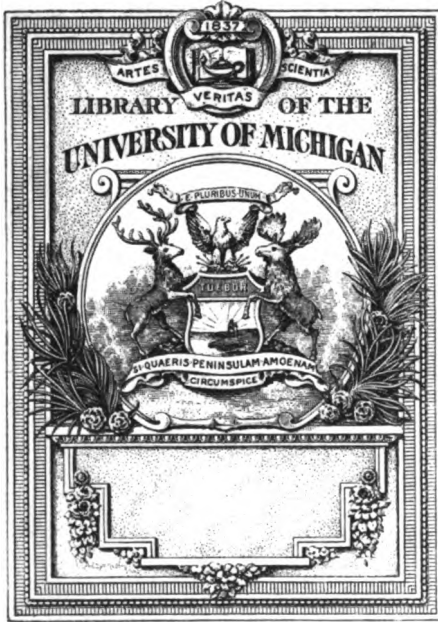
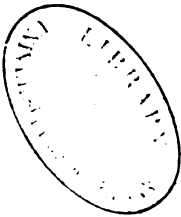
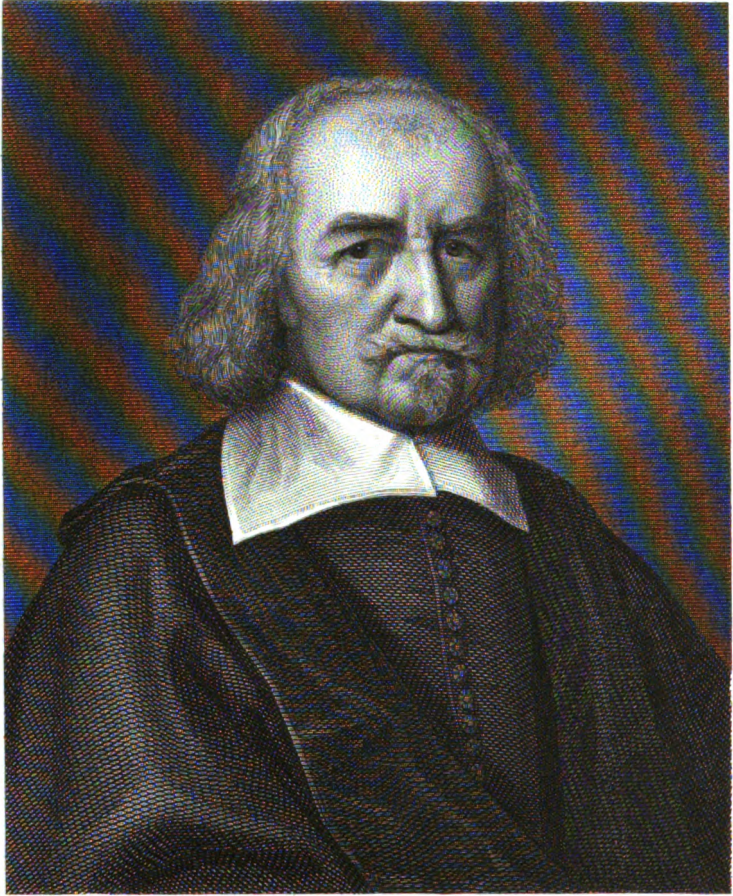


*The English works of Thomas
Hobbes of Malmesbury*

Thucydides, Homer





THOMAS HOBBES.

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1969

THE
ENGLISH WORKS
OF
THOMAS HOBBS

OF MALMESBURY;

NOW FIRST COLLECTED AND EDITED

BY

SIR WILLIAM MOLESWORTH, BART.

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To the Right Honorable
my very good Lord
the Earl of Devonshire

London

TO
GEORGE GROTE, ESQ.

M.P. FOR THE CITY OF LONDON.

DEAR GROTE,

I dedicate to you this edition of the Works of Hobbes ; first, because I know you will be well pleased to see a complete collection of all the writings of an Author for whom you have so high an admiration. Secondly, because I am indebted to you for my first acquaintance with the speculations of one of the greatest and most original thinkers in the English language, whose works, I have often heard you regret, were so scarce, and so much less read and studied than they deserved to be. It now, therefore, gives me great satis-

DEDICATION.

faction to be able to gratify a wish, you have frequently expressed, that some person, who had time and due reverence for that illustrious man, would undertake to edit his works, and bring his views again before his countrymen, who have so long and so unjustly neglected him. And likewise, I am desirous, in some way, to express the sincere regard and respect that I feel for you, and the gratitude that I owe you for the valuable instruction, that I have obtained from your society, and from the friendship with which you have honoured me, during the many years we have been companions in political life.

Yours, truly,

WILLIAM MOLESWORTH.

February 25th, 1839.
79, Eaton Square, London.

ELEMENTS OF PHILOSOPHY.

THE FIRST SECTION,

CONCERNING BODY,

WRITTEN IN LATIN

BY

THOMAS HOBBS OF MALMESBURY,

AND

TRANSLATED INTO ENGLISH.

THE
TRANSLATOR TO THE READER.

IF, when I had finished my translation of this first section of the Elements of Philosophy, I had presently committed the same to the press, it might have come to your hands sooner than now it doth. But as I undertook it with much diffidence of my own ability to perform it well; so I thought fit, before I published it, to pray Mr. Hobbes to view, correct, and order it according to his own mind and pleasure. Wherefore, though you find some places enlarged, others altered, and two chapters, xviii and xx, almost wholly changed, you may nevertheless remain assured, that as now I present it to you, it doth not at all vary from the author's own sense and meaning. As for the Six Lessons to the Savilian Professors at Oxford, they are not of my translation, but were written, as here you have them in English, by Mr. Hobbes himself; and are joined to this book, because they are chiefly in defence of the same.*

* They will be published in a separate volume, with other works of a similar description. W. M.

THE AUTHOR'S EPISTLE DEDICATORY,
TO THE
RIGHT HONORABLE, MY MOST HONORED LORD,
WILLIAM, EARL OF DEVONSHIRE.

THIS first section of the Elements of Philosophy, the monument of my service and your Lordship's bounty, though, after the Third Section published, long deferred, yet at last finished, I now present, my most excellent Lord, and dedicate to your Lordship. A little book, but full; and great enough, if men count well for great; and to an attentive reader versed in the demonstrations of mathematicians, that is, to your Lordship, clear and easy to understand, and almost new throughout, without any offensive novelty. I know that that part of philosophy, wherein are considered lines and figures, has been delivered to us notably improved by the ancients; and withal a most perfect pattern of the logic by which they were enabled to find out and demonstrate such excellent theorems as they have done. I know also that the

hypothesis of the earth's diurnal motion was the invention of the ancients; but that both it, and astronomy, that is, celestial physics, springing up together with it, were by succeeding philosophers strangled with the snares of words. And therefore the beginning of astronomy, except observations, I think is not to be derived from farther time than from Nicolaus Copernicus; who in the age next preceding the present revived the opinion of Pythagoras, Aristarchus, and Philolaus. After him, the doctrine of the motion of the earth being now received, and a difficult question thereupon arising concerning the descent of heavy bodies, Galileus in our time, striving with that difficulty, was the first that opened to us the gate of natural philosophy universal, which is the knowledge of the nature of *motion*. So that neither can the age of natural philosophy be reckoned higher than to him. Lastly, the science of *man's body*, the most profitable part of natural science, was first discovered with admirable sagacity by our countryman Doctor Harvey, principal Physician to King James and King Charles, in his books of the *Motion of the Blood*, and of the *Generation of Living Creatures*; who is the only man I know, that conquering envy, hath established a new doctrine in his life-time. Before these, there was nothing certain in natural philosophy

but every man's experiments to himself, and the natural histories, if they may be called certain, that are no certainer than civil histories. But since these, astronomy and natural philosophy in general have, for so little time, been extraordinarily advanced by Joannes Keplerus, Petrus Gassendus, and Marinus Mersennus; and the science of human bodies in special by the wit and industry of physicians, the only true natural philosophers, especially of our most learned men of the College of Physicians in London. Natural Philosophy is therefore but young; but Civil Philosophy yet much younger, as being no older (I say it provoked, and that my detractors may know how little they have wrought upon me) than my own book *De Cive*. But what? were there no philosophers natural nor civil among the ancient Greeks? There were men so called; witness Lucian, by whom they are derided; witness divers cities, from which they have been often by public edicts banished. But it follows not that there was *philosophy*. There walked in old Greece a certain phantasm, for superficial gravity, though full within of fraud and filth, a little like philosophy; which unwary men, thinking to be it, adhered to the professors of it, some to one, some to another, though they disagreed among themselves, and with great salary put their children to

them to be taught, instead of wisdom, nothing but to dispute, and, neglecting the laws, to determine every question according to their own fancies. The first doctors of the Church, next the Apostles, born in those times, whilst they endeavoured to defend the Christian faith against the Gentiles by natural reason, began also to make use of philosophy, and with the decrees of Holy Scripture to mingle the sentences of heathen philosophers; and first some harmless ones of Plato, but afterwards also many foolish and false ones out of the physics and metaphysics of Aristotle; and bringing in the enemies, betrayed unto them the citadel of Christianity. From that time, instead of the worship of God, there entered a thing called *school divinity*, walking on one foot firmly, which is the Holy Scripture, but halted on the other rotten foot, which the Apostle Paul called *vain*, and might have called *pernicious philosophy*; for it hath raised an infinite number of controversies in the Christian world concerning religion, and from those controversies, wars. It is like that *Empusa* in the Athenian comic poet, which was taken in Athens for a ghost that changed shapes, having one brazen leg, but the other was the leg of an ass, and was sent, as was believed, by Hecate, as a sign of some approaching evil fortune. Against this *Empusa* I think

there cannot be invented a better exorcism, than to distinguish between the rules of religion, that is, the rules of honouring God, which we have from the laws, and the rules of philosophy, that is, the opinions of private men; and to yield what is due to religion to the Holy Scripture, and what is due to philosophy to natural reason. And this I shall do, if I but handle the Elements of Philosophy truly and clearly, as I endeavour to do. Therefore having in the Third Section, which I have published and dedicated to your Lordship, long since reduced all power ecclesiastical and civil by strong arguments of reason, without repugnance to God's word, to one and the same sovereign authority; I intend now, by putting into a clear method the true foundations of natural philosophy, to fright and drive away this metaphysical *Empusa*; not by skirmish, but by letting in the light upon her. For I am confident, if any confidence of a writing can proceed from the writer's fear, circumspection, and diffidence, that in the three former parts of this book all that I have said is sufficiently demonstrated from definitions; and all in the fourth part from suppositions not absurd. But if there appear to your Lordship anything less fully demonstrated than to satisfy every reader, the cause was this, that I professed to write not all to

all, but some things to geometricians only. But that your Lordship will be satisfied, I cannot doubt.

There remains the second section, which is concerning *Man*. That part thereof, where I handle the *Optics*, containing six chapters, together with the tables of the figures belonging to them, I have already written and engraven lying by me above these six years. The rest shall, as soon as I can, be added to it; though by the contumelies and petty injuries of some unskilful men, I know already, by experience, how much greater thanks will be due than paid me, for telling men the truth of what men are. But the burthen I have taken on me I mean to carry through; not striving to appease, but rather to revenge myself of envy, by encreasing it. For it contents me that I have your Lordship's favour, which, being all you require, I acknowledge; and for which, with my prayers to Almighty God for your Lordship's safety, I shall, to my power, be always thankful.

Your Lordship's most humble servant,

THOMAS HOBBS.

London,
April 23, 1655.

THE

AUTHOR'S EPISTLE TO THE READER.

THINK not, Courteous Reader, that the philosophy, the elements whereof I am going to set in order, is that which makes philosophers' stones, nor that which is found in the metaphysic codes; but that it is the natural reason of man, busily flying up and down among the creatures, and bringing back a true report of their order, causes and effects. Philosophy, therefore, the child of the world and your own mind, is within yourself; perhaps not fashioned yet, but like the world its father, as it was in the beginning, a thing confused. Do, therefore, as the statuaries do, who, by hewing off that which is superfluous, do not make but find the image. Or imitate the creation: if you will be a philosopher in good earnest, let your reason move upon the deep of your own cogitations and experience; those things that lie in confusion must be set asunder, distinguished, and every one stamped with its own name set in order; that is to say, your method must resemble that of the creation. The order of the creation was, *light, distinction of day and night, the firmament, the luminaries, sensible creatures, man*; and, after the creation, the *commandment*. Therefore the order of contemplation will be, *reason, definition, space, the stars, sensible quality, man*; and after man is grown up, *subjection to command*. In the first part of this section, which is entitled *Logic*, I set up the light of reason. In the second, which hath for title

AUTHOR'S EPISTLE TO THE READER.

the Grounds of Philosophy, I distinguish the most common notions by accurate definition, for the avoiding of confusion and obscurity. The third part concerns the expansion of space, that is Geometry. The fourth contains the Motion of the Stars, together with the doctrine of sensible qualities.

In the second section, if it please God, shall be handled *Man*. In the third section, the doctrine of *Subjection* is handled already. This is the method I followed; and if it like you, you may use the same; for I do but propound, not commend to you anything of mine. But whatsoever shall be the method you will like, I would very fain commend philosophy to you, that is to say, the study of wisdom, for want of which we have all suffered much damage lately. For even they, that study wealth, do it out of love to wisdom; for their treasures serve them but for a looking-glass, wherein to behold and contemplate their own wisdom. Nor do they, that love to be employed in public business, aim at anything but place wherein to show their wisdom. Neither do voluptuous men neglect philosophy, but only because they know not how great a pleasure it is to the mind of man to be ravished in the vigorous and perpetual embraces of the most beauteous world. Lastly, though for nothing else, yet because the mind of man is no less impatient of empty time than nature is of empty place, to the end you be not forced for want of what to do, to be troublesome to men that have business, or take hurt by falling into idle company, but have somewhat of your own wherewith to fill up your time, I recommend unto you to study philosophy. Farewell.

T. H.

COMPUTATION OR LOGIC.

CHAPTER I.

OF PHILOSOPHY.

1. The Introduction.—2. The Definition of Philosophy explained.—3. Ratiocination of the Mind.—4. Properties, what they are.—5. How Properties are known by Generation, and contrarily.—6. The Scope of Philosophy.—7. The Utility of it.—8. The Subject.—9. The Parts of it.—10. The Epilogue.

PHILOSOPHY seems to me to be amongst men now, in the same manner as corn and wine are said to have been in the world in ancient time. For from the beginning there were vines and ears of corn growing here and there in the fields; but no care was taken for the planting and sowing of them. Men lived therefore upon acorns; or if any were so bold as to venture upon the eating of those unknown and doubtful fruits, they did it with danger of their health. In like manner, every man brought Philosophy, that is, Natural Reason, into the world with him; for all men can reason to some degree, and concerning some things: but where there is need of a long series of reasons, there most men wander out of the way, and fall into error for want of method, as it were for want

PART I.

1.

Introduction.

PART I.

1.

Introduction.

of sowing and planting, that is, of improving their reason. And from hence it comes to pass, that they who content themselves with daily experience, which may be likened to feeding upon acorns, and either reject, or not much regard philosophy, are commonly esteemed, and are, indeed, men of sounder judgment than those who, from opinions, though not vulgar, yet full of uncertainty, and carelessly received, do nothing but dispute and wrangle, like men that are not well in their wits. I confess, indeed, that that part of philosophy by which magnitudes and figures are computed, is highly improved. But because I have not observed the like advancement in the other parts of it, my purpose is, as far forth as I am able, to lay open the few and first Elements of Philosophy in general, as so many seeds from which pure and true Philosophy may hereafter spring up by little and little.

I am not ignorant how hard a thing it is to weed out of men's minds such inveterate opinions as have taken root there, and been confirmed in them by the authority of most eloquent writers; especially seeing true (that is, accurate) Philosophy professedly rejects not only the paint and false colours of language, but even the very ornaments and graces of the same; and the first grounds of all science are not only not beautiful, but poor, arid, and, in appearance, deformed. Nevertheless, there being certainly some men, though but few, who are delighted with truth and strength of reason in all things, I thought I might do well to take this pains for the sake even of those few. I proceed therefore to the matter, and take my beginning

from the very definition of philosophy, which is this. PART I.
1.

2. PHILOSOPHY is such knowledge of effects or appearances, as we acquire by true ratiocination from the knowledge we have first of their causes or generation: And again, of such causes or generations as may be from knowing first their effects. Definition of
Philosophy
explained.

For the better understanding of which definition, we must consider, first, that although Sense and Memory of things, which are common to man and all living creatures, be knowledge, yet because they are given us immediately by nature, and not gotten by ratiocination, they are not philosophy.

Secondly, seeing Experience is nothing but memory; and Prudence, or prospect into the future time, nothing but expectation of such things as we have already had experience of, Prudence also is not to be esteemed philosophy.

By RATIOCINATION, I mean *computation*. Now to compute, is either to collect the sum of many things that are added together, or to know what remains when one thing is taken out of another. *Ratiocination*, therefore, is the same with *addition* and *subtraction*; and if any man add *multiplication* and *division*, I will not be against it, seeing multiplication is nothing but addition of equals one to another, and division nothing but a subtraction of equals one from another, as often as is possible. So that all ratiocination is comprehended in these two operations of the mind, addition and subtraction.

3. But how by the *ratiocination* of our mind, we add and subtract in our silent thoughts, without the use of words, it will be necessary for me Ratiocination
of the Mind.

PART I.

1.

Ratiocination
of the Mind.

to make intelligible by an example or two. If therefore a man see something afar off and obscurely, although no appellation had yet been given to anything, he will, notwithstanding, have the same idea of that thing for which now, by imposing a name on it, we call it *body*. Again, when, by coming nearer, he sees the same thing thus and thus, now in one place and now in another, he will have a new idea thereof, namely, that for which we now call such a thing *animated*. Thirdly, when standing nearer, he perceives the figure, hears the voice, and sees other things which are signs of a rational mind, he has a third idea, though it have yet no appellation, namely, that for which we now call anything *rational*. Lastly, when, by looking fully and distinctly upon it, he conceives all that he has seen as one thing, the idea he has now is compounded of his former ideas, which are put together in the mind in the same order in which these three single names, *body*, *animated*, *rational*, are in speech compounded into this one name, *body-animated-rational*, or *man*. In like manner, of the several conceptions of *four sides*, *equality of sides*, and *right angles*, is compounded the conception of a *square*. For the mind may conceive a figure of four sides without any conception of their equality, and of that equality without conceiving a right angle; and may join together all these single conceptions into one conception or one idea of a square. And thus we see how the conceptions of the mind are compounded. Again, whosoever sees a man standing near him, conceives the whole idea of that man; and if, as he goes away, he follow him with his

eyes only, he will lose the idea of those things which were signs of his being rational, whilst, nevertheless, the idea of a body-animated remains still before his eyes, so that the idea of rational is subtracted from the whole idea of man, that is to say, of body-animated-rational, and there remains that of body-animated; and a while after, at a greater distance, the idea of animated will be lost, and that of body only will remain; so that at last, when nothing at all can be seen, the whole idea will vanish out of sight. By which examples, I think, it is manifest enough what is the internal ratiocination of the mind without words.

We must not therefore think that computation, that is, ratiocination, has place only in numbers, as if man were distinguished from other living creatures (which is said to have been the opinion of *Pythagoras*) by nothing but the faculty of numbering; for *magnitude, body, motion, time, degrees of quality, action, conception, proportion, speech and names* (in which all the kinds of philosophy consist) are capable of addition and subtraction. Now such things as we add or subtract, that is, which we put into an account, we are said to *consider*, in Greek λογίζεσθαι, in which language also συλλογίζεσθαι signifies to *compute, reason, or reckon*.

4. But *effects* and the *appearances* of things to sense, are faculties or powers of bodies, which make us distinguish them from one another; that is to say, conceive one body to be equal or unequal, like or unlike to another body; as in the example above, when by coming near enough to any body, we perceive the motion and going of the same, we distinguish it thereby from a tree, a

Properties,
what they are.

PART I.

1.

column, and other fixed bodies ; and so that motion or going is the *property* thereof, as being proper to living creatures, and a faculty by which they make us distinguish them from other bodies.

How Properties
are known by
Generation,
and contrarily.

5. How the knowledge of any effect may be gotten from the knowledge of the generation thereof, may easily be understood by the example of a circle: for if there be set before us a plain figure, having, as near as may be, the figure of a circle, we cannot possibly perceive by sense whether it be a true circle or no; than which, nevertheless, nothing is more easy to be known to him that knows first the generation of the propounded figure. For let it be known that the figure was made by the circumduction of a body whereof one end remained unmoved, and we may reason thus; a body carried about, retaining always the same length, applies itself first to one *radius*, then to another, to a third, a fourth, and successively to all; and, therefore, the same length, from the same point, toucheth the circumference in every part thereof, which is as much as to say, as all the *radii* are equal. We know, therefore, that from such generation proceeds a figure, from whose one middle point all the extreme points are reached unto by equal *radii*. And in like manner, by knowing first what figure is set before us, we may come by ratiocination to some generation of the same, though perhaps not that by which it was made, yet that by which it might have been made; for he that knows that a circle has the property above declared, will easily know whether a body carried about, as is said, will generate a circle or no.

6. The *end* or *scope* of philosophy is, that we may make use to our benefit of effects formerly seen; or that, by application of bodies to one another, we may produce the like effects of those we conceive in our mind, as far forth as matter, strength, and industry, will permit, for the commodity of human life. For the inward glory and triumph of mind that a man may have for the mastering of some difficult and doubtful matter, or for the discovery of some hidden truth, is not worth so much pains as the study of Philosophy requires; nor need any man care much to teach another what he knows himself, if he think that will be the only benefit of his labour. The end of knowledge is power; and the use of theorems (which, among geometricians, serve for the finding out of properties) is for the construction of problems; and, lastly, the scope of all speculation is the performing of some action, or thing to be done.

PART I.

1.

Scope of
Philosophy.

7. But what the *utility* of philosophy is, especially of natural philosophy and geometry, will be best understood by reckoning up the chief commodities of which mankind is capable, and by comparing the manner of life of such as enjoy them, with that of others which want the same. Now, the greatest commodities of mankind are the arts; namely, of measuring matter and motion; of moving ponderous bodies; of architecture; of navigation; of making instruments for all uses; of calculating the celestial motions, the aspects of the stars, and the parts of time; of geography, &c. By which sciences, how great benefits men receive is more easily understood than expressed. These benefits are enjoyed by almost all the people of

Utility of
Philosophy.

PART I.

I.

Utility of
Philosophy.

Europe, by most of those of Asia, and by some of Africa : but the Americans, and they that live near the Poles, do totally want them. But why? Have they sharper wits than these? Have not all men one kind of soul, and the same faculties of mind? What, then, makes this difference, except philosophy? Philosophy, therefore, is the cause of all these benefits. But the utility of moral and civil philosophy is to be estimated, not so much by the commodities we have by knowing these sciences, as by the calamities we receive from not knowing them. Now, all such calamities as may be avoided by human industry, arise from war, but chiefly from civil war; for from this proceed slaughter, solitude, and the want of all things. But the cause of war is not that men are willing to have it; for the will has nothing for object but good, at least that which seemeth good. Nor is it from this, that men know not that the effects of war are evil; for who is there that thinks not poverty and loss of life to be great evils? The cause, therefore, of civil war is, that men know not the causes neither of war nor peace, there being but few in the world that have learned those duties which unite and keep men in peace, that is to say, that have learned the rules of civil life sufficiently. Now, the knowledge of these rules is moral philosophy. But why have they not learned them, unless for this reason, that none hitherto have taught them in a clear and exact method? For what shall we say? Could the ancient masters of Greece, Egypt, Rome, and others, persuade the unskilful multitude to their innumerable opinions concerning the nature of their gods, which they

themselves knew not whether they were true or false, and which were indeed manifestly false and absurd; and could they not persuade the same multitude to civil duty, if they themselves had understood it? Or shall those few writings of geometricians which are extant, be thought sufficient for the taking away of all controversy in the matters they treat of, and shall those innumerable and huge volumes of *ethics* be thought insufficient, if what they teach had been certain and well demonstrated? What, then, can be imagined to be the cause that the writings of those men have increased science, and the writings of these have increased nothing but words, saving that the former were written by men that knew, and the latter by such as knew not, the doctrine they taught only for ostentation of their wit and eloquence? Nevertheless, I deny not but the reading of some such books is very delightful; for they are most eloquently written, and contain many clear, wholesome and choice sentences, which yet are not universally true, though by them universally pronounced. From whence it comes to pass, that the circumstances of times, places, and persons being changed, they are no less frequently made use of to confirm wicked men in their purposes, than to make them understand the precepts of civil duties. Now that which is chiefly wanting in them, is a true and certain rule of our actions, by which we might know whether that we undertake be just or unjust. For it is to no purpose to be bidden in every thing to do right, before there be a certain rule and measure of right established, which no man hitherto hath established. Seeing,

PART I.

1.

 Utility of
Philosophy.

PART I.

1.

therefore, from the not knowing of civil duties, that is, from the want of moral science, proceed civil wars, and the greatest calamities of mankind, we may very well attribute to such science the production of the contrary commodities. And thus much is sufficient, to say nothing of the praises and other contentment proceeding from philosophy, to let you see the utility of the same in every kind thereof.

Subject of
Philosophy.

8. The *subject* of Philosophy, or the matter it treats of, is every body of which we can conceive any generation, and which we may, by any consideration thereof, compare with other bodies, or which is capable of composition and resolution; that is to say, every body of whose generation or properties we can have any knowledge. And this may be deduced from the definition of philosophy, whose profession it is to search out the properties of bodies from their generation, or their generation from their properties; and, therefore, where there is no generation or property, there is no philosophy. Therefore it excludes *Theology*, I mean the doctrine of God, eternal, ingenerable, incomprehensible, and in whom there is nothing neither to divide nor compound, nor any generation to be conceived.

It excludes the doctrine of *angels*, and all such things as are thought to be neither bodies nor properties of bodies; there being in them no place neither for composition nor division, nor any capacity of more and less, that is to say, no place for ratiocination.

It excludes *history*, as well *natural* as *political*, though most useful (nay necessary) to philosophy;

because such knowledge is but experience, or authority, and not ratiocination.

PART I.
1.

It excludes all such knowledge as is acquired by Divine inspiration, or revelation, as not derived to us by reason, but by Divine grace in an instant, and, as it were, by some sense supernatural.

It excludes not only all doctrines which are false, but such also as are not well-grounded; for whatsoever we know by right ratiocination, can neither be false nor doubtful; and, therefore, *astrology*, as it is now held forth, and all such divinations rather than sciences, are excluded.

Lastly, the doctrine of *God's worship* is excluded from philosophy, as being not to be known by natural reason, but by the authority of the Church; and as being the object of faith, and not of knowledge.

9. The principal parts of philosophy are two. Parts of Philosophy. For two chief kinds of bodies, and very different from one another, offer themselves to such as search after their generation and properties; one whereof being the work of nature, is called a *natural body*, the other is called a *commonwealth*, and is made by the wills and agreement of men. And from these spring the two parts of philosophy, called *natural* and *civil*. But seeing that, for the knowledge of the properties of a commonwealth, it is necessary first to know the dispositions, affections, and manners of men, civil philosophy is again commonly divided into two parts, whereof one, which treats of men's dispositions and manners, is called *ethics*; and the other, which takes cognizance of their civil duties, is called *politics*, or simply *civil philosophy*. In the first place, there-

PART I.

1.

fore (after I have set down such premises as appertain to the nature of philosophy in general), I will discourse of *bodies natural*; in the second, of the *dispositions and manners of men*; and in the third, of the *civil duties of subjects*.

Epilogue.

10. To conclude; seeing there may be many who will not like this my definition of philosophy, and will say, that, from the liberty which a man may take of so defining as seems best to himself, he may conclude any thing from any thing (though I think it no hard matter to demonstrate that this definition of mine agrees with the sense of all men); yet, lest in this point there should be any cause of dispute betwixt me and them, I here undertake no more than to deliver the elements of that science by which the effects of anything may be found out from the known generation of the same, or contrarily, the generation from the effects; to the end that they who search after other philosophy, may be admonished to seek it from other principles.

CHAPTER II.

OF NAMES.

1. The necessity of sensible Moniments or Marks for the help of Memory: a Mark defined.—2. The necessity of Marks for the signification of the conceptions of the Mind.—3. Names supply both those necessities.—4. The Definition of a Name.—5. Names are Signs not of Things, but of our Cogitations.—6. What it is we give Names to.—7. Names Positive and Negative.—8. Contradictory Names.—9. A Common Name.—10. Names of the First and Second Intention.—11. Universal, Particular, Individual, and Indefinite Names.—12. Names Univocal and Equivocal.—13. Absolute and Relative Names.—14. Simple and Compound Names.—15. A Predicament described.—16. Some things to be noted concerning Predicaments.

1. How unconstant and fading men's thoughts are, and how much the recovery of them depends upon chance, there is none but knows by infallible experience in himself. For no man is able to remember quantities without sensible and present measures, nor colours without sensible and present patterns, nor number without the names of numbers disposed in order and learned by heart. So that whatsoever a man has put together in his mind by ratiocination without such helps, will presently slip from him, and not be revocable but by beginning his ratiocination anew. From which it follows, that, for the acquiring of philosophy, some sensible moniments are necessary, by which our past thoughts may be not only reduced, but

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2.

Necessity
of sensible
Moniments
or Marks
for the help
of Memory.

PART I.

2.

A Mark
defined.

also registered every one in its own order. These monuments I call **MARKS**, namely, sensible things taken at pleasure, that, by the sense of them, such thoughts may be recalled to our mind as are like those thoughts for which we took them.

Necessity of
Marks for the
signification of
the conceptions
of the Mind.

