin, vol. 21, 1910, pp. 497–503, 513–518.

Enthusiastic in support of the pyramid mysticism of Taylor, Piazzi Smyth, etc.; in mathematical treatment emphasizes that π and golden section are implied in related measurements of the pyramids. Surveys literature: O. Nairz, "Die Cheopspyramide, ein viertausendjähriges Räthsel," *Prometheus . . .*, vol. 17, 1906, pp. 305–311 (descriptive; see also Haedicke, pp. 732–734); H. Neikes, *Der goldene Schnitt und die "Geheimnisse der Cheops-Pyramide,"* Cologne, 1907, 20 pp. + 1 plate; M. Eyth, *Der Kampf um die Cheopspyramide. Eine Geschichte und Geschichten aus dem Leben eines Ingenieurs*, Heidelberg, 1902, 2 vols. 436 + 440 pp. (other editions in 1908 and 1909; chapter 14 of vol. 1 is entitled "3,14159 26535 89793 23846 26433 83279 50288 41971 = π"); and Jarolimek (1890).

A few out of many other possible references to recent literature are as follows: H. A. Naber, *Das Theorem des Pythagoras, wiederhergestellt in seiner ursprünglichen Form und betrachtet als Grundlage der ganzen Pythagoreischen Philosophie*, Haarlem, 1908, pp. 106–120, etc.; M. Eyth, *Lebendige Kräfte, Sieben Vorträge aus der Gebiete der Technik*, Berlin, 1904 (second ed., 1908; "Mathematik und Naturwissenschaften der Cheopspyramide," pp. 127–154); J. Edgar and M. Edgar, *The Great Pyramid Passages and Chambers*, 2 vols. + vol. plates, Glasgow, 1910–13 (vol. 1, second ed., London, 1923, 412 pp.); K. Kleppisch, *Die Cheopspyramide, ein Denkmal mathematischer Erkenntnis*, Munich and Berlin, 1921, 8 + 74 pp. + 1 pf.; F. Noetling, *Die kosmischen Zahlen der Cheopspyramide der mathematische Schlüssel zu den Einheits-Gesetzen im Aufbau des Weltalls*, 4. Auflage, Stuttgart, 1921, 15 + 181 pp. + 3 pls.; and D. Davidson and H. Aldersmith, *The Great Pyramid, its Divine Message. An original Coordination of historical Documents and archæological Evidences*, vol. 1, London, 1924, 32 + 568 pp., royal octavo.

SETHE, K. H., "Untersuchungen über die ägyptischen Zahlwörter," *Zeitschrift für Ägyptische Sprache . . .*, vol. 47, 1910, pp. 1–41.

1911

HOPPE, E., *Mathematik und Astronomie im klassischen Altertum*, (Bibliothek der klassischen Altertumswissenschaft, vol. 1), Heidelberg, 1911, 11 + 443 pp.

"Die Ägypter, pp. 35–46. Not important in this connection.

MILHAUD, G., *Nouvelles Études sur l'Histoire de la Pensée Scientifique*, Paris, 1911, pp. 58–66.

Digitized by Google

Original from UNIVERSITY OF MICHIGAN

General sketch with reference to Eisenlohr (1877), Rodet (1878, 1881), and Baillet (1892). These pages are included in the étude: "Les origines des sciences mathématiques dans les civilizations orientales et Égyptienne: l'apport de l'orient dans la science Grecque" (pp. 41–133). With regard to the chapter the author remarks: "Je reproduis ici, à très peu près telles que je les avis présentées il y a dix-huit ans, deux de mes 'Leçons sur les Origines de la Science grecque' dont le recueil est épuisé depuis longtemps." Compare Milhaud (1893).

MÖLLER, G., "Die Zeichen für die Bruchteile des Hohlmasses und das Uzatauge. Mit einem Nachtrag [Die sechs Teile des Horusauges und der 'sechste Tag']" von H. Junker. *Zeitschrift für Ägyptische Sprache* . . . , vol. 48, 1911, pp. 99–106.

Incidentally notes the Horus eye symbols in Daressy (1906), but wholly fails accurately to rectify him as Peet (1923, 1) and Sethe (1916) made clear. The principal unit for measure of capacity is the hekat; $\frac{1}{320}$ of a hekat is a ro. There are two ways of expressing parts of a hekat: (*a*) In ros and fractional parts of a ro if necessary; (*b*) In a series of fractions whose denominators are powers of 2 down to $\frac{1}{64}$, smaller portions being expressed in ros and parts of a ro in the ordinary fractional form. It is in expressing the fractions whose denominators are powers of two in this second method that the Horus-eye notation is used in hieratic writing of mathematical and medical content. This occurs in problems 35, 37, 38, 43, 47, 64, 66, 68, 69, 70, 71, 75, 80, 81, 82, 83, and 84 of the Rhind papyrus. According to mythological tradition the Uzat-eye or eye of Horus was lost in a battle with Seth and torn into parts, the following six of which were found: the right white of the eye, the pupil, the brow, the left[1] white of the eye, and the two markings below the eye, the curved and the perpendicular, corresponding respectively to the fractions $\frac{1}{2}$, $\frac{1}{4}$, . . ., and $\frac{1}{64}$. Added, this series gives $\frac{63}{64}$. The missing part was supplied in a wonderful manner. This description applies to writing from right to left.

Although all the pages of volume 48 of the *Zeitschrift* are dated 1910 the volume was published as a single number in May, 1911.

1912

BRUNSCHVICG, L., "Le calcul égyptien. Un problème d'Ahmès," *Les Étapes de la Philosophie Mathématique*, Paris, 1912, pp. 26–32.

Number 40 of Eisenlohr (1877), one of the two problems of the papyrus involving an arithmetic progression.

LÖFFLER, E., *Ziffern und Ziffernsysteme der Kulturvölker in Alten und Neuen Zeit*, (Mathematische Bibliothek, no. 1), Leipzig, 1912, 93 pp.

Zweite neu bearbeitete Auflage, Teil 1, 1918, 52 pp.

[1] "Right" and "left" are here used as a person looking at the eye of another would describe it.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

"Die Zahlzeichen der Babyloner und Assyrer," pp. 18–29 (second edition, 22–32); "Die Zahlzeichen der Ägypter," pp. 29–38 (second edition, 14–21). Compare Pihan (1860).

ZEUTHEN, H. G., *Die Mathematik im Altertum und im Mittelalter*. (Die Kultur der Gegenwart, Part 3, section 1.) Berlin and Leipzig, 1912. Pp. B9, 18B–B19, 24B–26B, B29.

1914

THOMPSON, H., "A Byzantine table of fractions," *Ancient Egypt*, London, 1914, pp. 52–54.

The outer leaf of a set of writing tablets with two lists of fractions with denominators 15 and 16, and numerators 2 to 15, 16 respectively, expressed as the sums of fractions with numerators unity. For example $\frac{1}{16}$ of 11 is $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{16}$. Headings of the columns indicate that this leaf is part of a series of tables giving the composition of various fractions. In Sethe (1916), p. 70, it is remarked that from the headings the fraction $\frac{1}{2}$ occurred on table 2 so that $\frac{2}{3}$ was probably on table 1. The leaf was bought in Egypt in 1913 and is now in University College, London.

1916

SCHUBART, W., "Mathematische Aufgaben auf Papyrus," *Amtliche Berichte aus den königlichen Kunstsammlungen*, Berlin, vol. 37, May, 1916, cols. 161–170.

This is a facsimile, transcription, and translation, with commentary, of one side of a Greek papyrus, P11529 (size 31.5 x 21 cm.) in the Berlin Museum. It is a document of the second century and is nearly related to the older (probably) Ayer papyrus; compare Goodspeed (1898). Some expressions used are the same as those of Heron of Alexandria in similar connections. The five problems are seemingly the work of a pupil, and in the first three the areas of a rectangle, of a right-angled triangle, and of an isosceles triangle are reckoned in arurae, the old Egyptian unit for measuring fields. In the first of these the square root of 164 is found in the form $12\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{26}$ $\frac{1}{32}$. The fourth problem is to find the volume of a stone in the shape of a rectangular parallelepiped, and the fifth problem suggests that the volume of a frustum of a cone [compare Grenfell (1903)] is being considered; but not all parts of the problem are clear. Schubart did not observe that the form of this problem is very similar to that of the first of the fifty problems in Baillet (1892).

In the Berlin Museum are also undeciphered fragments of Greek ostraca of mathematical interest; they are numbered P11999, P12000, P12002, P12007, and P12008.

SETHE, K. H., *Von Zahlen und Zahlworten bei den alten Ägyptern und was für andere Völker und Sprachen daraus zu lernen ist. Ein Beitrag*

Digitized by Google Original from UNIVERSITY OF MICHIGAN

zur Geschichte von Rechenkunst und Sprache, (*Schriften der wissenschaftlichen Gesellschaft in Strassburg*, Heft 25). Strasbourg, 1916, 8 + 147 pp. and 3 plates.

A work of prime importance in the field indicated by its title. There are definite references to the Rhind mathematical papyrus as edited by Eisenlohr, on pages 50, 60, 73, 85, 87, 99, 100, 119,—at least. Compare Crum (1902), and Sethe, *Göttingische Gelehrte Anzeigen*, 1916, pp. 476–490; review of his own book. Reviewed by B. Gunn, *Journal of Egyptian Archaeology*, vol. 3, 1916, pp. 279–286; critical and valuable.

UNGNAD, A., "Zur babylonischen Mathematik," *Orientalistische Literaturzeitung*, vol. 19, December, 1916, cols. 363–368.

Reference is made in Weidner (1916) and Zimmern (1916), as well as here, to two tablets 85194, 85210, transcribed in *Cuneiform Texts from Babylonian Tablets, &c., in the British Museum*, London, part IX, 1900, plates 8–15. The first of these contains about 35 problems and the second 12, but no one has yet been able to interpret the mathematics. Ungnad here includes a translation of the first problem of 85194 but he had to confess: "Auf eine Lösung der mathematischen Probleme, die hier vorliegen, muss ich leider verzichten." The problem is one regarding the volume of earth in a ramp[1] (inclined plane for entering, say, a vessel) and the amount of work one man could do as compared with a larger number. The fourth problem discusses circular canals.

WEIDNER, E. F., "Die Berechnung rechtwinkliger Dreiecke bei den Akkadern um 2000 v. Chr.," *Orientalistische Literaturzeitung*, vol. 19, September, 1916, cols. 257–263.

Interesting contribution to our knowledge of Babylonian geometry. It is the text of a table giving two methods for the calculation of the diagonal of a rectangle with sides 40 and 10 units. The first leads in numbers to the approximation $c = a + \frac{2ab^2}{3600}$, if a is the greater side, b the lesser, and c the diagonal.[2] The second, to $c = a + b^2/2a$ which is what one arrives at in calculation of $\sqrt{a^2 + b^2}$, if terms after the second are neglected. Compare W. Lietzmann, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, vol. 49, 1918, pp. 148–149.

ZIMMERN, H., "Zu den altakkadischen geometrischen Berechnungsaufgaben," *Orientalistische Literaturzeitung*, vol. 19, November, 1916, cols. 321–325.

[1] For this word in translation, unknown to Ungnad, I am indebted to Mr. Sidney Smith of the British Museum.

[2] The impossibility of such a non-homogeneous relation is obvious.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

1917

KARPINSKI, L. C., "Algebraical developments among the Egyptians and the Babylonians," *American Mathematical Monthly*, vol. 24, 1917, pp. 257–265.

"Algebraical ideas in Egypt," pp. 258–263.

TURAEV,[1] B. A., "The volume of the truncated pyramid in Egyptian mathematics," *Ancient Egypt*, London, 1917, pp. 100–102.

This paper exhibits one of "nineteen"[2] problems (four of which are geometrical) occurring in a hieratic mathematical papyrus, written about 1800 B.C., which was probably also the date of the original of the Rhind mathematical papyrus, the copy we possess having been written one or two hundred years later. This papyrus, formerly the property of Golenishchev, compare Cantor 1894 [1880], now professor of Egyptian philology at the Egyptian University, Cairo, was acquired about 1916 by the Museum of Fine Arts in Moscow. It appears to indicate a familiarity with the formula for the volume of the frustum of a square pyramid, $V = (h/3)(a_1^2 + a_2^2 + a_1 a_2)$, where h is the altitude of the frustum, the sides of whose bases are[3] a_1 and a_2.

This extraordinary result, and the facts here revealed by the late Professor Turaev, would suggest that accounts of Egyptian mathematics may have to be rewritten so soon as all of the contents of this Moscow papyrus are generally known. But Professor Peet who had access to a photograph of the papyrus has written, Peet (1923, 2), p. 6: "though the papyrus is of the highest interest owing to its early date and admirable state of preservation (in part at least) it contains nothing, with the exception of the problem of the truncated pyramid, which will greatly modify the conception of Egyptian mathematics given to us by the already published papyri and fragments."

[1] The name in connection with this article was transliterated into the form Touraeff.

[2] This is the number of the problems in the main body of the papyrus without any reference to what is to be found in the fragments. We refer later to a fifth geometrical problem on one of the fragments; the total number of problems is 26 since there are 7 problems on the fragments, as Professor Struve has kindly informed me.

[3] Brahmagupta (about 628 A.D.) gave the equivalent of a formula which reduces to this (*Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahmegupta and Bhâscara*, ed. by Colebrooke, London, 1817, pp. 312–313). So also Mahāvīrācārya (c. 850 A.D.). *The Gaṇita-Sāra-Saṅgraha* . . ., with English translation and notes by M. Raṅgācārya, Madras, 1912, p. 260 of the translation.

In the Moscow papyrus $a_1 = 4$, $a_2 = 2$, and $h = 6$. In al-Khowârizmî's algebra, the same problem is solved for $a_1 = 4$, $a_2 = 2$, and $h = 10$. (*The Algebra of Mohammed Ben Musa*. Edited and translated by F. Rosen, London, 1831, pp. 83–84; see also *Annali di Matematica*, vol. 7, pp. 279–280, 1866); the method of solution is: determine the height (20), and hence the volume, of the completed pyramid, from which is subtracted the volume of the pyramid of height 10 on the upper base. To Democritus (who flourished about 400 B.C.) is due the discovery that the volume of a pyramid is one-third that of the prism having the same base and equal height; the first rigorous proof of this result was given by Eudoxus.

The general formula for the volume of a frustum of a pyramid $(h/3)(A_1 + A_2 + \sqrt{A_1 A_2})$,

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Professor Turaev left at least the first draft of a hieroglyphic transcription of 36 out of the 38 columns of the Golenishchev papyrus; he had not attempted to deal with the 8 fragments. This transcription, so far as five of its problems are concerned, was published in Tsinserling (1925). Dr. V. Struve of the Hermitage Museum, Leningrad, is to edit and complete Turaev's transcription and prepare a German translation. This, together with a facsimile of the papyrus, and a mathematical commentary by Professor Tsinserling will probably be published by the Academy of Sciences, Leningrad, early in 1928.

Turaev remarked that paleographically the papyrus is "like some of the Illahun papyri, whilst the breadth of its leaves brings it near to the MSS. of Sinuhe, found in the Ramesseum." Hence an approximate date for the papyrus would be about 1850 B.C.

The results of Turaev (1917) were first brought to the attention of mathematicians in an address I delivered at Cleveland, Ohio, in September, 1922. This part of *Ancient Egypt* was there exhibited.

SEDGWICK, W. T., AND TYLER, H. W., *A Short History of Science*, New York, 1917.

"The Ahmes papyrus," pp. 30–34; of no importance in this connection.

1918

ARCHIBALD, R. C., "The oldest mathematical work extant," *American Mathematical Monthly*, vol. 25, January 1918, pp. 36–37.

Descriptive notes, with brief bibliography, of the Rhind papyrus; of no importance. While the statement in these notes that "any approximately complete bibliography should contain more than forty titles" is accurate, a more correct impression would have been conveyed by the substitution of the words "two hundred" for the word "forty." In the light of the chronological list of documents at the end of the present Bibliography it is clear that the title of the article is also misleading.

SETHE, K. H., "Ein altägyptischer Fingerzählreim," *Zeitschrift für Ägyptische Sprache . . .*, vol. 54, 1918, pp. 16–39.

Finger numbering in the Book of the Dead, introduction to no. 99, edited by E. Naville; the new edition of this part by H. Grapow (Urkunden des Aegyptischen Altertums, Leipzig, section 5, part 3, 1917, pp. 146–180) was based on XIIth

---

A_1 and A_2 being the areas of the bases, seems to have been first given by Leonardo Pisano in his 'Practica geometriae' of 1220 (*Scritti di Leonardo Pisano . . .*, Rome, vol. 2, 1862, p. 174). Of course the formula was there expressed in words, not in letters.

Heron of Alexandria (third century A.D.?) found the volume of a frustum of a pyramid for which $a_1 = 10$, $a_2 = 2$, $h = 7$ (*Heronis Alexandrini Opera quae supersunt omnia*, Leipzig, vol. 5, 1914, pp. 30–35). His operations are equivalent to substituting in the formula $V = h\left[\left(\frac{a_1 + a_2}{2}\right)^2 + \frac{1}{3}\left(\frac{a_1 - a_2}{2}\right)^2\right]$, which reduces to the formula of the Golenishchev papyrus. Compare Grenfell (1903).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

Dynasty documents and is the basis of Sethe's discussion. Compare Gunn (1922). A reference may be added to M. A. Murray, "Egyptian finger counting rhymes," *Folk Lore*, vol. 36, 1925, pp. 186–187; she gives a free English translation of the finger numbering spell. The first sentence of her article is as follows: "It has often been suggested that many children's games and rhymes may be derived from, or in imitation of religious ritual." Only disciples of Petrie will agree with Miss Murray's suggested date for the above mentioned finger numbering spell.

1919

KARPINSKI, L. C., "Origines et développement de l'algèbre," *Scientia*, vol. 26, 1919, pp. 89–101.

A translation by M. E. Philippi of the English original, published in *School Science and Mathematics*, vol. 23, 1923, pp. 54–65; Egyptian mathematics is referred to on pp. 54–57 (89–93 of the translation).

1921

HEATH, T. L., *A History of Greek Mathematics*, Oxford, 1921. Vol. 1, pp. 122–128 ("Egyptian geometry, i.e., mensuration"), 130, 131; vol. 2, pp. 440–441 ("'Hau'-calculations").

See also Baillet (1892). Attributes to Borchardt, instead of to the Revillouts (1881), the now generally accepted cotangent interpretation of seked. Quotation (p. 128): "But, lastly, the *se-qeṭ* in No. 56 is $1\frac{8}{25}$ and, if *se-qeṭ* is taken in the sense of cot *HFE* [defined in the previous discussion], this gives for the angle *HFE* the value of 54° 14′ 16″, which is *precisely*, to the seconds, the slope of the lower half of the southern stone pyramid of Dakshūr [*sic*]; in Nos. 57–9 the *se-qeṭ*, $\frac{3}{4}$, is the cotangent of an angle of 53° 7′ 48″, which again is exactly the slope of the second pyramid of Gizeh as measured by Flinders Petrie; and the *se-qeṭ* in No. 60, which is $\frac{1}{4}$, is the cotangent of an angle of 75° 57′ 50″, corresponding exactly to the slope of the Mastaba-tombs of the Ancient Empire and of the sides of the Mēdūm pyramid." [1]

This paragraph is based, apparently, on Borchardt (1893, p. 16) and Griffith (1894, p. 238); a similar series of statements occurs in the *Nature* review listed under Ahmes (1898). From what we have indicated under Petrie (1883), it is clear that the paragraph is replete with error and false suggestion. The angles 53° 7′ 48″ and 75° 57′ 50″ are, as we have seen, "theoretical angles"—quite different from even the *mean* of the observed angles; but apart from this we have noted that the angle 75° 57′ 50″ is more than 24° in error for the Medum pyramid! It is palpably absurd to employ in this connection such terms as: "*precisely*, to the seconds," "exactly the slope," and "exactly to the slope."

But again, the angle 54° 14′ 16″ is not given by either Petrie (1883) or Borchardt (1893); perhaps it is a misprint for 54° 14′ 46″, a value apparently

[1] "Flinders Petrie, *Pyramids and Temples of Gizeh*, p. 162."

Digitized by Google Original from UNIVERSITY OF MICHIGAN

assumed by Borchardt (page 16) so as to agree with a result of the Rhind papyrus! We have seen that Petrie's mean value here, available when Borchardt wrote, is 55° 1′.

1922

BORCHARDT, L., *Gegen die Zahlenmystik an der grossen Pyramide bei Gise. Vortrag gehalten in der Vorderasiatisch-ägyptischen Gesellschaft zu Berlin am 1. February, 1922.* Berlin, 1922, 40 pp.

Of considerable interest in connection with the study of problems 57–60 of the Rhind papyrus. There is a critical survey of literature in this connection, such as listed in Roeber (1854), and Jarolimek (1910). Compare Borchardt (1893).

CRUM, W. E., AND BELL, H. I., editors, *Wadi Saga, Coptic and Greek Texts from the Excavations undertaken by the Byzantine Research Account* (Coptica Consilio et imprensis Instituti Rask-Oerstediani edita, vol. 3), Copenhagen, 1922.

"Mathematical," nos. 22–28, pp. 53–57. Nos. 22–23 contain multiplication tables $6 \times 1 = 6$, $6 \times 2 = 12$, . . ., and $7 \times 1 = 7$, $7 \times 2 = 14$, . . . each up to 10; compare the table in Crum (1905). Nos. 24–26 contain the results of expressing $\frac{1}{7}$ and $\frac{1}{11}$ of various numbers from 1 to 9; the results are identical with those given in the Akhmîm papyrus, Baillet (1892), pp. 26–29. No. 27 contains the results of $\frac{1}{25}$ of 11, $\frac{1}{49}$ of 7, $\frac{1}{7}$ of 7, and of 8, and of 9. Nos. 22–28 are included in Papyrus 2241 of the British Museum.

GADD, C. J., "Forms and colours. I. Forms," *Revue d'Assyriologie et d'Archéologie Orientale*, vol. 19, 1922, pp. 149–158.

An important contribution to our knowledge of Babylonian geometry of about 2000 B.C. It deals with a large fragment 22.3 x 15.2 cm., no. 15285, in the British Museum. In the second paragraph of the article the author states very emphatically that the "purpose of these geometrical constructions was to facilitate the surveying or parcelling out of land." I learn (1) that this statement was mainly based on a word in each problem translated as "field;" and (2) that the author now thinks that the translation "area" is also possible, and indeed preferable in this document.

Of the figures, six are practically perfect on the tablet while for two others, of great interest, sufficient is given to indicate what the completed figures were. These figures certainly suggest that the Babylonians were interested in a geometry which might be the basis of designs. (a) A square is divided into 16 squares, (b) into four equal isosceles right triangles, and (c) into eight such triangles. (d) A square is inscribed in a square and (e) a third square is inscribed in this. (f) A circle is inscribed in a square and (g) part of another figure is made up of four arcs of circles forming a figure somewhat resembling a "four-cusped" hypocycloid. (h) Another figure involves arcs of three tangent and three intersecting circles within a square.

The exact drawings of the printed text do not in any wise misrepresent

Digitized by Google Original from UNIVERSITY OF MICHIGAN

what one finds on the tablet. Plates 28 (2000–745 B.C.) and 34 (663–525 B.C.) of A. C. T. E. Prisse D'Arenne's *Histoire de l'Art Égyptien d'après les Monuments*, Paris, *Atlas*, vol. 1, 1878, present designs suggesting similar figures.

GUNN, B. G., " 'Finger-numbering' in the pyramid texts," *Zeitschrift für Ägyptische Sprache . . .* , vol. 57, 1922, pp. 71–72.

Inspired by Sethe (1918). Of interest in connection with Horus-eye notation; compare note under Möller (1911).

JÉQUIER, G., "Le système numérique en égyptien," pp. 467–482 of *Recueil d'Études Égyptologiques dediés à la Mémoire de Jean François Champollion*, Paris, 1922. (Bibliothèque de L'École des Hautes Études, Sciences historiques et philologiques, vol. 234).

Refers to Eisenlohr (1877), Goodwin (1867), Sethe (1916).

SLOLEY, R. W., "Ancient Egyptian mathematics," *Ancient Egypt*, 1922, pp. 111–117.

Of no value.

VETTER, Q., "Egyptiské zlomky," *Časopis pro Pěstování Matematiky a Fysiky*, vol. 52, 1922, pp. 169–176; résumé in French, "Les fractions égyptiennes," pp. 176–177.

References to Eisenlohr (1877, 1891), Rodet (1881), Brugsch (1891), Hultsch (1897), Sethe (1916). Unimportant.

1923

FETTWEIS, E., *Wie man einstens rechnete*, (Mathematisch-physikalische Bibliothek, no. 49), Leipzig and Berlin, 1923.

"Das Rechnen bei den vom Griechentum unabhängigen Kulturvölkern," pp. 11–19; Egyptian, pp. 11–15, Babylonian, pp. 17–19. Unimportant.