2. Again, though some one man, of how excellent a wit soever, should spend all his time partly in reasoning, and partly in inventing marks for the help of his memory, and advancing himself in learning; who sees not that the benefit he reaps to himself will not be much, and to others none at all? For unless he communicate his notes with others, his science will perish with him. But if the same notes be made common to many, and so one man's inventions be taught to others, sciences will thereby be increased to the general good of mankind. It is therefore necessary, for the acquiring of philosophy, that there be certain signs, by which what one man finds out may be manifested and made known to others. Now, those things we call **SIGNS** are the *antecedents of their consequents, and the consequents of their antecedents, as often as we observe them to go before or follow after in the same manner*. For example, a thick cloud is a sign of rain to follow, and rain a sign that a cloud has gone before, for this reason only, that we seldom see clouds without the consequence of rain, nor rain at any time but when a cloud has gone before. And of signs, some are *natural*, whereof I have already given an example, others are *arbitrary*, namely, those we make choice of at our own pleasure, as a bush hung up, signifies that wine is to be sold there; a stone set in the ground signifies the bound of a field; and

words so and so connected, signify the cogitations and motions of our mind. The difference, therefore, betwixt marks and signs is this, that we make those for our own use, but these for the use of others.

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2.

3. Words so connected as that they become signs of our thoughts, are called **SPEECH**, of which every part is a *name*. But seeing (as is said) both marks and signs are necessary for the acquiring of philosophy, (marks by which we may remember our own thoughts, and signs by which we may make our thoughts known to others), names do both these offices ; but they serve for marks before they be used as signs. For though a man were alone in the world, they would be useful to him in helping him to remember ; but to teach others, (unless there were some others to be taught) of no use at all. Again, names, though standing singly by themselves, are marks, because they serve to recal our own thoughts to mind ; but they cannot be signs, otherwise than by being disposed and ordered in speech as parts of the same. For example, a man may begin with a word, whereby the hearer may frame an idea of something in his mind, which, nevertheless, he cannot conceive to be the idea which was in the mind of him that spake, but that he would say something which began with that word, though perhaps not as by itself, but as part of another word. So that the nature of a name consists principally in this, that it is a mark taken for memory's sake ; but it serves also by accident to signify and make known to others what we remember ourselves, and, therefore, I will define it thus :

Names
supply
both those
necessities.

PART I.

2.

Definition
of a Name.

4. *A NAME is a word taken at pleasure to serve for a mark, which may raise in our mind a thought like to some thought we had before, and which being pronounced to others, may be to them a sign of what thought the speaker had, or had not before in his mind.* And it is for brevity's sake that I suppose the original of names to be arbitrary, judging it a thing that may be assumed as unquestionable. For considering that new names are daily made, and old ones laid aside; that diverse nations use different names, and how impossible it is either to observe similitude, or make any comparison betwixt a name and a thing, how can any man imagine that the names of things were imposed from their natures? For though some names of living creatures and other things, which our first parents used, were taught by God himself; yet they were by him arbitrarily imposed, and afterwards, both at the Tower of Babel, and since, in process of time, growing everywhere out of use, are quite forgotten, and in their room have succeeded others, invented and received by men at pleasure. Moreover, whatsoever the common use of words be, yet philosophers, who were to teach their knowledge to others, had always the liberty, and sometimes they both had and will have a necessity, of taking to themselves such names as they please for the signifying of their meaning, if they would have it understood. Nor had mathematicians need to ask leave of any but themselves to name the figures they invented, *parabolas, hyperboles, cissoeides, quadratices, &c.* or to call one magnitude A, another B.

5. But seeing names ordered in speech (as is defined) are signs of our conceptions, it is manifest they are not signs of the things themselves; for that the sound of this word *stone* should be the sign of a stone, cannot be understood in any sense but this, that he that hears it collects that he that pronounces it thinks of a stone. And, therefore, that disputation, whether names signify the matter or form, or something compounded of both, and other like subtleties of the *metaphysics*, is kept up by erring men, and such as understand not the words they dispute about.

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2.

Names
are signs
not of things,
but of our
cogitations.

6. Nor, indeed, is it at all necessary that every name should be the name of something. For these, a *man*, a *tree*, a *stone*, are the names of the things themselves, so the images of a man, of a tree, and of a stone, which are represented to men sleeping, have their names also, though they be not things, but only fictions and phantasms of things. For we can remember these; and, therefore, it is no less necessary that they have names to mark and signify them, than the things themselves. Also this word *future* is a name, but no future thing has yet any being, nor do we know whether that which we call future, shall ever have a being or no. Nevertheless, seeing we use in our mind to knit together things past with those that are present, the name *future* serves to signify such knitting together. Moreover, that which neither is, nor has been, nor ever shall, or ever can be, has a name, namely, *that which neither is nor has been*, &c.; or more briefly this, *impossible*. To conclude; this word *nothing* is a name, which yet cannot be the name of any thing: for when, for

What it is
we give
names to.

L

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2.

example, we subtract 2 and 3 from 5, and so nothing remaining, we would call that subtraction to mind, this speech *nothing remains*, and in it the word *nothing* is not unuseful. And for the same reason we say truly, *less than nothing* remains, when we subtract more from less; for the mind feigns such remains as these for doctrine's sake, and desires, as often as is necessary, to call the same to memory. But seeing every name has some relation to that which is named, though that which we name be not always a thing that has a being in nature, yet it is lawful for doctrine's sake to apply the word *thing* to whatsoever we name; as if it were all one whether that thing be truly existent, or be only feigned.

Names Positive
and Negative.

7. The first distinction of names is, that some are *positive*, or *affirmative*, others *negative*, which are also called *privative* and *indefinite*. *Positive* are such as we impose for the likeness, equality, or identity of the things we consider; *negative*, for the diversity, unlikeness, or inequality of the same. Examples of the former are, *a man*, *a philosopher*; for *a man* denotes any one of a multitude of men, and *a philosopher*, any one of many philosophers, by reason of their similitude; also, *Socrates* is a positive name, because it signifies always one and the same man. Examples of *negatives* are such positives as have the negative particle *not* added to them, as *not-man*, *not-philosopher*. But positives were before negatives; for otherwise there could have been no use at all of these. For when the name of *white* was imposed upon certain things, and afterwards upon other things, the names of *black*, *blue*, *trans-*

parent, &c. the infinite dissimilitudes of these with *white* could not be comprehended in any one name, save that which had in it the negation of white, that is to say, the name *not-white*, or some other equivalent to it, in which the word *white* is repeated, such as *unlike to white, &c.* And by these *negative names*, we take notice ourselves, and signify to others what we have not thought of.

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2.

8. Positive and negative names are *contradictory* to one another, so that they cannot both be the name of the same thing. Besides, of contradictory names, one is the name of anything whatsoever ; for whatsoever is, is either man, or not-man, white or not-white, and so of the rest. And this is so manifest, that it needs no farther proof or explication ; for they that say *the same thing cannot both be, and not be*, speak obscurely ; but they that say, *whatsoever is, either is, or is not*, speak also absurdly and ridiculously. The certainty of this axiom, viz. *of two contradictory names, one is the name of anything whatsoever, the other not*, is the original and foundation of all ratiocination, that is, of all philosophy ; and therefore it ought to be so exactly propounded, that it may be of itself clear and perspicuous to all men ; as indeed it is, saving to such, as reading the long discourses made upon this subject by the writers of *metaphysics* (which they believe to be some egregious learning) think they understand not, when they do.

Contradictory
names.

9. Secondly, of names, some are *common* to many things, as a *man, a tree* ; others *proper* to one thing, as *he that writ the Iliad, Homer, this man, that man*. And a common name, being the

A common
name.

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2.

name of many things severally taken, but not collectively of all together (as man is not the name of all mankind, but of every one, as of Peter, John, and the rest severally) is therefore called an *universal name*; and therefore this word *universal* is never the name of any thing existent in nature, nor of any idea or phantasm formed in the mind, but always the name of some word or name; so that when a *living creature*, a *stone*, a *spirit*, or any other thing, is said to be *universal*, it is not to be understood, that any man, stone, &c. ever was or can be universal, but only that these words, *living creature*, *stone*, &c. are *universal names*, that is, names common to many things; and the conceptions answering them in our mind, are the images and phantasms of several living creatures, or other things. And therefore, for the understanding of the extent of an universal name, we need no other faculty but that of our imagination, by which we remember that such names bring sometimes one thing, sometimes another, into our mind. Also of common names, some are more, some less common. *More common*, is that which is the name of more things; *less common*, the name of fewer things; as *living creature* is more common than *man*, or *horse*, or *lion*, because it comprehends them all: and therefore a more common name, in respect of a less common, is called the *genus*, or a *general name*; and this in respect of that, the *species*, or a *special name*.

Names
of the first
and second
intention.

10. And from hence proceeds the third distinction of names, which is, that some are called names of the *first*, others of the *second intention*.

Of the *first intention* are the names of things, *a man, stone, &c.*: of the *second* are the names of names and speeches, as *universal, particular, genus, species, syllogism*, and the like. But it is hard to say why those are called names of the *first*, and these of the *second intention*, unless perhaps it was first intended by us to give names to those things which are of daily use in this life, and afterwards to such things as appertain to science, that is, that our second intention was to give names to names. But whatsoever the cause hereof may be, yet this is manifest, that *genus, species, definition, &c.* are names of words and names only; and therefore to put *genus* and *species* for things, and *definition* for the nature of any thing, as the writers of *metaphysics* have done, is not right, seeing they be only significations of what we think of the nature of things.

11. Fourthly, some names are of *certain* and *determined*, others of *uncertain* and *undetermined* signification. Of *determined* and *certain* signification is, first, that name which is given to any one thing by itself, and is called an *individual name*; as *Homer, this tree, that living creature, &c.* Secondly that which has any of these words, *all, every, both, either*, or the like added to it; and it is therefore called an universal name, because it signifies every one of those things to which it is common; and of *certain* signification for this reason, that he which hears, conceives in his mind the same thing that he which speaks would have him conceive. Of *indefinite* signification is, first, that name which has the word *some*, or the like added to it, and is called a *particular*

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2.

Universal,
particular,
individual,
and indefinite
names.

PART I.
2.

name ; secondly, a common name set by itself without any note either of universality or particularity, as *man*, *stone*, and is called an *indefinite name* ; but both *particular* and *indefinite* names are of uncertain signification, because the hearer knows not what thing it is the speaker would have him conceive ; and therefore in speech, particular and indefinite names are to be esteemed equivalent to one another. But these words, *all*, *every*, *some*, &c. which denote universality and particularity, are not names, but parts only of names ; so that *every man*, and *that man which the hearer conceives in his mind*, are all one ; and *some man*, and *that man which the speaker thought of*, signify the same. From whence it is evident, that the use of signs of this kind, is not for a man's own sake, or for his getting of knowledge by his own private meditation (for every man has his own thoughts sufficiently determined without such helps as these) but for the sake of others ; that is, for the teaching and signifying of our conceptions to others ; nor were they invented only to make us remember, but to make us able to discourse with others.

Names
univocal
and equivocal.

12. Fifthly, names are usually distinguished into *univocal* and *equivocal*. *Univocal* are those which in the same train of discourse signify always the same thing ; but *equivocal* those which mean sometimes one thing and sometimes another. Thus, the name *triangle* is said to be *univocal*, because it is always taken in the same sense ; and *parabola* to be *equivocal*, for the signification it has sometimes of allegory or similitude, and sometimes of a certain geometrical figure. Also every

metaphor is by profession *equivocal*. But this distinction belongs not so much to names, as to those that use names, for some use them properly and accurately for the finding out of truth ; others draw them from their proper sense, for ornament or deceit.

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2.

13. Sixthly, of names, some are *absolute*, others *relative*. *Relative* are such as are imposed for some comparison, as *father, son, cause, effect, like, unlike, equal, unequal, master, servant, &c.* And those that signify no comparison at all are *absolute names*. But, as it was noted above, that universality is to be attributed to words and names only, and not to things, so the same is to be said of other distinctions of names ; for no things are either *univocal* or *equivocal*, or *relative* or *absolute*. There is also another distinction of names into *concrete* and *abstract* ; but because abstract names proceed from proposition, and can have no place where there is no affirmation, I shall speak of them hereafter.

Absolute
and relative
names.

14. Lastly, there are *simple* and *compounded names*. But here it is to be noted, that a name is not taken in philosophy, as in grammar, for one single word, but for any number of words put together to signify one thing ; for among philosophers *sentient animated body* passes but for one name, being the name of every living creature, which yet, among grammarians, is accounted three names. Also a *simple name* is not here distinguished from a *compounded name* by a preposition, as in grammar. But I call a *simple name*, that which in every kind is the most common or most universal ; and that a *compounded name*, which

Simple and
compounded
names.

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2.

Simple and
compounded
names.

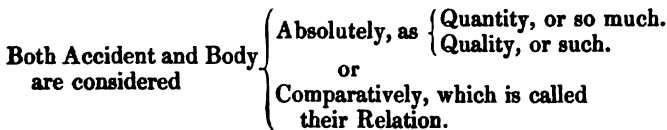
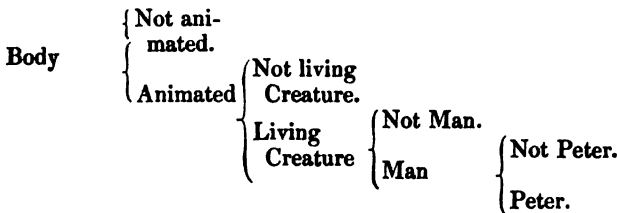
by the joining of another name to it, is made less universal, and signifies that more conceptions than one were in the mind, for which that latter name was added. For example, in the conception of *man* (as is shown in the former chapter.) First, he is conceived to be something that has extension, which is marked by the word *body*. *Body*, therefore, is a *simple name*, being put for that first single conception; afterwards, upon the sight of such and such motion, another conception arises, for which he is called an *animated body*; and this I here call a *compounded name*, as I do also the name *animal*, which is equivalent to an *animated body*. And, in the same manner, an *animated rational body*, as also a *man*, which is equivalent to it, is a more compounded name. And by this we see how the composition of conceptions in the mind is answerable to the composition of names; for, as in the mind one idea or phantasm succeeds to another, and to this a third; so to one name is added another and another successively, and of them all is made one *compounded* name. Nevertheless we must not think bodies which are without the mind, are compounded in the same manner, namely, that there is in nature a body, or any other imaginable thing existent, which at the first has no magnitude, and then, by the addition of magnitude, comes to have quantity, and by more or less quantity to have density or rarity; and again, by the addition of figure, to be figurate, and after this, by the injection of light or colour, to become lucid or coloured; though such has been the philosophy of many.

15. The writers of *logic* have endeavoured to digest the names of all the kinds of things into certain *scales* or degrees, by the continual subordination of names less common, to names more common. In the scale of *bodies* they put in the first and highest place *body* simply, and in the next place under it less common names, by which it may be more limited and determined, namely *animated* and *inanimated*, and so on till they come to *individuals*. In like manner, in the scale of *quantities*, they assign the first place to *quantity*, and the next to *line*, *superficies*, and *solid*, which are names of less latitude; and these orders or scales of names they usually call *predicaments* and *categories*. And of this ordination not only positive, but negative names also are capable; which may be exemplified by such forms of the *predicaments* as follow :

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2.
A predicament described.

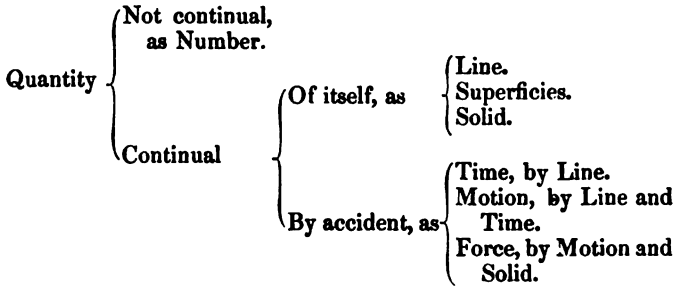
THE FORM OF THE PREDICAMENT OF BODY.

Not-Body, or
Accident.



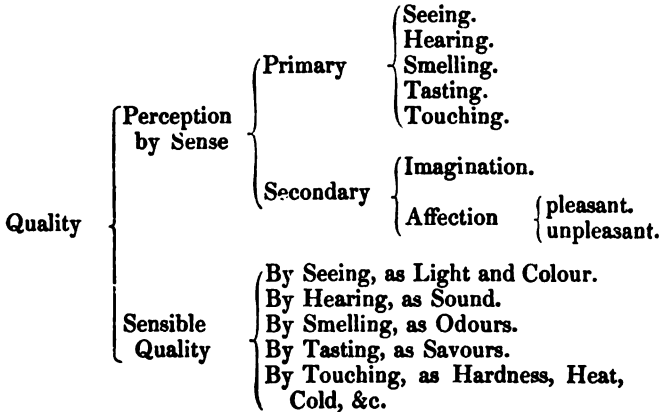
PART I.
2.

THE FORM OF THE PREDICAMENT OF QUANTITY.

A predicament
described.

Where, it is to be noted, that *line*, *superficies*, and *solid*, may be said to be of such and such quantity, that is, to be originally and of their own nature capable of equality and inequality; but we cannot say there is either majority or minority, or equality, or indeed any quantity at all, in *time*, without the help of *line* and *motion*; nor in *motion*, without *line* and *time*; nor in *force*, otherwise than by *motion* and *solid*.

THE FORM OF THE PREDICAMENT OF QUALITY.



THE FORM OF THE PREDICAMENT OF RELATION.

Relation of	{	Magnitudes, as Equality and Inequality.			
		Qualities, as Likeness and Unlikeness.			
		Order	Together	{	In Place.
			Not together	{	In Time.
		{	In Place { Former.		
		{	In Time { Former.		
			{ Later.		
			{ Later.		

16. Concerning which predicaments it is to be noted, in the first place, that as the division is made in the first predicament into contradictory names, so it might have been done in the rest. For, as there, *body* is divided into *animated* and *not-animated*, so, in the second predicament, *continual quantity* may be divided into *line* and *not-line*, and again, *not-line* into *superficies* and *not-superficies*, and so in the rest; but it was not necessary.

Secondly, it is to be observed, that of *positive* names the former comprehends the latter; but of *negatives* the former is comprehended by the latter. For example, *living-creature* is the name of every man, and therefore it comprehends the name *man*; but, on the contrary, *not-man* is the name of everything which is not-living-creature, and therefore the name *not-living-creature*, which is put first, is comprehended by the latter name, *not-man*.

Thirdly, we must take heed that we do not think, that as names, so the diversities of things themselves may be searched out and determined by such distinctions as these; or that arguments

PART I. may be taken from hence (as some have done
 2. ridiculously) to prove that the kinds of things are
 Predicaments. not infinite.

Fourthly, I would not have any man think I deliver the forms above for a true and exact ordination of names ; for this cannot be performed as long as philosophy remains imperfect ; nor that by placing (for example) *light* in the predicament of *qualities*, while another places the same in the predicament of bodies, I pretend that either of us ought for this to be drawn from his opinion ; for this is to be done only by arguments and ratiocination, and not by disposing of words into *classes*.

Lastly, I confess I have not yet seen any great use of the predicaments in philosophy. I believe *Aristotle* when he saw he could not digest the things themselves into such orders, might nevertheless desire out of his own authority to reduce words to such forms, as I have done ; but I do it only for this end, that it may be understood what this ordination of words is, and not to have it received for true, till it be demonstrated by good reason to be so.

CHAPTER III.

OF PROPOSITION.

1. Divers kinds of speech.—2. Proposition defined.—3. Subject, predicate, and copula, what they are; and abstract and concrete what. The use and abuse of names abstract.—5. Proposition, universal and particular.—6. Affirmative and negative.—7. True and false.—8. True and false belongs to speech, and not to things.—9. Proposition, primary, not primary, definition, axiom, petition.—10. Proposition, necessary and contingent.—11. Categorical and hypothetical.—12. The same proposition diversely pronounced.—13. Propositions that may be reduced to the same categorical proposition, are equipollent.—14. Universal propositions converted by contradictory names, are equipollent.—15. Negative propositions are the same, whether negation be before or after the copula.—16. Particular propositions simply converted, are equipollent.—17. What are subaltern, contrary, subcontrary, and contradictory propositions.—18. Consequence, what it is.—19. Falsity cannot follow from truth.—20. How one proposition is the cause of another.

1. FROM the connexion or contexture of names arise divers kinds of speech, whereof some signify the desires and affections of men; such are, first, *interrogations*, which denote the desire of knowing: as, *Who is a good man?* In which speech there is one name expressed, and another desired and expected from him of whom we ask the same. Then *prayers*, which signify the desire of having something; *promises, threats, wishes, commands, complaints*, and other significations of other affections. Speech may also be absurd and insignificant; as when there is a succession of

PART I.
3.

Divers kinds
of speech.

PART I.
3.

words, to which there can be no succession of thoughts in mind to answer them ; and this happens often to such, as, understanding nothing in some subtle matter, do, nevertheless, to make others believe they understand, speak of the same incoherently ; for the connection of incoherent words, though it want the end of speech (which is signification) yet it is speech ; and is used by writers of *metaphysics* almost as frequently as speech significative. In philosophy, there is but one kind of speech useful, which some call in Latin *dictum*, others *enuntiatum et pronunciatum* ; but most men call it *proposition*, and is the speech of those that affirm or deny, and expresseth truth or falsity.

Proposition
defined.

2. A PROPOSITION is a speech consisting of two names copulated, by which he that speaketh signifies he conceives the latter name to be the name of the same thing whereof the former is the name ; or (which is all one) that the former name is comprehended by the latter. For example, this speech, *man is a living creature*, in which two names are copulated by the verb *is*, is a *proposition*, for this reason, that he that speaks it conceives both *living creature* and *man* to be names of the same thing, or that the former name, *man*, is comprehended by the latter name, *living creature*. Now the former name is commonly called the *subject*, or *antecedent*, or the *contained name*, and the latter the *predicate*, *consequent*, or *containing name*. The sign of connection amongst most nations is either some word, as the word *is* in the proposition *man is a living creature*, or some case or termination of a word, as in this

proposition, *man walketh* (which is equivalent to this, *man is walking*); the termination by which it is said he *walketh*, rather than he *is walking*, signifieth that those two are understood to be copulated, or to be names of the same thing.

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3.

But there are, or certainly may be, some nations that have no word which answers to our verb *is*, who nevertheless form propositions by the position only of one name after another, as if instead of *man is a living creature*, it should be said *man a living creature*; for the very order of the names may sufficiently show their connection; and they are as apt and useful in philosophy, as if they were copulated by the verb *is*.

3. Wherefore, in every proposition three things are to be considered, *viz.* the two names, which are the *subject*, and the *predicate*, and their *copulation*; both which names raise in our mind the thought of one and the same thing; but the copulation makes us think of the cause for which those names were imposed on that thing. . As, for example, when we say *a body is moveable*, though we conceive the same thing to be designed by both those names, yet our mind rests not there, but searches farther what it is *to be a body*, or *to be moveable*, that is, wherein consists the difference betwixt these and other things, for which these are so called, others are not so called. They, therefore, that seek what it is *to be* any thing, as *to be moveable*, *to be hot*, &c. seek in things the causes of their names.

Subject,
predicate,
and copula,
what they are,
and abstract
and concrete
what.

And from hence arises that distinction of names (touched in the last chapter) into *concrete* and *abstract*. For *concrete* is the name of any thing

PART I.
3.

Subject.

which we suppose to have a being, and is therefore called the *subject*, in Latin *suppositum*, and in Greek *ὑποκείμενον*; as *body*, *moveable*, *moved*, *figurate*, *a cubit high*, *hot*, *cold*, *like*, *equal*, *Appius*, *Lentulus*, and the like; and, *abstract* is that which in any subject denotes the cause of the concrete name, as *to be a body*, *to be moveable*, *to be moved*, *to be figurate*, *to be of such quantity*, *to be hot*, *to be cold*, *to be like*, *to be equal*, *to be Appius*, *to be Lentulus*, &c. Or names equivalent to these, which are most commonly called *abstract names*, as *corporiety*, *mobility*, *motion*, *figure*, *quantity*, *heat*, *cold*, *likeness*, *equality*, and (as Cicero has it) *Appiety* and *Lentulity*. Of the same kind also are infinitives; for *to live* and *to move* are the same with *life* and *motion*, or *to be living* and *to be moved*. But *abstract names* denote only the causes of *concrete names*, and not the things themselves. For example, when we see any thing, or conceive in our mind any visible thing, that thing appears to us, or is conceived by us, not in one point, but as having parts distant from one another, that is, as being extended and filling some space. Seeing therefore we call the thing so conceived *body*, the cause of that name is, that that thing is *extended*, or the *extension* or *corporiety* of it. So when we see a thing appear sometimes here, sometimes there, and call it *moved* or *removed*, the cause of that name is that it *is moved* or the *motion* of the same.

And these causes of names are the same with the causes of our conceptions, namely, some power of action, or affection of the thing con-

ceived, which some call the manner by which any thing works upon our senses, but by most men they are called *accidents*; I say accidents, not in that sense in which accident is opposed to necessary; but so, as being neither the things themselves, nor parts thereof, do nevertheless accompany the things in such manner, that (saving extension) they may all perish, and be destroyed, but can never be abstracted.

PART I.
3.

4. There is also this difference betwixt *concrete* and *abstract* names, that those were invented before propositions, but these after; for these could have no being till there were propositions, from whose *copula* they proceed. Now in all matters that concern this life, but chiefly in philosophy, there is both great use and great abuse of *abstract names*; and the use consists in this, that without them we cannot, for the most part, either reason, or compute the properties of bodies; for when we would multiply, divide, add, or subtract heat, light, or motion, if we should double or add them together by concrete names, saying (for example) hot is double to hot, light double to light, or moved double to moved, we should not double the properties, but the bodies themselves that are hot, light, moved, &c. which we would not do. But the abuse proceeds from this, that some men seeing they can consider, that is (as I said before) bring into account the increasings and decreasings of quantity, heat and other accidents, without considering their bodies or subjects (which they call *abstracting*, or making to exist apart by themselves) they speak of *accidents*, as if they might be separated from all bodies. And

The use
and abuse of
names abstract.

PART I.
3.

from hence proceed the gross errors of writers of metaphysics; for, because they can consider thought without the consideration of body, they infer there is no need of a thinking-body; and because quantity may be considered without considering body, they think also that quantity may be without body, and body without quantity; and that a body has quantity by the addition of quantity to it. From the same fountain spring those insignificant words, *abstract substance, separated essence*, and the like; as also that confusion of words derived from the Latin verb *est*, as *essence, essentiality, entity, entitative*; besides *reality, aliquidity, quiddity, &c.* which could never have been heard of among such nations as do not copulate their names by the verb *is*, but by adjective verbs, as *runneth, readeth, &c.* or by the mere placing of one name after another; and yet seeing such nations compute and reason, it is evident that philosophy has no need of those words *essence, entity*, and other the like barbarous terms.

Proposition,
universal and
particular.

5. There are many distinctions of propositions, whereof the first is, that some are *universal*, others *particular*, others *indefinite*, and others *singular*; and this is commonly called the distinction of *quantity*. An *universal* proposition is that whose subject is affected with the sign of an universal name, as *every man is a living creature*. *Particular*, that whose subject is affected with the sign of a particular name, as *some man is learned*. An *indefinite* proposition has for its subject a common name, and put without any sign, as *man is a living creature, man is learned*.

And a *singular* proposition is that whose subject is a singular name, as *Socrates is a philosopher*, *this man is black*. PART I.
3.

6. The second distinction is into *affirmative* and *negative*, and is called the distinction of *quality*. An *affirmative* proposition is that whose predicate is a positive name, as *man is a living creature*. *Negative*, that whose predicate is a negative name, as *man is not a stone*. Affirmative
and negative.

7. The third distinction is, that one is *true*, another *false*. A *true* proposition is that, whose predicate contains, or comprehends its subject, or whose predicate is the name of every thing, of which the subject is the name; as *man is a living creature* is therefore a true proposition, because whatsoever is called *man*, the same is also called *living creature*; and *some man is sick*, is true, because *sick* is the name of *some man*. That which is not true, or that whose predicate does not contain its subject, is called a *false* proposition, as *man is a stone*. True & false.

Now these words *true*, *truth*, and *true proposition*, are equivalent to one another; for truth consists in speech, and not in the things spoken of; and though *true* be sometimes opposed to *apparent* or *feigned*, yet it is always to be referred to the truth of proposition; for the image of a man in a glass, or a ghost, is therefore denied to be a very man, because this proposition, *a ghost is a man*, is not true; for it cannot be denied but that a ghost is a very ghost. And therefore truth or verity is not any affection of the thing, but of the proposition concerning it. As for that which the writers of metaphysics say, that *a thing, one*

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3.

thing, and *a very thing*, are equivalent to one another, it is but trifling and childish ; for who does not know, that *a man*, *one man*, and *a very man*, signify the same.

True & false
belongs to
speech, and
not to things.

8. And from hence it is evident, that truth and falsity have no place but amongst such living creatures as use speech. For though some brute creatures, looking upon the image of a man in a glass, may be affected with it, as if it were the man himself, and for this reason fear it or fawn upon it in vain ; yet they do not apprehend it as true or false, but only as like ; and in this they are not deceived. Wherefore, as men owe all their true ratiocination to the right understanding of speech ; so also they owe their errors to the misunderstanding of the same ; and as all the ornaments of philosophy proceed only from man, so from man also is derived the ugly absurdity of false opinions. For speech has something in it like to a spider's web, (as it was said of old of *Solon's* laws) for by contexture of words tender and delicate wits are ensnared and stopped ; but strong wits break easily through them.

From hence also this may be deduced, that the first truths were arbitrarily made by those that first of all imposed names upon things, or received them from the imposition of others. For it is true (for example) that *man is a living creature*, but it is for this reason, that it pleased men to impose both those names on the same thing.

Proposition,
primary,
not primary,
definition,
axiom,
petition.