PEET, T. E., "Arithmetic in the middle kingdom," *The Journal of Egyptian Archæology*, vol. 9, 1923, pp. 91–95.

Definitive interpretation of Cairo tablets translated by Daressy (1906) but later corrected by Möller (1911); an error by Möller is corrected by Sethe (1916), p. 74, n. 2, in which Peet finds a slip. The gist of this article is set forth on page 7 of the book described in the next title. In his review, listed below, Gunn (1926) [1923] adds, page 124, important comment in this connection. The tablets indicate the values of a third, a seventh, a tenth, an eleventh, and a thirteenth of a hekat, in terms of the parts of a hekat used in everyday transactions, namely $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, and the $\frac{1}{320}$ part called the ro. Thus one seventh of a hekat is found to be $(\frac{1}{8} + \frac{1}{64})$ hekat $+ (\frac{1}{2} + \frac{1}{7} + \frac{1}{14})$ ro. Although hekat is not a liquid measure Gunn has translated it by "gallon."

Digitized by Google Original from UNIVERSITY OF MICHIGAN

PEET, T. E., *The Rhind Mathematical Papyrus, British Museum 10057 and 10058. Introduction, Transcription, Translation and Commentary.* London, 1923, 2 + 136 pp. + 24 plates, folio.

A thoroughly scholarly work in which the most recent results of research are incorporated. Hieroglyphic transcription (slips may be found) and a free English translation of the original hieratic are given. For this purpose the papyrus itself was used, and not the B. M. "Facsimile" (1898) which has been shown to be unreliable in certain details. Twenty-five of the fragments, in the library of the New York Historical Society, which belong in the 18 cm. gap between the papyri B. M. 10057 and 10058, are put in place; other unimportant fragments in the library may also belong in the gap. This Bibliography (to the end of 1921) was placed at Peet's disposal in preparing his work.

Review by A. Agostini, *Periodico di Matematiche*, s. 4, vol. 4, March, 1924, p. 139.

Review by R. C. Archibald, *American Mathematical Monthly*, vol. 31, May, 1924, pp. 246–251. One finds here the first published announcement of the discoverer, Percy E. Newberry, of the New York fragments of the Rhind papyrus.

Review by A. B. Chace, *Science*, n. s., vol. 59, Feb. 29, 1924, pp. 215–216.

Review by W. R. Dawson, *Science Progress*, vol. 19, July, 1924, pp. 50–59.

Review by H. Fehr, *L'Enseignement Mathématique*, vol. 23, April, 1924, p. 234.

Review by [W. J. Greenstreet], *Mathematical Gazette*, vol. 12, May, 1924, pp. 130–132.

Review by F. L. Griffith, *University of Liverpool. Annals of Archaeology and Anthropology*, vol. 11, [June], 1924, pp. 103–104. Quotation: "Altogether we accept the volume with a deep sense of satisfaction. Concise, yet elaborate and complete, the treatment of the papyrus itself is practically final, and future discovery can do little more than adjust its relationship to other documents as they may appear." For the casual reader the following sentence of the review might well mislead: "For a copy of the hieratic the student will, of course, go to the facsimile, issued by the British Museum or to the Eisenlohr's copy;" mislead because Peet has corrected the "facsimile" in more than one particular.

Review by B. G. Gunn, *Journal of Egyptian Archæology*, vol. 12, April, 1926, pp. 123–137. Brilliant review by an acknowledged expert in Egyptian of the Middle Kingdom. Peet's book is characterized as "a very able piece of work." Nearly 14 of the closely printed pages of the review are occupied with most valuable detailed criticisms or remarks of a more or less technical character. For example his discussion of *mryt* (p. 133) in no. 51 of the Rhind papyrus leaves little doubt that the equivalent of our formula for the area of a triangle, namely, one half the product of the lengths of its base and altitude, was here used.[1] This removes a blot on the geometric-arithmetic part of the Rhind papyrus which was placed there by Eisenlohr[2] (1877); it makes more plausible the correctness of the remarkable geometric result presented in Turaev (1917).

[1] The same expression occurs also on one of the fragments of the Golenishchev papyrus.

[2] Eisenlohr (and later writers) contended that in no. 51 the area of an isosceles triangle was considered, and expressed as one half the product of the lengths of the base and a side. This method of calculation is used in the Edfu inscription, see Lepsius (1856), and is given in *Heronis Alexandrini Opera quae supersunt omnia*, Leipzig, vol. 5, ed. by Heiberg, 1914,

Digitized by Google Original from UNIVERSITY OF MICHIGAN

On page 125 Gunn states: "The word for 'two thirds' is shown by evidence at Saqqara, shortly to be published, to be not *r 2*, 'two parts,' but *rwỉ*, 'the two parts.'" I learn that this will appear in either *Annales du Service des Antiquités de l'Égypte*, or in a separate publication, ranking as a volume (5 or 6) of the series called *Excavations at Saqqara*, both issued by the Service des Antiquités de l'Égypte.

Review by W. Lietzmann, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, vol. 55, November, 1924, p. 231.

Review by S. A. B. Mercer, *Journal of the Society of Oriental Research*, Toronto, vol. 9, January, 1925, pp. 77–79.

Review by O. Neugebauer, *Matematisk Tidsskrift, A*, 1925, pp. 66–70.

Review by G. Sarton, *Isis*, vol. 6, May, 1924, pp. 553–557.

Review by K. Sethe, *Jahresbericht der deutschen Mathematiker Vereinigung*, vol. 33, January, 1925, pp. 139–143. Valuable review; last paragraph: "Auf Einzelheiten philologischer Natur, über die man etwas anders denken könnte als Prof. Peet, einzugehen, ist hier nicht der Ort. Sie sind nicht derart, dass sie das Gesamturteil über sein Buch beeinträchtigen könnten; dieses Urteil ist dahin zusammenzufassen, dass hier eine durchaus solide Grundlage für alle weitere Forschung gelegt ist."

Review by D. E. Smith, *Bulletin of the American Mathematical Society*, vol. 30, November-December, 1924, pp. 557–559.

Review by W. Spiegelberg, *Orientalistische Literatur-Zeitung*, vol. 27, July, 1924, cols. 319–320. Quotations: "Sie ist jetzt von Peet unternommen und so vortrefflich durchgeführt worden, dass sie überall zu einer erheblichen Verbesserung der früheren Übersetzung und zu einer in der Hauptsache sicheren Erklärung des Textes geführt hat. . . . Den wichtigsten Teil der Arbeit bildet die neue Umschrift und Übersetzung des Textes mit einem eingehenden Kommentar. Seine Beurteilung setzt eine besondere Begabung für mathematische Probleme voraus, die dem Referenten abgeht. Ich kann lediglich als Philologe aussprechen, dass mir die Begründungen der Übersetzung sehr einleuchtend sind. Überall ist Peet über die erste Veröffentlichung hinausgekommen und die Mathematiker werden sich in Zukunft an diese neue Bearbeitung zu halten haben, die eine sehr solide Grundlage hat."

Review by D. W. Thompson, *Nature*, vol. 15, June 13, 20, 1924, pp. 899–902, 935–937.

Review by Q. Vetter, *Časopis pro Pěstováni Mathematiky a Fysiky*, vol. 54, 1925, pp. 192-195.

Review by H. Wieleitner, *Mitteilungen zur Geschichte der Medizin und der Naturwissenschaften*, vol. 23, July, 1924, p. 121.

Review by H. Wieleitner, "Zur ägyptischen Mathematik," *Zeitschrift für*

pp. 206–207. But Heron gave also two correct results for the area of any triangle, one in terms of the base and altitude (known to the Egyptians) *Opera*, vol. 4, ed. Heiberg, 1912, p. 238; the other in terms of the three sides (probably due to Archimedes) *Opera*, vol. 3, ed. Schöne, 1903, pp. 20–27; vol. 4, pp. 248–249, etc. It may be noted that Hilprecht (1906), does not claim that the Babylonians, even later than the Rhind papyrus, knew any other formula for the area of a triangle than that the area of a right triangle was equal to one half the product of the lengths of the sides about the right angle.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

mathematischen und naturwissenschaftlichen Unterricht, vol. 56, 1925, pp. 129–137. Thoughtfully informing and critical. References include Eisenlohr (1877), Simon (1905), Cantor (1907), Weidner (1916), Zimmern (1916), Heath (1921), Tropfke (1921), Borchardt (1922), Wolff 1924 [1923], Archibald 1924 [1923]. Points out that in Peet's references to what is known of Babylonian mathematics the papers of Weidner, Zimmern and Ungnad had been overlooked.

Review by G. Wolff, *Unterrichtsblätter für Mathematik und Naturwissenschaften*, vol. 30, 1924, pp. 107–108.

Review (anonymous) *Discovery*, vol. 5, June, 1924, pp. 106–107.

Review (anonymous) *The Times Literary Supplement*, vol. 23, March 20, 1924, p. 175.

ROBBINS, F. E., "A Greco-Egyptian mathematical papyrus," *Classical Philology*, vol. 18, October, 1923, pp. 328–333.

An account of a Greek papyrus at the University of Michigan which came originally from the Fayûm and dates from approximately the fourth century A.D. It is earlier than the Akhmîm papyrus, Baillet (1892), but contains tables similar to those in this, as well as in the earlier Rhind papyrus. Robbins states that the account of this papyrus by L. C. Karpinski was based on information furnished by him; this account is: "Michigan mathematical papyrus, no. 621," *Isis*, vol. 5, October, 1922, pp. 20–25 + 1 plate.

SMITH, D. E., *History of Mathematics*, Boston, Mass., vol. 1, 1923, pp. 41–53; vol. 2, 1925, pp. 45–47, 209–211, 270, 386, 410, 431–432, 435–437, 498–501, 634–635.

VASCONCELLOS, FERNANDO DE, "A numeração fraccionária no papiro de Rhind e em Herão de Alexandria," *Associação Portuguesa para o Progresso das Sciências, Congresso do Porto, Primero Congresso . . . 1921*, Secções de Matemática, Astronomía e Sciências Físico-Químicas, Coimbre, 1923, pp. 43–93.

Also in *Anais do Instituto Superior de Agronomia*, Lisbon, vol. 2, 1924, 50 pp.

With references to Eisenlohr (1877), Tannery (1884), Loria (1914) Baillet (1892).

VETTER, Q., "Egyptiské dělení" [Egyptian division], *Jahresberichte der königlich-böhmischen Gesellschaft der Wissenschaften*, Class II, Prague, for 1921–22, no. 14, 1923, pp. 1–23; résumé in French, pp. 23–25.

First paragraph of the résumé. "L'article présent cherche à démontrer que les Égyptiens divisaient d'après le système, suivi dans la grande tabelle d'Ahmes [2: $(2n + 1)$], c'est-à-dire en cherchant pour le quotient $a : b$ un nombre c qui, multiplié par b, donne a. On commence par démontrer la manière de procéder des Égyptiens lorsque a, b, c sont des nombres entiers."

Digitized by Google Original from UNIVERSITY OF MICHIGAN

VETTER, Q., "Le progressioni arithmetiche presso gli Egiziani," *Bollettino di Matematica, Sezione Storico-bibliografia*, 1923, pp. XCVII–XCIX.

References to Eisenlohr (1877), Rodet (1881), Cantor (1898).

1924

PEET, T. E., *The Cambridge Ancient History*, vol. 2, *The Egyptian and Hittite Empires to c. 1000 B.C.*, Cambridge, 1924.

Chapter IX, "Contemporary life and thought in Egypt," pp. 196–226, is by Professor Peet. Mathematics and astronomy are treated on pp. 215–218.

REYMOND, A., *Histoire des Sciences Exactes et Naturelles dans l'Antiquité Gréco-Romaine*, Paris, 1924, 8 + 238 pp.

"L'Égypte et la Chaldée," pp. 1–16. Of no importance in this connection.

VETTER, Q., "Poznámka k t. zv. trigonometrii Ahmõseově a krozměrům pyramidy chufuovy," *Časopis pro Pěstování Matematiky a Fysiky*, vol. 54, 1924, pp. 281–283. (A French summary, "Remarque sur la trigonométrie d'Ahmõse et sur les dimensions de la pyramide de Chufu," p. 283.)

The author contends that the seked of problems 56 and 59 is not a trigonometric function.

1925

KARPINSKI, L. C., *The History of Arithmetic*, Chicago, 1925, pp. 2–7, 36, 61, 121–2, 169.

Of no importance. References to Heath (1921), Peet (1923).

MEISSNER, B., *Babylonien und Assyrien* (Kulturgeschichtliche Bibliothek, Ethnologische Bibliothek no. 4), Heidelberg, vol. 2, 1925.

This volume contains the most important recent survey of results and the literature of Babylonian mathematics (pp. 385–394), and Astronomy and Calendar (pp. 394–418). In particular some details, with two of the figures, are given of F. Thureau-Dangin, "Un cadastre Chaldéen," *Revue d'Assyriologie et d'Archéologie Orientale*, vol. 4, 1897, pp. 13–27. No reference is given by Meissner to A. Eisenlohr's pamphlet dealing with the same text: *Ein Altbabylonischer Feldplan*, Leipzig, 1896, 18 pp. + 3 plates. Apparently also, Meissner has not taken account of J. Oppert's article. "Quelques mots sur le cadastre Chal-

Digitized by Google Original from UNIVERSITY OF MICHIGAN

déen," occupying[1] pages 28–33 of the above mentioned volume of the *Revue*, which develops new facts and points out slips due to false assumptions in these publications of both of the authors. The tablet, found at Tello, Arabia, and now in the Ottoman Museum, Constantinople, is a plan of a great field. The figure is divided into 15 parts: 7 right-angled triangles, 4 rectangles (nearly), and 4 trapezia (one side always perpendicular to the parallel sides), along the sides of which lengths are written; areas are also indicated in each figure. For the area of a trapezium the equivalent of the following formula is evidently used: one half the sum of the lengths of the parallel sides times the distance between them.

TSINSERLING, D. P., "Geometriya u drevnikh egiptyan" [Geometry in ancient Egypt], *Bulletin de l'Académie des Sciences de l'Union des Republiques Soviétiques Socialistes*, Leningrad, series 6, vol. 19, 15 September–1 November, 1925, pp. 541–568.

This paper was presented to the Academy by Turaev on April 16, 1919, probably in somewhat different form. In rather long-drawn-out fashion some earlier work in connection with the topic of the paper,[2] chiefly by Eisenlohr, Cantor, Bobynin, and Lepsius, is summarized. No account is taken of Peet's book or of other earlier and later important material indicated in this Bibliography. About 13 pages of Tsinserling's paper are occupied with a discussion of nos. 51–52 of the Rhind papyrus and an attempt is made to prove that Revillout (1881) was correct in believing that we must here consider that a right triangle and a right-angled trapezium are in question. It is incorrectly stated (p. 544) that Schack-Schackenburg (1899) suggested that no. 43 of the Rhind papyrus dealt with the frustum of a cone. A wrong reference is given to Turaev (1917), and there are various other slips.

The only new things to be found in the article are: (*a*) Turaev's hieroglyphic transcription of four problems (1, 2, 12, 15) of the Golenishchev papyrus, three of which are geometrical, and the Russian translation of the same; (*b*) Turaev's description of this papyrus containing 38 columns of writing and drawings on 8 pieces and 8 fragments totalling 5.44 meters in length; (*c*) facsimiles of the parts of the Golenishchev papyrus discussing problems 1 and 12; (*d*) the Russian translation of the truncated pyramid problem (9) given in Turaev (1917); (*e*) notes to these problems (1, 2, 9, 12, 15) by V. V. Struve; (*f*) the statement that on a fragment of the Golenishchev papyrus there is a triangle problem similar to that of no. 51 of the Rhind papyrus.

We have already referred, Turaev (1917), to the Golenishchev papyrus giving

[1] This article is practically the same as the one published by Oppert a year earlier: "Un cadastre chaldéen du quatrième millénium avant l'ère chrétienne," Académie des Inscriptions et Belles-Lettres, *Comptes Rendus*, series 4, vol. 24, 1896, pp. 331–348; there are here interesting details not in the later article.

[2] There are references to: Baillet (1892), Blume 1848 [1856], Bobynin (1882, 1905, 1909), Borchardt (1893, 1897), Cantor 1907 [1880], Eisenlohr (1877), Griffith (1897), Lepsius (1856), Revillout (1881), Schack-Schackenburg (1899, 1900), Simon (1909), Tropfke 1903 [1902], Turaev (1917), Weyr (1884).

Digitized by Google Original from UNIVERSITY OF MICHIGAN

the volume of the frustum of a square pyramid. Tsinserling professes to give Turaev's transcription, and a translation of five problems, only four of which are geometrical. Since the fifth problem has nothing whatever to do with geometry[1] it is not clear why it was included in this paper. On the other hand, as already noted, a geometrical problem for finding the area of a triangle occurs on one of the fragments. The transcription is incorrect, incomplete, and probably misprinted; obviously a piece of unfinished work on the part of Turaev.

The first of the geometrical problems, which is number 1 in the Golenishchev papyrus, is as follows: Given that the area of a rectangle is 12 arurae and the ratio of the lengths of the sides 1 : ½ ¼, find the sides. It will be noted that the solution of this problem leads to the equations $xy = 12$, $x : y = 1 : \frac{3}{4}$ which are identical with those occurring in a problem of the Kahun papyri; compare Griffith (1897).

The second and fourth problems (second and twelfth of the Golenishchev papyrus) are really the same as the first; but the areas are here right triangles. In the first of them the ratio of the "length" to the "breadth" is 2½ : 1, in the second[2] ⅓ 1/16 1/240 : 1. In each case the area is given as 20. These three problems[3] seem to prove that the Egyptians of 1850 B.C. were familiar with the following results: The area of a rectangle is the product of the lengths of a pair of adjacent sides; the area of a right triangle is one half the area of the rectangle with one right angle the same as that of the triangle and with a diagonal coincident with the hypotenuse of the triangle.[4]

The third of the problems given by Tsinserling (ninth in the Golenishchev papyrus) is that one which we have already discussed under Turaev (1917).

VASCONCELLOS, FERNANDO DE, *História das Matemáticas na Antiquidade*, Paris and Lisbon, 1925.

Egyptian mathematics, pp. 59–89. Refers to Eisenlohr (1877), Revillouts (1881), Baillet (1892), Milhaud (1911), Cantor (1880), and Vasconcellos (1923).

[1] This problem, the fifteenth of the Golenishchev papyrus, deals with the mixture of two lots of material, possibly corn. There is no problem just like it in the Rhind papyrus. At the time of his death, July, 1920, Turaev had prepared the Russian translation of only four problems of the Golenishchev papyrus, and an English translation of a fifth. It is these five problems which Tsinserling has published.

[2] It is to be noted that ⅓ 1/16 1/240 is much more complicated than ⅓ 1/15 given in the Kahun, Rhind, and Akhmîm papyri tables as the equivalent of 2 divided by 5.

[3] Turaev (1917), p. 100, gave the following unintelligible and misleading description, quoted by Peet (1923, 2), of these problems: "The first shows how to define the length of the sides of a quadrilateral, when the relation of the sides and the area of the quadrilateral are known. The two next give a method of calculating the area of a triangle: a method already known to us."

[4] By implication the area of a right triangle in the second or fourth problems is regarded as one half the product of its "length" and "breadth." In the fragment problem for a triangle these are called the *tp-r* and *mryt*, as in the Rhind papyrus, and the operations of forming their product and halving the result are explicitly carried out.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

VETTER, Q., "Quelques remarques sur le papyrus mathématique no. 621 de la Michigan collection," *Classical Philology*, vol. 20, October, 1925, pp. 309–312.

Suggested by Robbins (1923) and somewhat of a repetition of Vetter (1922).

WEILL, R., "L'unité de valeur *shat* et le papyrus de Boulaq No. 11," *Revue de l'Égypte Ancienne*, Paris, vol. 1, 1925, pp. 45-87 + 2 plates. Supplementary note pp. 243-244.

Apparently without any knowledge of this paper Gunn 1926 [1923] inquired if "*shat*" in the Rhind (problem 62) and Bulak 11 papyri is not "a non-concrete unit of value." This is the well-documented contention of Weill.

WIELEITNER, H., "Zur muslimischen und aegyptischen Gleichungsaufloesung," *Archivio di Storia della Scienza*, vol. 6, March, 1925, pp. 46–48.

1926

GILLAIN, O., "A propos du papyrus mathématique de Rhind," *Chronique d'Égypte*, Brussels, Année 1, no. 2, June, 1926, pp. 98–102.

Of no importance.

KARPINSKI, L. C., *Nicomachus of Gerasa. Introduction to Arithmetic, Translated into English by Martin Luther D'Ooge, with Studies in Greek Arithmetic* by F. E. Robbins and L. C. Karpinski, (University of Michigan Studies, Humanistic series, vol. 16), New York, 1926.

Babylonian and Egyptian mathematics are discussed in chapter 1, "The sources of Greek mathematics," pp. 3–15. According to the preface Karpinski is the author of the chapter.

NEUGEBAUER, O., *Die Grundlagen der ägyptischen Bruchrechnung*, Berlin, 1926, 3 + 45 pages + 8 folding plates.

Thoughtful and interesting monograph[1] dealing chiefly with the table at the beginning of the Rhind papyrus, and showing considerable insight into the spirit of Egyptian mathematics; but it does not appear to indicate the most natural explanation of the way in which this table was formed.

In discussing the two parts of the separation of 2/n, Neugebauer calls the first fraction, as it is given in the papyrus, the "completion-term" (Ergänzungsterm), and the rest of the separation the "principal-part" (Hauptglied), as if

[1] It is divided into two chapters: I, "Die begrifflichen Grundlagen der ägyptischen Mathematik," pp. 3–17; II, "Die ägyptische Bruchrechnung," pp. 18–44 + 8 plates, which was the author's Göttingen doctoral dissertation in 1926.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

the latter were the first to be thought of, and indeed he speaks of it as first. Then he adopts Peet's idea that the Egyptian began by separating 2 into two parts. He gives the same names to the two parts of 2 and gets the products of 1/n by them as the two parts of 2/n.

He suggests that the Egyptian might take for the principal part in the separation of 2 one of the "natural fractions," ½, ¼, ⅛, . . . ⅓, ⅙, . . ., and shows that ½.1/n will be a suitable principal part for 2/n if n is a multiple of 3, and ½.1/n if n is a multiple of 5, and that these and two or three other combinations of this kind explain all but fifteen cases of the table. But in the case of any one of the fifteen "exception numbers," if not in most of the other cases, the Egyptian actually got first the completion-term, and so, after all, placed that first as it is in the papyrus.

The multiplication required to get the completion-term of ⅔ is given in problem No. 11 of the Rhind papyrus, and so our author sees in problems 7, 9–15, the completion-table for ⅔. Less clearly he associates problems 8, 16–20 with the completion-term of ⅔. In this way he justifies the use of the word "completion" with No. 7, and supposes that the Egyptian may have had other completion-tables for the separation of 2/n in other cases.

Incidentally Neugebauer gives (pp. 14–15) a new explanation of problem 79 of the Rhind papyrus, which seems very natural and does away with speculation as to whether the Egyptians knew our algebraic formula for the sum of a number of terms of a geometric progression.

NEUGEBAUER, O., "Über die Konstruktion von *sp* "Mal" im mathematischen Papyrus Rhind," *Zeitschrift für Ägyptische Sprache . . .*, vol. 62, 1926, pp. 61–62.

SETHE, K. H., "Das Zahlwort 'fünf'," *Zeitschrift für Ägyptische Sprache . . .*, vol. 62, 1926, pp. 60–61.

VETTER, Q., *Jak se počitalo a Měřilo na Usvitě Kultury* [How one reckoned and measured at the dawn of culture], (Lidová Universita [people's university] series, vol. 15), Prague, 1926, 141 pp.

After a number of paragraphs dealing with Babylonian reckoning and measuring, pages 63–138 are occupied with an account of such things among the ancient Egyptians.

WIELEITNER, H., "Kannten die Ägypter den Begriff eines allgemeinen Bruches?", *Mitteilungen zur Geschichte der Medizin und der Naturwissenschaften*, vol. 25, 1926, pp. 1–4.

1927

GARDINER, A. H., *Egyptian Grammar, being an Introduction to the Study of Hieroglyphs*, Oxford, 1927, 28 + 595 pp., quarto.

Monumental work entirely superseding earlier works of the kind. Numbers are

Digitized by Google Original from UNIVERSITY OF MICHIGAN

treated on pages 191–206. There are more than one hundred quotations from the Rhind papyrus (pages 65–458) in illustration of grammatical questions, and most of them are translated into English.