9. Fourthly, propositions are distinguished into *primary* and *not primary*. *Primary* is that wherein the subject is explicated by a predicate of many names, as *man is a body, animated,*

rational; for that which is comprehended in the name *man*, is more largely expressed in the names *body*, *animated*, and *rational*, joined together; and it is called *primary*, because it is first in ratiocination; for nothing can be proved, without understanding first the name of the thing in question. Now *primary* propositions are nothing but definitions, or parts of definitions, and these only are the principles of demonstration, being truths constituted arbitrarily by the inventors of speech, and therefore not to be demonstrated. To these propositions, some have added others, which they call *primary* and *principles*, namely, *axioms*, and *common notions*; which, (though they be so evident that they need no proof) yet, because they may be proved, are not truly principles; and the less to be received for such, in regard propositions not intelligible, and sometimes manifestly false, are thrust on us under the name of *principles* by the clamour of men, who obtrude for evident to others, all that they themselves think true. Also certain petitions are commonly received into the number of principles; as, for example, *that a straight line may be drawn between two points*, and other petitions of the writers of geometry; and these are indeed the principles of art or construction, but not of science and demonstration.

10. Fifthly, propositions are distinguished into *necessary*, that is, necessarily true; and true, but not necessarily, which they call *contingent*. A *necessary* proposition is when nothing can at any time be conceived or feigned, whereof the subject is the name, but the predicate also is the name of

Proposition
necessary &
contingent.

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3.

Proposition
necessary &
contingent.

the same thing ; as *man is a living creature* is a necessary proposition, because at what time soever we suppose the name *man* agrees with any thing, at that time the name *living-creature* also agrees with the same. But a *contingent* proposition is that, which at one time may be true, at another time false ; as *every crow is black* ; which may perhaps be true now, but false hereafter. Again, in every *necessary* proposition, the predicate is either equivalent to the subject, as in this, *man is a rational living creature* ; or part of an equivalent name, as in this, *man is a living creature*, for the name *rational-living-creature*, or *man*, is compounded of these two, *rational* and *living-creature*. But in a *contingent* proposition this cannot be ; for though this were true, *every man is a liar*, yet because the word *liar* is no part of a compounded name equivalent to the name *man*, that proposition is not to be called *necessary*, but *contingent*, though it should happen to be true always. And therefore those propositions only are *necessary*, which are of sempiternal truth, that is, true at all times. From hence also it is manifest, that truth adheres not to things, but to speech only, for some truths are eternal ; for it will be eternally true, *if man, then living-creature* ; but that any *man*, or *living-creature*, should exist eternally, is not necessary.

Categorical &
hypothetical.

11. A sixth distinction of propositions is into *categorical* and *hypothetical*. A *categorical* proposition is that which is simply or absolutely pronounced, as *every man is a living-creature*, *no man is a tree* ; and *hypothetical* is that which is pronounced conditionally, as, *if any thing be a*

man, the same is also a living-creature, if any-thing be a man, the same is also not-a-stone.

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3.

7 A *categorical* proposition, and an *hypothetical* answering it, do both signify the same, if the propositions be necessary ; but not if they be contingent. For example, if this, *every man is a living-creature*, be true, this also will be true, *if any thing be a man, the same is also a living-creature* ; but in contingent propositions, though this be true, *every crow is black*, yet this, *if any thing be a crow, the same is black*, is false. But an *hypothetical* proposition is then rightly said to be true, when the consequence is true, as *every man is a living-creature*, is rightly said to be a true proposition, because of whatsoever it is truly said *that is a man*, it cannot but be truly said also, *the same is a living creature*. And therefore whensoever an *hypothetical* proposition is true, the *categorical* answering it, is not only true, but also necessary ; which I thought worth the noting, as an argument, that philosophers may in most things reason more solidly by *hypothetical* than *categorical* propositions.

Categorical &
hypothetical.

12. But seeing every proposition may be, and uses to be, pronounced and written in many forms, and we are obliged to speak in the same manner as most men speak, yet they that learn philosophy from masters, had need to take heed they be not deceived by the variety of expressions. And therefore, whensoever they meet with any obscure proposition, they ought to reduce it to its most simple and categorical form ; in which the copulative word *is* must be expressed by itself, and not mingled in any manner either with the subject or

The same
proposition
diversely
pronounced.

PART I.
3.

predicate, both which must be separated and clearly distinguished one from another. For example, if this proposition, *man can not sin*, be compared with this, *man cannot sin*, their difference will easily appear if they be reduced to these, *man is able not to sin*, and, *man is not able to sin*, where the predicates are manifestly different. But they ought to do this silently by themselves, or betwixt them and their masters only; for it will be thought both ridiculous and absurd, for a man to use such language publicly. Being therefore to speak of *equipollent* propositions, I put in the first place all those for *equipollent*, that may be reduced purely to one and the same categorical proposition.

Propositions that may be reduced to the same categorical proposition, are equipollent.

13. Secondly, that which is categorical and necessary, is *equipollent* to its hypothetical proposition; as this categorical, *a right-lined triangle has its three angles equal to two right angles*, to this hypothetical, *if any figure be a right-lined triangle, the three angles of it are equal to two right angles*.

Universal propositions converted by contradictory names, are equipollent.

14. Also, any two universal propositions, of which the terms of the one (that is, the subject and predicate) are contradictory to the terms of the other, and their order inverted, as these, *every man is a living creature*, and *every thing that is not a living-creature is not a man*, are *equipollent*. For seeing *every man is a living creature* is a true proposition, the name *living creature* contains the name *man*; but they are both positive names, and therefore (by the last article of the precedent chapter) the negative name *not man*, contains the negative name *not living-creature*;

wherefore *every thing that is not a living-creature, is not a man*, is a true proposition. Likewise these, *no man is a tree, no tree is a man*, are equipollent. For if it be true that *tree* is not the name of any *man*, then no one thing can be signified by the two names *man* and *tree*, wherefore *no tree is a man* is a true proposition. Also to this, *whatsoever is not a living-creature is not a man*, where both the terms are negative, this other proposition is equipollent, *only a living creature is a man*.

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8.

15. Fourthly, negative propositions, whether the particle of negation be set after the copula as some nations do, or before it, as it is in Latin and Greek, if the terms be the same, are equipollent: as, for example, *man is not a tree*, and, *man is not-a-tree*, are equipollent, though Aristotle deny it. Also these, *every man is not a tree*, and *no man is a tree*, are equipollent, and that so manifestly, as it needs not be demonstrated.

Negative propositions are the same, whether the negation be before or after the copula.

16. Lastly, all particular propositions that have their terms inverted, as these, *some man is blind*, *some blind thing is a man*, are equipollent; for either of the two names, is the name of some one and the same man; and therefore in which soever of the two orders they be connected, they signify the same truth.

Particular propositions simply converted, are equipollent.

17. Of propositions that have the same terms, and are placed in the same order, but varied either by quantity or quality, some are called *subaltern*, others *contrary*, others *subcontrary*, and others *contradictory*.

What are subaltern, contrary, subcontrary, and contradictory propositions.

Subaltern, are universal and particular propositions of the same quality; as, *every man is a*

PART I. *living creature, some man is a living creature ;*
 3.
 or, *no man is wise, some man is not wise.* Of
 Propositions. these, if the universal be true, the particular will
 be true also.

Contrary, are universal propositions of different quality ; as, *every man is happy, no man is happy.* And of these, if one be true, the other is false : also, they may both be false, as in the example given.

Subcontrary, are particular propositions of different quality ; as, *some man is learned, some man is not learned ;* which cannot be both false, but they may be both true.

Contradictory are those that differ both in quantity and quality ; as, *every man is a living creature, some man is not a living-creature ;* which can neither be both true, nor both false.

Consequence,
 what it is.

18. A proposition is said to *follow* from two other propositions, when these being granted to be true, it cannot be denied but the other is true also. For example, let these two propositions, *every man is a living creature*, and, *every living creature is a body*, be supposed true, that is, that *body* is the name of *every living creature*, and *living creature* the name of *every man*. Seeing therefore, if these be understood to be true, it cannot be understood that *body* is not the name of *every man*, that is, that *every man is a body* is false, this proposition will be said to *follow* from those two, or to be necessarily *inferred* from them.

Falsity
 cannot follow
 from truth.

19. That a true proposition may follow from false propositions, may happen sometimes ; but false from true, never. For if these, *every man is a stone*, and *every stone is a living creature*,

(which are both false) be granted to be true, it is granted also that *living creature* is the name of every stone, and stone of every man, that is, that *living creature* is the name of every man; that is to say, this proposition *every man is a living creature*, is true, as it is indeed true. Wherefore a true proposition may sometimes follow from false. But if any two propositions be true, a false one can never follow from them. For if true follow from false, for this reason only, that the false are granted to be true, then truth from two truths granted will follow in the same manner.

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3.

20. Now, seeing none but a true proposition will follow from true, and that the understanding of two propositions to be true, is the cause of understanding that also to be true which is deduced from them; the two antecedent propositions are commonly called the causes of the inferred proposition, or conclusion. And from hence it is that logicians say, the *premises* are causes of the *conclusion*; which may pass, though it be not properly spoken; for though understanding be the cause of understanding, yet speech is not the cause of speech. But when they say, the cause of the properties of any thing, is the thing itself, they speak absurdly. For example, if a figure be propounded which is triangular; seeing every triangle has all its angles together equal to two right angles, from whence it follows that all the angles of that figure are equal to two right angles, they say, for this reason, that that figure is the cause of that equality. But seeing the figure does not itself make its angles, and therefore cannot be said to be the *efficient-cause*, they

How one proposition is the cause of another.

PART I.
3.

How one proposition is the cause of another.

call it the *formal-cause*; whereas indeed it is no cause at all; nor does the property of any figure follow the figure, but has its being at the same time with it; only the knowledge of the figure goes before the knowledge of the properties; and one knowledge is truly the cause of another knowledge, namely the *efficient cause*.

And thus much concerning *proposition*; which in the progress of philosophy is the first step, like the moving towards of one foot. By the due addition of another step I shall proceed to *syllogism*, and make a complete pace. Of which in the next chapter.

 CHAPTER IV.

OF SYLLOGISM.

1. The definition of syllogism.—2. In a syllogism there are but three terms.—3. Major, minor, and middle term; also major and minor proposition, what they are.—4. The middle term in every syllogism ought to be determined in both the propositions to one and the same thing.—5. From two particular propositions nothing can be concluded.—6. A syllogism is the collection of two propositions into one sum.—7. The figure of a syllogism, what it is.—8. What is in the mind answering to a syllogism.—9. The first indirect figure, how it is made.—10. The second indirect figure, how made.—11. How the third indirect figure is made.—12. There are many moods in every figure, but most of them useless in philosophy.—13. An hypothetical syllogism when equipollent to a categorical.

Definition of syllogism.

1. A **SPEECH**, consisting of three propositions, from two of which the third follows, is called a **SYLLOGISM**; and that which follows is called the *conclusion*; the other two *premises*. For example,

this speech, *every man is a living creature, every living creature is a body*, therefore, *every man is a body*, is a *syllogism*, because the third proposition follows from the two first ; that is, if those be granted to be true, this must also be granted to be true. PART I.
4.

2. From two propositions which have not one term common, no conclusion can follow ; and therefore no *syllogism* can be made of them. For let any two premises, *a man is a living creature, a tree is a plant*, be both of them true, yet because it cannot be collected from them that *plant* is the name of a *man*, or *man* the name of a *plant*, it is not necessary that this conclusion, *a man is a plant*, should be true. Corollary : therefore, in the *premises* of a *syllogism* there can be but three *terms*. In a syllogism there are but three terms.

Besides, there can be no term in the *conclusion*, which was not in the *premises*. For let any two premises be, *a man is a living creature, a living creature is a body*, yet if any other term be put in the conclusion, as *man is two-footed* ; though it be true, it cannot follow from the premises, because from them it cannot be collected, that the name *two-footed* belongs to a *man* ; and therefore, again, in every *syllogism* there can be but three *terms*.

3. Of these terms, that which is the *predicate* in the conclusion, is commonly called the *major* ; that which is the *subject* in the conclusion, the *minor*, and the other is the *middle term* ; as in this *syllogism*, *a man is a living creature, a living creature is a body*, therefore, *a man is a body*, *body* is the *major*, *man* the *minor*, and Major, minor and middle term ; also major & minor proposition, what they are.

PART I.
4

living creature the *middle term*. Also of the premises, that in which the *major term* is found, is called the *major proposition*, and that which has the *minor term*, the *minor proposition*.

The middle term in every syllogism to be determined in both propositions to one and the same thing.

4. If the middle term be not in both the premises determined to one and the same singular thing, no conclusion will follow, nor syllogism be made. For let the minor term be *man*, the middle term *living creature*, and the major term *lion*; and let the premises be, *man is a living creature*, *some living creature is a lion*, yet it will not follow that *every* or *any man is a lion*. By which it is manifest, that in every syllogism, that proposition which has the middle term for its *subject*, ought to be either *universal* or *singular*, but not *particular* nor *indefinite*. For example, this syllogism, *every man is a living creature*, *some living creature is four-footed*, therefore *some man is four-footed*, is therefore faulty, because the middle term, *living creature*, is in the first of the premises determined only to *man*, for there the name of *living creature* is given to man only, but in the latter premise it may be understood of some other living creature besides man. But if the latter premise had been *universal*, as here, *every man is a living creature*, *every living creature is a body*, therefore *every man is a body*, the syllogism had been true; for it would have followed that *body* had been the name of *every living creature*, that is of *man*; that is to say, the conclusion *every man is a body* had been true. Likewise, when the middle term is a *singular* name, a syllogism may be made, I say a true syllogism, though useless in philosophy, as this, *some man is Socrates*, *Socrates*

is a philosopher, therefore, *some man is a philosopher* ; for the premises being granted, the conclusion cannot be denied. PART I.
4.

5. And therefore of two premises, in both which the middle term is particular, a syllogism cannot be made ; for whether the middle term be the *subject* in both the premises, or the *predicate* in both, or the *subject* in one, and the *predicate* in the other, it will not be necessarily determined to the same thing. For let the premises be,

Some man is blind, } In both which the middle
Some man is learned, } term is the *subject*,

it will not follow that *blind* is the name of any *learned man*, or *learned* the name of any *blind man*, seeing the name *learned* does not contain the name *blind*, nor this that ; and therefore it is not necessary that both should be names of the same man. So from these premises,

Every man is a living-creature, } In both which the middle
Every horse is a living-creature, } term is the *predicate*,

nothing will follow. For seeing *living creature* is in both of them *indefinite*, which is equivalent to *particular*, and that *man* may be one kind of *living creature*, and *horse* another kind, it is not necessary that *man* should be the name of *horse*, or *horse* of *man*. Or if the premises be,

Every man is a living-creature, } In one of which the middle-
Some living creature is } term is the *subject*, and in
four-footed, } the other the *predicate*,

the conclusion will not follow, because the name

PART I. *living creature* being not determined, it may in
 4. one of them be understood of *man*, in the other of
not-man.

A syllogism is
 the collection
 of two propo-
 sitions into
 one sum.

6. Now it is manifest from what has been said,
 that a syllogism is nothing but a collection of the
 sum of two propositions, joined together by a
 common term, which is called the *middle term*.
 And as proposition is the addition of two names,
 so syllogism is the adding together of three.

The figure of
 a syllogism
 what it is.

7. Syllogisms are usually distinguished according
 to their diversity of *figures*, that is, by the diverse
 position of the middle term. And again in
 figure there is a distinction of certain *moods*,
 which consist of the differences of propositions in
quantity and *quality*. The first figure is that, in
 which the terms are placed one after another
 according to their latitude of signification; in
 which order the *minor term* is first, the *middle*
term next, and the *major* last; as, if the minor
 term be *man*, the middle term, *living creature*,
 and the major term, *body*, then, *man is a living-*
creature, is a body, will be a syllogism in the first
 figure: in which, *man is a living creature* is the
 minor proposition; the major, *living creature is*
a body, and the conclusion, or sum of both, *man is*
a body. Now this figure is called *direct*, because
 the terms stand in direct order; and it is varied
 by *quantity* and *quality* into four *moods*: of
 which the first is that wherein all the terms are
positive, and the minor term *universal*, as *every*
man is a living creature, every living creature is
a body: in which all the propositions are affirma-
 tive, and universal. But if the major term be a
 negative name, and the minor an universal name,

the *figure* will be in the second *mood*, as, *every man is a living creature, every living creature is not a tree*, in which the major proposition and conclusion are both universal and negative. To these two, are commonly added two more, by making the minor term particular. Also it may happen that both the major and middle terms are negative terms, and then there arises another *mood*, in which all the propositions are negative, and yet the syllogism will be good; as, if the minor term be *man*, the middle term *not a stone*, and the major term *not a flint*, this syllogism, *no man is a stone, whatsoever is not a stone is not a flint, therefore, no man is a flint*, is true, though it consist of three negatives. But in philosophy, the profession whereof is to establish universal rules concerning the properties of things, seeing the difference betwixt negatives and affirmatives is only this, that in the former the subject is affirmed by a negative name, and by a positive in the latter, it is superfluous to consider any other *mood* in direct *figure*, besides that, in which all the propositions are both universal and affirmative.

8. The thoughts in the mind answering to a direct syllogism, proceed in this manner; first, there is conceived a phantasm of the thing named, with that accident or quality thereof, for which it is in the minor proposition called by that name which is the subject; next, the mind has a phantasm of the same thing with that accident, or quality, for which it hath the name, that in the same proposition is the predicate; thirdly, the thought returns of the same thing as having that

What is
in the mind
answering to
a syllogism.

PART I.

4.

accident in it, for which it is called by the name, that is the predicate of the major proposition ; and lastly, remembering that all those are the accidents of one and the same thing, it concludes that those three names are also names of one and the same thing ; that is to say, the conclusion is true. For example, when this syllogism is made, *man is a living creature, a living creature is a body, therefore, man is a body*, the mind conceives first an image of a man speaking or discoursing, and remembers that that, which so appears, is called *man* ; then it has the image of the same man moving, and remembers that that, which appears so, is called *living creature* ; thirdly, it conceives an image of the same man, as filling some place or space, and remembers that what appears so is called *body* ; and lastly, when it remembers that that thing, which was extended, and moved and spake, was one and the same thing, it concludes that the three names, *man, living creature, and body*, are names of the same thing, and that therefore *man is a living creature* is a true proposition. From whence it is manifest, that living creatures that have not the use of speech, have no conception or thought in the mind, answering to a syllogism made of universal propositions ; seeing it is necessary to think not only of the thing, but also by turns to remember the divers names, which for divers considerations thereof are applied to the same.

The first indirect figure how made.

9. The rest of the figures arise either from the inflexion, or inversion of the first or direct figure ; which is done by changing the major, or minor, or both the propositions, into converted propositions equipollent to them.

From whence follow three other figures; of which, two are *inflected*, and the third *inverted*. PART I.
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 The first of these three is made by the conversion of the major proposition. For let the minor, middle, and major terms stand in direct order, thus, *man is a living creature, is not a stone*, which is the first or direct figure; the inflection will be by converting the major proposition in this manner, *man is a living creature, a stone is not a living creature*; and this is the second figure, or the first of the indirect figures; in which the conclusion will be, *man is not a stone*. For (having shown in the last chapter, art. 14, that universal propositions, converted by contradiction of the terms, are equipollent) both those syllogisms conclude alike; so that if the major be read (like Hebrew) backwards, thus, *a living creature is not a stone*, it will be direct again, as it was before. In like manner this direct syllogism, *man is not a tree, is not a pear-tree*, will be made indirect by converting the major proposition (by contradiction of the terms) into another equipollent to it, thus, *man is not a tree, a pear-tree is a tree*; for the same conclusion will follow, *man is not a pear-tree*.

The first indirect figure how made.

But for the conversion of the direct figure into the first indirect figure, the major term in the direct figure ought to be negative. For though this direct, *man is a living creature, is a body*, be made indirect, by converting the major proposition, thus,

*Man is a living creature,
Not a body is not a living creature,
Therefore, Every man is a body;*

Yet this conversion appears so obscure, that

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this mood is of no use at all. By the conversion of the major proposition, it is manifest, that in this figure, the middle term is always the predicate in both the premises.

Second indirect figure how made.

10. The second indirect figure is made by converting the minor proposition, so as that the middle term is the subject in both. But this never concludes universally, and therefore is of no use in philosophy. Nevertheless I will set down an example of it; by which this direct

*Every man is a living creature,
Every living creature is a body,*

by conversion of the minor proposition, will stand thus,

*Some living creature is a man,
Every living creature is a body,
Therefore, Some man is a body.*

For *every man s a living creature* cannot be converted into this, *every living creature is a man*: and therefore if this syllogism be restored to its direct form, the minor proposition will be *some man is a living creature*, and consequently the conclusion will be *some man is a body*, seeing the minor term *man*, which is the subject in the conclusion, is a particular name.

How the third indirect figure is made.

11. The third indirect or inverted figure, is made by the conversion of both the premises. For example, this direct syllogism,

*Every man is a living creature,
Every living creature is not a stone,
Therefore, Every man is not a stone,*

being inverted, will stand thus,

*Every stone is not a living creature,
 Whatsoever is not a living creature, is not a man,
 Therefore, Every stone is not a man ;*

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which conclusion is the converse of the direct conclusion, and equipollent to the same.

The figures, therefore, of syllogisms, if they be numbered by the diverse situation of the middle term only, are but three ; in the first whereof, the middle term has the middle place ; in the second, the last ; and in the third, the first place. But if they be numbered according to the situation of the terms simply, they are four ; for the first may be distinguished again into two, namely, into direct and inverted. From whence it is evident, that the controversy among logicians concerning the fourth figure, is a mere *λογόμαχια*, or contention about the name thereof ; for, as for the thing itself, it is plain that the situation of the terms (not considering the quantity or quality by which the moods are distinguished) makes four differences of syllogisms, which may be called figures, or have any other name at pleasure.

12. In every one of these figures there are many moods, which are made by varying the premises according to all the differences they are capable of, by quantity and quality ; as namely, in the direct figure there are six moods ; in the first indirect figure, four ; in the second, fourteen ; and in the third, eighteen. But because from the direct figure I rejected as superfluous all moods besides that which consists of universal propositions, and whose minor proposition is affirmative, I do, together with it, reject the moods of the rest

There are many moods in every figure, but most of them useless in philosophy.

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An hypothet-
ical syllogism
when equipol-
lent to a cate-
gorical.

of the figures which are made by conversion of the premises in the direct figure.

13. As it was showed before, that in necessary propositions a categorical and hypothetical proposition are equipollent; so likewise it is manifest that a categorical and hypothetical syllogism are equivalent. For every categorical syllogism, as this,

*Every man is a living creature,
Every living creature is a body,
Therefore, Every man is a body,*

is of equal force with this hypothetical syllogism :

*If any thing be a man, the same is also a living creature,
If any thing be a living creature, the same is a body,
Therefore, If any thing be a man, the same is a body.*

In like manner, this categorical syllogism in an indirect figure,

*No stone is a living creature,
Every man is a living creature,
Therefore, No man is a stone,
Or, No stone is a man,*

is equivalent to this hypothetical syllogism :

*If any thing be a man, the same is a living creature,
If any thing be a stone, the same is not a living creature,
Therefore, If any thing be a stone, the same is not a man,
Or, If any thing be a man, the same is not a stone.*

And thus much seems sufficient for the nature of syllogisms; (for the doctrine of moods and figures is clearly delivered by others that have written largely and profitably of the same). Nor are precepts so necessary as practice for the attaining of true ratiocination; and they that study the demonstrations of mathematicians, will

sooner learn true logic, than they that spend time in reading the rules of syllogizing which logicians have made; no otherwise than little children learn to go, not by precepts, but by exercising their feet. This, therefore, may serve for the first pace in the way to Philosophy.

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In the next place I shall speak of the faults and errors into which men that reason unwarily are apt to fall; and of their kinds and causes.

CHAPTER V.

OF ERRING, FALSITY, AND CAPTIONS.

1. Erring and falsity how they differ. Error of the mind by itself without the use of words, how it happens.—2. A sevenfold incoherency of names, every one of which makes always a false proposition.—3. Examples of the first manner of incoherency.—4. Of the second.—5. Of the third.—6. Of the fourth.—7. Of the fifth.—8. Of the sixth.—9. Of the seventh.
10. Falsity of propositions detected by resolving the terms with definitions continued till they come to simple names, or names that are the most general of their kind.—11. Of the fault of a syllogism consisting in the implication of the terms with the copula.—12. Of the fault which consists in equivocation.—13. Sophistical captions are oftener faulty in the matter than in the form of syllogisms.

1. MEN are subject to *err* not only in affirming and denying, but also in perception, and in silent cogitation. In affirming and denying, when they call any thing by a name, which is not the name thereof; as if from seeing the sun first by reflection in water, and afterwards again directly in the

Erring & falsity how they differ. Error of the mind by itself, without the use of words, how it happens.

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Erring and
falsity how
they differ.

firmament, we should to both those appearances give the name of sun, and say there are two suns ; which none but men can do, for no other living creatures have the use of names. This kind of error only deserves the name of *falsity*, as arising, not from sense, nor from the things themselves, but from pronouncing rashly ; for names have their constitution, not from the species of things, but from the will and consent of men. And hence it comes to pass, that men pronounce *falsely*, by their own negligence, in departing from such appellations of things as are agreed upon, and are not deceived neither by the things, nor by the sense ; for they do not perceive that the thing they see is called sun, but they give it that name from their own will and agreement. Tacit errors, or the errors of sense and cogitation, are made, by passing from one imagination to the imagination of another different thing ; or by feigning that to be past, or future, which never was, nor ever shall be ; as when, by seeing the image of the sun in water, we imagine the sun itself to be there ; or by seeing swords, that there has been or shall be fighting, because it uses to be so for the most part ; or when from promises we feign the mind of the promiser to be such and such ; or lastly, when from any sign we vainly imagine something to be signified, which is not. And errors of this sort are common to all things that have sense ; and yet the deception proceeds neither from our senses, nor from the things we perceive ; but from ourselves while we feign such things as are but mere images to be something more than images. But neither things, nor imaginations of

things, can be said to be false, seeing they are truly what they are ; nor do they, as signs, promise any thing which they do not perform ; for they indeed do not promise at all, but we from them ; nor do the clouds, but we, from seeing the clouds, say it shall rain. The best way, therefore, to free ourselves from such errors as arise from natural signs, is first of all, before we begin to reason concerning such conjectural things, to suppose ourselves ignorant, and then to make use of our ratiocination ; for these errors proceed from the want of ratiocination ; whereas, errors which consist in affirmation and negation, (that is, the falsity of propositions) proceed only from reasoning amiss. Of these, therefore, as repugnant to philosophy, I will speak principally.

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2. Errors which happen in reasoning, that is, in syllogizing, consist either in the falsity of the premises, or of the inference. In the first of these cases, a syllogism is said to be faulty in the *matter* of it ; and in the second case, in the *form*. I will first consider the matter, namely, how many ways a proposition may be false ; and next the form, and how it comes to pass, that when the premises are true, the inference is, notwithstanding, false.

A sevenfold incoherency of names, all of which make always a false proposition.

Seeing, therefore, that proposition only is true, (chap. III, art. 7) in which are copulated two names of one and the same thing ; and that always false, in which names of different things are copulated, look how many ways names of different things may be copulated, and so many ways a false proposition may be made.

Now, all things to which we give names, may be

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reduced to these four kinds, namely, *bodies*, *accidents*, *phantasms*, and *names* themselves; and therefore, in every true proposition, it is necessary that the names copulated, be both of them names of *bodies*, or both names of *accidents*, or both names of *phantasms*, or both names of *names*. For names otherwise copulated are incoherent, and constitute a false proposition. It may happen, also, that the name of a *body*, of an *accident*, or of a *phantasm*, may be copulated with the name of a *speech*. So that copulated names may be incoherent seven manner of ways.

1. If the name of a Body	} be copulated with	the name of an Accident.
2. If the name of a Body		the name of a Phantasm.
3. If the name of a Body		the name of a Name.
4. If the name of an Accident		the name of a Phantasm.
5. If the name of an Accident		the name of a Name.
6. If the name of a Phantasm		the name of a Name.
7. If the name of a Body, Accident, or Phantasm		the name of a Speech.

Of all which I will give some examples.

Examples
of the first
manner of
incoherency.

3. After the first of these ways propositions are false, when abstract names are copulated with concrete names; as (in Latin and Greek) *esse est ens*, *essentia est ens*, τὸ τί ἦν εἶναι (*i.*); *quidditas est ens*, and many the like, which are found in Aristotle's *Metaphysics*. Also, the *understanding worketh*, the *understanding understandeth*, the *sight seeth*; a *body is magnitude*, a *body is quantity*, a *body is extension*; to *be a man is a man*, *whiteness is a white thing*, &c.; which is as if one should say, the *runner is the running*, or the *walk walketh*. Moreover, *essence is separated*, *substance is abstracted*: and others like these, or derived from these, (with which common

philosophy abounds.) For seeing no *subject* of PART I.
5. an *accident* (that is, no *body*) is an *accident*: no name of an *accident* ought to be given to a *body*, nor of a *body* to an *accident*.

4. False, in the second manner, are such propo- The second. sitions as these; a *ghost is a body*, or a *spirit*, that is, a thin body; *sensible species fly up and down in the air*, or *are moved hither and thither*, which is proper to bodies; also, a *shadow is moved*, or *is a body*; *light is moved*, or *is a body*; *colour is the object of sight*, *sound of hearing*; *space or place is extended*; and innumerable others of this kind. For seeing ghosts, sensible species, a shadow, light, colour, sound, space, &c. appear to us no less sleeping than waking, they cannot be things without us, but only phantasms of the mind that imagines them; and therefore the names of these, copulated with the names of bodies, cannot constitute a true proposition.