Two notable earlier works are: (1) H. K. Brugsch, *Grammaire Hieroglyphique contenant les principes généraux de la langue et de l'écriture sacrées des anciens Égyptiens*, Leipzig, 1872, 136 pp.; "Du système de la numération," pp. 32–35; (2) A. Erman, *Ägyptische Grammatik mit Schrifttafel, Lieteratur, Lesestücken und Wörterverzeichnis*, dritte völlig umgestaltete Auflage, Leipzig, 1911, 16 + 324 + 24* pp.; "Zahlworte," pp. 129–133. For a supplement to this see P. Lacau, "Notes de Grammaire à propos de la Grammaire Égyptienne de M. Erman," *Recueil de Travaux relatifs à la Philologie et à l'Archéologie . . .*, vol. 35, 1913, pp. 80–81.

GILLAIN, O., *La Science Egyptienne: l'Arithmétique au Moyen Empire*. Paris, 1927, 8vo, 264 pp.

WIELEITNER, H., "War die Wissenschaft der alten Ægypter wirklich nur praktisch?", *Isis*, vol. 9, February, 1927, pp. 11–28.

The question in the title is answered by the author in the negative. In support of his thoroughly documented argument he refers to work of the Egyptians in art, in medicine, in astronomy, but especially in mathematics.

Digitized by Google Original from UNIVERSITY OF MICHIGAN

CHRONOLOGICAL LIST OF DOCUMENTS DISCUSSED

In more than one case the dates assigned below can be only very approximate; for example, since the Akhmîm papyrus is probably not earlier than 600, nor later than 900, the year half way between these years has been given. When printed sources gave no dates the leading living authorities were consulted for determining them. An attempt has been made to adjust to recent scholarship the dates in printed sources.

Approximate Dates B. C.	*Documents*	*Locations*
3500	Hierakonopolis mace (hieroglyphic)	Oxford
2200	Babylonian tablet 10201 from Nippur[1] (?) (cuneiform)	Philadelphia
2200	Tello tablet survey (cuneiform)	Constantinople
2150	Babylonian tablet 12648 from Nippur (?) (cuneiform)	Philadelphia
2070	Babylonian tablets (25) from Nippur (?) (cuneiform)	Philadelphia
2000	Akkadian clay tablet (cuneiform)	Berlin (?)
2000	Akhmîm tablets 25367, 25368 (hieratic)	Cairo
2000	Geometrical tablet 15285 (cuneiform)	London
2000	Mathematical tablets 85194, 85210 (cuneiform)	London
1900	Babylonian tablet CBS 8536 (cuneiform)	Philadelphia
1900	Book of the Dead, Introduction to Section 99, Berlin 9 and Cairo 28023 (hieratic)	Berlin and Cairo
1900	Kahun counting stick 372–59 (hieroglyphic)	London
1850	Golenishchev mathematical papyrus (hieratic)	Moscow
1850	Kahun papyri (hieratic)	London
1850	Thebes papyrus 6619 (hieratic)	Berlin
1800	Babylonian tablet 19797 from Nippur[2] (?) (cuneiform)	Philadelphia
1700	Bulak papyrus 11, 18 (hieratic)	Cairo
1650	Leather roll 10250[3] (hieratic)	London
1650	Rhind mathematical papyrus 10057, 10058 (hieratic)	London
1650	Rhind papyrus fragments 265 to go between 10057, 10058, above (hieratic)	New York
1650	Tablet 7798 (hieroglyphic)	Berlin
1465	Papyri 9784, 9785 (hieratic)	Berlin

[1] A question mark is here added because many assyriologists are in doubt as to whether some tablets, claimed by Hilprecht to have come from Nippur, were ever there.

[2] Transcribed but not interpreted.

[3] Wholly unpublished.

Digitized by Google

Original from UNIVERSITY OF MICHIGAN

Approximate Dates	*Documents*	*Locations*
B. C.		
1400	Karnak water clock (hieroglyphic)	Cairo
1350	Babylonian tablets (22) from Nippur (?) (cuneiform)	Philadelphia
1350	Rollin papyri (hieratic)	Paris
1315	Anastasy papyrus I 350 (hieratic)	Leyden
1000	Temple inscription, an ellipse	Luxor
280 to 90	Papyri XVI, XX, XXII, XXVII, XXVIII, XXX, XXXVII, XXXVIII, XL (demotic)	Manchester
250	Elephantine ostraca P 11999, P 12000, P 12002, P 12007, P 12008 (Greek)[1]	Berlin
210	Bilingual papyrus contract sale 10463 (demotic and Greek)	London
175	Tebtunis papyrus 87 (Greek)	Oxford[2]
100	Temple inscriptions (hieroglyphic)	Edfu
A. D.		
100	Papyrus CCLXVII (Greek)	London
140	Papyrus table of fractions (demotic)	London
150	Ayer papyrus (Greek)	Chicago
150	Papyrus 11529 (Greek)	Berlin
250	Oxyrhynchus papyrus 470 (Greek)	Dublin
350	Byzantine tablet of fractions 374–92 (Greek)	London
350	Michigan mathematical papyrus 621 (Greek)	Ann Arbor
350	Oxyrhynchus papyrus 186 (Greek)	Florence
350	Papyrus Gr 19996 (Greek)[1]	Vienna
550	Ostracon 480 (Coptic)	London
550	Ostracon 29750 (Coptic)	London
600	Papyrus 2241, nos. 22–28 (Greek)	London
600	Ostracon table of numbers, 6221 (Coptic)	Manchester
750	Akhmîm papyrus (Greek)	Cairo
1000	Parchment palimpsest Or 5707, no. 528 in catalogue (demotic and Greek)[3]	London

[1] Wholly unpublished.
[2] Later to be Berkeley.
[3] Mostly untranslated and only partially transcribed.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

INDEX OF AUTHORS

An entry such as "1918" after a name is the date of a main-entry under which reference to the author's, editor's, or copyist's contribution is to be found. An entry such as "[1918]" refers to a sub-entry for that date, fuller details being given elsewhere in the Bibliography. An entry such as "1887 [1883]" means that details regarding a publication of 1887 are listed under 1883.

Agostini, A. 1924 [1923]
Ahmes (A'h-mosè) [1877], 1898, [1899], [1921], [1922]
Akerblad, J. D. 1802 [1819]
Aldersmith, H. 1925 [1910]
al-Khowârizmî [1917]
Archibald, R. C. 1921 [1854], 1918 [1897], [1898], 1918, [1923], 1924 [1923]
Aurès, A. [1878], 1880
Babelon, E. 1882 [1878]
Baillet, J. 1892, [1893], [1895], [1897], [1899], [1901], [1903], [1905], [1906], [1911], [1916], [1921], [1922], [1923], [1925]
Barros Raja Gabaglia, E. de 1899
Bede the Venerable 1612 [1856]
Bell, H. I., 1922
Birch, S. 1868, [1870], [1875], [1898], [1899]
Blume, F. 1848 [1856], [1925]
Bobynin, V. V. [1857], [1881], 1882, [1882], 1887 [1883], 1889, 1890, 1895 [1892], 1894, [1894], 1896, 1897, 1898, 1899, [1899], 1905, [1905], 1906, 1909, [1909], [1925]
Borchardt, L. [1737], 1890 [1872], [1881], [1883], 1893, [1893], 1896, 1897, [1903], 1920 [1903], [1904], [1905], [1921], 1922, [1922], [1923], [1925]
Bortolotti, P. 1878
Brahmagupta 1879 [1856], [1917]
Braunmühl, A. von 1900
Bretschneider, C. A. 1870, [1873]
Brugsch, H. K. 1849, [1855], 1884 [1856], 1865, 1864 [1867], 1871 [1867], 1874, [1875], 1891, [1891], 1893 [1891], [1895], [1898], [1899], [1901], 1855 [1909], [1922], 1872 [1927]
Brunschvicg, L. 1912
Bubnov, N. M. 1899 [1856]
Budge, E. A. T. W. Prefatory note, [1898]
Cajori, F. 1897, 1917 [1897], 1922 [1906]
Calice, F. v. [1893], 1903, [1904]
Cantor, M. 1900 [1857], 1875, 1877, [1877], 1880, [1880], 1892 [1880], 1894 [1880], 1900 [1880], 1907 [1880], 1882 [1881], [1883], [1884], 1885, [1891], 1893 [1892], [1893], [1894], [1895], [1897], 1898, [1898], [1899], [1906], [1917], [1923], [1925]

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Chabas, F. 1869 [1868]
Chace, A. B. [1909], 1924 [1923]
Colebrooke, H. T. 1817 [1917]
Collignon, E. [1857], 1881, [1894]
Crum, W. E. 1902, 1921 [1902], 1905, [1916], 1922, [1922]
Curtiss, D. R. 1922 [1882]
Curtze, M. 1898 [1909]
Daressy, G. [1891], 1901, [1901], 1916 [1903], 1906, [1906], [1911], [1923]
Davidson, D. 1924 [1910]
Davies, T. S. 1850
Dawson, W. R. 1924 [1923]
Demme, C. 1886
De Morgan, A. 1872 [1854]
Devéria, T. [1819], 1862
Diophantus of Alexandria 1900
Dümichen, J. 1870 [1868], 1875
Ebers, G. 1878 [1877], [1898]
Edgar, J. E. 1910 [1910], 1913 [1910], 1923 [1910]
Edgar M. 1910 [1910], 1913 [1910], 1923 [1910]
Eisenlohr A. Prefatory note, 1897 [1868], [1874], 1875, [1875], 1876, [1876], 1877, [1877], [1878], 1881, [1881], 1882 [1881], [1883], [1884], [1885], 1891 [1891], [1891], 1892, [1892], [1893], [1894], [1895], 1896, [1896], [1897], [1898], 1899, [1899], [1901], [1903], [1906], [1911], [1912], [1916], [1922], [1923], [1925], 1896 [1925]
Eisenlohr, F. [1877]
Erman, A. 1885, 1891, [1891], [1897], [1898], 1899 [1900], 1904, 1911 [1927]
Everitt, C. 1910
Eyth, M. 1902 [1910], 1904 [1910], 1908 [1910], 1909 [1910]
Fagnano, G. C. *see* Toschi di Fagnano, G. C.
Favaro, A. 1876 [1875], [1878], 1879, [1881], [1899]
Fazzari, G. 1903, 1907 [1903]
Fehr, H. 1924 [1923]
Fettweis, E. [1906], 1923
Fibonacci *see* Leonardo Pisano (Fibonacci)
Gadd, C. J. [1906], 1922
Garbett, E. L. 1866 [1854]
Gardiner, A. H. 1906 [1868], [1872], 1906, 1927
Gegenbauer, L. 1894, 1895
Gerbert, Sylvester II 1867 [1856], 1899 [1856]
Gertz, M. C. 1892 [1706]
Gillain, O. 1926, 1927
Girard, P. S. 1809, 1822 [1809]
Goodspeed, E. J. 1898, [1916]
Goodwin, C. W. 1864 [1867], 1871 [1867], 1867, 1868, [1868], [1922]
Gow, J. 1884, [1899]
Gram, H. 1706
Grapow, H. 1917 [1918]
Graves, R. P. 1889 [1854]

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Greaves, J. 1737, [1737]
Greenstreet, W. J. 1924 [1923]
Grenfell, B. P. [1892], 1902 [1856], 1903, [1916], [1917]
Griffith, F. L. 1891 [1872], [1881], 1891, [1891], 1894 [1891], 1892, [1892], [1893], 1897. [1897], 1898 [1897], [1898], 1899 [1898], [1899], [1900], [1903], [1905], [1906], 1909, 1910, [1921], 1924 [1923], [1925]
Günther, S. 1876, 1878, 1881 [1880], 1908
Gunn, B. G. [1856], [1904], 1906], 1916 [1916], [1918], 1922, [1923], 1926 [1923], [1925]
Haas, A. 1894 [1903]
Haas, K. 1898
Haedicke, 1906 [1910]
Hall, H. R. 1905 [1856]
Hamilton, W. R. 1864 [1854]
Heath, T. L. [1881], [1883], [1884], [1892], [1893], [1898], [1900], 1910, 1921, [1923], [1925]
Herodotus [1854]
Heron of Alexandria 1912 [1856], [1856], [1903], 1914 [1903], 1914 [1917], 1903 [1923], 1914 [1923]
Herschel, J. 1860 [1854]
Hilprecht, H. V. 1906, [1906], [1923]
Hoppe, E. 1911
Hultsch, F. [1857], 1881 [1880], [1881], 1894 [1892], 1894, [1894], 1895, [1895], 1901, [1901], 1903, [1922], [1926]
Hunt, A. S. 1902 [1856], 1903
Ibrahim-Hilmy 1888 [1882]
Isely, L. 1895
Jacobi, C. G. J. 1849
Jarolimek, A. 1890, [1890], 1910, [1922]
Jéquier, G. 1922
Jomard, E. F. 1809, 1817 [1809], 1822 [1809], 1818, 1829 [1818], 1819
Junker, H. 1911
Karpinski, L. C. [1906], 1917, 1919, 1922 [1923], 1923 [1919], 1925, 1926
Kellogg, O. D. 1921 [1882]
Kenyon, F. G. 1898 [1855]
Kleppisch, K. 1921 [1910]
Krebs, F. 1899 [1900]
Lecau, P. 1913 [1927]
Lachmann, K. 1848 [1856]
Lauritzen, C. J. C. 1899
Lauth, F. J. 1868 [1868], 1877 [1877]
Leemans, C. 1846 [1868]
Lenormant, F. 1867, [1867]
Leonardo Pisano (Fibonacci) 1857, [1881], [1890], [1894], [1899], [1906], [1917]
Lepsius, K. R. 1856, 1865, 1866, [1875], [1878], 1884, [1884], [1891], [1893], [1898], [1923], [1925]
Lietzmann, W. 1918 [1916], 1924 [1923]
Limognelli, D. 1916 [1903]

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Lindemann, F. 1897
Löffler, F. [1906], 1912, 1918 [1912]
Loria, G. [1881], 1892, [1892], 1893, 1914 [1893], 1894, [1897], [1899], [1903], [1923]
Lutz, H. F. 1920 [1906]
Mahler, E. 1904
Mansion, P. [1881], 1888, 1898 [1888]
Mahāvirācārya [1917]
Mariette, A. [1868], 1872
Maspero, G. C. C. 1896 [1862]
Maspero, H. 1905 [1856]
Matthiessen, L. 1878, [1897]
Meissner, B. [1906], 1925
Mercer, S. A. B., 1925 [1923]
Meyerson, E. 1883
Michelsen, J. 1899 [1899]
Milhaud, G. 1893, [1893], 1911, [1911], [1925]
Miller, G. A. 1905
Möller, G. 1909 [1877], [1891], [1901], [1906], 1909, 1912 [1909], 1911, [1922], [1923]
Mommsen, T. 1884 [1884]
Muir, T. 1882 [1882]
Murray, M. A. 1925 [1918]
Naber, H. A. 1908 [1910]
Nairz, O. 1906 [1910]
Naville, E. [1918]
Neikes, H. 1907 [1910]
Neugebauer, O. 1925 [1923], 1926
Newton, I. 1737, 1744 [1737]
Noetling, F. 1921 [1910]
Olleris, A. 1867 [1856]
Oppert, J. 1896 [1925], 1897 [1925]
Peet, T. E. [1856], [1877], [1891], [1892], [1897], [1898], [1901], [1903], [1904], [1906], [1911], [1917], 1923, [1923], 1924, [1925]
Petrie, W. M. F. [1881], 1883, [1883], 1885 [1883], 1888 [1883], 1892 [1883], 1911 [1891], [1893], [1897], 1891 [1897], [1921]
Peyron, V. A. 1827 [1884]
Piehl, K. 1891
Pihan, A. P. 1860, [1912]
Pleyte, W. [1867], 1868, 1867 [1868], [1868], [1896]
Prisse D'Arenne, A. C. T. E. 1878 [1922]
Proctor, R. A. 1883 [1854]
Quaritch [1898], [1899]
Quibell, J. E. 1900
Quintino, G. di Saint 1825
Raṅgācārya [1917]
Ranke, H. 1923 [1885]
Revillout, E. 1891 [1892], 1895, [1901]

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Revillout, E. and V. 1881, 1881 [1881], [1881], [1883], [1885], [1893], [1894], [1898], [1899], [1921], [1925]
Reymond, A. 1924
Robbins, F. E. 1923, [1923], [1925], 1926
Rodet L. [1857], 1878, 1881, [1881], 1882, [1882], [1894], [1895], [1898], [1899], [1911], [1922], [1923]
Röber, F. 1854, [1854], 1855 [1854], [1922]
Röber, F. G. [1854], 1861 [1854], [1922]
Rosen, F. 1831 [1917]
Rougé, O. C. C. E. de 1868, [1872]
Rudio, F. [1901], 1907, 1908 [1895]
Rudorff, A. 1848 [1856]
Ruska, J. 1908
Sarton, G. 1924 [1923]
Schack-Schackenburg, H. [1881], 1882, [1882], [1897], [1898], 1899, 1900, 1902, [1903], 1904, [1904], [1925]
Scharff, A. 1922 [1872]
Schiaparelli, G. V. 1876 [1875], 1926 [1875]
Schubart, W. [1892], [1898], [1903], 1916
Sedgwick, W. T. 1917
Sethe, K. H. 1893, 1896, [1898], 1900, [1902], [1906], [1909], 1910, [1911], [1914], 1916, [1916], 1918, [1922], [1923], 1925 [1923], 1926
Seyffarth, G. 1829
Simon, M. [1881], 1904, 1905, 1907, 1909, [1923], [1925]
Sloley, R. W. 1922
Smith, D. E. 1907 [1906], 1923, 1924 [1923]
Smyly, J. J. 1902 [1856]
Smyth, C. P. 1867 [1737], [1854], 1864 [1854], 1867 [1854], 1874 [1854], 1877 [1854], 1880 [1854], 1890 [1854], [1910]
Spiegelberg, W. 1895 [1868], [1868], [1872], [1893], 1896, 1898, 1925 [1909], 1924 [1923]
Sterner, M. 1891
Struve, V. V. [1925]
Studnička, F. I. 1881 [1880]
Suter, H. 1873, [1898]
Sylvester II, *see* Gerbert
Sylvester, J. J. 1880, [1880], 1909 [1880], [1881], 1882, [1882], [1897], [1903]
Tannery, P. 1880, [1881], 1884, 1894 [1892], 1920 [1892], [1893], [1895], 1912 [1884], [1923]
Taylor, J. 1859 [1854], 1864 [1854], [1910]
Thompson, D. W. 1924 [1923]
Thompson, H. 1914
Thureau-Dangin, F. 1909 [1906], 1921 [1906], 1897 [1925]
Thurnell, G. 1866 [1854]
Toschi di Fagnano, G. C. 1715 [1903]
Touraeff, B. A. *see* Turaev, B. A.
Treutlein, J. P. 1881
Tropfke, J. 1902, 1903 [1902], 1921–1924 [1902], [1923], [1925]

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Tsinserling, D. P. [1917], 1925, [1925]
Turaev, B. A. [1880], [1898], 1917, [1917], [1923], [1925]
Tyler, H. W. 1917
Ungnad, A. [1906], 1916, [1923]
Vacca, G. 1909
Van den Berg, F. J. 1892 [1891]
Vaschchenko-Zakharchenko, M. E. 1883
Vasconcellos, F. de [1881], 1923, 1924 [1923], 1925, [1925]
Versluys, J. (?) 1881
Vetter, Q. [1881], [1906], 1922 [1906], 1923 [1906], 1922, 1923, 1925 [1923], 1924, 1925, [1925], 1926
Warren, C. 1900
Weidner, E. F. [1906], 1916, [1916], [1923]
Weill, R. 1925
Weissenborn, H. 1879 [1856]
Wessely, K. F. J. 1887
Weyr, E. 1884, [1885], [1925]
Wiedemann, A. 1884, [1898]
Wieleitner, H. 1924 [1923], 1925, 1926, 1927
Wilcken, U. 1884
Wilske, K. 1894
Wolff, G. [1923], 1924 [1923]
Zeuthen, H. G. 1893, 1896 [1893], 1902 [1893], 1912
Zimmern, H. [1906], 1916, [1916], [1923]
Zinserling, D. P., *see* Tsinserling, D. P.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

SUBJECT INDEX

The Index which follows should not be regarded as complete, but only as additional information intended to promote facility in the use of the Bibliography. References are only to the years under which the item occurs; a number enclosed in parentheses after a year indicates the number of references under the year in question.

Abhandlung zur Geschichte der Mathematik 1856, 1899, 1909
Abraham 1706
Académie de Nimes, *Mémoires* 1880
Académie des Inscriptions et Belles-Lettres, Paris, *Comptes Rendus* 1925
Académie des Sciences, Paris, *Comptes Rendus* 1867
Académie des Sciences, Leningrad, 1917; *Bulletin* 1925
Accademia di Scienze, Lettere, ed Arti, Modena, *Memorie* 1878, 1879, 1893
Accounts, *see* Papyri: *Bulak 18*, *Manchester*, and *Rollin*
Akademie der Wissenschaften, Berlin, *Abhandlungen* 1856, 1866; *Bericht über die . . . Verhandlungen* 1849
Akademie der Wissenschaften, Munich, *Sitzungsberichte* 1897
Akademie der Wissenschaften, Vienna, *Almanach* 1884; *Sitzungsberichte* 1885
Akhmîm 1906. *See also* Papyri, *Akhmîm*
Allgemeine Zeitung, Beilage 1877 (2), 1880
American Journal of Mathematics 1880
American Journal of Philology 1898
American Journal of Semitic Languages and Literatures 1906
American Mathematical Monthly 1854, 1882 (2), 1897, 1898, 1906, 1917, 1918, 1923
American Mathematical Society, *Bulletin* 1906, 1923
Amtliche Berichte aus den kōniglichen Kunstsammlungen, Berlin 1916
Ancient Egypt 1914, 1917 (2), 1922
Annales du Service des Antiquités de l'Égypte 1923
Annali di Matematica 1917
Annals of Archaeology and Anthropology 1923
Archiv der Mathematik und Physik 1905, 1907
Archiv für Papyrusforschung und verwandte Gebiete 1903
Archivio di Storia della Scienze 1906, 1925
Areas, of circles, *see* π; of quadrilaterals and triangles 1856 (3), 1875, 1898, 1916, 1923, 1925
Associação Portuguesa para o Progresso das Sciencias 1923
Ateneum 1883
Athenaeum 1854
Aula, Die 1895
Babylonian Expedition of the University of Pennsylvania 1906
Babylonian mathematics 1893, 1906, 1912, 1916 (3), 1917, 1922, 1923, 1925, 1926 (2)