5. False propositions of the third kind, are such The third. as these; *genus est ens*, *universale est ens*, *ens de ente predicatur*. For *genus*, and *universale*, and *predicare*, are names of names, and not of things. Also, *number is infinite*, is a false proposition; for no number can be infinite, but only the word *number* is then called an indefinite name when there is no determined number answering to it in the mind.

6. To the fourth kind belong such false propo- The fourth. sitions as these, *an object is of such magnitude or figure as appears to the beholders*; *colour, light, sound, are in the object*; and the like. For the same object appears sometimes greater, sometimes

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lesser, sometimes square, sometimes round, according to the diversity of the distance and medium; but the true magnitude and figure of the thing seen is always one and the same; so that the magnitude and figure which appears, is not the true magnitude and figure of the object, nor anything but phantasm; and therefore, in such propositions as these, the names of accidents are copulated with the names of phantasms.

The fifth.

7. Propositions are false in the fifth manner, when it is said that *the definition is the essence of a thing; whiteness, or some other accident, is the genus, or universal.* For definition is not the essence of any thing, but a speech signifying what we conceive of the essence thereof; and so also not whiteness itself, but the word whiteness, is a genus, or an universal name.

The sixth.

8. In the sixth manner they err, that say the *idea of anything is universal*; as if there could be in the mind an image of a man, which were not the image of some one man, but a man simply, which is impossible; for every idea is one, and of one thing; but they are deceived in this, that they put the *name* of the thing for the *idea* thereof.

The seventh.

9. They err in the seventh manner, that make this distinction between things that have being, that some of them *exist by themselves*, others by *accident*; namely, because *Socrates is a man* is a necessary proposition, and *Socrates is a musician* a contingent proposition, therefore they say some things exist necessarily or by themselves, others contingently or by accident; whereby, seeing *necessary, contingent, by itself, by accident*, are not names of things, but of propositions, they

that say *any thing that has being, exists by accident*, copulate the name of a proposition with the name of a thing. In the same manner also, they err, which place some ideas in the understanding, others in the fancy ; as if from the understanding of this proposition, *man is a living creature*, we had one idea or image of a man derived from sense to the memory, and another to the understanding ; wherein that which deceives them is this, that they think one idea should be answerable to a name, another to a proposition, which is false ; for proposition signifies only the order of those things one after another, which we observe in the same idea of man ; so that this proposition, *man is a living creature* raises but one idea in us, though in that idea we consider that first, for which he is called man, and next that, for which he is called living creature. The falsities of propositions in all these several manners, is to be discovered by the definitions of the copulated names.

10. But when names of bodies are copulated with names of bodies, names of accidents with names of accidents, names of names with names of names, and names of phantasms with names of phantasms, if we, nevertheless, remain still doubtful whether such propositions are true, we ought then in the first place to find out the definition of both those names, and again the definitions of such names as are in the former definition, and so proceed by a continual resolution till we come to a simple name, that is, to the most general or most universal name of that kind ; and if after all

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Falsity of propositions detected by resolving the terms with definitions.))

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this, the truth or falsity thereof be not evident, we must search it out by philosophy, and ratiocination, beginning from definitions. For every proposition, universally true, is either a definition, or part of a definition, or the evidence of it depends upon definitions.

Of the fault of a syllogism consisting in implication of the terms with the copula.

11. That fault of a syllogism which lies hid in the form thereof, will always be found either in the implication of the copula with one of the terms, or in the equivocation of some word; and in either of these ways there will be four terms, which (as I have shewn) cannot stand in a true syllogism. Now the implication of the copula with either term, is easily detected by reducing the propositions to plain and clear predication; as (for example) if any man should argue thus,

*The hand toucheth the pen,
The pen toucheth the paper,
Therefore, The hand toucheth the paper;*

the fallacy will easily appear by reducing it, thus:

*The hand, is, touching the pen,
The pen, is, touching the paper,
Therefore, The hand, is, touching the paper;*

where there are manifestly these four terms, *the hand, touching the pen, the pen, and touching the the paper*. But the danger of being deceived by sophisms of this kind, does not seem to be so great, as that I need insist longer upon them.

Of the fault which consists in equivocation.

12. And though there may be fallacy in equivocal terms, yet in those that be manifestly such, there is none at all; nor in metaphors, for they profess the transferring of names from one thing

to another. Nevertheless, sometimes equivocals (and those not very obscure) may deceive; as in this argumentation:—*It belongs to metaphysics to treat of principles; but the first principle of all, is, that the same thing cannot both exist and not exist at the same time; and therefore it belongs to metaphysics to treat whether the same thing may both exist and not exist at the same time*; where the fallacy lies in the equivocation of the word principle; for whereas Aristotle in the beginning of his *Metaphysics*, says, that *the treating of principles belongs to primary science*, he understands by principles, causes of things, and certain existences which he calls primary; but where he says *a primary proposition is a principle*, by principle, there, he means the beginning and cause of knowledge, that is, the understanding of words, which, if any man want, he is incapable of learning.

13. But the *captions* of sophists and sceptics, by which they were wont, of old, to deride and oppose truth, were faulty for the most part, not in the form, but in the matter of syllogism; and they deceived not others oftener than they were themselves deceived. For the force of that famous argument of Zeno against motion, consisted in this proposition, *whatsoever may be divided into parts, infinite in number, the same is infinite*; which he, without doubt, thought to be true, yet nevertheless is false. For to be divided into infinite parts, is nothing else but to be divided into as many parts as any man will. But it is not necessary that a line should have parts infinite in

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number, or be infinite, because I can divide and subdivide it as often as I please; for how many parts soever I make, yet their number is finite; but because he that says parts, simply, without adding how many, does not limit any number, but leaves it to the determination of the hearer, therefore we say commonly, a line may be divided infinitely; which cannot be true in any other sense.

Conclusion.

And thus much may suffice concerning syllogism, which is, as it were, the first pace towards philosophy; in which I have said as much as is necessary to teach any man from whence all true argumentation has its force. And to enlarge this treatise with all that may be heaped together, would be as superfluous, as if one should (as I said before) give a young child precepts for the teaching of him to go; for the art of reasoning is not so well learned by precepts as by practice, and by the reading of those books in which the conclusions are all made by severe demonstration. And so I pass on to the way of philosophy, that is, to the method of study.

CHAPTER VI.

OF METHOD.

1. Method and science defined.—2. It is more easily known concerning singular, than universal things, that they are ; and contrarily, it is more easily known concerning universal, than singular things, why they are, or what are their causes.—
3. What it is philosophers seek to know.—4. The first part, by which principles are found out, is purely analytical.—5. The highest causes, and most universal in every kind, are known by themselves.—6. Method from principles found out, tending to science simply, what it is.—7. That method of civil and natural science, which proceeds from sense to principles, is analytical ; and again, that, which begins at principles, is synthetical.—8. The method of searching out, whether any thing propounded be matter or accident.—9. The method of seeking whether any accident be in this, or in that subject.
10. The method of searching after the cause of any effect propounded.—11. Words serve to invention, as marks ; to demonstration, as signs.—12. The method of demonstration is synthetical.—13. Definitions only are primary and universal propositions.—14. The nature and definition of a definition.
15. The properties of a definition.—16. The nature of a demonstration.—17. The properties of a demonstration, and order of things to be demonstrated.—18. The faults of a demonstration.—19. Why the analytical method of geometriicians cannot be treated of in this place.

1. FOR the understanding of *method*, it will be necessary for me to repeat the definition of philosophy, delivered above (Chap. I, art. 2.) in this manner, *Philosophy is the knowledge we acquire, by true ratiocination, of appearances, or apparent effects, from the knowledge we have of some possible production or generation of the same ; and*

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Method and science defined.

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6.Method and
science defined

of such production, as has been or may be, from the knowledge we have of the effects. METHOD, therefore, in the study of philosophy, *is the shortest way of finding out effects by their known causes, or of causes by their known effects.* But we are then said to know any effect, when we know *that there be causes of the same, and in what subject those causes are, and in what subject they produce that effect, and in what manner they work the same.* And this is the science of causes, or, as they call it, of the *δίότι*. All other science, which is called the *ὄτι*, is either perception by sense, or the imagination, or memory remaining after such perception.

The first beginnings, therefore, of knowledge, are the phantasms of sense and imagination; and that there be such phantasms we know well enough by nature; but to know why they be, or from what causes they proceed, is the work of ratiocination; which consists (as is said above, in the 1st Chapter, Art. 2) in *composition*, and *division* or *resolution*. There is therefore no method, by which we find out the causes of things, but is either *compositive* or *resolutive*, or *partly compositive*, and *partly resolutive*. And the resolutive is commonly called *analytical* method, as the compositive is called *synthetical*.

2. It is common to all sorts of method, to proceed from known things to unknown; and this is manifest from the cited definition of philosophy. But in knowledge by sense, the whole object is more known, than any part thereof; as when we see a man, the conception or whole idea of that man is first or more known, than the particular

It is easier known concerning singular than universal things, that they are; and contrarily it is easier known concerning universal than sin-

ideas of his being *figurate*, *animate*, and *rational*; that is, we first see the whole man, and take notice of his being, before we observe in him those other particulars. And therefore in any knowledge of the *ὅτι*, or that any thing *is*, the beginning of our search is from the whole idea; and contrarily, in our knowledge of the *δίότι*, or of the causes of any thing, that is, in the sciences, we have more knowledge of the causes of the parts than of the whole. For the cause of the whole is compounded of the causes of the parts; but it is necessary that we know the things that are to be compounded, before we can know the whole compound. Now, by parts, I do not here mean parts of the thing itself, but parts of its nature; as, by the parts of man, I do not understand his head, his shoulders, his arms, &c. but his figure, quantity, motion, sense, reason, and the like; which accidents being compounded or put together, constitute the whole nature of man, but not the man himself. And this is the meaning of that common saying, namely, that some things are more known to us, others more known to nature; for I do not think that they, which so distinguish, mean that something is known to nature, which is known to no man; and therefore, by those things, that are more known to us, we are to understand things we take notice of by our senses, and, by more known to nature, those we acquire the knowledge of by reason; for in this sense it is, that the *whole*, that is, those things that have universal names, (which, for brevity's sake, I call *universal*) are more known to us than the *parts*, that is, such things as have names less universal,

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gular things,
why they are,
or what are
their causes.

PART I. (which I therefore call *singular*); and the causes
 6. of the parts are more known to nature than the
 cause of the whole; that is, universals than
 singulars.

What it is
 philosophers
 seek to know.

3. In the study of philosophy, men search after science either simply or indefinitely; that is, to know as much as they can, without propounding to themselves any limited question; or they enquire into the cause of some determined appearance, or endeavour to find out the certainty of something in question, as what is the cause of *light*, of *heat*, of *gravity*, of a *figure* propounded, and the like; or in what *subject* any propounded *accident* is inherent; or what may conduce most to the *generation* of some propounded *effect* from many *accidents*; or in what manner particular causes ought to be compounded for the production of some certain effect. Now, according to this variety of things in question, sometimes the *analytical method* is to be used, and sometimes the *synthetical*.

The first part,
 by which prin-
 ciples are dis-
 covered, is
 purely ana-
 lytical.

4. But to those that search after science indefinitely, which consists in the knowledge of the causes of all things, as far forth as it may be attained, (and the causes of singular things are compounded of the causes of universal or simple things) it is necessary that they know the causes of universal things, or of such accidents as are common to all bodies, that is, to all matter, before they can know the causes of singular things, that is, of those accidents by which one thing is distinguished from another. And, again, they must know what those universal things are, before they can know their causes. Moreover, seeing universal

things are contained in the nature of singular things, the knowledge of them is to be acquired by reason, that is, by resolution. For example, if there be propounded a conception or *idea* of some singular thing, as of a *square*, this square is to be resolved into a *plain, terminated with a certain number of equal and straight lines and right angles*. For by this resolution we have these things universal or agreeable to all matter, namely, *line, plain*, (which contains *superficies*) *terminated, angle, straightness, rectitude, and equality*; and if we can find out the causes of these, we may compound them altogether into the cause of a square. Again, if any man propound to himself the conception of *gold*, he may, by resolving, come to the ideas of *solid, visible, heavy*, (that is, tending to the centre of the earth, or downwards) and many other more universal than gold itself; and these he may resolve again, till he come to such things as are most universal. And in this manner, by resolving continually, we may come to know what those things are, whose causes being first known severally, and afterwards compounded, bring us to the knowledge of singular things. I conclude, therefore, that the method of attaining to the universal knowledge of things, is purely *analytical*.

5. But the causes of universal things (of those, at least, that have any cause) are manifest of themselves, or (as they say commonly) known to nature; so that they need no method at all; for they have all but one universal cause, which is motion. For the variety of all figures arises out of the variety of those motions by which they are

The highest causes, and most universal in every kind, are known by themselves.

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made; and motion cannot be understood to have any other cause besides motion; nor has the variety of those things we perceive by sense, as of *colours, sounds, savours, &c.* any other cause than motion, residing partly in the objects that work upon our senses, and partly in ourselves, in such manner, as that it is manifestly some kind of motion, though we cannot, without ratiocination, come to know what kind. For though many cannot understand till it be in some sort demonstrated to them, that all mutation consists in motion; yet this happens not from any obscurity in the thing itself, (for it is not intelligible that anything can depart either from rest, or from the motion it has, except by motion), but either by having their natural discourse corrupted with former opinions received from their masters, or else for this, that they do not at all bend their mind to the enquiring out of truth.

Method from principles found out, tending to science simply, what it is.

6. By the knowledge therefore of universals, and of their causes (which are the first principles by which we know the *διόρι* of things) we have in the first place their definitions, (which are nothing but the explication of our simple conceptions.) For example, he that has a true conception of *place*, cannot be ignorant of this definition, *place is that space which is possessed or filled adequately by some body*; and so, he that conceives *motion* aright, cannot but know that *motion is the privation of one place, and the acquisition of another*. In the next place, we have their generations or descriptions; as (for example) that *a line is made by the motion of a point, superficies by the motion of a line, and one motion by another*

motion, &c. It remains, that we enquire what motion begets such and such effects; as, what motion makes a straight line, and what a circular; what motion thrusts, what draws, and by what way; what makes a thing which is seen or heard, to be seen or heard sometimes in one manner, sometimes in another. Now the method of this kind of enquiry, is *compositive*. For first we are to observe what effect a body moved produceth, when we consider nothing in it besides its motion; and we see presently that this makes a line, or length; next, what the motion of a long body produces, which we find to be superficies; and so forwards, till we see what the effects of simple motion are; and then, in like manner, we are to observe what proceeds from the addition, multiplication, subtraction, and division, of these motions, and what effects, what figures, and what properties, they produce; from which kind of contemplation sprung that part of philosophy which is called *geometry*.

From this consideration of what is produced by simple motion, we are to pass to the consideration of what effects one body moved worketh upon another; and because there may be motion in all the several parts of a body, yet so as that the whole body remain still in the same place, we must enquire first, what motion causeth such and such motion in the whole, that is, when one body invades another body which is either at rest or in motion, what way, and with what swiftness, the invaded body shall move; and, again, what motion this second body will generate in a third, and so forwards. From which contemplation shall be

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Method from principles found out, tending to science simply, what it is.

PART I. drawn that part of philosophy which treats of
 6. motion.

Method from
 principles
 found out,
 tending to
 science simply,
 what it is.

In the third place we must proceed to the enquiry of such effects as are made by the motion of the parts of any body, as, how it comes to pass, that things when they are the same, yet seem not to be the same, but changed. And here the things we search after are sensible qualities, such as *light, colour, transparency, opacity, sound, odour, savour, heat, cold*, and the like; which because they cannot be known till we know the causes of sense itself, therefore the consideration of the causes of *seeing, hearing, smelling, tasting, and touching*, belongs to this third place; and all those qualities and changes, above mentioned, are to be referred to the fourth place; which two considerations comprehend that part of philosophy which is called *physics*. And in these four parts is contained whatsoever in natural philosophy may be explicated by demonstration, properly so called. For if a cause were to be rendered of natural appearances in special, as, what are the motions and influences of the heavenly bodies, and of their parts, the reason hereof must either be drawn from the parts of the sciences above mentioned, or no reason at all will be given, but all left to uncertain conjecture.

After *physics* we must come to *moral philosophy*; in which we are to consider the motions of the mind, namely, *appetite, aversion, love, benevolence, hope, fear, anger, emulation, envy, &c.*; what causes they have, and of what they be causes. And the reason why these are to be considered after *physics* is, that they have

(their causes in sense and imagination, which are the subject of *physical* contemplation. Also the reason, why all these things are to be searched after in the order above-said, is, that physics cannot be understood, except we know first what motions are in the smallest parts of bodies; nor such motion of parts, till we know what it is that makes another body move; nor this, till we know what simple motion will effect. And because all appearance of things to sense is determined, and made to be of such and such quality and quantity by compounded motions, every one of which has a certain degree of velocity, and a certain and determined way; therefore, in the first place, we we are to search out the ways of motion simply (in which geometry consists); next the ways of such generated motions as are manifest; and, lastly, the ways of internal and invisible motions (which is the enquiry of natural philosophers). And, therefore, they that study natural philosophy, study in vain, except they begin at geometry; and such writers or disputers thereof, as are ignorant of geometry, do but make their readers and hearers lose their time.

7. *Civil and moral philosophy* do not so adhere to one another, but that they may be severed. For the causes of the motions of the mind are known, not only by ratiocination, but also by the experience of every man that takes the pains to observe those motions within himself. And, therefore, not only they that have attained the knowledge of the passions and perturbations of the mind, by the *synthetical method*, and from the very first principles of philosophy, may by

That method of civil and natural science, proceeding from sense to principles, is analytical; and again, that which begins at principles is synthetical.

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That method
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tural science,
proceeding
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to principles,
is analytical;
and again that
which begins
at principles,
is synthetical.

proceeding in the same way, come to the causes and necessity of constituting commonwealths, and to get the knowledge of what is natural right, and what are civil duties; and, in every kind of government, what are the rights of the commonwealth, and all other knowledge appertaining to civil philosophy; for this reason, that the principles of the politics consist in the knowledge of the motions of the mind, and the knowledge of these motions from the knowledge of sense and imagination; but even they also that have not learned the first part of philosophy, namely, *geometry* and *physics*, may, notwithstanding, attain the principles of civil philosophy, by the *analytical method*. For if a question be propounded, as, *whether such an action be just or unjust*; if that *unjust* be resolved into *fact against law*, and that notion *law* into the *command* of him or them that have *coercive power*; and that *power* be derived from the *wills* of men that constitute such power, to the end they may live in peace, they may at last come to this, that the appetites of men and the passions of their minds are such, that, unless they be restrained by some power, they will always be making war upon one another; which may be known to be so by any man's experience, that will but examine his own mind. And, therefore, from hence he may proceed, by compounding, to the determination of the justice or injustice of any propounded action. So that it is manifest, by what has been said, that the method of philosophy, to such as seek science simply, without propounding to themselves the solution of any particular question, is partly

analytical, and partly synthetical ; namely, that which proceeds from sense to the invention of principles, analytical ; and the rest synthetical. PART I.
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8. To those that seek the cause of some certain and propounded appearance or effect, it happens, sometimes, that they know not whether the thing, whose cause is sought after, be matter or body, or some accident of a body. For though in geometry, when the cause is sought of magnitude, or proportion, or figure, it be certainly known that these things, namely magnitude, proportion, and figure, are accidents ; yet in natural philosophy, where all questions are concerning the causes of the phantasms of sensible things, it is not so easy to discern between the things themselves, from which those phantasms proceed, and the appearances of those things to the sense ; which have deceived many, especially when the phantasms have been made by light. For example, a man that looks upon the sun, has a certain shining idea of the magnitude of about a foot over, and this he calls the sun, though he know the sun to be truly a great deal bigger ; and, in like manner, the phantasm of the same thing appears sometimes round, by being seen afar off, and sometimes square, by being nearer. Whereupon it may well be doubted, whether that phantasm be matter, or some body natural, or only some accident of a body ; in the examination of which doubt we may use this method. The properties of matter and accidents already found out by us, by the synthetical method, from their definitions, are to be compared with the idea we have before us ; and if it agree with the properties of matter or body, then it is a body ; other-

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wise it is an accident. Seeing, therefore, matter cannot by any endeavour of ours be either made or destroyed, or increased, or diminished, or moved out of its place, whereas that idea appears, vanishes, is increased and diminished, and moved hither and thither at pleasure; we may certainly conclude that it is not a body, but an accident only. And this method is *synthetical*.

The method of seeking whether any accident be in this or in that subject.

9. But if there be a doubt made concerning the subject of any known accident (for this may be doubted sometimes, as in the precedent example, doubt may be made in what subject that splendour and apparent magnitude of the sun is), then our enquiry must proceed in this manner. First, matter in general must be divided into parts, as, into object, medium, and the sentient itself, or such other parts as seem most conformable to the thing propounded. Next, these parts are severally to be examined how they agree with the definition of the subject; and such of them as are not capable of that accident are to be rejected. For example, if by any true ratiocination the sun be found to be greater than its apparent magnitude, then that magnitude is not in the sun; if the sun be in one determined straight line, and one determined distance, and the magnitude and splendour be seen in more lines and distances than one, as it is in reflection or refraction, then neither that splendour nor apparent magnitude are in the sun itself, and, therefore, the body of the sun cannot be the subject of that splendour and magnitude. And for the same reasons the air and other parts will be rejected, till at last nothing remain which can be the subject of that splendour and mag-

nitude but the sentient itself. And this method, in regard the subject is divided into parts, is analytical; and in regard the properties, both of the subject and accident, are compared with the accident concerning whose subject the enquiry is made, it is synthetical.

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10. But when we seek after the cause of any propounded effect, we must in the first place get into our mind an exact notion or idea of that which we call cause, namely, that *a cause is the sum or aggregate of all such accidents, both in the agents and the patient, as concur to the producing of the effect propounded; all which existing together, it cannot be understood but that the effect existeth with them; or that it can possibly exist if any one of them be absent.* This being known, in the next place we must examine singly every accident that accompanies or precedes the effect, as far forth as it seems to conduce in any manner to the production of the same, and see whether the propounded effect may be conceived to exist, without the existence of any of those accidents; and by this means separate such accidents, as do not concur, from such as concur to produce the said effect; which being done, we are to put together the concurring accidents, and consider whether we can possibly conceive, that when these are all present, the effect propounded will not follow; and if it be evident that the effect will follow, then that aggregate of accidents is the entire cause, otherwise not; but we must still search out and put together other accidents. For example, if the cause of light be propounded to be sought

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the cause of
any effect,
propounded.

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out; first, we examine things without us, and find that whensoever light appears, there is some principal object, as it were the fountain of light, without which we cannot have any perception of light; and, therefore, the concurrence of that object is necessary to the generation of light. Next we consider the medium, and find, that unless it be disposed in a certain manner, namely, that it be transparent, though the object remain the same, yet the effect will not follow; and, therefore, the concurrence of transparency is also necessary to the generation of light. Thirdly, we observe our own body, and find that by the indisposition of the eyes, the brain, the nerves, and the heart, that is, by obstructions, stupidity, and debility, we are deprived of light, so that a fitting disposition of the organs to receive impressions from without is likewise a necessary part of the cause of light. Again, of all the accidents inherent in the object, there is none that can conduce to the effecting of light, but only action (or a certain motion), which cannot be conceived to be wanting, whensoever the effect is present; for, that anything may shine, it is not requisite that it be of such or such magnitude or figure, or that the whole body of it be moved out of the place it is in (unless it may perhaps be said, that in the sun, or other body, that which causes light is the light it hath in itself; which yet is but a trifling exception, seeing nothing is meant thereby but the cause of light; as if any man should say that the cause of light is that in the sun which produceth it); it remains, therefore, that the action, by which light is generated, is motion only in the parts of the

object. Which being understood, we may easily conceive what it is the medium contributes, namely, the continuation of that motion to the eye; and, lastly, what the eye and the rest of the organs of the sentient contribute, namely, the continuation of the same motion to the last organ of sense, the heart. And in this manner the cause of light may be made up of motion continued from the original of the same motion, to the original of vital motion, light being nothing but the alteration of vital motion, made by the impression upon it of motion continued from the object. But I give this only for an example, for I shall speak more at large of light, and the generation of it, in its proper place. In the mean time it is manifest, that in the searching out of causes, there is need partly of the analytical, and partly of the synthetical method; of the analytical, to conceive how circumstances conduce severally to the production of effects; and of the synthetical, for the adding together and compounding of what they can effect singly by themselves. And thus much may serve for the method of invention. It remains that I speak of the method of teaching, that is, of demonstration, and of the means by which we demonstrate.

11. In the method of invention, the use of words consists in this, that they may serve for marks, by which, whatsoever we have found out may be recalled to memory; for without this all our inventions perish, nor will it be possible for us to go on from principles beyond a syllogism or two, by reason of the weakness of memory. For example, if any man, by considering a triangle

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Words serve
to invention
as marks; to
demonstration
as signs.

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set before him, should find that all its angles together taken are equal to two right angles, and that by thinking of the same tacitly, without any use of words either understood or expressed; and it should happen afterwards that another triangle, unlike the former, or the same in different situation, should be offered to his consideration, he would not know readily whether the same property were in this last or no, but would be forced, as often as a different triangle were brought before him (and the difference of triangles is infinite) to begin his contemplation anew; which he would have no need to do if he had the use of names, for every universal name denotes the conceptions we have of infinite singular things. Nevertheless, as I said above, they serve as *marks* for the help of our memory, whereby we register to ourselves our own inventions; but not as *signs* by which we declare the same to others; so that a man may be a philosopher alone by himself, without any master; Adam had this capacity. But to teach, that is, to demonstrate, supposes two at the least, and syllogistical speech.

The method of demonstration is synthetical.

12. And seeing teaching is nothing but leading the mind of him we teach, to the knowledge of our inventions, in that track by which we attained the same with our own mind; therefore, the same method that served for our invention, will serve also for demonstration to others, saving that we omit the first part of method which proceeded from the sense of things to universal principles, which, because they are principles, cannot be demonstrated; and seeing they are known by nature, (as was said above in the 5th article) they

need no demonstration, though they need explanation. The whole method, therefore, of demonstration, is *synthetical*, consisting of that order of speech which begins from primary or most universal propositions, which are manifest of themselves, and proceeds by a perpetual composition of propositions into syllogisms, till at last the learner understand the truth of the conclusion sought after.

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13. Now, such principles are nothing but definitions, whereof there are two sorts; one of names, that signify such things as have some conceivable cause, and another of such names as signify things of which we can conceive no cause at all. Names of the former kind are, *body*, or *matter*, *quantity*, or *extension*, *motion*, and whatsoever is common to all matter. Of the second kind, are *such a body*, *such and so great motion*, *so great magnitude*, *such figure*, and whatsoever we can distinguish one body from another by. And names of the former kind are well enough defined, when, by speech as short as may be, we raise in the mind of the hearer perfect and clear ideas or conceptions of the things named, as when we define motion to be *the leaving of one place, and the acquiring of another continually*; for though no thing moved, nor any cause of motion be in that definition, yet, at the hearing of that speech, there will come into the mind of the hearer an *idea* of motion clear enough. But definitions of things, which may be understood to have some cause, must consist of such names as express the cause or manner of their generation; as when we define a circle to be a figure made by

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only are
primary,
& universal
propositions.

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the circumduction of a straight line in a plane, &c. Besides definitions, there is no other proposition that ought to be called primary, or (according to severe truth) be received into the number of principles. For those *axioms of Euclid*, seeing they may be demonstrated, are no principles of demonstration, though they have by the consent of all men gotten the authority of principles, because they need not be demonstrated. Also, those *petitions*, or *postulata*, (as they call them) though they be principles, yet they are not principles of demonstration, but of construction only; that is, not of science, but of power; or (which is all one) not of *theorems*, which are speculations, but of *problems*, which belong to practice, or the doing of something. But as for those common received opinions, *Nature abhors vacuity*, *Nature doth nothing in vain*, and the like, which are neither evident in themselves, nor at all to be demonstrated, and which are oftener false than true, they are much less to be acknowledged for principles.

To return, therefore, to definitions; the reason why I say that the cause and generation of such things, as have any cause or generation, ought to enter into their definitions, is this. The end of science is the demonstration of the causes and generations of things; which if they be not in the definitions, they cannot be found in the conclusion of the first syllogism, that is made from those definitions; and if they be not in the first conclusion, they will not be found in any further conclusion deduced from that; and, therefore, by proceeding in this manner, we shall never come to

science ; which is against the scope and intention of demonstration. PART I.
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14. Now, seeing definitions (as I have said) are principles, or primary propositions, they are therefore speeches ; and seeing they are used for the raising of an *idea* of some thing in the mind of the learner, whensoever that thing has a name, the definition of it can be nothing but the explication of that name by speech ; and if that name be given it for some compounded conception, the definition is nothing but a resolution of that name into its most universal parts. As when we define man, saying *man is a body animated, sentient, rational*, those names, *body animated, &c.* are parts of that whole name *man* ; so that definitions of this kind always consist of *genus* and *difference* ; the former names being all, till the last, *general* ; and the last of all, *difference*. But if any name be the most universal in its kind, then the definition of it cannot consist of *genus* and *difference*, but is to be made by such circumlocution, as best explicateth the force of that name. Again, it is possible, and happens often, that the *genus* and *difference* are put together, and yet make no definition ; as these words, *a straight line*, contain both the *genus* and *difference* ; but are not a definition, unless we should think a straight line may be thus defined, *a straight line is a straight line* : and yet if there were added another name, consisting of different words, but signifying the same thing which these signify, then these might be the definition of that name. From what has been said, it may be understood how a definition ought to be defined, namely, *that it is a*

PART I. *proposition, whose predicate resolves the subject,*
 6. *when it may ; and when it may not, it exemplifies*
the same.

Properties of
 a definition.

15. The properties of a definition are :

First, that it takes away equivocation, as also all that multitude of distinctions, which are used by such as think they may learn philosophy by disputation. For the nature of a definition is to define, that is, to determine the signification of the defined name, and to pare from it all other signification besides what is contained in the definition itself ; and therefore one definition does as much, as all the distinctions (how many soever) that can be used about the name defined.

Secondly, that it gives an universal notion of the thing defined, representing a certain universal picture thereof, not to the eye, but to the mind. For as when one paints a man, he paints the image of some man ; so he, that defines the name man, makes a representation of some man to the mind.

Thirdly, that it is not necessary to dispute whether definitions are to be admitted or no. For when a master is instructing his scholar, if the scholar understand all the parts of the thing defined, which are resolved in the definition, and yet will not admit of the definition, there needs no further controversy betwixt them, it being all one as if he refused to be taught. But if he understand nothing, then certainly the definition is faulty ; for the nature of a definition consists in this, that it exhibit a clear idea of the thing defined ; and principles are either known by themselves, or else they are not principles.

Fourthly, that, in philosophy, definitions are

before defined names. For in teaching philosophy, the first beginning is from definitions; and all progression in the same, till we come to the knowledge of the thing compounded, is compositive. Seeing, therefore, definition is the explication of a compounded name by resolution, and the progression is from the parts to the compound, definitions must be understood before compounded names; nay, when the names of the parts of any speech be explicated, it is not necessary that the definition should be a name compounded of them. For example, when these names, *equilateral*, *quadrilateral*, *right-angled*, are sufficiently understood, it is not necessary in geometry that there should be at all such a name as *square*; for defined names are received in philosophy for brevity's sake only.

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a definition.

Fifthly, that compounded names, which are defined one way in some one part of philosophy, may in another part of the same be otherwise defined; as a *parabola* and an *hyperbole* have one definition in geometry, and another in rhetoric; for definitions are instituted and serve for the understanding of the doctrine which is treated of. And, therefore, as in one part of philosophy, a definition may have in it some one fit name for the more brief explanation of some proposition in geometry; so it may have the same liberty in other parts of philosophy; for the use of names is particular (even where many agree to the settling of them) and arbitrary.

Sixthly, that no name can be defined by any one word; because no one word is sufficient for the resolving of one or more words.

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Seventhly, that a defined name ought not to be repeated in the definition. For a defined name is the whole compound, and a definition is the resolution of that compound into parts ; but no total can be part of itself.

Nature of a demonstration.

16. Any two definitions, that may be compounded into a syllogism, produce a conclusion ; which, because it is derived from principles, that is, from definitions, is said to be demonstrated ; and the derivation or composition itself is called a demonstration. In like manner, if a syllogism be made of two propositions, whereof one is a definition, the other a demonstrated conclusion, or neither of them is a definition, but both formerly demonstrated, that syllogism is also called a demonstration, and so successively. The definition therefore of a demonstration is this, *a demonstration is a syllogism, or series of syllogisms, derived and continued, from the definitions of names, to the last conclusion.* And from hence it may be understood, that all true ratiocination, which taketh its beginning from true principles, produceth science, and is true demonstration. For as for the original of the name, although that, which the Greeks called ἀποδείξις, and the Latins *demonstratio*, was understood by them for that sort only of ratiocination, in which, by the describing of certain lines and figures, they placed the thing they were to prove, as it were before men's eyes, which is properly ἀποδεικνύειν, or to *shew* by the figure ; yet they seem to have done it for this reason, that unless it were in geometry, (in which only there is place for such figures) there was no ratiocination certain, and ending in

science, their doctrines concerning all other things being nothing but controversy and clamour; which, nevertheless, happened, not because the truth to which they pretended could not be made evident without figures, but because they wanted true principles, from which they might derive their ratiocination; and, therefore, there is no reason but that if true definitions were premised in all sorts of doctrines, the demonstrations also would be true.

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17. It is proper to methodical demonstration, First, that there be a true succession of one reason to another, according to the rules of syllogizing delivered above.

Properties of a demonstration, and order of things to be demonstrated.

Secondly, that the premises of all syllogisms be demonstrated from the first definitions.

Thirdly, that after definitions, he that teaches or demonstrates any thing, proceed in the same method by which he found it out; namely, that in the first place those things be demonstrated, which immediately succeed to universal definitions (in which is contained that part of philosophy which is called *philosophia prima*). Next, those things which may be demonstrated by simple motion (in which geometry consists). After geometry, such things as may be taught or shewed by manifest action, that is, by thrusting from, or pulling towards. And after these, the motion or mutation of the invisible parts of things, and the doctrine of sense and imaginations, and of the internal passions, especially those of men, in which are comprehended the grounds of civil duties, or civil philosophy; which takes up the last place. And that this method ought to be kept in all sorts of philosophy, is evident from hence, that such things

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as I have said are to be taught last, cannot be demonstrated, till such as are propounded to be first treated of, be fully understood. Of which method no other example can be given, but that treatise of the elements of philosophy, which I shall begin in the next chapter, and continue to the end of the work.

Faults of a
demonstration.

18. Besides those *paralogisms*, whose fault lies either in the falsity of the premises, or the want of true composition, of which I have spoken in the precedent chapter, there are two more, which are frequent in demonstration; one whereof is commonly called *petitio principii*; the other is the supposing of a *false cause*; and these do not only deceive unskilful learners, but sometimes masters themselves, by making them take that for well demonstrated, which is not demonstrated at all. *Petitio principii* is, when the conclusion to be proved is disguised in other words, and put for the definition or principle from whence it is to be demonstrated; and thus, by putting for the cause of the thing sought, either the thing itself or some effect of it, they make a circle in their demonstration. As for example, he that would demonstrate that the earth stands still in the centre of the world, and should suppose the earth's gravity to be the cause thereof, and define gravity to be a quality by which every heavy body tends towards the centre of the world, would lose his labour; for the question is, what is the cause of that quality in the earth? and, therefore, he that supposes gravity to be the cause, puts the thing itself for its own cause.

Of a *false cause* I find this example in a certain treatise where the thing to be demonstrated

is the motion of the earth. He begins, therefore, with this, that seeing the earth and the sun are not always in the same situation, it must needs be that one of them be locally moved, which is true; next, he affirms that the vapours, which the sun raises from the earth and sea, are, by reason of this motion, necessarily moved, which also is true; from whence he infers the winds are made, and this may pass for granted; and by these winds he says, the waters of the sea are moved, and by their motion the bottom of the sea, as if it were beaten forwards, moves round; and let this also be granted; wherefore, he concludes, the earth is moved; which is, nevertheless, a paralogism. For, if that wind were the cause why the earth was, from the beginning, moved round, and the motion either of the sun or the earth were the cause of that wind, then the motion of the sun or the earth was before the wind itself; and if the earth were moved, before the wind was made, then the wind could not be the cause of the earth's revolution; but, if the sun were moved, and the earth stand still, then it is manifest the earth might remain unmoved, notwithstanding that wind; and therefore that motion was not made by the cause which he allegeth. But paralogisms of this kind are very frequent among the writers of *physics*, though none can be more elaborate than this in the example given.

19. It may to some men seem pertinent to treat in this place of that art of the geometricians, which they call *logistica*, that is, the art, by which, from supposing the thing in question to be true, they proceed by ratiocination, till either they come to something known, by which they may

Why the analytical method of geometricians cannot be treated of in this place.

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Why the analytical method of geometricians cannot be treated of in this place.

demonstrate the truth of the thing sought for ; or to something which is impossible, from whence they collect that to be false, which they supposed true. But this art cannot be explicated here, for this reason, that the method of it can neither be practised, nor understood, unless by such as are well versed in geometry ; and among geometricians themselves, they, that have most theorems in readiness, are the most ready in the use of this *logistica* ; so that, indeed, it is not a distinct thing from geometry itself ; for there are, in the method of it, three parts ; the first whereof consists in the finding out of equality betwixt known and unknown things, which they call equation ; and this equation cannot be found out, but by such as know perfectly the nature, properties, and transpositions of proportion, as also the addition, subtraction, multiplication, and division of lines and superficies, and the extraction of roots ; which are the parts of no mean geometrician. The second is, when an equation is found, to be able to judge whether the truth or falsity of the question may be deduced from it, or no ; which yet requires greater knowledge. And the third is, when such an equation is found, as is fit for the solution of the question, to know how to resolve the same in such manner, that the truth or falsity may thereby manifestly appear ; which, in hard questions, cannot be done without the knowledge of the nature of crooked-lined figures ; but he that understands readily the nature and properties of these, is a complete geometrician. It happens besides, that for the finding out of equations, there is no certain method, but he is best able to do it, that has the best natural wit.

PART II.

THE FIRST GROUNDS OF PHILOSOPHY.

CHAPTER VII.

OF PLACE AND TIME.

1. Things that have no existence, may nevertheless be understood and computed.—2. What is Space.—3. Time.—4. Part.
5. Division.—6. One.—7. Number.—8. Composition.—
9. The whole.—10. Spaces and times contiguous, and continual.—11. Beginning, end, way, finite, infinite.—12. What is infinite in power. Nothing infinite can be truly said to be either whole, or one; nor infinite spaces or times, many.—
13. Division proceeds not to the least.

1. IN the teaching of natural philosophy, I cannot begin better (as I have already shewn) than from *privation*; that is, from feigning the world to be annihilated. But, if such annihilation of all things be supposed, it may perhaps be asked, what would remain for any man (whom only I except from this universal annihilation of things) to consider as the subject of philosophy, or at all to reason upon; or what to give names unto for ratiocination's sake.

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Things that have no existence, may nevertheless be understood and computed.

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have no ex-
istence, may
nevertheless
be understood
and computed.

I say, therefore, there would remain to that man ideas of the world, and of all such bodies as he had, before their annihilation, seen with his eyes, or perceived by any other sense; that is to say, the memory and imagination of magnitudes, motions, sounds, colours, &c. as also of their order and parts. All which things, though they be nothing but ideas and phantasms, happening internally to him that imagineth; yet they will appear as if they were external, and not at all depending upon any power of the mind. And these are the things to which he would give names, and subtract them from, and compound them with one another. For seeing, that after the destruction of all other things, I suppose man still remaining, and namely that he thinks, imagines, and remembers, there can be nothing for him to think of but what is past; nay, if we do but observe diligently what it is we do when we consider and reason, we shall find, that though all things be still remaining in the world, yet we compute nothing but our own phantasms. For when we calculate the magnitude and motions of heaven or earth, we do not ascend into heaven that we may divide it into parts, or measure the motions thereof, but we do it sitting still in our closets or in the dark. Now things may be considered, that is, be brought into account, either as internal accidents of our mind, in which manner we consider them when the question is about some faculty of the mind; or as species of external things, not as really existing, but appearing only to exist, or to have a being without us. And in this manner we are now to consider them.

2. If therefore we remember, or have a phantasm of any thing that was in the world before the supposed annihilation of the same; and consider, not that the thing was such or such, but only that it had a being without the mind, we have presently a conception of that we call *space*: an imaginary space indeed, because a mere phantasm, yet that very thing which all men call so. For no man calls it space for being already filled, but because it may be filled; nor does any man think bodies carry their places away with them, but that the same space contains sometimes one, sometimes another body; which could not be if space should always accompany the body which is once in it. And this is of itself so manifest, that I should not think it needed any explaining at all, but that I find space to be falsely defined by certain philosophers, who infer from thence, one, that the world is infinite (for taking *space* to be the extension of bodies, and thinking extension may encrease continually, he infers that bodies may be infinitely extended); and, another, from the same definition, concludes rashly, that it is impossible even to God himself to create more worlds than one; for, if another world were to be created, he says, that seeing there is nothing without this world, and therefore (according to his definition) no space, that new world must be placed in nothing; but in nothing nothing can be placed; which he affirms only, without showing any reason for the same; whereas the contrary is the truth: for more cannot be put into a place already filled, so much is empty space fitter than that, which is full, for the receiving of new bodies.

PART II.
7.What
is Space.

PART II. ^{7.} Having therefore spoken thus much for these men's sakes, and for theirs that assent to them, I return to my purpose, and define *space* thus: *SPACE is the phantasm of a thing existing without the mind simply*; that is to say, that phantasm, in which we consider no other accident, but only that it appears without us.

Time. 3. As a body leaves a phantasm of its magnitude in the mind, so also a moved body leaves a phantasm of its motion, namely, an idea of that body passing out of one space into another by continual succession. And this idea, or phantasm, is that, which (without receding much from the common opinion, or from *Aristotle's* definition) I call *Time*. For seeing all men confess a year to be time, and yet do not think a year to be the accident or affection of any body, they must needs confess it to be, not in the things without us, but only in the thought of the mind. So when they speak of the times of their predecessors, they do not think after their predecessors are gone, that their times can be any where else than in the memory of those that remember them. And as for those that say, days, years, and months are the motions of the sun and moon, seeing it is all one to say, motion *past* and motion *destroyed*, and that *future* motion is the same with motion which *is not yet begun*, they say that, which they do not mean, that there neither is, nor has been, nor shall be any time: for of whatsoever it may be said, *it has been* or *it shall be*, of the same also it might have been said heretofore, or may be said hereafter, *it is*. What then can days, months, and years, be, but the names of such

computations made in our mind? *Time* therefore is a phantasm, but a phantasm of motion, for if we would know by what moments time passes away, we make use of some motion or other, as of the sun, of a clock, of the sand in an hour-glass, or we mark some line upon which we imagine something to be moved, there being no other means by which we can take notice of any time at all. And yet, when I say *time* is a phantasm of motion, I do not say this is sufficient to define it by; for this word *time* comprehends the notion of *former* and *latter*, or of *succession* in the motion of a body, in as much as it is first *here* then *there*. Wherefore a complete definition of *time* is such as this, *TIME is the phantasm of before and after in motion*; which agrees with this definition of *Aristotle*, *time is the number of motion according to former and latter*; for that numbering is an act of the mind; and therefore it is all one to say, *time is the number of motion according to former and latter*; and *time is a phantasm of motion numbered*. But that other definition, *time is the measure of motion*, is not so exact, for we measure time by motion and not motion by time.

PART II.

7.

Time.

4. One space is called *part* of another space, and one time *part* of another time, when this contains that and something besides. From whence it may be collected, that nothing can rightly be called a PART, but that which is compared with something that contains it.

Part.

5. And therefore to *make parts*, or to *part* or *DIVIDE* space or *time*, is nothing else but to consider one and another within the same; so that

Division.

PART II. ⁷ if any man *divide* space or time, the diverse
 Division. conceptions he has are more, by one, than the
 parts he makes; for his first conception is of that
 which is to be divided, then of some part of it,
 and again of some other part of it, and so
 forwards as long as he goes on in dividing.

But it is to be noted, that here, by *division*, I
 do not mean the severing or pulling asunder of
 one space or time from another (for does any
 man think that one hemisphere may be separated
 from the other hemisphere, or the first hour from
 the second?) but diversity of consideration; so
 that division is not made by the operation of the
 hands but of the mind.

One. 6. When space or time is considered among
 other spaces or times, it is said to be ONE, namely
one of them; for except one space might be
 added to another, and subtracted from another
 space, and so of time, it would be sufficient to
 say space or time simply, and superfluous to say
 one space or one time, if it could not be conceived
 that there were another. The common definition
 of *one*, namely, that *one is that which is undivided*,
 is obnoxious to an absurd consequence; for it may
 thence be inferred, that whatsoever is divided is
 many things, that is, that every divided thing, is
 divided things, which is insignificant.

Number. 7. NUMBER is *one* and *one*, or *one one* and *one*,
 and so forwards; namely, *one* and *one* make the
 number *two*, and *one one* and *one* the number
three; so are all other numbers made; which is
 all one as if we should say, *number is unities*.

Composition. 8. To COMPOUND space of spaces, or time of
 times, is first to consider them one after another,

and then altogether as one; as if one should reckon first the head, the feet, the arms, and the body, severally, and then for the account of them all together put *man*. And that which is so put for all the severals of which it consists, is called the **WHOLE**; and those severals, when by the division of the whole they come again to be considered singly, are parts thereof; and therefore the *whole* and *all the parts taken together* are the same thing. And as I noted above, that in *division* it is not necessary to pull the parts asunder; so in *composition*, it is to be understood, that for the making up of a whole there is no need of putting the parts together, so as to make them touch one another, but only of collecting them into one sum in the mind. For thus all men, being considered together, make up the whole of mankind, though never so much dispersed by time and place; and twelve hours, though the hours of several days, may be compounded into one number of twelve.

9. This being well understood, it is manifest, ^{The whole.} that nothing can rightly be called a whole, that is not conceived to be compounded of parts, and that it may be divided into parts; so that if we deny that a thing has parts, we deny the same to be a whole. For example, if we say the soul can have no parts, we affirm that no soul can be a whole soul. Also it is manifest, that nothing has parts till it be divided; and when a thing is divided, the parts are only so many as the division makes them. Again, that a part of a part is a part of the whole; and thus any part of the number *four*, as *two*, is a part of the number *eight*; for *four* is

PART II. made of *two* and *two*; but *eight* is compounded
 7. of *two*, *two*, and *four*, and therefore *two*, which
 is a part of the part *four*, is also a part of the
 whole *eight*.

Spaces and
 times con-
 tiguous and
 continual.

10. Two spaces are said to be CONTIGUOUS, when there is no other space betwixt them. But two times, betwixt which there is no other time, are called immediate, as A B, B C.

And any two spaces, as well as $\begin{array}{ccc} \text{A} & & \text{B} & & \text{C} \\ \hline & & & & \end{array}$ times, are said to be CONTINUAL, when they have one common part, as A C, B D, $\begin{array}{cccc} & & \text{A} & & \text{B} & & \text{C} & & \text{D} \\ \hline & & & & & & & & \end{array}$ where the part B C is common;

and more spaces and times are continual, when every two which are next one another are continual.

Beginning,
 end, way,
 finite, infinite.

11. That part which is between two other parts, is called a MEAN; and that which is not between two other parts, an EXTREME. And of extremes, that which is first reckoned is the BEGINNING, and that which last, the END; and all the means together taken are the WAY. Also, *extreme parts* and *limits* are the same thing. And from hence it is manifest, that *beginning* and *end* depend upon the order in which we number them; and that to *terminate* or *limit* space and time, is the same thing with *imagining their beginning and end*; as also that every thing is FINITE or INFINITE, according as we imagine or not imagine it *limited* or *terminated* every way; and that the *limits* of any number are *unities*, and of these, that which is the first in our numbering is the *beginning*, and that which we number last, is the *end*. When we say number is *infinite*, we mean only that no number is expressed; for when we

speak of the numbers *two, three, a thousand, &c.* they are always *finite*. But when no more is said but this, *number is infinite*, it is to be understood as if it were said, this name *number* is an *indefinite* name.

PART II.

7.

12. Space or time is said to be *finite in power*, or *terminable*, when there may be assigned a number of finite spaces or times, as of paces or hours, than which there can be no greater number of the same measure in that space or time; and *infinite in power* is that space or time, in which a greater number of the said paces or hours may be assigned, than any number that can be given. But we must note, that, although in that space or time which is infinite in power, there may be numbered more paces or hours than any number that can be assigned, yet their number will always be finite; for every number is finite. And therefore his ratiocination was not good, that undertaking to prove the world to be finite, reasoned thus; *If the world be infinite, then there may be taken in it some part which is distant from us an infinite number of paces: but no such part can be taken; wherefore the world is not infinite;* because that consequence of the major proposition is false; for in an infinite space, whatsoever we take or design in our mind, the distance of the same from us is a finite space; for in the very designing of the place thereof, we put an end to that space, of which we ourselves are the beginning; and whatsoever any man with his mind cuts off both ways from infinite, he determines the same, that is, he makes it finite.

What is infinite in power. Nothing infinite can be truly said to be either whole or one; nor infinite spaces or times, many.

Of infinite space or time, it cannot be said that

PART II.
7.

it is a *whole* or *one*: not a *whole*, because not compounded of parts; for seeing parts, how many soever they be, are severally finite, they will also, when they are all put together, make a whole finite: nor *one*, because nothing can be said to be one, except there be another to compare it with; but it cannot be conceived that there are two spaces, or two times, infinite. Lastly, when we make question whether the world be finite or infinite, we have nothing in our mind answering to the name *world*; for whatsoever we imagine, is therefore finite, though our computation reach the fixed stars, or the ninth or tenth, nay, the thousandth sphere. The meaning of the question is this only, whether God has actually made so great an addition of body to body, as we are able to make of space to space.

Division
proceeds not
to the least.

13. And, therefore, that which is commonly said, that space and time may be divided infinitely, is not to be so understood, as if there might be any infinite or eternal division; but rather to be taken in this sense, *whatsoever is divided, is divided into such parts as may again be divided*; or thus, *the least divisible thing is not to be given*; or, as geometricians have it, *no quantity is so small, but a less may be taken*; which may easily be demonstrated in this manner. Let any space or time, that which was thought to be the least divisible, be divided into two equal parts, A and B. I say either of them, as A, may be divided again. For suppose the part A to be contiguous to the part B of one side, and of the other side to some other space equal to B. This whole space, therefore, being greater than the

space given, is divisible. Wherefore, if it be divided into two equal parts, the part in the middle, which is A, will be also divided into two equal parts; and therefore A was divisible.

PART II.
7.

CHAPTER VIII.

OF BODY AND ACCIDENT.

1. Body defined.—2. Accident defined.—3. How an accident may be understood to be in its subject.—4. Magnitude, what it is.—5. Place, what it is, and that it is immovable.—6. What is full and empty.—7. Here, there, somewhere, what they signify.—8. Many bodies cannot be in one place, nor one body in many places.—9. Contiguous and continual, what they are.—10. The definition of motion. No motion intelligible but with time.—11. What it is to be at rest, to have been moved, and to be moved. No motion to be conceived, without the conception of past and future.—12. A point, a line, superficies and solid, what they are.—13. Equal, greater, and less in bodies and magnitudes, what they are.—14. One and the same body has always one and the same magnitude. 15. Velocity, what it is.—16. Equal, greater, and less in times, what they are.—17. Equal, greater, and less, in velocity, what. 18. Equal, greater, and less, in motion, what.—19. That which is at rest, will always be at rest, except it be moved by some external thing; and that which is moved, will always be moved, unless it be hindered by some external thing.—20. Accidents are generated and destroyed, but bodies not so. 21. An accident cannot depart from its subject.—22. Nor be moved.—23. Essence, form, and matter, what they are. 24. First matter, what.—25. That the whole is greater than any part thereof, why demonstrated.

1. HAVING understood what imaginary space is, Body defined. in which we supposed nothing remaining without us, but all those things to be destroyed, that, by

PART II.
8.

existing heretofore, left images of themselves in our minds ; let us now suppose some one of those things to be placed again in the world, or created anew. It is necessary, therefore, that this new-created or replaced thing do not only fill some part of the space above mentioned, or be coincident and coextended with it, but also that it have no dependance upon our thought. And this is that which, for the extension of it, we commonly call *body* ; and because it depends not upon our thought, we say is *a thing subsisting of itself* ; as also *existing*, because without us ; and, lastly, it is called the *subject*, because it is so placed in and *subjected* to imaginary space, that it may be understood by reason, as well as perceived by sense. The definition, therefore, of *body* may be this, *a body is that, which having no dependance upon our thought, is coincident or coextended with some part of space.*

Accident
defined.

2. But what an *accident* is cannot so easily be explained by any definition, as by examples. Let us imagine, therefore, that a body fills any space, or is coextended with it ; that coextension is not the coextended body : and, in like manner, let us imagine that the same body is removed out of its place ; that removing is not the removed body : or let us think the same not removed ; that not removing or rest is not the resting body. What, then, are these things ? They are *accidents* of that body. But the thing in question is, *what is an accident ?* which is an enquiry after that which we know already, and not that which we should enquire after. For who does not always and in the same manner understand him that says any

thing is extended, or moved, or not moved? But PART II.
8. most men will have it be said that *an accident is something*, namely, some part of a natural thing, when, indeed, it is no part of the same. To satisfy these men, as well as may be, they answer best that define an *accident* to be *the manner by which any body is conceived*; which is all one as if they should say, *an accident is that faculty of any body, by which it works in us a conception of itself*. Which definition, though it be not an answer to the question propounded, yet it is an answer to that question which should have been propounded, namely, *whence does it happen that one part of any body appears here, another there?* For this is well answered thus: *it happens from the extension of that body*. Or, *how comes it to pass that the whole body, by succession, is seen now here, now there?* and the answer will be, *by reason of its motion*. Or, lastly, *whence is it that any body possesseth the same space for sometime?* and the answer will be, *because it is not moved*. For if concerning the name of a body, that is, concerning a concrete name, it be asked, *what is it?* the answer must be made by definition; for the question is concerning the signification of the name. But if it be asked concerning an abstract name, *what is it?* the cause is demanded why a thing appears so or so. As if it be asked, *what is hard?* The answer will be, *hard is that, whereof no part gives place, but when the whole gives place*. But if it be demanded, *what is hardness?* a cause must be shewn why a part does not give place, except the

Accident
defined.

PART II.
8.

How an accident may be understood to be in its subject.

whole give place. Wherefore, I define an *accident* to be *the manner of our conception of body*.

3. When an *accident* is said to be in a body, it is not so to be understood, as if any thing were contained in that body; as if, for example, redness were in blood, in the same manner, as blood is in a bloody cloth, that is, as a part in the whole; for so, an accident would be a body also. But, as magnitude, or rest, or motion, is in that which is great, or which resteth, or which is moved, (which, how it is to be understood, every man understands) so also, it is to be understood, that every other accident *is in* its subject. And this, also, is explicated by *Aristotle* no otherwise than negatively, namely, that *an accident is in its subject, not as any part thereof, but so as that it may be away, the subject still remaining*; which is right, saving that there are certain accidents which can never perish except the body perish also; for no body can be conceived to be without extension, or without figure. All other accidents, which are not common to all bodies, but peculiar to some only, as *to be at rest, to be moved, colour, hardness*, and the like, do perish continually, and are succeeded by others; yet so, as that the body never perisheth. And as for the opinion that some may have, that all other accidents are not in their bodies in the same manner that extension, motion, rest, or figure, are in the same; for example, that colour, heat, odour, virtue, vice, and the like, are otherwise in them, and, as they say, *inherent*; I desire they would suspend their judgment for the present, and expect a little, till it be found out

by ratiocination, whether these very accidents are not also certain motions either of the mind of the perceiver, or of the bodies themselves which are perceived; for in the search of this, a great part of natural philosophy consists.

PART II.
8.

4. The *extension* of a body, is the same thing with the *magnitude* of it, or that which some call *real space*. But this *magnitude* does not depend upon our cogitation, as imaginary space doth; for this is an effect of our imagination, but *magnitude* is the cause of it; this is an accident of the mind, that of a body existing out of the mind.

Magnitude,
what it is.

5. That space, by which word I here understand imaginary space, which is coincident with the magnitude of any body, is called the *place* of that body; and the body itself is that which we call the *thing placed*. Now *place*, and the *magnitude* of the thing placed, differ. First in this, that a body keeps always the same *magnitude*, both when it is at rest, and when it is moved; but when it is moved, it does not keep the same *place*. Secondly in this, that *place* is a phantasm of any body of such and such quantity and figure; but *magnitude* is the peculiar accident of every body; for one body may at several times have several places, but has always one and the same magnitude. Thirdly in this, that *place* is nothing out of the mind, nor *magnitude* any thing within it. And lastly, *place* is feigned extension, but *magnitude* true extension; and a placed body is not extension, but a thing extended. Besides, *place is immovable*; for, seeing that which is moved, is understood to be carried from place to place, if place were moved, it would also be carried from place to

Place, what it
is, and that it
is immovable.

PART II.
8.

Place, what it
is, and that it
is immovable.

place, so that one place must have another place, and that place another place, and so on infinitely, which is ridiculous. And as for those, that, by making *place* to be of the same nature with *real space*, would from thence maintain it to be immovable, they also make place, though they do not perceive they make it so, to be a mere phantasm. For whilst one affirms that place is therefore said to be immovable, because space in general is considered there; if he had remembered that nothing is general or universal besides names or signs, he would easily have seen that that space, which he says is considered in general, is nothing but a phantasm, in the mind or the memory, of a body of such magnitude and such figure. And whilst another says: real space is made immovable by the understanding; as when, under the superficies of running water, we imagine other and other water to come by continual succession, that superficies fixed there by the understanding, is the *immovable place* of the river: what else does he make it to be but a phantasm, though he do it obscurely and in perplexed words? Lastly, the nature of *place* does not consist in the *superficies of the ambient*, but in *solid space*; for the whole placed body is coextended with its whole place, and every part of it with every answering part of the same place; but seeing every placed body is a solid thing, it cannot be understood to be coextended with superficies. Besides, how can any whole body be moved, unless all its parts be moved together with it? Or how can the internal parts of it be moved, but by leaving their place? But the internal parts of a body cannot leave the

superficies of an external part contiguous to it ; and, therefore, it follows, that if place be the superficies of the ambient, then the parts of a body moved, that is, bodies moved, are not moved.

PART II.
8.

6. Space, or place, that is possessed by a body, is called *full*, and that which is not so possessed, is called *empty*.

What is full
and empty.

7. *Here, there, in the country, in the city*, and other the like names, by which answer is made to the question *where is it ?* are not properly names of place, nor do they of themselves bring into the mind the place that is sought ; for *here* and *there* signify nothing, unless the thing be shewn at the same time with the finger or something else ; but when the eye of him that seeks, is, by pointing or some other sign, directed to the thing sought, the place of it is not hereby defined by him that answers, but found out by him that asks the question. Now such shewings as are made by words only, as when we say, *in the country*, or *in the city*, are some of greater latitude than others, as when we say, *in the country, in the city, in such a street, in a house, in the chamber, in bed, &c.* For these do, by little and little, direct the seeker nearer to the proper place ; and yet they do not determine the same, but only restrain it to a lesser space, and signify no more, than that the place of the thing is within a certain space designed by those words, as a part is in the whole. And all such names, by which answer is made to the question *where ?* have, for their highest *genus*, the name *somewhere*. From whence it may be understood, that whatsoever is somewhere, is in some place properly so called, which place is part of

Here, there,
somewhere,
what they
signify.