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Baking calculations 1868
Baudhâyna 1886
Bayerische Akademie der Wissenschaften, *see* Akademie der Wissenschaften, Munich
Berlin Museen 1881, 1900, 1906, 1916
Berliner Philologische Wochenschrift 1892
Bibliotheca Mathematica 1883, 1889, 1890, 1892, 1893, 1894, 1895, 1896, 1901
Bibliothek der klassischen Altertumswissenschaften 1911
Bibliothèque de l'École des Hautes Études 1922
Bibliothèque Egyptologique 1862
Bibliothèque Nationale 1868
Binary scale 1897
Bobynin, V. V. Prefatory note
Böhmische Gesellschaft der Wissenschaften, *see* Gesellschaft der Wissenschaft, Prague
Bollettino di Bibliografia e di Storia delle Scienze Mathematiche (Loria) 1909
Bollettino di Matematica, Sezione storico-bibliografia 1923
Boncompagni, B 1857
Book of the Dead 1918
British Museum. Prefatory note, 1856, 1867, 1874, 1876, 1877, 1878, 1892, 1898, 1905, 1906, 1922, 1923
British School of Archaeology in Egypt, *Studies* 1891
Builder, The 1854
Bulak 1872
Bulletin Critique 1878
Bulletin de l'Institut Égyptien 1903
Bulletin des Sciences Mathématiques 1884
Bullettino di Bibliografia e di Storia della Scienze Matematiche e Fisiche (Boncompagni) 1875
Byzantine mathematics 1892, 1914
Cairo tablets, *see* Egyptian Museum
Cambridge Ancient History 1924
Časopis pro Pěstování Mathematiky a Fysiky 1880, 1906, 1922, 1923, 1924
Chronique d'Égypte 1926
Circle, area, *see* π
Classical Philology 1923, 1925
Clepsydra 1903
Cole, J. H. 1856
Cone frustum, volume 1892, 1903 (3)
Coptic documents, *see* Greek and Coptic documents
Counting stick 1897
Cuneiform Texts from Babylonian Tablets, &c., in the British Museum 1916
Cylinder 1897
Dansk Biografisk Lexikon 1706
Demotic 1829, 1892, 1895, 1909
Description de l'Égypte . . . 1809 (4), 1818 (2)
Deutsche Gesellschaft für Geschichte der Medizin und der Naturwissenschaften. *Mitteilungen zur Geschichte der Medizin und Naturwissenschaften, see Mitteilungen,* etc.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Deutsche Mathematiker-Vereinigung, *Jahresberichte* 1923
Deutsche Morganländischen Gesellschaft, *Zeitschrift* 1855
Discovery 1923
Division 1895, 1906, 1923
Dnevnik ix-go Syezda Russkikh Estestvoispuitatelei i vrachei 1905
Documents, *see* Clepsydra, Counting stick, Edfu, Leather roll, Hierakonopolis, Luxor, Ostraca, Papyri, Parchment, Tablets
Dowries 1909
Dublin, University, Trinity College 1903
Edfu Temple 1854 (2), 1856
Egypt Exploration Fund 1903
Egypt Exploration Society 1895
Egyptian angle and rule 1854
Egyptian arithmetic 1882, 1892, 1894, 1926, 1927
Egyptian Museum 1891, 1900, 1901, 1903, 1906, 1916, 1923
Egyptian University 1917
Ellipse, Egyptian construction 1896
Encyclopædia Britannica 1910 (2)
Eneström, G. Prefatory note
Enseignement Mathématique, L' 1906, 1923
Equations 1878, 1897, 1898, 1900, 1902, 1925; *see also* false position
False position 1878, 1882, 1900, 1906
Fayûm, The. Prefatory note, 1881, 1898, 1923
Finger-numbering 1918, 1922
Fiziko-mathematicheskiya Nauki v ikh Nastoyashchem i Proshedshem 1892, 1897, 1898
Folk Lore 1918
Fractions 1849, 1857, 1865, 1868, 1880, 1881, 1882 (2), 1884 (2), 1888, 1890, 1892, 1894, 1895, 1896, 1898, 1899, 1901 (2), 1905, 1906, 1907, 1909, 1911, 1914, 1916, 1921, 1922, 1923, 1926 (2)
Geometry 1706, 1850, 1854, 1856, 1867, 1868, 1870, 1876, 1881, 1883, 1884, 1893, 1894, 1925; *see also* Areas, Cone frustum, Ellipse, Golden section, Heptagon, π, Quadrilaterals
Gesellschaft der Wissenschaft, Leipzig, *Abhandlungen* 1895
Gesellschaft der Wissenschaft, Prague, *Abhandlungen* 1878; *Jahresberichte* 1923
Giornale de' Letterati d'Italia 1903
Giornale di Matematiche 1894
Gizeh pyramids, temples, matabas 1854, 1883, 1890, 1916, 1921
Golden section 1910
Gol'denberg, A. F. 1882
Golenishchev, V. S. 1880, 1917, 1925
Grammar, Demotic, Egyptian 1868, 1909, 1927
Greek and Coptic documents, *see* Ostraca, Papyrus-*Akhmîm*, Papyrus-*Ayer*, Papyrus-*Berlin* (11529), Papyrus-*Bilingual*, Papyrus-*London* (267, 2241), Papyrus-*Tebtunis*, Papyrus-*Vienna*, Parchment palimpsest
Greek debt to Egypt 1880, 1884, 1893, 1921, 1926
Griffith, F. L. Prefatory note
Hemisphere, volume 1897, 1899

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Heptagon 1854
Hermes 1884
Hermitage Museum 1880
Heron of Alexandria 1884, 1893, 1903, 1916, 1917, 1923
Hierakonopolis mace 1900
Hieratic documents, *see* Leather roll, Papyri-*Berlin* (9, 6619, 9784, 9785), Papyri-*Bulak*, Papyri-*Cairo*, Papyri-*Golenishchev*, Papyri-*Kahun*, Papyri-*Leyden*, Papyri-*Rhind*, Papyri-*Rollin*, Tablets-*Akhmîm*
Hieroglyphic documents, *see* Clepsydra, Counting stick, Edfu, Hierakonopolis, Tablets-*Berlin*
Historische Gesellschaft zu Dresden, *Historische Untersuchungen Ernst Förstemann zum fünfzigjährigen Doctorjubiläum gewidmet* 1894
History of Science Society, *Isis*, see *Isis*
Horus eye notation 1911, 1922
Infinity 1897
L'Institut Égyptien, *Bulletin* 1903
Instituto Superiôr de Agronomia, *Anais* 1924 [1921]
Isis 1923 (2), 1927
Istituto Lombardo di Scienze e Lettere, *Rendiconti* 1875
Jahrbücher für Classische Philologie 1880
John Rylands Library 1909
Journal Asiatique 1881 (2), 1906
Journal of Egyptian Archaeology 1916, 1923
Kultur der Gegenwart 1912
Leather roll (hieratic). Prefatory note
Leipziger Literatur-Zeitung 1829
Liber Abaci (Leonardo) 1857, 1881, 1890, 1899, 1906
Lidová Universita 1926
Literarisches Centralblatt für Deutschland 1877
Liverpool, University, *Annals of Archaeology and Anthropology*, see *Annals*, etc.
London, University College 1897, 1902, 1914
Luxor 1867, 1896
Manchester University Museum, 1902
Manning, H. P. Prefatory note, 1882
Manuali Hoepli 1893
Mastabas 1883, 1921
Matematicheskiya Listok 1882
Matematisk Tidsskrift 1923
Mathematical Association of America, *American Mathematical Monthly, see American*, etc.
Mathematical Gazette 1923
Mathematical Institute, Mittag-Leffler's 1882
Mathematical Questions . . . from the 'Educational Times' 1882
Mathematicians, International Congress, *Verhandlung* 1904
Mathematikai és Physikai Lapok 1904
Mathematische Bibliothek 1912
Mathematisch-Physikalische Bibliothek 1923

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Measures 1737, 1809 (5), 1856 (2), 1866, 1875 (2), 1876, 1878, 1880, 1884, 1892, 1895, 1896 (2), 1900, 1903, 1905, 1909, 1911, 1926
Mechanics Magazine, The 1850
Mémoires publiés par les membres de la Mission Archéologique Française au Caire 1892
Michigan, University, *Studies* 1926
Mitteilungen aus der Sammlung der Papyrus Erzherzog Rainer 1887
Mitteilungen zur Geschichte der Medizin und Naturwissenschaften 1908, 1923, 1926
Mittelschule 1894
Multiplication 1895, 1897, 1905, 1906, 1922
Museum of Fine Arts, Moscow 1917
Museum, Egyptian, *see* Egyptian Museum
National-Bibliothek, Vienna. Prefatory note
Nature 1898, 1921, 1923
Naturwissenschaftlicher Verein in Karlsruhe, *Verhandlungen* 1881 (2)
New York Historical Society 1923
Newberry, P. E. 1898, 1923
Nieuw Archief voor Wiskunde 1891
Nippur 1906
Numerals and their signs 1818 (2), 1819 (2), 1825, 1829, 1855, 1860, 1862, 1867, 1868 (2), 1874, 1881, 1887, 1891, 1895, 1896, 1897, 1898, 1900 (2), 1909 (2), 1910, 1912, 1916, 1922, 1926 (2), 1927
Nyt Tidsskrift for Matematik 1899
Oesterreichische Mittelschule 1898
Österreichischer Ingineur- und Architekten-Verein, *Wochenschrift* 1890; *Zeitschrift* 1894
Orientalistische Litteratur-Zeitung (*Literaturzeitung*) 1898 (2), 1916 (3), 1923
Orientalists, International Congress, *Transactions* 1876
Ostraca 6221 (Coptic)—1902; 480 (Coptic)—1902; 29750 (Coptic)—1856; Elephantine (P11199, P12000, P12002, P12007, P12008)—1916; see also under Tablets
Ottoman Museum 1925
Oxyrhynchus, *see* Papyri-*Oxyrhynchus*
π approximation 1875, 1878, 1886, 1896, 1903, 1905, 1907, 1909, 1910
Paleography, Egyptian 1909
Palestine Exploration Fund, Quarterly Statement 1900
Papyri
Akhmîm 1892, 1893, 1894, 1895 (2), 1899, 1906, 1922, 1923, 1925
Ayer 1898, 1916
Berlin 9—1918; 6619—1897, 1900, 1902, 1904, 1916; 7798—1881; 9784, 9785—1906; P11529—1916
Bilingual 10463—1892
Bulak 11—1925; 18—1868, 1872, 1877, 1909
Cairo 28023—1918
Golenishchev 1880, 1897, 1917, 1923, 1925
Illahun 1897, 1917
Kahun 1897, 1898, 1899, 1900, 1904, 1906, 1925 (2)
Leyden I, 350—1868
London 267 (Greek)—1856; (demotic)———1895; 2241—1922
Manchester XVI, XX, etc. (demotic)—1909

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Michigan 621—1923, 1925
Oxyrhynchus 186—1903; 470—1892, 1903
Petrie 1897
Rhind 10057, 10058—Prefatory note, 1867, 1874, 1875, 1876, 1877, 1878 (2), 1879, 1881 (4), 1882; 1883 (4), 1884 (3), 1886, 1888, 1890, 1891 (2), 1892 (4), 1893, 1894 (3), 1895 (2), 1897 (2), 1898 (2), 1899, 1900 (2), 1903, 1904 (3), 1906 (2), 1909 (2), 1911, 1912, 1916, 1917, 1918, 1919, 1923 (6), 1924, 1925, 1926
Rhind, particular problems (52) 1856; (79) 1857; (49–52) 1877; (35–38, 79) 1881; (28, 35–38, 82) 1882; (56–60) 1883; (86, 87) 1891; (56–60) 1893; (48) 1896; (40, 43, 64) 1897; (43) 1899; (24, 28, 33, 69) 1900; (43, 60) 1903; (26, 35–38, 41, 60) 1904; (24, 31–34, 40, 62) 1906; (35, 37, 38, 43, 47, 64, 66, 68, 69, 70, 71, 75, 80, 81, 82, 83, 84) 1911; (40) 1912; (56–60) 1921; (56–60) 1922; (51) 1923; (56, 59) 1924; (43, 51, 52, 62) 1925; (79) 1926
Rollin 1868, 1872, 1896, 1909
Tebtunis 87—1856
Vienna Gr. 19996—Prefatory note
Parchment manuscript Or 5707—1905
Paterius 1901
Peet, T. E. Prefatory note
Pennsylvania, University 1906
Periodico di Matematiche 1923
Petrie, W. M. F. 1897
Philosophical Magazine 1854
Pitagora, Il 1903
Plato's geometrical number 1906
Progressions, Arithmetic 1897, 1923; Geometric 1857, 1881, 1926 (problem no. 79)
Prometheus 1910 (2)
Pyramids 1737, 1854, 1883, 1885, 1890, 1899; *frustum volume*—1917, 1925; *mysticism*—1890, 1910 (9), 1922; *seked*—1881, 1883, 1884, 1885, 1893, 1896, 1903, 1921, 1924. *See also* Gizeh
Quadrilateral figures 1856 (3), 1878
Reclam's Universal Bibliothek 1891
Recueil de Travaux relatifs à la Philologie et à l'Archéologie Égyptiennes et Assyriennes 1868, 1882, 1896, 1906, 1927
Revue Archéologique 1862
Revue d'Assyriologie et d'Archéologie Orientale 1906, 1922, 1925
Revue de l'Égypte Ancienne 1925
Revue des Études Grecques 1892
Revue Égyptologique 1881 (2)
Revue Philosophique de la France et de l'Étranger 1880
Revue Scientifique 1903
Rhind, A. H. 1867. *See also* Papyrus-*Rhind*
Russian literature. Prefatory note, 1882, 1883, 1893, 1897, 1898, 1905 (2), 1909, 1925
Sächsische Gesellschaft der Wissenschaft, *see* Gesellschaft der Wissenschaft, Leipzig
School Science and Mathematics 1905, 1919
Science 1923
Science Progress 1923

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Scientia 1919
Seked, *see* Pyramids—*seked*
Service des Antiquités en Égypte, *Annales* 1923
"Seven" problems 1857, 1881. *See also* Heptagon
Società Italiana per la Ricerca dei Papiri Greci e Latini in Egitto 1903
Société des Sciences Naturelles de Neuchâtel, *Bulletin* 1895; *Bibliothèque Universelle, Archives des Sciences Physiques et Naturelles* 1895
Société Mathématique de France, *Bulletin* 1878
Société Philologique, *Bulletin des Séances* 1882
Société Philomathique de France, *Bulletin* 1881
Société Scientifique de Bruxelles, *Annales* 1888
Society of Biblical Archæology, *Proceedings* 1868, 1881, 1891 (3), 1892 (3), 1898, 1899
Society of Oriental Research, *Journal* 1923
Sorites 1880, 1882
Square root 1854, 1884, 1897, 1908, 1916
Struve, V. V. 1917
Surveying 1856, 1867, 1875, 1909, 1925
Tablets *Akhmîm* (25367, 25368)—1891, 1901, 1923; *Babylonian* (CBS 8536)—1906; (85120, 85194)—1916; (15285) 1922; *Berlin* (7798)—1881; *Byzantine*—1914; *Constantinople* (Tello)—1925, *Nippur* (?)—1906
Tebtunis, *see* Papyri-*Tebtunis*
Thales 1880
Thesaurus Inscriptionum Aegyptiacarum (Brugsch) 1856
Tijdschrift voor Vormleer, Rekenkunde en de Beginseln der Wiskunde 1881
Time reckoning, *see* Clepsydra
Times Literary Supplement 1923
Triangular fields, *see* Areas
Trinity College, Dublin, 1903
Unterrichtsblätter für Mathematik und Naturwissenschaften 1923
Urkunden des aegyptischen Altertums 1918
Valentin, G. Prefatory note
Volumes 1892, 1897, 1899, 1903, 1916
Vorderasiatisch-ägyptische Gesellschaft zu Berlin 1922
Vossische Zeitung 1891
Wissenschaftliche Gesellschaft in Strassburg, *Schriften* 1916
Zeitschrift für Ägyptische Sprache und Altertumskunde 1865 (2), 1867 (3), 1868 (6), 1872 (3), 1874 (2), 1875 (4), 1884, 1891, 1893 (2), 1896 (2), 1897, 1898, 1899 (2), 1900 (2), 1902, 1903, 1904 (2), 1906, 1910, 1911 (2), 1918, 1922, 1926 (2)
Zeitschrift für Mathematik und Physik 1856, 1881, 1886, 1892, 1899, 1909
Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht 1916, 1923 (2)
Zhurnal Ministerstva Narodnago Prosvycshcheniya 1905, 1909

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

GENERAL INDEX

Headings of the subdivisions of the Introduction and Free Translation are not given here. A few references are given to matters of interest in the Bibliography, but names of authors and publications and a fuller list of subjects will be found in the indices to the Bibliography itself.