PART II. that greater space that is signified by some of these
 8. names, *in the country, in the city,* or the like.

Many bodies
 cannot be in
 one place, nor
 one body in
 many places.

8. A body, and the magnitude, and the place thereof, are divided by one and the same act of the mind ; for, to divide an extended body, and the extension thereof, and the idea of that extension, which is place, is the same with dividing any one of them ; because they are coincident, and it cannot be done but by the mind, that is by the division of space. From whence it is manifest, that neither two bodies can be together in the same place, nor one body be in two places at the same time. Not two bodies in the same place ; because when a body that fills its whole place is divided into two, the place itself is divided into two also, so that there will be two places. Not one body in two places ; for the place that a body fills being divided into two, the placed body will be also divided into two ; for, as I said, a place and the body that fills that place, are divided both together ; and so there will be two bodies.

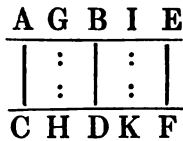
Contiguous
 and continual,
 what they are.

9. Two bodies are said to be *contiguous* to one another, and *continual*, in the same manner as spaces are ; namely, *those are contiguous, between which there is no space.* Now, by space I understand, here as formerly, an idea or phantasm of a body. Wherefore, though between two bodies there be put no other body, and consequently no magnitude, or, as they call it, real space, yet if another body may be put between them, that is, if there intercede any imagined space which may receive another body, then those bodies are not contiguous. And this is so easy to be understood, that I should wonder at some men, who being

otherwise skilful enough in philosophy, are of a different opinion, but that I find that most of those that affect metaphysical subtleties wander from truth, as if they were led out of their way by an *ignis fatuus*. For can any man that has his natural senses, think that two bodies must therefore necessarily touch one another, because no other body is between them? Or that there can be no *vacuum*, because *vacuum* is nothing, or as they call it, *non ens*? Which is as childish, as if one should reason thus; no man can fast, because to fast is to eat nothing; but nothing cannot be eaten. *Continual, are any two bodies that have a common part; and more than two are continual, when every two, that are next to one another, are continual.*

10. MOTION is a continual relinquishing of one place, and acquiring of another; and that place which is relinquished is commonly called the *terminus a quo*, as that which is acquired is called the *terminus ad quem*; I say a continual relinquishing, because no body, how little soever, can totally and at once go out of its former place into another, so, but that some part of it will be in a part of a place which is common to both, namely, to the relinquished and the acquired places. For example, let any body be in the place A C B D; the same body cannot come into the place B D E F, but it must first be in G H I K, whose part G H B D is common to both the places A C B D, and G H I K, and whose part B D I K, is common to both the places G H I K, and B D E F. Now it cannot be con-

The definition of motion. No motion intelligible but with time.



PART II.
8.

ceived that any thing can be moved without time; for time is, by the definition of it, a phantasm, that is, a conception of motion; and, therefore, to conceive that any thing may be moved without time, were to conceive motion without motion, which is impossible.

What it is to be at rest, to have been moved, and to be moved. No motion to be conceived without the conception of past and future.

11. *That is said to be at rest, which, during any time, is in one place; and that to be moved, or to have been moved, which, whether it be now at rest or moved, was formerly in another place than that which it is now in.* From which definitions it may be inferred, first, that *whatsoever is moved, has been moved*; for if it be still in the same place in which it was formerly, it is at rest, that is, it is not moved, by the definition of *rest*; but if it be in another place, it has been moved, by the definition of *moved*. Secondly, that *what is moved, will yet be moved*; for that which is moved, leaveth the place where it is, and therefore will be in another place, and consequently will be moved still. Thirdly, that *whatsoever is moved, is not in one place during any time, how little soever that time be*; for by the definition of rest, that which is in one place during any time, is at rest.

There is a certain sophism against motion, which seems to spring from the not understanding of this last proposition. For they say, that, *if any body be moved, it is moved either in the place where it is, or in the place where it is not; both which are false; and therefore nothing is moved.* But the falsity lies in the major proposition; for that which is moved, is neither moved in the place where it is, nor in the place where is not; but

from the place where it is, to the place where it is not. Indeed it cannot be denied but that whatsoever is moved, is moved somewhere, that is, within some space; but then the place of that body is not that whole space, but a part of it, as is said above in the seventh article. From what is above demonstrated, namely, that whatsoever is moved, has also been moved, and will be moved, this also may be collected, that there can be no conception of motion, without conceiving past and future time.

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12. Though there be no body which has not some magnitude, yet if, when any body is moved, the magnitude of it be not at all considered, the way it makes is called a *line*, or one single dimension; and the space, through which it passeth, is called *length*; and the body itself, a *point*; in which sense the earth is called a *point*, and the way of its yearly revolution, the *ecliptic line*. But if a body, which is moved, be considered as *long*, and be supposed to be so moved, as that all the several parts of it be understood to make several lines, then the way of every part of that body is called *breadth*, and the space which is made is called *superficies*, consisting of two dimensions, one whereof to every several part of the other is applied whole. Again, if a body be considered as having *superficies*, and be understood to be so moved, that all the several parts of it describe several lines, then the way of every part of that body is called *thickness* or *depth*, and the space which is made is called *solid*, consisting of three dimensions, any two whereof are applied whole to every several part of the third.

A point, a line,
superficies,
and solid,
what they are.

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But if a body be considered as *solid*, then it is not possible that all the several parts of it should describe several lines; for what way soever it be moved, the way of the following part will fall into the way of the part before it, so that the same solid will still be made which the foremost superficies would have made by itself. And therefore there can be no other dimension in any body, as it is a body, than the three which I have now described; though, as it shall be shewed hereafter, *velocity*, which is motion according to *length*, may, by being applied to all the parts of a *solid*, make a magnitude of motion, consisting of four dimensions; as the goodness of gold, computed in all the parts of it, makes the price and value thereof.

Equal, great, greater, and less, in bodies and magnitudes, what they are.

13. *Bodies*, how many soever they be, that can fill every one the place of every one, are said to be *equal* every one to every other. Now, one body may fill the same place which another body filleth, though it be not of the same figure with that other body, if so be that it may be understood to be reducible to the same figure, either by flexion or transposition of the parts. And *one body is greater than another body, when a part of that is equal to all this; and less, when all that is equal to a part of this*. Also, *magnitudes are equal, or greater, or lesser, than one another, for the same consideration, namely, when the bodies, of which they are the magnitudes, are either equal, or greater, or less, &c.*

One and the same body has always one and the same magnitude.

14. One and the same body is always of one and the same magnitude. For seeing a body and the magnitude and place thereof cannot be comprehended in the mind otherwise than as they

are coincident, if any body be understood to be at rest, that is, to remain in the same place during some time, and the magnitude thereof be in one part of that time greater, and in another part less, that body's place, which is one and the same, will be coincident sometimes with greater, sometimes with less magnitude, that is, the same place will be greater and less than itself, which is impossible. But there would be no need at all of demonstrating a thing that is in itself so manifest, if there were not some, whose opinion concerning bodies and their magnitudes is, that a body may exist separated from its magnitude, and have greater or less magnitude bestowed upon it, making use of this principle for the explication of the nature of *rarum* and *densum*.

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15. Motion, in as much as a certain length may in a certain time be transmitted by it, is called VELOCITY or *swiftness* : &c. For though *swift* be very often understood with relation to *slower* or *less swift*, as great is in respect of less, yet nevertheless, as magnitude is by philosophers taken absolutely for extension, so also *velocity* or *swiftness* may be put absolutely for motion according to length.

Velocity,
what it is.

16. Many motions are said to be made in equal times, when every one of them begins and ends together with some other motion, or if it had begun together, would also have ended together with the same. For time, which is a phantasm of motion; as in dials by the motion of the sun or of the hand; and if two or more motions begin and end with this motion, they are said to be made in

Equal,
greater, and
less, in times,
what they are.

PART II. equal times ; from whence also it is easy to understand what it is to be moved in greater or longer time, and in less time or not so long ; namely, that that is longer moved, which beginning with another, ends later ; or ending together, began sooner.

Equal, greater, and less, in velocity, what.

17. Motions are said to be equally swift, when equal lengths are transmitted in equal times ; and greater swiftness is that, wherein greater length is passed in equal time, or equal length in less time. Also that swiftness by which equal lengths are passed in equal parts of time, is called *uniform* swiftness or motion ; and of motions *not uniform*, such as become swifter or slower by equal increasings or decreasings in equal parts of time, are said to be *accelerated* or *retarded uniformly*.

Equal, greater, and less, in motion, what.

18. But motion is said to be greater, less, and equal, not only in regard of the length which is transmitted in a certain time, that is, in regard of swiftness only, but of swiftness applied to every smallest particle of magnitude ; for when any body is moved, every part of it is also moved ; and supposing the parts to be halves, the motions of those halves have their swiftness equal to one another, and severally equal to that of the whole ; but the motion of the whole is equal to those two motions, either of which is of equal swiftness with it ; and therefore it is one thing for two motions to be equal to one another, and another thing for them to be equally swift. And this is manifest in two horses that draw abreast, where the motion of both the horses together is of equal swiftness with the motion of either of them singly ; but the motion of both is greater than the motion of one

of them, namely, double. Wherefore *motions are said to be simply equal to one another, when the swiftness of one, computed in every part of its magnitude, is equal to the swiftness of the other computed also in every part of its magnitude: and greater than one another, when the swiftness of one computed as above, is greater than the swiftness of the other so computed; and less, when less.* Besides, the magnitude of motion computed in this manner is that which is commonly called **FORCE**.

19. *Whatsoever is at rest, will always be at rest, unless there be some other body besides it, which, by endeavouring to get into its place by motion, suffers it no longer to remain at rest.*

For suppose that some finite body exist and be at rest, and that all space besides be empty; if now this body begin to be moved, it will certainly be moved some way; seeing therefore there was nothing in that body which did not dispose it to rest, the reason why it is moved this way is in something out of it; and in like manner, if it had been moved any other way, the reason of motion that way had also been in something out of it; but seeing it was supposed that nothing is out of it, the reason of its motion one way would be the same with the reason of its motion every other way, wherefore it would be moved alike all ways at once; which is impossible.

In like manner, *whatsoever is moved, will always be moved, except there be some other body besides it, which causeth it to rest.* For if we suppose nothing to be without it, there will be no reason why it should rest now, rather than at

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That which is at rest will always be at rest, except it be moved by some external thing.

That which is moved will always be moved, unless it be hindered by some external thing.

PART II. another time ; wherefore its motion would cease
 8. in every particle of time alike ; which is not
 intelligible.

Accidents are
 generated and
 destroyed, but
 bodies not so.

20. When we say a living creature, a tree, or any other specified body is *generated* or *destroyed*, it is not to be so understood as if there were made a body of that which is not-body, or not a body of a body, but of a living creature not a living creature, of a tree not a tree, &c. that is, that those accidents for which we call one thing a living creature, another thing a tree, and another by some other name, are generated and destroyed ; and that therefore the same names are not to be given to them now, which were given them before. But that magnitude for which we give to any thing the name of body is neither generated nor destroyed. For though we may feign in our mind that a point may swell to a huge bulk, and that this may again contract itself to a point ; that is, though we may imagine something to arise where before was nothing, and nothing to be there where before was something, yet we cannot comprehend in our mind how this may possibly be done in nature. And therefore philosophers, who tie themselves to natural reason, suppose that a body can neither be generated nor destroyed, but only that it may appear otherwise than it did to us, that is, under different *species*, and consequently be called by other and other names ; so that that which is now called *man*, may at another time have the name of not-man ; but that which is once called body, can never be called not-body. But it is manifest, that all other accidents besides magnitude or extension may be generated and destroyed ;

as when a white thing is made black, the whiteness that was in it perisheth, and the blackness that was not in it is now generated; and therefore bodies, and the accidents under which they appear diversely, have this difference, that bodies are things, and not generated; accidents are generated, and not things.

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21. And therefore, when any thing appears otherwise than it did by reason of other and other accidents, it is not to be thought that an accident goes out of one subject into another, (for they are not, as I said above, in their subjects as a part in the whole, or as a contained thing in that which contains it, or as a master of a family in his house,) but that one accident perisheth, and another is generated. For example, when the hand, being moved, moves the pen, motion does not go out of the hand into the pen; for so the writing might be continued though the hand stood still; but a new motion is generated in the pen, and is the pen's motion.

An accident cannot depart from its subject

22. And therefore also it is improper to say, an accident is moved; as when, instead of saying, *figure is an accident of a body carried away*, we say, *a body carries away its figure*.

Nor be moved.

23. Now that accident for which we give a certain name to any body, or the accident which denominates its subject, is commonly called the **ESSENCE** thereof; as rationality is the essence of a man; whiteness, of any white thing, and extension the essence of a body. And the same essence, in as much as it is generated, is called the **FORM**. Again, a body, in respect of any accident, is called the **SUBJECT**, and in respect of the form it is called the **MATTER**.

Essence, form, and matter, what they are.

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Also, the production or perishing of any accident makes its subject be said *to be changed*; only the production or perishing of form makes it be said it is *generated* or *destroyed*; but in all generation and mutation, the name of *matter* still remains. For a table made of wood is not only wooden, but wood; and a statue of brass is brass as well as brazen; though Aristotle, in his *Metaphysics*, says, that whatsoever is made of any thing ought not to be called *ἐκείνὸν*, but *ἐκείνινον*; as that which is made of wood, not *ξύλον*, but *ξύλινον*, that is, not wood, but wooden.

First matter, what.

24. And as for that matter which is common to all things, and which philosophers, following Aristotle, usually call *materia prima*, that is, *first matter*, it is not any body distinct from all other bodies, nor is it one of them. What then is it? A mere name; yet a name which is not of vain use; for it signifies a conception of body without the consideration of any form or other accident except only magnitude or extension, and aptness to receive form and other accident. So that whensoever we have use of the name *body in general*, if we use that of *materia prima*, we do well. For as when a man not knowing which was first, water or ice, would find out which of the two were the matter of both, he would be fain to suppose some third matter which were neither of these two; so he that would find out what is the matter of all things, ought to suppose such as is not the matter of anything that exists. Wherefore *materia prima* is nothing; and therefore they do not attribute to it either form or any other accident besides quantity; whereas all singular things have their forms and accidents certain.

Materia prima, therefore, is body in general, that is, body considered universally, not as having neither form nor any accident, but in which no form nor any other accident but quantity are at all considered, that is, they are not drawn into argumentation. PART II.
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25. From what has been said, those axioms may be demonstrated, which are assumed by Euclid in the beginning of his first element, about the equality and inequality of magnitudes; of which, omitting the rest, I will here demonstrate only this one, *the whole is greater than any part thereof*; to the end that the reader may know that those axioms are not indemonstrable, and therefore not principles of demonstration; and from hence learn to be wary how he admits any thing for a principle, which is not at least as evident as these are. *Greater* is defined to be that, whose part is equal to the whole of another. Now if we suppose any whole to be A, and a part of it to be B; seeing the whole B is equal to itself, and the same B is a part of A; therefore a part of A will be equal to the whole B. Wherefore, by the definition above, A is greater than B; which was to be proved. That the whole is greater than any part thereof, why demonstrated.

CHAPTER IX.

OF CAUSE AND EFFECT.

1. Action and passion, what they are.—2. Action and passion mediate and immediate.—3. Cause simply taken. Cause without which no effect follows, or cause necessary by supposition.—4. Cause efficient and material.—5. An entire cause is always sufficient to produce its effect. At the same instant that the cause is entire, the effect is produced. Every effect has a necessary cause.—6. The generation of effects is continual. What is the beginning in causation.—7. No cause of motion but in a body contiguous and moved.—8. The same agents and patients, if alike disposed, produce like effects though at different times.—9. All mutation is motion.
10. Contingent accidents, what they are.

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9.

Action
and passion,
what they are.

1. A BODY is said to work upon or *act*, that is to say, *do* something to another body, when it either generates or destroys some accident in it: and the body in which an accident is generated or destroyed is said to *suffer*, that is, to have something *done* to it by another body; as when one body by putting forwards another body generates motion in it, it is called the AGENT; and the body in which motion is so generated, is called the PATIENT; so fire that warms the hand is the *agent*, and the hand, which is warmed, is the *patient*. That accident, which is generated in the patient, is called the EFFECT.

Action and
passion, me-
diate, and
immediate.

2. When an agent and patient are contiguous to one another, their action and passion are then said to be *immediate*, otherwise, *mediate*; and when another body, lying betwixt the agent and patient, is contiguous to them both, it is then itself both an

agent and a patient; an agent in respect of the body next after it, upon which it works, and a patient in respect of the body next before it, from which it suffers. Also, if many bodies be so ordered that every two which are next to one another be contiguous, then all those that are betwixt the first and the last are both agents and patients, and the first is an agent only, and the last a patient only.

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3. An agent is understood to *produce* its determined or certain effect in the patient, according to some certain accident or accidents, with which both it and the patient are affected; that is to say, the agent hath its effect precisely such, not because it is a body, but because such a body, or so moved. For otherwise all agents, seeing they are all bodies alike, would produce like effects in all patients. And therefore the fire, for example, does not warm, because it is a body, but because it is hot; nor does one body put forward another body because it is a body, but because it is moved into the place of that other body. The cause, therefore, of all effects consists in certain accidents both in the agents and in the patients; which when they are all present, the effect is produced; but if any one of them be wanting, it is not produced; and that accident either of the agent or patient, without which the effect cannot be produced, is called *causa sine qua non*, or *cause necessary by supposition*, as also the *cause requisite for the production of the effect*. But a *CAUSE* simply, or an *entire cause*, is the aggregate of all the accidents both of the agents how many soever they be, and of the patient, put together; which when they

Cause simply taken.

Cause without which no effect follows, or cause necessary by supposition.

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9.

are all supposed to be present, it cannot be understood but that the effect is produced at the same instant; and if any one of them be wanting, it cannot be understood but that the effect is not produced.

Cause efficient,
and material.

4. The aggregate of accidents in the agent or agents, requisite for the production of the effect, the effect being produced, is called the *efficient cause* thereof; and the aggregate of accidents in the patient, the effect being produced, is usually called the *material cause*; I say the effect being produced; for where there is no effect, there can be no cause; for nothing can be called a cause, where there is nothing that can be called an effect. But the efficient and material causes are both but partial causes, or parts of that cause, which in the next precedent article I called an entire cause. And from hence it is manifest, that the effect we expect, though the agents be not defective on their part, may nevertheless be frustrated by a defect in the patient; and when the patient is sufficient, by a defect in the agents.

An entire cause
is always sufficient to
produce its effect.

5. An entire cause is always sufficient for the production of its effect, if the effect be at all possible. For let any effect whatsoever be propounded to be produced; if the same be produced, it is manifest that the cause which produced it was a sufficient cause; but if it be not produced, and yet be possible, it is evident that something was wanting either in some agent, or in the patient, without which it could not be produced; that is, that some accident was wanting which was requisite for its production; and therefore, that cause was not *entire*, which is contrary to what was supposed.

It follows also from hence, that in whatsoever instant the cause is entire, in the same instant the effect is produced. For if it be not produced, something is still wanting, which is requisite for the production of it; and therefore the cause was not entire, as was supposed.

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At the same instant that the cause is entire, the effect is produced.

And seeing a necessary cause is defined to be that, which being supposed, the effect cannot but follow; this also may be collected, that whatsoever effect is produced at any time, the same is produced by a necessary cause. For whatsoever is produced, in as much as it is produced, had an entire cause, that is, had all those things, which being supposed, it cannot be understood but that the effect follows; that is, it had a necessary cause. And in the same manner it may be shewn, that whatsoever effects are hereafter to be produced, shall have a necessary cause; so that all the effects that have been, or shall be produced, have their necessity in things antecedent.

Every effect has a necessary cause.

6. And from this, that whensoever the cause is entire, the effect is produced in the same instant, it is manifest that causation and the production of effects consist in a certain continual progress; so that as there is a continual mutation in the agent or agents, by the working of other agents upon them, so also the patient, upon which they work, is continually altered and changed. For example: as the heat of the fire increases more and more, so also the effects thereof, namely, the heat of such bodies as are next to it, and again, of such other bodies as are next to them, increase more and more accordingly; which is already no little argument that all mutation consists in motion

The generation of effects is continual. What is the beginning in causation.

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9.

only; the truth whereof shall be further demonstrated in the ninth article. But in this progress of causation, that is, of action and passion, if any man comprehend in his imagination a part thereof, and divide the same into parts, the first part or beginning of it cannot be considered otherwise than as action or cause; for, if it should be considered as effect or passion, then it would be necessary to consider something before it, for its cause or action; which cannot be, for nothing can be before the beginning. And in like manner, the last part is considered only as effect; for it cannot be called cause, if nothing follow it; but after the last, nothing follows. And from hence it is, that in all action the beginning and cause are taken for the same thing. But every one of the intermediate parts are both action and passion, and cause and effect, according as they are compared with the antecedent or subsequent part.

No cause of motion but in a body contiguous and moved.

7. There can be no cause of motion, except in a body contiguous and moved. For let there be any two bodies which are not contiguous, and betwixt which the intermediate space is empty, or, if filled, filled with another body which is at rest; and let one of the propounded bodies be supposed to be at rest; I say it shall always be at rest. For if it shall be moved, the cause of that motion, by the 8th chapter, article 19, will be some external body; and, therefore, if between it and that external body there be nothing but empty space, then whatsoever the disposition be of that external body or of the patient itself, yet if it be supposed to be now at rest, we may conceive it will continue so till it be touched by some other body.

But seeing cause, by the definition, is the aggregate of all such accidents, which being supposed to be present, it cannot be conceived but that the effect will follow, those accidents, which are either in external bodies, or in the patient itself, cannot be the cause of future motion. And in like manner, seeing we may conceive that whatsoever is at rest will still be at rest, though it be touched by some other body, except that other body be moved; therefore in a contiguous body, which is at rest, there can be no cause of motion. Wherefore there is no cause of motion in any body, except it be contiguous and moved.

The same reason may serve to prove that whatsoever is moved, will always be moved on in the same way and with the same velocity, except it be hindered by some other contiguous and moved body; and consequently that no bodies, either when they are at rest, or when there is an interposition of vacuum, can generate or extinguish or lessen motion in other bodies. There is one that has written that things moved are more resisted by things at rest, than by things contrarily moved; for this reason, that he conceived motion not to be so contrary to motion as rest. That which deceived him was, that the words *rest* and *motion* are but contradictory names; whereas motion, indeed, is not resisted by rest, but by contrary motion.

8. But if a body work upon another body at one time, and afterwards the same body work upon the same body at another time, so that both the agent and patient, and all their parts, be in all things as they were; and there be no difference, except only in time, that is, that one action be former, the

The same agents and patients, if alike disposed, produce like effects, though at different times.

PART II. other later in time; it is manifest of itself, that the
 9. effects will be equal and like, as not differing in anything besides time. And as effects themselves proceed from their causes, so the diversity of them depends upon the diversity of their causes also.

All mutation
is motion.

9. This being true, it is necessary that mutation can be nothing else but motion of the parts of that body which is changed. For first, we do not say anything is changed, but that which appears to our senses otherwise than it appeared formerly. Secondly, both those appearances are effects produced in the sentient; and, therefore, if they be different, it is necessary, by the preceding article, that either some part of the agent, which was formerly at rest, is now moved, and so the mutation consists in this motion; or some part, which was formerly moved, is now otherwise moved, and so also the mutation consists in this new motion; or which, being formerly moved, is now at rest, which, as I have shewn above, cannot come to pass without motion; and so again, mutation is motion; or lastly, it happens in some of these manners to the patient, or some of its parts; so that mutation, howsoever it be made, will consist in the motion of the parts, either of the body which is perceived, or of the sentient body, or of both. Mutation therefore is motion, namely, of the parts either of the agent or of the patient; which was to be demonstrated. And to this it is consequent, that rest cannot be the cause of anything, nor can any action proceed from it; seeing neither motion nor mutation can be caused by it.

Contingent
accidents.

10. Accidents, in respect of other accidents which precede them, or are before them in time.

and upon which they do not depend as upon their causes, are called *contingent* accidents; I say, in respect of those accidents by which they are not generated; for, in respect of their causes, all things come to pass with equal necessity; for otherwise they would have no causes at all; which, of things generated, is not intelligible.

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CHAPTER X.

OF POWER AND ACT.

1. Power and cause are the same thing.—2. An act is produced at the same instant in which the power is plenary.—3. Active and passive power are parts only of plenary power.—4. An act, when said to be possible.—5. An act necessary and contingent, what.—6. Active power consists in motion.—7. Cause, formal and final, what they are.

1. CORRESPONDENT to *cause* and *effect*, are POWER and ACT; nay, those and these are the same things; though, for divers considerations, they have divers names. For whensoever any agent has all those accidents which are necessarily requisite for the production of some effect in the patient, then we say that agent has *power* to produce that effect, if it be applied to a patient. But, as I have shewn in the precedent chapter, those accidents constitute the efficient cause; and therefore the same accidents, which constitute the efficient cause, constitute also the *power* of the agent. Wherefore the *power of the agent* and the *efficient cause* are the same thing. But they are considered with this difference, that *cause* is

Power and
cause are the
same thing.

PART II.
10.

so called in respect of the effect already produced, and power in respect of the same effect to be produced hereafter ; so that *cause* respects the past, *power* the future time. Also, the *power of the agent* is that which is commonly called *active power*.

In like manner, whensoever any patient has all those accidents which it is requisite it should have, for the production of some effect in it, we say it is in the *power* of that patient to produce that effect, if it be applied to a fitting agent. But those accidents, as is defined in the precedent chapter, constitute the material cause ; and therefore the *power of the patient*, commonly called *passive power*, and *material cause*, are the same thing ; but with this different consideration, that in cause the past time, and in power the future, is respected. Wherefore the power of the agent and patient together, which may be called entire or *plenary power*, is the same thing with *entire cause* ; for they both consist in the sum or aggregate of all the accidents, as well in the agent as in the patient, which are requisite for the production of the effect. Lastly, as the accident produced is, in respect of the cause, called an effect, so in respect of the power, it is called an *act*.

An act is produced at the same instant in which the power is plenary.

2. As therefore the effect is produced in the same instant in which the cause is entire, so also every act that may be produced, is produced in the same instant in which the power is plenary. And as there can be no effect but from a sufficient and necessary cause, so also no act can be produced but by sufficient power, or that power by which it could not but be produced.

3. And as it is manifest, as I have shewn, that the efficient and material causes are severally and by themselves parts only of an entire cause, and cannot produce any effect but by being joined together, so also power, active and passive, are parts only of plenary and entire power ; nor, except they be joined, can any act proceed from them ; and therefore these powers, as I said in the first article, are but conditional, namely, *the agent has power, if it be applied to a patient ; and the patient has power, if it be applied to an agent ;* otherwise neither of them have power, nor can the accidents, which are in them severally, be properly called powers ; nor any action be said to be possible for the power of the agent alone or of the patient alone.

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Active and passive power are parts only of plenary power.

4. For that is an impossible act, for the production of which there is no power plenary. For seeing plenary power is that in which all things concur, which are requisite for the production of an act, if the power shall never be plenary, there will always be wanting some of those things, without which the act cannot be produced ; wherefore that act shall never be produced ; that is, that act is IMPOSSIBLE : and every act, which is not impossible, is POSSIBLE. Every act, therefore, which is possible, shall at some time be produced ; for if it shall never be produced, then those things shall never concur which are requisite for the production of it ; wherefore that act is *impossible*, by the definition ; which is contrary to what was supposed.

An act, when said to be possible.

5. A *necessary act* is that, the production whereof it is impossible to hinder ; and therefore

An act necessary and contingent, what.

PART II.
10.

An act necessary and contingent, what.

every act, that shall be produced, shall necessarily be produced ; for, that it shall not be produced, is impossible ; because, as is already demonstrated, every possible act shall at some time be produced ; nay, this proposition, *what shall be, shall be*, is as necessary a proposition as this, *a man is a man*.

But here, perhaps, some man may ask whether those future things, which are commonly called *contingents*, are necessary. I say, therefore, that generally all contingents have their necessary causes, as is shewn in the preceding chapter ; but are called contingents in respect of other events, upon which they do not depend ; as the rain, which shall be tomorrow, shall be necessary, that is, from necessary causes ; but we think and say it happens by chance, because we do not yet perceive the causes thereof, though they exist now ; for men commonly call that *casual* or *contingent*, whereof they do not perceive the necessary cause ; and in the same manner they used to speak of things past, when not knowing whether a thing be done or no, they say it is possible it never was done.

Wherefore, all propositions concerning future things, contingent or not contingent, as this, *it will rain tomorrow*, or this, *tomorrow the sun will rise*, are either necessarily true, or necessarily false ; but we call them contingent, because we do not yet know whether they be true or false ; whereas their verity depends not upon our knowledge, but upon the foregoing of their causes. But there are some, who though they confess this whole proposition, *tomorrow it will either rain, or not rain*, to be true, yet they will not acknowledge the parts of it, as, *tomorrow it will rain, or, tomorrow*

it will not rain, to be either of them true by itself; because they say neither this nor that is true *determinately*. But what is this *determinately true*, but true upon our knowledge, or evidently true? And therefore they say no more but that it is not yet known whether it be true or no; but they say it more obscurely, and darken the evidence of the truth with the same words, with which they endeavour to hide their own ignorance.

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6. In the 9th article of the preceding chapter, I have shewn that the efficient cause of all motion and mutation consists in the motion of the agent, or agents; and in the first article of this chapter, that the power of the agent is the same thing with the efficient cause. From whence it may be understood, that all active power consists in motion also; and that power is not a certain accident, which differs from all acts, but is, indeed, an act, namely, motion, which is therefore called power, because another act shall be produced by it afterwards. For example, if of three bodies the first put forward the second, and this the third, the motion of the second, in respect of the first which produceth it, is the act of the second body; but, in respect of the third, it is the active power of the same second body.

Active power
consists in
motion.

7. The writers of metaphysics reckon up two other causes besides the *efficient* and *material*, namely, the ESSENCE, which some call the *formal cause*, and the END, or *final cause*; both which are nevertheless efficient causes. For when it is said the essence of a thing is the cause thereof, *as to be rational is the cause of man*, it is not intelligible; for it is all one, as if it were said, *to be a*

Cause, formal
and final,
what they are.

PART II.
10.

man is the cause of man; which is not well said. And yet the knowledge of the *essence* of anything, is the cause of the knowledge of the thing itself; for, if I first know that a thing is *rational*, I know from thence, that the same is *man*; but this is no other than an efficient cause. A *final cause* has no place but in such things as have sense and will; and this also I shall prove hereafter to be an efficient cause.

CHAPTER XI.

OF IDENTITY AND DIFFERENCE.

1. What it is for one thing to differ from another.—2. To differ in number, magnitude, species, and genus, what.—3. What is relation, proportion, and relatives.—4. Proportionals, what.—5. The proportion of magnitudes to one another, wherein it consists.—6. Relation is no new accident, but one of those that were in the relative before the relation or comparison was made. Also the causes of accidents in the correlatives, are the cause of relation.—7. Of the beginning of individuation.

What it is
for one thing
to differ from
another.

1. HITHERTO I have spoken of body simply, and accidents common to all bodies, as *magnitude, motion, rest, action, passion, power, possible, &c.*; and I should now descend to those accidents by which one body is distinguished from another, but that it is first to be declared what it is to be *distinct* and *not distinct*, namely, what are the **SAME** and **DIFFERENT**; for this also is common to all bodies, that they may be distinguished and differed from one another. Now, two bodies are said to *differ* from one another, when something may be said of one of them, which cannot be said of the other at the same time.

2. And, first of all, it is manifest that no two bodies are the *same* ; for seeing they are two, they are in two places at the same time ; as that, which is the *same*, is at the same time in one and the same place. All bodies therefore differ from one another in *number*, namely, as one and another ; so that the *same* and *different in number*, are names opposed to one another by contradiction.

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To differ
in number,
magnitude,
species, and
genus, what.

In *magnitude* bodies differ when one is greater than another, as *a cubit long*, and *two cubits long*, of *two pound weight*, and of *three pound weight*. And to these, *equals* are opposed.

Bodies, which differ more than in magnitude, are called *unlike* ; and those, which differ only in magnitude, *like*. Also, of unlike bodies, some are said to differ in the *species*, others in the *genus* ; in the *species*, when their difference is perceived by one and the same sense, as *white* and *black* ; and in the *genus*, when their difference is not perceived but by divers senses, as *white* and *hot*.

3. And the *likeness*, or *unlikeness*, *equality*, or *inequality* of one body to another, is called their **RELATION** ; and the bodies themselves *relatives* or *correlatives* ; *Aristotle* calls them τὰ πρὸς τὶ ; the first whereof is usually named the *antecedent*, and the second the *consequent* ; and the *relation* of the antecedent to the consequent, according to magnitude, namely, the equality, the excess or defect thereof, is called the **PROPORTION** of the antecedent to the consequent ; so that *proportion* is nothing but the equality or inequality of the magnitude of the antecedent compared to the magnitude of the consequent by their difference only, or compared also with their difference. For ex-

What is
relation,
proportion,
and relatives.

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11.

ample, the *proportion* of three to two consists only in this, that three *exceeds* two by unity ; and the proportion of two to five in this, that two, compared with five, is *deficient* of it by three, either simply, or compared with the numbers different ; and therefore in the proportion of unequals, the proportion of the less to the greater, is called DEFECT ; and that of the greater to the less, EXCESS.

Proportionals, what.

4. Besides, of unequals, some are more, some less, and some equally unequal ; so that there is *proportion of proportions*, as well as of *magnitudes* ; namely, where two unequals have relation to two other unequals ; as, when the inequality which is between 2 and 3, is compared with the inequality which is between 4 and 5. In which comparison there are always four magnitudes ; or, which is all one, if there be but three, the middlemost is twice numbered ; and if the proportion of the first to the second, be equal to the proportion of the third to the fourth, then the four are said to be *proportionals* ; otherwise they are not proportionals.

The proportion of magnitudes to one another, wherein it consists.

5. The proportion of the antecedent to the consequent consists in their difference, not only simply taken, but also as compared with one of the relatives ; that is, either in that part of the greater, by which it exceeds the less, or in the remainder, after the less is taken out of the greater ; as the proportion of two to five consists in the three by which five exceeds two, not in three simply only, but also as compared with five or two. For though there be the same difference between two and five, which is between nine and twelve, namely three, yet there is not the same inequality ;

and therefore the proportion of two to five is not in all relation the same with that of nine to twelve, but only in that which is called arithmetical.

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6. But we must not so think of relation, as if it were an accident differing from all the other accidents of the relative; but one of them, namely, that by which the comparison is made. For example, the likeness of one *white* to another *white*, or its unlikeness to *black*, is the same accident with its *whiteness*; and *equality* and *inequality*, the same accident with the *magnitude* of the thing compared, though under another name: for that which is called *white* or *great*, when it is not compared with something else, the same when it is compared, is called *like* or *unlike*, *equal* or *unequal*. And from this it follows that the causes of the accidents, which are in relatives, are the causes also of *likeness*, *unlikeness*, *equality* and *inequality*; namely, that he, that makes two unequal bodies, makes also their inequality; and he, that makes a rule and an action, makes also, if the action be congruous to the rule, their congruity; if incongruous, their incongruity. And thus much concerning *comparison* of one body with another.

Relation is no new accident, but one of those that were in the relative, before the relation or comparison was made. Also the causes of accidents in correlatives are the cause of relation.

7. But the same body may at different times be compared with itself. And from hence springs a great controversy among philosophers about the *beginning of individuation*, namely, in what sense it may be conceived that a body is at one time the same, at another time not the same it was formerly. For example, whether a man grown old be the same man he was whilst he was young, or another man; or whether a city be in different ages the same, or another city. Some place *individuity* in

Of the beginning of individuation.

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11.

Of the begin-
ning of indi-
viduation.

the unity of *matter* ; others, in the unity of *form* ; and one says it consists in the unity of the *aggregate of all the accidents together*. For *matter*, it is pleaded that a lump of wax, whether it be spherical or cubical, is the same wax, because the same matter. For *form*, that when a man is grown from an infant to be an old man, though his matter be changed, yet he is still the same numerical man ; for that *identity*, which cannot be attributed to the matter, ought probably to be ascribed to the form. For the *aggregate of accidents*, no instance can be made ; but because, when any new accident is generated, a new name is commonly imposed on the thing, therefore he, that assigned this cause of *individuity*, thought the thing itself also was become another thing. According to the first opinion, he that sins, and he that is punished, should not be the same man, by reason of the perpetual flux and change of man's body ; nor should the city, which makes laws in one age and abrogates them in another, be the same city ; which were to confound all civil rights. According to the second opinion, two bodies existing both at once, would be one and the same numerical body. For if, for example, that ship of Theseus, concerning the difference whereof made by continual reparation in taking out the old planks and putting in new, the sophisters of Athens were wont to dispute, were, after all the planks were changed, the same numerical ship it was at the beginning ; and if some man had kept the old planks as they were taken out, and by putting them afterwards together in the same order, had again made a ship of them, this, without doubt, had also been the same nume-

rical ship with that which was at the beginning ; and so there would have been two ships numerically the same, which is absurd. But, according to the third opinion, nothing would be the same it was ; so that a man standing would not be the same he was sitting ; nor the water, which is in the vessel, the same with that which is poured out of it. Wherefore the beginning of *individuation* is not always to be taken either from matter alone, or from form alone.

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11.

Of the beginning of individuation.

But we must consider by what name anything is called, when we inquire concerning the *identity* of it. For it is one thing to ask concerning Socrates, whether he be the same man, and another to ask whether he be the same body ; for his body, when he is old, cannot be the same it was when he was an infant, by reason of the difference of magnitude ; for one body has always one and the same magnitude ; yet, nevertheless, he may be the same man. And therefore, whensoever the name, by which it is asked whether a thing be the same it was, is given it for the matter only, then, if the matter be the same, the thing also is *individually* the same ; as the water, which was in the sea, is the same which is afterwards in the cloud ; and any body is the same, whether the parts of it be put together, or dispersed ; or whether it be congealed, or dissolved. Also, if the name be given for such form as is the beginning of motion, then, as long as that motion remains, it will be the same *individual* thing ; as that man will be always the same, whose actions and thoughts proceed all from the same beginning of motion, namely, that which was in his generation ; and that will be the same river

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11.

Of the beginning of individuation.

which flows from one and the same fountain, whether the same water, or other water, or something else than water, flow from thence; and that the same city, whose acts proceed continually from the same institution, whether the men be the same or no. Lastly, if the name be given for some accident, then the *identity* of the thing will depend upon the matter; for, by the taking away and supplying of matter, the accidents that were, are destroyed, and other new ones are generated, which cannot be the same numerically; so that a ship, which signifies matter so figured, will be the same as long as the matter remains the same; but if no part of the matter be the same, then it is numerically another ship; and if part of the matter remain and part be changed, then the ship will be partly the same, and partly not the same.

 CHAPTER XII.

OF QUANTITY.

1. The definition of quantity.—2. The exposition of quantity, what it is.—3. How line, superficies, and solid, are exposed.
4. How time is exposed.—5. How number is exposed.—6. How velocity is exposed.—7. How weight is exposed.—8. How the proportion of magnitudes is exposed.—9. How the proportion of times and velocities is exposed.

Definition of quantity.

1. WHAT and how manifold *dimension* is, has been said in the 8th chapter, namely, that there are three dimensions, line or length, superficies, and solid; every one of which, if it be determined, that is, if the limits of it be made known, is commonly called *quantity*; for by *quantity* all [men under-

stand that which is signified by that word, by which answer is made to the question, *How much is it?* Whensoever, therefore, it is asked, for example, *How long is the journey?* it is not answered indefinitely, *length*; nor, when it is asked, *How big is the field?* is it answered indefinitely, *superficies*; nor, if a man ask, *How great is the bulk?* indefinitely, *solid*: but it is answered determinately, the journey is a hundred miles; the field is a hundred acres; the bulk is a hundred cubical feet; or at least in some such manner, that the magnitude of the thing enquired after may by certain limits be comprehended in the mind. QUANTITY, therefore, cannot otherwise be defined, than to be *a dimension determined, or a dimension, whose limits are set out, either by their place, or by some comparison.*

2. And *quantity* is determined two ways; one, by the sense, when some sensible object is set before it; as when a line, a superficies or solid, of a foot or cubit, marked out in some matter, is objected to the eyes; which way of determining, is called *exposition*, and the quantity so known is called *exposed quantity*; the other by memory, that is, by comparison with some exposed quantity. In the first manner, when it is asked of what quantity a thing is, it is answered, of such quantity as you see exposed. In the second manner, answer cannot be made but by comparison with some exposed quantity; for if it be asked, how long is the way? the answer is, so many thousand paces; that is, by comparing the way with a pace, or some other measure, determined and known by exposition; or the quantity of it is to some other quan-

The exposition of quantity, what it is.

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12.

Exposition of
quantity,
what it is.

tity known by exposition, as the diameter of a square is to the side of the same, or by some other the like means. But it is to be understood, that the quantity exposed must be some standing or permanent thing, such as is marked out in consistent or durable matter; or at least something which is revocable to sense; for otherwise no comparison can be made by it. Seeing, therefore, by what has been said in the next preceding chapter, comparison of one magnitude with another is the same thing with proportion; it is manifest, that quantity determined in the second manner is nothing else but the proportion of a dimension not exposed to another which is exposed; that is, the comparison of the equality or inequality thereof with an exposed quantity.

How line,
superficies,
and solids,
are exposed.

3. *Lines, superficies, and solids*, are exposed, first, by *motion*, in such manner as in the 8th chapter I have said they are generated; but so as that the marks of such motion be permanent; as when they are designed upon some matter, as a line upon paper; or graven in some durable matter. Secondly, by *apposition*; as when one line or length is applied to another line or length, one breadth to another breadth, and one thickness to another thickness; which is as much as to describe a line by points, a superficies by lines, and a solid by superficies; saving that by points in this place are to be understood very short lines; and, by superficies, very thin solids. Thirdly, lines and superficies may be exposed by *section*, namely, a line may be made by cutting an exposed superficies; and a superficies, by the cutting of an exposed solid.

4. *Time* is exposed, not only by the exposition of a line, but also of some moveable thing, which is moved uniformly upon that line, or at least is supposed so to be moved. For, seeing time is an idea of motion, in which we consider former and latter, that is succession, it is not sufficient for the exposition of time that a line be described; but we must also have in our mind an imagination of some moveable thing passing over that line; and the motion of it must be uniform, that time may be divided and compounded as often as there shall be need. And, therefore, when philosophers, in their demonstrations, draw a line, and say, *Let that line be time*, it is to be understood as if they said, *Let the conception of uniform motion upon that line, be time*. For though the circles in dials be lines, yet they are not of themselves sufficient to note time by, except also there be, or be supposed to be, a motion of the shadow or the hand.

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12.

How time
is exposed.

5. *Number* is exposed, either by the exposition of points, or of the names of number, *one, two, three, &c.*; and those points must not be contiguous, so as that they cannot be distinguished by notes, but they must be so placed that they may be *discerned* one from another; for, from this it is, that number is called *discreet quantity*, whereas all quantity, which is designed by motion, is called *continual quantity*. But that number may be exposed by the names of number, it is necessary that they be recited by heart and in order, as one, two, three, &c.; for by saying one, one, one, and so forward, we know not what number we are at beyond two or three; which also appear to us in this manner, not as number, but as figure.

How number
is exposed.

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How velocity
is exposed.

6. For the exposition of *velocity*, which, by the definition thereof, is a motion which, in a certain time, passeth over a certain space, it is requisite, not only that time be exposed, but that there be also exposed that space which is transmitted by the body, whose velocity we would determine; and that a body be understood to be moved in that space also; so that there must be exposed two lines, upon one of which uniform motion must be understood to be made, that the time may be determined; and, upon the other, the velocity is to be computed. As if $\frac{A}{C} \frac{B}{D}$ we would expose the velocity of the body A, we draw two lines A B and C D, and place a body in C also; which done, we say the velocity of the body A is so great, that it passeth over the line A B in the same time in which the body C passeth over the line C D with uniform motion.

How weight
is exposed.

7. *Weight* is exposed by any heavy body, of what matter soever, so it be always alike heavy.

How the pro-
portion of
magnitudes
is exposed.

8. The *proportion* of two magnitudes is then exposed, when the magnitudes themselves are exposed, namely, the proportion of equality, when the magnitudes are equal; and of inequality, when they are unequal. For seeing, by the 5th article of the preceding chapter, the proportion of two unequal magnitudes consists in their difference, compared with either of them; and when two unequal magnitudes are exposed, their difference is also exposed; it follows, that when magnitudes, which have proportion to one another, are exposed, their proportion also is exposed with them; and, in like manner, the proportion of equals,

which consists in this, that there is no difference of magnitude betwixt them, is exposed at the same time when the equal magnitudes themselves are exposed. For example, if the exposed lines

A B and C D be equal, the proportion of equality is exposed in them ; $\frac{A}{C} = \frac{B}{D}$
 and if the exposed lines, E F and E G be unequal, the proportion which E F has to E G, and that which E G

has to E F are also exposed in them ; for not only the lines themselves, but also their difference, G F, is exposed. The proportion of unequals is quantity ; for the difference, G F, in which it consists, is quantity. But the proportion of equality is not quantity ; because, between equals, there is no difference ; nor is one equality greater than another, as one inequality is greater than another inequality.

9. The proportion of two *times*, or of two *uniform velocities*, is then exposed, when two lines are exposed by which two bodies are understood to be moved uniformly ; and therefore the same two lines serve to exhibit both their own proportion, and that of the times and velocities, according as they are considered to be exposed for the magnitudes themselves, or for the times or velocities. For let the two lines A and B be exposed ; their proportion therefore (by the last foregoing article) is exposed ; and if they be considered as drawn with equal and uniform velocity, then, seeing their times are greater, or equal, or less, according as the same spaces are transmitted in greater, or equal, or less time, the lines A and B will exhibit the equality or inequality, that is, the proportion

How the proportion of times and velocities is exposed.

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12.

of the times. To conclude, if the same lines, A and B, be considered as drawn in the same time, then, seeing their velocities are greater, or equal, or less, according as they pass over in the same time longer, or equal, or shorter lines, the same lines, A and B, will exhibit the equality, or inequality, that is, the proportion of their velocities.

CHAPTER XIII.

OF ANALOGISM, OR THE SAME PROPORTION.

1, 2, 3, 4. The nature and definition of proportion, arithmetical and geometrical.—5. The definition, and some properties of the same arithmetical proportion.—6, 7. The definition and transmutations of analogism, or the same geometrical proportion.—8, 9. The definitions of hyperlogism and hypologism, that is, of greater and less proportion, and their transmutations.—10, 11, 12. Comparison of analogical quantities, according to magnitude.—13, 14, 15. Composition of proportions. 16, 17, 18, 19, 20, 21, 22, 23, 24, 25. The definition and properties of continual proportion.—26, 27, 28, 29. Comparison of arithmetical and geometrical proportions.

[Note, that in this chapter the sign + signifies that the quantities betwixt which it is put, are added together; and this sign — the remainder after the latter quantity is taken out of the former. So that $A+B$ is equal to both A and B together; and where you see $A-B$, there A is the whole, B the part taken out of it, and $A-B$ the remainder. Also, two letters, set together without any sign, signify, unless they belong to a figure, that one of the quantities is multiplied by the other; as $A B$ signifies the product of A multiplied by B.]

The nature
and definition
of proportion,
arithmetical
& geometrical.

1. **GREAT** and little are not intelligible, but by comparison. Now that, to which they are compared, is something exposed; that is, some magnitude either perceived by sense, or so defined by words, that it may be comprehended by the mind. Also that, to which any magnitude is compared, is either

greater or less, or equal to it. And therefore proportion (which, as I have shewn, is the estimation or comprehension of magnitudes by comparison,) is threefold, namely, proportion of *equality*, that is, of equal to equal; or of *excess*, which is of the greater to the less; or of *defect*, which is the proportion of the less to the greater.

Again, every one of these proportions is twofold; for if it be asked concerning any magnitude given, how great it is, the answer may be made by comparing it two ways; first, by saying it is greater or less than another magnitude, by so much; as seven is less than ten, by three unities; and this is called *arithmetical proportion*. Secondly, by saying it is greater or less than another magnitude, by such a part or parts thereof; as seven is less than ten, by three tenth parts of the same ten. And though this proportion be not always explicable by number, yet it is a determinate proportion, and of a different kind from the former, and called *geometrical proportion*, and most commonly *proportion simply*.

2. Proportion, whether it be arithmetical or geometrical, cannot be exposed but in two magnitudes, (of which the former is commonly called the *antecedent*, and the latter the *consequent* of the proportion) as I have shewn in the 8th article of the preceding chapter. And, therefore, if two proportions be to be compared, there must be four magnitudes exposed, namely, two antecedents and two consequents; for though it happen sometimes that the consequent of the former proportion be the same with the antecedent of the latter, yet in that double comparison it must of necessity be

The nature and definition of proportion, &c.

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The nature
and defini-
tion of pro-
portion, &c.

twice numbered ; so that there will be always four terms.

3. Of two proportions, whether they be arithmetical or geometrical, when the magnitudes compared in both (which Euclid, in the fifth definition of his sixth book, calls the *quantities of proportions*;) are equal, then one of the proportions cannot be either greater or less than the other ; for one equality is neither greater nor less than another equality. But of two proportions of inequality, whether they be proportions of excess or of defect, one of them may be either greater or less than the other, or they may both be equal ; for though there be propounded two magnitudes that are unequal to one another, yet there may be other two more unequal, and other two equally unequal, and other two less unequal than the two which were propounded. And from hence it may be understood, that the proportions of excess and defect are quantity, being capable of more and less ; but the proportion of equality is not quantity, because not capable neither of *more*, nor of *less*. And therefore proportions of inequality may be added together, or subtracted from one another, or be multiplied or divided by one another, or by number ; but proportions of equality not so.

4. Two equal proportions are commonly called *the same proportion* ; and, it is said, that the proportion of the first antecedent to the first consequent is the *same* with that of the second antecedent to the second consequent. And when four magnitudes are thus to one another in geometrical proportion, they are called *proportionals* ; and by some, more briefly, *analogism*. And *greater*

proportion is the proportion of a greater antecedent to the same consequent, or of the same antecedent to a less consequent; and when the proportion of the first antecedent to the first consequent is greater than that of the second antecedent to the second consequent, the four magnitudes, which are so to one another, may be called *hyperlogism*.

Less proportion is the proportion of a less antecedent to the same consequent, or of the same antecedent to a greater consequent; and when the proportion of the first antecedent to the first consequent is less than that of the second to the second, the four magnitudes may be called *hypologism*.

5. One arithmetical proportion is the *same* with another arithmetical proportion, when one of the antecedents exceeds its consequent, or is exceeded by it, as much as the other antecedent exceeds its consequent, or is exceeded by it. And therefore, in four magnitudes, arithmetically proportional, the sum of the extremes is equal to the sum of the means. For if $A. B :: C. D$ be arithmetically proportional, and the difference on both sides be the same excess, or the same defect, E , then $B+C$ (if A be greater than B) will be equal to $A-E+C$; and $A+D$ will be equal to $A+C-E$; but $A-E+C$ and $A+C-E$ are equal. Or if A be less than B , then $B+C$ will be equal to $A+E+C$; and $A+D$ will be equal to $A+C+E$; but $A+E+C$ and $A+C+E$ are equal.

The definition and some properties of the same arithmetical proportion.

Also, if there be never so many magnitudes, arithmetically proportional, the sum of them all will be equal to the product of half the number of the terms multiplied by the sum of the extremes.

PART II. For if $A. B :: C. D :: E. F$ be arithmetically proportional, the couples $A+F$, $B+E$, $C+D$ will be equal to one another; and their sum will be equal to $A+F$, multiplied by the number of their combinations, that is, by half the number of the terms.

13.
The definition
and some pro-
perties of, &c.

If, of four unequal magnitudes, any two, together taken, be equal to the other two together taken, then the greatest and the least of them will be in the same combination. Let the unequal magnitudes be A, B, C, D ; and let $A+B$ be equal to $C+D$; and let A be the greatest of them all; I say B will be the least. For, if it may be, let any of the rest, as D , be the least. Seeing therefore A is greater than C , and B than D , $A+B$ will be greater than $C+D$; which is contrary to what was supposed.

If there be any four magnitudes, the sum of the greatest and least, the sum of the means, the difference of the two greatest, and the difference of the two least, will be arithmetically proportional. For, let there be four magnitudes, whereof A is the greatest, D the least, and B and C the means; I say $A+D. B+C :: A-B. C-D$ are arithmetically proportional. For the difference between the first antecedent and its consequent is this, $A+D-B-C$; and the difference between the second antecedent and its consequent this, $A-B-C+D$; but these two differences are equal; and therefore, by this 5th article, $A+D. B+C :: A-B. C-D$ are arithmetically proportional.

If, of four magnitudes, two be equal to the other two, they will be in reciprocal arithmetical proportion. For let $A+B$ be equal to $C+D$, I say $A. C :: D. B$ are arithmetically proportional. For

if they be not, let $A. C :: D. E$ (supposing E to be greater or less than B) be arithmetically proportional, and then $A + E$ will be equal to $C + D$; wherefore $A + B$ and $C + D$ are not equal; which is contrary to what was supposed.

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13.

6. One geometrical proportion is the same with another geometrical proportion; when the same cause, producing equal effects in equal times, determines both the proportions.

The definition and transmutations of analogism, or the same geometrical proportion.

If a point uniformly moved describe two lines, either with the same, or different velocity, all the parts of them which are contemporary, that is, which are described in the same time, will be two to two, in geometrical proportion, whether the antecedents be taken in the same line, or not. For, from the point A (in the 10th figure at the end of the 14th chapter) let the two lines, $A D$, $A G$, be described with uniform motion; and let there be taken in them two parts $A B$, $A E$, and again, two other parts, $A C$, $A F$; in such manner, that $A B$, $A E$, be contemporary, and likewise $A C$, $A F$ contemporary. I say first (taking the antecedents $A B$, $A C$ in the line $A D$, and the consequents $A E$, $A F$ in the line $A G$) that $A B. A C :: A E. A F$ are proportionals. For seeing (by the 8th chap. and the 15th art.) velocity is motion considered as determined by a certain length or line, in a certain time transmitted by it, the quantity of the line $A B$ will be determined by the velocity and time by which the same $A B$ is described; and for the same reason, the quantity of the line $A C$ will be determined by the velocity and time, by which the same $A C$ is described; and therefore the proportion of $A B$ to $A C$, whe-

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gism, &c.

ther it be proportion of equality, or of excess or defect, is determined by the velocities and times by which A B, A C are described ; but seeing the motion of the point A upon A B and A C is uniform, they are both described with equal velocity ; and therefore whether one of them have to the other the proportion of majority or of minority, the sole cause of that proportion is the difference of their times ; and by the same reason it is evident, that the proportion of A E to A F is determined by the difference of their times only. Seeing therefore A B, A E, as also A C, A F are contemporary, the difference of the times in which A B and A C are described, is the same with that in which A E and A F are described. Wherefore the proportion of A B to A C, and the proportion of A E to A F are both determined by the same cause. But the cause, which so determines the proportion of both, works equally in equal times, for it is uniform motion ; and therefore (by the last precedent definition) the proportion of A B to A C is the same with that of A E to A F ; and consequently A B. A C :: A E. A F are proportionals ; which is the first.

Secondly, (taking the antecedents in different lines) I say, A B. A E :: A C. A F are proportionals ; for seeing A B, A E are described in the same time, the difference of the velocities in which they are described is the sole cause of the proportion they have to one another. And the same may be said of the proportion of A C to A F. But seeing both the lines A D and A G are passed over by uniform motion, the difference of the velocities in which A B, A E are described, will be the same

with the difference of the velocities, in which A C, A F are described. Wherefore the cause which determines the proportion of A B to A E, is the same with that which determines the proportion of A C to A F; and therefore A B. A E :: A C. A F, are proportionals; which remained to be proved.

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Coroll. I. If four magnitudes be in geometrical proportion, they will also be proportionals by *permutation*, that is, by transposing the middle terms. For I have shown, that not only A B. A C :: A E. A F, but also that, by *permutation*, A B. A E :: A C. A F are proportionals.

Coroll. II. If there be four proportionals, they will also be proportionals by *inversion* or *conversion*, that is, by turning the antecedents into consequents. For if in the last *analogism*, I had for A B, A C, put by inversion A C, A B, and in like manner converted A E, A F into A F, A E, yet the same demonstration had served. For as well A C, A B, as A B, A C are of equal velocity; and A C, A F, as well as A F, A C are contemporary.

Coroll. III. If proportionals be added to proportionals, or taken from them, the aggregates, or remainders, will be proportionals. For contemporaries, whether they be added to contemporaries, or taken from them, make the aggregates or remainders contemporary, though the addition or subtraction be of all the terms, or of the antecedents alone, or of the consequents alone.

Coroll. IV. If both the antecedents of four proportionals, or both the consequents, or all the terms, be multiplied or divided by the same number or quantity, the products or quotients will be proportionals. For the multiplication and division

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of proportionals, is the same with the addition and subtraction of them.

Coroll. v. If there be four proportionals, they will also be proportionals by *composition*, that is, by compounding an antecedent of the antecedent and consequent put together, and by taking for consequent either the consequent singly, or the antecedent singly. For this composition is nothing but addition of proportionals, namely, of consequents to their own antecedents, which by supposition are proportionals.

Coroll. vi. In like manner, if the antecedent singly, or consequent singly, be put for antecedent, and the consequent be made of both put together, these also will be proportionals. For it is the *inversion of proportion by composition*,

Coroll. vii. If there be four proportionals, they will also be proportionals by division, that is, by taking the remainder after the consequent is subtracted from the antecedent, or the difference between the antecedent and consequent for antecedent, and either the whole or the subtracted for consequent; as if $A.B :: C.D$ be proportionals, they will by division be $A-B.B :: C-D.D$, and $A-B.A :: C-D.C$; and when the consequent is greater than the antecedent, $B-A.A :: D-C.C$, and $B-A.B :: D-C.D$. For in all these divisions, proportionals are, by the very supposition of the analogism $A.B :: C.D$, taken from A and B, and from C and D.

Coroll. viii. If there be four proportionals, they will also be proportionals by the *conversion of proportion*, that is, by inverting the divided proportion, or by taking the whole for antecedent, and the difference or remainder for consequent.

As, if $A. B :: C. D$ be proportionals, then $A. A-B :: C. C-D$, as also $B. A-B :: D. C-D$ will be proportionals. For seeing these inverted be proportionals, they are also themselves proportionals.

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Coroll. IX. If there be two analogisms which have their quantities equal, the second to the second, and the fourth to the fourth, then either the sum or difference of the first quantities will be to the second, as the sum or difference of the third quantities is to the fourth. Let $A. B :: C. D$ and $E. B :: F. D$ be analogisms; I say $A + E. B :: C + F. D$ are proportionals. For the said analogisms will by permutation be $A. C :: B. D$, and $E. F :: B. D$; and therefore $A. C :: E. F$ will be proportionals, for they have both the proportion of B to D common. Wherefore, if in the permutation of the first analogism, there be added E and F to A and C , which E and F are proportional to A and C , then (by the third coroll.) $A + E. B :: C + F. D$ will be proportionals; which was to be proved.