'*aha*', quantity, 25.
akhet, season of inundation, 44.
A'h-mosè, 1, 41.
Akhmîm papyrus, 7, 9.
Amenemhet III, 1.
Amenhotep I, 46.
Arabs, used false position, 10, 13; unit fractions, 10.
Archibald, on the Egyptian's interest in mathematics for its own sake, 43.
Areas, of a circle, trapezoid, triangle, 36; the rectangle equivalent to a given triangle or trapezoid, 37; question of altitude or side of a triangle or trapezoid, Sethe's opinion, Gunn's theory, 36, 37, 183; area of a quadrilateral, 132, 162; areas in the Golenischev papyrus, 187.
Arithmetic, integers and reciprocal numbers, 3; concrete and abstract numbers, see Multiplication; see also Completion, Division, False position, Fractions.
'A-user-Rê', 1.

Babylon, tables, 40, see 171; geometry, diagonal of a rectangle, 177; geometrical figures, 181.
Bloc extractif, 9.
Bobynin, on the Rhind papyrus, 172.
Borchardt, argument on the lines in a pyramid, 37.

Circle, area, 35, 36.
Completion, 13; second step in multiplication of the second kind, 5; term used by mistake before Problems 7–20; 13, 23.
Coptic writing, 1.
Cubit, determined by Sir Isaac Newton, 127; see Measures of length; cubit-strip, 33.
Cylinder, volume, 35; two methods of determining it in khar, 88; see also 159, 167.

Demotic writing, 1.
Diophantus, used false position, 13.
Division, multiplication problems of the second kind, in which the multiplicand and product are given, 5; problems in which the multiplier and product are given, 6, 7; see Multiplication.
Division of 2 by odd numbers, the same as multiplying their reciprocals by 2: 13, 18; is a multiplication of the second kind, 14; solution generally regarded as proof, theory of Peet, Hultsch, 14; classification of solutions, 16; discussion by Sylvester, Loria, 15; third step in the solution, Gunn's theory of the way of writing this, 17; the process of Leonardo of Pisa, 134, 150; the word "find" used in the more difficult cases, 20.
Division of the numbers 1–9 by 10, table, how derived, 22–23.

Ebers papyrus, 46.
Egyptians compared with the Greeks, 39; their mathematical knowledge, 39; they

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

used a method of trial, had rules, 39, tables, 40; generalized from particular cases, 40; their mistakes, mistakes of copying in the Rhind papyrus, 40; their incorrect methods, 41; correction by taking away 1/10, 42, 94; not merely practical, many problems practical only in form, 42; their scientific interest shown in the Edwin Smith papyrus, 43; their mathematics not a study of mystery, 15–16.

Eisenlohr, numbering the problems, 71; discussion of Problem 43, 88.

Epagnomenal days, 43.

False position, 10–13, 164, 170; with the Arabs, 10, 13; in Problem 35, 11; in Problem 40, 12; the nature of quantities kept in mind by it, 7, 11.

"Find" word put in with the more difficult cases of the division of 2 by odd numbers, 20; "finding" used in expressions stating multiplications of the second kind, 5.

Finger, 1/4 of a palm, 34, 167.

Fractions, unit, or reciprocal numbers, 3; complementary fractions, 7; uneven fractions, 25; method of applying fractions to a particular number 7–10; views of Rodet, Hultsch, Peet, 9; method used once for division and once for multiplication, but generally for addition or subtraction, 9, 79; multiplication of fractions, two forms of statement, 100; 2/3 halved to get 1/3, 4; the reciprocal of 1 1/2, 4, 7; 2/3 of a fraction, rule, 25; "Horus eye" fractions, 31, see Measures of capacity; expression of a quantity in a series of unit fractions, the series called *sorites* by Sylvester, 142, 146; the fractions called *quantièmes* by P. Tannery, 149.

Gardiner calls the *khet* a rod, 33.

Geometrical figures, plan of a field, 187; figures on a Babylonian tablet, 181.

Geometry, origin according to Herodotus, 127; of the great pyramid, 131.

Golenishchev papyrus, 43, 93, 178–179, 187–188.

Griffith, theory about 2/3, 7.

Gunn, about 2/3, 4 and 184; expression for division, 5; for taking a reciprocal, 6; on the way of writing the third step in the division of 2 by an odd number, 17; on uneven fractions, 25; calls the *hekat* a gallon, 31, 182; theory in regard to *meret*, 37, 183; interpretation of Problems 54 and 55, 96, of Problem 80, 113; on enigmatic writing, 118.

Hau, same as *'aha'*, 25; *hau*-reckoning, 43.

Hekat, hînu, 31; see Measures of capacity.

Herodotus on Egyptian geometry, 127.

Heptagon, regular, construction by Röber, examined by Hamilton, 131.

Hieratic, hieroglyphic, 1.

"Horus eye" fractions, 31; mythological tradition, 175; see Measures of capacity.

Hultsch, views on applying fractions to a particular number, 9; on the method of dividing 2 by odd numbers, 15.

Hyksos kings, 1, 43.

Khar, 32; see Measures of capacity.

Khet, 33; see Measures of length.

Leonardo of Pisa, problem of geometrical progression, 112, 134; process of separating 2 divided by an odd number into unit fractions, 134, 150.

Loria, on the method of dividing 2 by odd numbers, 15.

Measures of area, *setat*, cubit-strip, 33; method of writing expressions for an area, 33.

Measures of capacity, *hekat, henu, ro*, "Horus eye" fractions, 31; Peet calls the *hekat* a bushel, Gunn calls it a gallon, 31; *hînu* represented by certain vases, 31; double *hekat*, quadruple *hekat, khar*, 32; method of writing expressions for quantity, 32; tables of

Digitized by Google

Original from UNIVERSITY OF MICHIGAN

fractional parts of a *hekat*, Cairo tablets, 170, 182; Problems 47, 80, and 81: 40.
Measures of length, cubit, 32; *khet*, called rod, chain, 33; palm, finger, 34.
Meret, side of a triangle, trapezoid, 36.
Moscow papyrus, 142, 178; see Golenishchev papyrus.
Multiplication, mostly by 2, 10, ⅔, ½, and 1/10, 3, 4; checks used in multiplications, 4, 50; multiplicand and product may be things of some kind (concrete numbers), multiplier must be a mere number (abstract number), 6; this distinction is sometimes made clear by the notation, 30; multiplication by ½ to infinity, 159.
Multiplication of the second kind, when the multiplicand and product are given (division), 5, see Vetter, 185; the three steps, 5; rule for the third step, 6; Egyptian method of stating the problem, 5.
Multiplication problems when the multiplier and product are given, Problems 24–38, 25; first method, interchange of multiplier and multiplicand, 6, 25; second method, false position, 7, 25; see False position; proofs of certain of these problems, 27.

Newberry, discoverer of the New York fragments of the Rhind papyrus, 2, 161, 183.
Neugebauer, theory of Problems 7–20, 23; on Problem 79, 30.
Numbers and their signs, 136, 172.

Palm, ¼ of a cubit, 34.
Papyri, Akhmîm, 7, 9; Ebers, 46; Golenishchev (Moscow), 43, 93; Rhind, see Rhind papyrus; Smith, Edwin, 2, 43.
Peet, arranged the New York fragments, 2; views on applying fractions to a number, 9; on Problem 40, 12; on the method of dividing 2 by odd numbers, 15; calls the *hekat* a bushel, 31, the *khet* a chain, 33, the *seked* batter, 37; explains mistakes of copying, 40; the omission in Problem 28, 70; discussion of the mistake in Problem 43, 88; on the division of *setat* by *setat*, 96; two forms of statement for multiplication of fractions, 100; on the misplaced column in Problem 81, 114.
Pefsu, 105.
Prôt, season of going forth, 44.
Problems, '*aha*' problems, 25; typical, 28, 29; inverse, 35; numbering by Eisenlohr, 71; problems equivalent to simultaneous equations, 13, 159, 162, 164, 166, 188.
Progression, arithmetical, explanation of Problem 40, 12; an example in the Petrie papyri, 159; a geometrical progression with ratio 7; 112, 134, 144.
Pyramid, great, its geometry, 131; pyramid, angles, 147, 180; mysticism, 174; lines in a pyramid, Borchardt's argument, 37; *seked*, batter, 37.

Quantièmes, unit fractions, 149.

Reciprocal numbers, see fractions.
Rectangular parallelepiped, volume, 35.
Rhind papyrus, date, description, history, 1; corrections, alterations, patches, 41; New York fragments, 1; first announcement of its existence, 135; supposed by E. and V. Revillout to be a pupil's copybook, 39, 143; as characterized by Bobynin, 172; three divisions, 2; references to the calendar, 43; the expression, "I return filled," in Problems 35–38, 169; Problem 43: 159, 164, 168, 187; Problem 60, 168; Problem 62, 170; Numbers 86 and 87, 151.
Ro, 31; see Measures of capacity.
Rodet, on applying fractions, 9; on Problem 79, 112.

Saint Ives, children's rhyme about, 112.
Schack-Schackenburg, explained Problem 43, 88.
Seasons, see Year.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Seked, ratio in a pyramid, 37; see Pyramid.
Setat, 33; see Measures of area.
Sethe, complementary fractions, 7; on Peet's view of fractions, 10; view in regard to *meret*, 37.
Sha'ty, unit of value, 101, 189.
Shômu, summer season, 44.
Sirius, heliacal rising, 44; used to determine dates, 46.
Smith, Edwin, papyrus, 2, 43.
Sobhy, *hînu* determined from vases, 31.
Sorites, series of unit fractions, 142, 146.
Square root, approximate determination, 177.
Sylvester, method of dividing 2 by odd numbers, 15.

Tables, division of 2 by odd numbers, 21; division of the numbers 1–9 by 10, 22; fractions of a hekat, Problems 47, 80, 81: 40; see 170, 182.
Trapezium, 138, 187; see Trapezoid.
Trapezoid, area, 36; see Areas.
Triangle, area, 36; areas of sections, 94; see Areas; similar triangles, relation of sides, 37.
Two-thirds, see Fractions.

Volume of a rectangular parallelepiped, 35; of a cylinder, 35, 159; method of determining the volume of a cylinder directly in khar, 88, 159; volume of a frustum of a pyramid, Golenischev papyrus, 178; of a frustum of a cone, 167.

Wieleitner, on the Egyptian's interest in mathematics for its own sake, 43, 191.
Writing, Egyptian, four forms, 1; enigmatic writing, 118.

Year, Egyptian, the three seasons, epagomenal days, 43; "Wandering year," 44; Julian year, 44; siderial year, 45; Sirius year, 44–45; solar year, 45; equation for a star year, 45–46.

Digitized by Google
Original from UNIVERSITY OF MICHIGAN

Digitized by Google

Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN

Digitized by Google
Original from
UNIVERSITY OF MICHIGAN