Also in the same manner it may be shown, that $A - E. B :: C - F. D$ are proportionals.

7. If there be two analogisms, where four antecedents make an analogism, their consequents also shall make an analogism; as also the sums of their antecedents will be proportional to the sums of their consequents. For if $A. B :: C. D$ and $E. F :: G. H$ be two analogisms, and $A. E :: C. G$ be proportionals, then by permutation $A. C :: E. G$, and $E. G :: F. H$, and $A. C :: B. D$ will be proportionals; wherefore $B. D :: E. G$, that is, $B. D :: F. H$, and by permutation $B. F :: D. H$ are proportionals; which is the first. Secondly, I say $A + E. B + F :: C + G. D + H$ will be proportionals. For seeing

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A. E :: C. G are proportionals, A + E. E :: C + G. G will also by composition be proportionals, and by permutation A + E. C + G :: E. G will be proportionals; wherefore, also A + E. C + G :: F. H will be proportionals. Again, seeing, as is shown above, B. F :: D. H are proportionals, B + F. F :: D + H. H will also by composition be proportionals; and by permutation B + F. D + H :: F. H will also be proportionals; wherefore A + E. C + G :: B + F. D + H are proportionals; which remained to be proved.

Coroll. By the same reason, if there be never so many analogisms, and the antecedents be proportional to the antecedents, it may be demonstrated also that the consequents will be proportional to the consequents, as also the sum of the antecedents to the sum of the consequents.

The definition of hyperlogism and hypologism, that is, of greater and less proportion, and their transmutations.

8. In an hyperlogism, that is, where the proportion of the first antecedent to its consequent is greater than the proportion of the second antecedent to its consequent, the permutation of the proportionals, and the addition of proportionals to proportionals, and subtraction of them from one another, as also their composition and division, and their multiplication and division by the same number, produce always an hyperlogism. For suppose A. B :: C. D and A. C :: E. F be analogisms, A + E. B :: C + F. D will also be an analogism; but A + E. B :: C. D will be an hyperlogism; wherefore by permutation, A + E. C :: B. D is an hyperlogism, because A. B :: C. D is an analogism. Secondly, if to the hyperlogism A + E. B :: C. D the proportionals G and H be added, A + E + G. B :: C + H. D will be an hyperlogism, by reason A + E + G. B :: C + F + H. D is an analogism. Also, if G and

H be taken away, $A + E - G. B :: C - H. D$ will be an hyperlogism; for $A + E - G. B :: C + F - H. D$ is an analogism. Thirdly, by composition $A + E + B. B :: C + D. D$ will be an hyperlogism, because $A + E + B. B :: C + F + D. D$ is an analogism, and so it will be in all the varieties of composition. Fourthly, by division, $A + E - B. B :: C - D. D$ will be an hyperlogism, by reason $A + E - B. B :: C + F - D. D$ is an analogism. Also $A + E - B. A + E :: C - D. C$ is an hyperlogism; for $A + E - B. A + E :: C + F - D. C$ is an analogism. Fifthly, by multiplication $4 A + 4 E. B :: 4 C. D$ is an hyperlogism, because $4 A. B :: 4 C. D$ is an analogism; and by division $\frac{1}{4} A + \frac{1}{4} E. B :: \frac{1}{4} C. D$ is an hyperlogism, because $\frac{1}{4} A. B :: \frac{1}{4} C. D$ is an analogism.

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9. But if $A + E. B :: C. D$ be an hyperlogism, then by inversion $B. A + E :: D. C$ will be an hypologism, because $B. A :: D. C$ being an analogism, the first consequent will be too great. Also, by conversion of proportion, $A + E. A + E - B :: C. C - D$ is an hypologism, because the inversion of it, namely $A + E - B. A + E :: C - D. C$ is an hyperlogism, as I have shown but now. So also $B. A + E - B :: D. C - D$ is an hypologism, because, as I have newly shown, the inversion of it, namely $A + E - B. B :: C - D. D$ is an hyperlogism. Note that this hypologism $A + E. A + E - B :: C. C - D$ is commonly thus expressed; if the proportion of the whole, $(A + E)$ to that which is taken out of it (B) , be greater than the proportion of the whole (C) to that which is taken out of it (D) , then the proportion of the whole $(A + E)$ to the remainder $(A + E - B)$ will be less than the proportion of the whole (C) to the remainder $(C - D)$.

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of analogical
quantities,
according
to magnitude.

10. If there be four proportionals, the difference of the two first, to the difference of the two last, will be as the first antecedent is to the second antecedent, or as the first consequent to the second consequent. For if $A.B :: C.D$ be proportionals, then by division $A-B.B :: C-D.D$ will be proportionals; and by permutation $A-B.C-D :: B.D$; that is, the differences are proportional to the consequents, and therefore they are so also to the antecedents.

11. Of four proportionals, if the first be greater than the second, the third also shall be greater than the fourth. For seeing the first is greater than the second, the proportion of the first to the second is the proportion of excess; but the proportion of the third to the fourth is the same with that of the first to the second; and therefore also the proportion of the third to the fourth is the proportion of excess; wherefore the third is greater than the fourth. In the same manner it may be proved, that whensoever the first is less than the second, the third also is less than the fourth; and when those are equal, that these also are equal.

12. If there be four proportionals whatsoever, $A.B :: C.D$, and the first and third be multiplied by any one number, as by 2; and again the second and fourth be multiplied by any one number, as by 3; and the product of the first 2 A, be greater than the product of the second 3 B; the product also of the third 2 C, will be greater than the product of the fourth 3 D. But if the product of the first be less than the product of the second, then the product of the third will be less than that of the fourth. And lastly, if the products of the first

and second be equal, the products of the third and fourth shall also be equal. Now this theorem is all one with Euclid's definition of *the same proportion*; and it may be demonstrated thus. Seeing $A. B :: C. D$ are proportionals, by permutation also (art. 6, coroll. 1.) $A. C :: B. D$ will be proportionals; wherefore (by coroll. IV. art. 6) $2 A. 2 C :: 3 B. 3 D$ will be proportionals; and again, by permutation, $2 A. 3 B :: 2 C. 3 D$ will be proportionals; and therefore, by the last article, if $2 A$ be greater than $3 B$, then $2 C$ will be greater than $3 D$; if less, less; and if equal, equal; which was to be demonstrated.

13. If any three magnitudes be propounded, or three things whatsoever that have any proportion one to another, as three numbers, three times, three degrees, &c.; the proportions of the first to the second, and of the second to the third, together taken, are equal to the proportion of the first to the third. Let there be three lines, for any proportion may be reduced to the proportion of lines, $A B, A C, A D$; and in the first place, let the proportion as well of the first $A B$ to the second $A C$, $A \quad B \quad C \quad D$ as of the second $A C$ to the third $A D$, be the proportion of defect, or of less to greater; I say the proportions together taken of $A B$ to $A C$, and of $A C$ to $A D$, are equal to the proportion of $A B$ to $A D$. Suppose the point A to be moved over the whole line $A D$ with uniform motion; then the proportions as well of $A B$ to $A C$, as of $A C$ to $A D$, are determined by the difference of the times in which they are described; that is, $A B$ has to $A C$ such proportion as is determined by the different times of their description; and $A C$ to $A D$ such propor-

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tion as is determined by their times. But the proportion of $A B$ to $A D$ is such as is determined by the difference of the times in which $A B$ and $A D$ are described; and the difference of the times in which $A B$ and $A C$ are described, together with the difference of the times in which $A C$ and $A D$ are described, is the same with the difference of the times in which $A B$ and $A D$ are described. And therefore, the same cause which determines the two proportions of $A B$ to $A C$ and of $A C$ to $A D$, determines also the proportion of $A B$ to $A D$. Wherefore, by the definition of the same proportion, delivered above in the 6th article, the proportion of $A B$ to $A C$ together with the proportion of $A C$ to $A D$, is the same with the proportion of $A B$ to $A D$.

In the second place, let $A D$ be the first, $A C$ the second, and $A B$ the third, and let their proportion be the proportion of excess, or the greater to less; then, as before, the proportions of $A D$ to $A C$, and of $A C$ to $A B$, and of $A D$ to $A B$, will be determined by the difference of their times; which in the description of $A D$ and $A C$, and of $A C$ and $A B$ together taken, is the same with the difference of the times in the description of $A D$ and $A B$. Wherefore the proportion of $A D$ to $A B$ is equal to the two proportions of $A D$ to $A C$ and of $A C$ to $A B$.

In the last place. If one of the proportions, namely of $A D$ to $A B$, be the proportion of excess, and another of them, as of $A B$ to $A C$ be the proportion of defect, thus also the proportion of $A D$ to $A C$ will be equal to the two proportions together taken of $A D$ to $A B$, and of $A B$ to $A C$. For the difference of the times in which $A D$ and $A B$

are described, is excess of time; for there goes more time to the description of AD than of AB ; and the difference of the times in which AB and AC are described, is defect of time, for less time goes to the description of AB than of AC ; but this excess and defect being added together, make $DB - BC$, which is equal to DC , by which the first AD exceeds the third AC ; and therefore the proportions of the first AD to the second AB , and of the second AB to the third AC , are determined by the same cause which determines the proportion of the first AD to the third AC . Wherefore, if any three magnitudes, &c.

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Coroll. I. If there be never so many magnitudes having proportion to one another, the proportion of the first to the last is compounded of the proportions of the first to the second, of the second to the third, and so on till you come to the last; or, the proportion of the first to the last is the same with the sum of all the intermediate proportions. For any number of magnitudes having proportion to one another, as A, B, C, D, E being propounded, the proportion of A to E , as is newly shown, is compounded of the proportions of A to D and of D to E ; and again, the proportion of A to D , of the proportions of A to C , and of C to D ; and lastly, the proportion of A to C , of the proportions of A to B , and of B to C .

Coroll. II. From hence it may be understood how any two proportions may be compounded. For if the proportions of A to B , and of C to D , be propounded to be added together, let B have to something else, as to E , the same proportion which C has to D , and let them be set in this order,

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A, B, E; for so the proportion of A to E will evidently be the sum of the two proportions of A to B, and of B to E, that is, of C to D. Or let it be as D to C, so A to something else, as to E, and let them be ordered thus, E, A, B; for the proportion of E to B will be compounded of the proportions of E to A, that is, of C to D, and of A to B. Also, it may be understood how one proportion may be taken out of another. For if the proportion of C to D be to be subtracted out of the proportion of A to B, let it be as C to D, so A to something else, as E, and setting them in this order, A, E, B, and taking away the proportion of A to E, that is, of C to D, there will remain the proportion of E to B.

Coroll. III. If there be two orders of magnitudes which have proportion to one another, and the several proportions of the first order be the same and equal in number with the proportions of the second order; then, whether the proportions in both orders be successively answerable to one another, which is called *ordinate proportion*, or not successively answerable, which is called *perturbed proportion*, the first and the last in both will be proportionals. For the proportion of the first to the last is equal to all the intermediate proportions; which being in both orders the same, and equal in number, the aggregates of those proportions will also be equal to one another; but to their aggregates, the proportions of the first to the last are equal; and therefore the proportion of the first to the last in one order, is the same with the proportion of the first to the last in the other order. Wherefore the first and the last in both are proportionals.

14. If any two quantities be made of the mutual multiplication of many quantities, which have proportion to one another, and the efficient quantities on both sides be equal in number, the proportion of the products will be compounded of the several proportions, which the efficient quantities have to one another.

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First, let the two products be AB and CD , whereof one is made of the multiplication of A into B , and the other of the multiplication of C into D . I say the proportion of AB to CD is compounded of the proportions of the efficient A to the efficient C , and of the efficient B to the efficient D . For let AB , CB and CD be set in order; and as B is to D , so let C be to another quantity as E ; and let A , C , E be set also in order. Then (by coroll. iv. of the 6th art.) it will be as AB the first quantity to CB

$AB.$	$A.$
$CB.$	$C.$
$CD.$	$E.$

the second quantity in the first order, so A to C in the second order; and again, as CB to CD in the first order, so B to D , that is, by construction, so C to E in the second order; and therefore (by the last corollary) $AB.CD :: A.E$ will be proportionals. But the proportion of A to E is compounded of the proportions of A to C , and of B to D ; wherefore also the proportion of AB to CD is compounded of the same.

Secondly, let the two products be ABF , and CDG , each of them made of three efficient, the first of A , B and F , and the second of C , D and G ; I say, the proportion of ABF to CDG is compounded of the proportions of A to C , of B to D , and of F to G . For let them be set in order as

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before; and as B is to D, so let C be to another quantity E; and again, as F is to G, so let E be to another, H; and let the first order stand thus, ABF, CBF, CDF and CDG; and the second order thus, A, C, E, H. Then the proportion of ABF to CBF in the first order, will be as A to C in

ABF.	A.
CBF.	C.
CDF.	E.
CDG.	H.

the second; and the proportion of CBF to CDF in the first order, as B to D, that is, as C to E (by construction) in the second order; and the proportion of CDF to CDG in the first, as F to G, that is, as E to H (by construction) in the second order; and therefore ABF. CDG :: A. H will be proportionals. But the proportion of A to H is compounded of the proportions of A to C, B to D, and F to G. Wherefore the proportion of the product ABF to CDG is also compounded of the same. And this operation serves, how many soever the efficient be that make the quantities given.

From hence ariseth another way of compounding many proportions into one, namely, that which is supposed in the 5th definition of the 6th book of Euclid; which is, by multiplying all the antecedents of the proportions into one another, and in like manner all the consequents into one another. And from hence also it is evident, in the first place, that the cause why parallelograms, which are made by the duction of two straight lines into one another, and all solids which are equal to figures so made, have their proportions compounded of the proportions of the efficient; and in the second place, why the multiplication of two or more fractions into one another is the same thing

with the composition of the proportions of their several numerators to their several denominators. For example, if these fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ be to be multiplied into one another, the numerators 1, 2, 3, are first to be multiplied into one another, which make 6; and next the denominators 2, 3, 4, which make 24; and these two products make the fraction $\frac{6}{24}$. In like manner, if the proportions of 1 to 2, of 2 to 3, and of 3 to 4, be to be compounded, by working as I have shown above, the same proportion of 6 to 24 will be produced.

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15. If any proportion be compounded with itself inverted, the compound will be the proportion of equality. For let any proportion be given, as of A to B, and let the inverse of it be that of C to D; and as C to D, so let B be to another quantity; for thus they will be compounded (by the second coroll. of the 12th art.) Now seeing the proportion of C to D is the inverse of the proportion of A to B, it will be as C to D, so B to A; and therefore if they be placed in order, A, B, A, the proportion compounded of the proportions of A to B, and of C to D, will be the proportion of A to A, that is, the proportion of equality. And from hence the cause is evident why two equal products have their efficient reciprocally proportional. For, for the making of two products equal, the proportions of their efficient must be such, as being compounded may make the proportion of equality, which cannot be except one be the inverse of the other; for if betwixt A and A any other quantity, as C, be interposed, their order will be A, C, A, and the later proportion of C to A will be the inverse of the former proportion of A to C.

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proportion.

16. A proportion is said to be multiplied by a number, when it is so often taken as there be unities in that number; and if the proportion be of the greater to the less, then shall also the quantity of the proportion be increased by the multiplication; but when the proportion is of the less to the greater, then as the number increaseth, the quantity of the proportion diminisheth; as in these three numbers, 4, 2, 1, the proportion of 4 to 1 is not only the duplicate of 4 to 2, but also twice as great; but inverting the order of those numbers thus, 1, 2, 4, the proportion of 1 to 2 is greater than that of 1 to 4; and therefore though the proportion of 1 to 4 be the duplicate of 1 to 2, yet it is not twice so great as that of 1 to 2, but contrarily the half of it. In like manner, a proportion is said to be divided, when between two quantities are interposed one or more means in continual proportion, and then the proportion of the first to the second is said to be subduplicate of that of the first to the third, and subtriplicate of that of the first to the fourth, &c.

This mixture of proportions, where some are proportions of excess, others of defect, as in a merchant's account of debtor and creditor, is not so easily reckoned as some think; but maketh the composition of proportions sometimes to be addition, sometimes subtraction; which soundeth absurdly to such as have always by composition understood addition, and by diminution subtraction. Therefore to make this account a little clearer, we are to consider (that which is commonly assumed, and truly) that if there be never so many quantities, the proportion of the first to

the last is compounded of the proportions of the first to the second, and of the second to the third, and so on to the last, without regarding their equality, excess, or defect; so that if two proportions, one of inequality, the other of equality, be added together, the proportion is not thereby made greater nor less; as for example, if the proportions of A to B and of B to B be compounded, the proportion of the first to the second is as much as the sum of both, because proportion of equality, being not quantity, neither augmenteth quantity nor lesseneth it. But if there be three quantities, A, B, C, unequal, and the first be the greatest, the last least, then the proportion of B to C is an addition to that of A to B, and makes it greater; and on the contrary, if A be the least, and C the greatest quantity, then doth the addition of the proportion of B to C make the compounded proportion of A to C less than the proportion of A to B, that is, the whole less than the part. The composition therefore of proportions is not in this case the augmentation of them, but the diminution; for the same quantity (Euclid v. 8) compared with two other quantities, hath a greater proportion to the lesser of them than to the greater. Likewise, when the proportions compounded are one of excess, the other of defect, if the first be of excess, as in these numbers, 8, 6, 9, the proportion compounded, namely, of 8 to 9, is less than the proportion of one of the parts of it, namely, of 8 to 6; but if the proportion of the first to the second be of defect, and that of the second to the third be of excess, as in these numbers, 6, 8, 4, then shall the proportion of the first to the third

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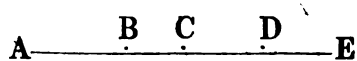
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be greater than that of the first to the second, as 6 hath a greater proportion to 4 than to 8; the reason whereof is manifestly this, that the less any quantity is deficient of another, or the more one exceedeth another, the proportion of it to that other is the greater.

Suppose now three quantities in continual proportion, A B 4, A C 6, A D 9. Because therefore A D is greater than A C, but not greater than A B, the proportion of A D to A C will be (by Euclid, v. 8) greater than that of A D to A B; and likewise, because the proportions of A D to A C, and of A C to A B are the same, the proportions of A D to A C and of A C to A B, being both proportions of excess, make the whole proportion of A D to A B, or of 9 to 4, not only the duplicate of A D to A C, that is, of 9 to 6, but also the double, or twice so great. On the other side, because the proportion of A D to A D, or 9 to 9, being proportion of equality, is no quantity, and yet greater than that of A C to A D, or 6 to 9, it will be as 0—9 to 0—6, so A C to A D, and again, as 0—9 to 0—6, so 0—6 to 0—4; but 0—4, 0—6, 0—9 are in continual proportion; and because 0—4 is greater than 0—6, the proportion of 0—4 to 0—6 will be double to the proportion of 0—4 to 0—9, double I say, and yet not duplicate, but subduplicate.

If any be unsatisfied with this ratiocination, let him first consider that (by Euclid v. 8) the proportion of A B to A C is greater than that of A B to A D, wheresoever D be placed in the line A C prolonged; and the further off the point



D is from C, so much the greater is the proportion of A B to A C than that of A B to A D. There is therefore some point (which suppose be E) in such distance from C, as that the proportion of A B to A C will be twice as great as that of A B to A E. That considered, let him determine the length of the line A E, and demonstrate, if he can, that A E is greater or less than A D.

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By the same method, if there be more quantities than three, as A, B, C, D, in continual proportion, and A be the least, it may be made appear that the proportion of A to B is triple magnitude, though subtriple in multitude, to the proportion of A to D.

17. If there be never so many quantities, the number whereof is odd, and their order such, that from the middlemost quantity both ways they proceed in continual proportion, the proportion of the two which are next on either side to the middlemost is subduplicate to the proportion of the two which are next to these on both sides, and subtriple of the proportion of the two which are yet one place more remote, &c. For let the magnitudes be C, B, A, D, E, and let A, B, C, as also A, D, E be in continual proportion; I say the proportion of D to B is subduplicate of the proportion of E to C. For the proportion of D to B is compounded of the proportions of D to A, and of A to B once taken; but the proportion of E to C is compounded of the same twice taken; and therefore the proportion of D to B is subduplicate of the proportion of E to C. And in the same manner, if there were three terms on either side, it might be demonstrated that the proportion of

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D to B would be subtriplicate of that of the extremes, &c.

18. If there be never so many continual proportionals, as the first, second, third, &c. their differences will be proportional to them. For the second, third, &c. are severally consequents of the preceding, and antecedents of the following proportion. But (by art. x.) the difference of the first antecedent and consequent, to difference of the second antecedent and consequent, is as the first antecedent to the second antecedent, that is, as the first term to the second, or as the second to the third, &c. in continual proportionals.

19. If there be three continual proportionals, the sum of the extremes, together with the mean twice taken, the sum of the mean and either of the extremes, and the same extreme, are continual proportionals. For let A. B. C be continual proportionals. Seeing, therefore, $A : B :: B : C$ are proportionals, by composition also $A + B : B :: B + C : C$ will be proportionals; and by permutation $A + B : B + C :: B : C$ will also be proportionals; and again, by composition $A + 2B + C : B + C :: B + C : C$; which was to be proved.

20. In four continual proportionals, the greatest and the least put together is a greater quantity than the other two put together. Let $A : B :: C : D$ be continual proportionals; whereof let the greatest be A, and the least be D; I say $A + D$ is greater than $B + C$. For by art. 10, $A - B : C - D :: A : C$ are proportionals; and therefore $A - B$ is, by art. 11, greater than $C - D$. Add B on both sides, and A will be greater than $C + B - D$. And again, add D on both sides, and $A + D$ will be greater than $B + C$; which was to be proved.

21. If there be four proportionals, the extremes multiplied into one another, and the means multiplied into one another, will make equal products. Let $A : B :: C : D$ be proportionals; I say $A D$ is equal to $B C$. For the proportion of $A D$ to $B C$ is compounded, by art. 13, of the proportions of A to B , and D to C , that is, its inverse B to A ; and therefore, by art. 14, this compounded proportion is the proportion of equality; and therefore also, the proportion of $A D$ to $B C$ is the proportion of equality. Wherefore they are equal.

The definition and properties of continual proportion.

22. If there be four quantities, and the proportion of the first to the second be duplicate of the proportion of the third to the fourth, the product of the extremes to the product of the means, will be as the third to the fourth. Let the four quantities be A, B, C and D ; and let the proportion of A to B be duplicate of the proportion of C to D , I say $A D$, that is, the product of A into D is to $B C$, that is, to the product of the means, as C to D . For seeing the proportion of A to B is duplicate of the proportion of C to D , if it be as C to D , so D to another, E , then $A : B :: C : E$ will be proportionals; for the proportion of A to B is by supposition duplicate of the proportion of C to D ; and C to E duplicate also of that of C to D by the definition, art. 15. Wherefore, by the last article, $A E$ or A into E is equal to $B C$ or B into C ; but, by coroll. iv. art. 6, $A D$ is to $A E$ as D to E , that is, as C to D ; and therefore $A D$ is to $B C$, which as I have shown is equal to $A E$, as C to D ; which was to be proved.

Moreover, if the proportion of the first A to the second B be triplicate of the proportion of

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the third C to the fourth D, the product of the extremes to the product of the means will be duplicate of the proportion of the third to the fourth. For if it be as C to D so D to E, and again, as D to E so E to another, F, then the proportion of C to F will be triplicate of the proportion of C to D; and consequently, $A. B :: C. F$ will be proportionals, and $A F$ equal to BC . But as $A D$ to $A F$, so is D to F ; and therefore, also, as $A D$ to BC , so D to F , that is, so C to E ; but the proportion of C to E is duplicate of the proportion of C to D ; wherefore, also, the proportion of $A D$ to BC is duplicate of that of C to D , as was propounded.

23. If there be four proportionals, and a mean be interposed betwixt the first and second, and another betwixt the third and fourth, the first of these means will be to the second, as the first of the proportionals is to the third, or as the second of them is to the fourth. For let $A. B :: C. D$ be proportionals, and let E be a mean betwixt A and B , and F a mean betwixt C and D ; I say $A. C :: E. F$ are proportionals. For the proportion of A to E is subduplicate of the proportion of A to B , or of C to D . Also, the proportion of C to F is subduplicate of that of C to D ; and therefore $A. E :: C. F$ are proportionals; and by permutation $A. C :: E. F$ are also proportionals; which was to be proved.

24. Any thing is said to be divided into extreme and mean proportion, when the whole and the parts are in continual proportion. As for example, when $A + B. A. B$ are continual proportionals; or when the straight line $A C$ is so divided in B , that

A C. A B. B C are in continual proportion. And if the same line A C be again divided in D, so as that A C. C D. A D be $\frac{A}{D} \mid \frac{B}{C}$ continual proportionals; then also

A C. A B. A D will be continual proportionals; and in like manner, though in contrary order, C A. C D. C B will be continual proportionals; which cannot happen in any line otherwise divided.

25. If there be three continual proportionals, and again, three other continual proportions, which have the same middle term, their extremes will be in reciprocal proportion. For let A. B. C and D. B. E be continual proportionals, I say A. D :: E. C shall be proportionals. For the proportion of A to D is compounded of the proportions of A to B, and of B to D; and the proportion of E to C is compounded of those of E to B, that is, of B to D, and of B to C, that is, of A to B. Wherefore, by equality, A. D :: E. C are proportionals.

26. If any two unequal quantities be made extremes, and there be interposed betwixt them any number of means in geometrical proportion, and the same number of means in arithmetical proportion, the several means in geometrical proportion will be less than the several means in arithmetical proportion. For betwixt A the lesser, and E the greater extreme, let there be interposed three means, B, C, D, in geometrical proportion, and as many more, F, G, H, in arithmetical proportion; I say B will be less than F, C than G, and D than H. For first, the difference between A and F is the same with that between F and G, and with that between G and H, by the definition of arithme-

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tical proportion ; and therefore, the difference of the proportionals which stand next to one another, to the difference of the extremes, is, when there is but one mean, half their difference ; when two, a third part of it ; when three, a quarter, &c. ; so that in this example it is a quarter. But the difference between D and E, by art. 17, is more than a quarter of the difference between the extremes, because the proportion is geometrical, and therefore the difference between A and D is less than three quarters of the same difference of the extremes. In

$$\begin{array}{cc} \frac{A}{B} & \frac{A}{F} \\ \frac{C}{D} & \frac{G}{H} \\ \hline E & E \end{array}$$

like manner, if the difference between A and D be understood to be divided into three equal parts, it may be proved, that the difference between A and C is less than two quarters of the difference of the extremes A and E. And lastly, if the difference between A and C be divided into two equal parts, that the difference between A and B is less than a quarter of the difference of the extremes A and E.

From the consideration hereof, it is manifest, that B, that is A together with something else which is less than a fourth part of the difference of the extremes A and E, is less than F, that is, than the same A with something else which is equal to the said fourth part. Also, that C, that is A with something else which is less than two fourth parts of the said difference, is less than G, that is, than A together with the said two-fourths. And lastly, that D, which exceeds A by less than three-fourths of the said difference, is less than H, which ex-

ceeds the same A by three entire fourths of the said difference. And in the same manner it would be if there were four means, saving that instead of fourths of the difference of the extremes we are to take fifth parts ; and so on.

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cal proportions:

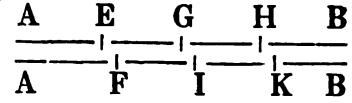
27. LEMMA. If a quantity being given, first one quantity be both added to it and subtracted from it, and then another greater or less, the proportion of the remainder to the aggregate, is greater where the less quantity is added and subtracted, than where the greater quantity is added and subtracted. Let B be added to and subtracted from the quantity A ; so that $A-B$ be the remainder, and $A+B$ the aggregate ; and again, let C, a greater quantity than B, be added to and subtracted from the same A, so that $A-C$ be the remainder and $A+C$ the aggregate ; I say $A-B. A+B :: A-C. A+C$ will be an hyperlogism. For $A-B. A :: A-C. A$ is an hyperlogism of a greater antecedent to the same consequent ; and therefore $A-B. A+B :: A-C. A+C$ is a much greater hyperlogism, being made of a greater antecedent to a less consequent.

28. If unequal parts be taken from two equal quantities, and betwixt the whole and the part of each there be interposed two means, one in geometrical, the other in arithmetical proportion ; the difference betwixt the two means will be greatest, where the difference betwixt the whole and its part is greatest. For let A B and A B be two equal quantities, from which let two unequal parts be taken, namely, A E the less, and A F the greater ; and betwixt A B and A E let A G be a mean in geometrical proportion, and A H a mean in arithme-

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tical proportion. Also

betwixt A B and A F let



A I be a mean in geo-
 metrical proportion, and

A K a mean in arithmetical proportion; I say
 H G is greater than K I.

For in the first place we have
 this analogism A B. A G :: B G. G E, by
 article 18.

Then by composition we have
 this A B + A G. A B :: B G + G E
 that is, B E. B G.

And by taking the halves of
 the antecedents this third . $\frac{1}{2}$ A B + $\frac{1}{2}$ A G. A B :: $\frac{1}{2}$ B G +
 $\frac{1}{2}$ G E, that is, B H. B G.

And by conversion a fourth . A B. $\frac{1}{2}$ A B + $\frac{1}{2}$ A G :: B G. B H.

And by division this fifth . $\frac{1}{2}$ A B - $\frac{1}{2}$ A G. $\frac{1}{2}$ A B + $\frac{1}{2}$ A G
 :: H G. B H.

And by doubling the first an-
 tecedent and the first con-
 sequent A B - A G. A B + A G :: H G. B H.

Also by the same method may
 be found out this analogism A B - A I. A B + A I :: K I. B K.

Now seeing the proportion of A B to A E is
 greater than that of A B to A F, the proportion of
 A B to A G, which is half the greater proportion,
 is greater than the proportion of A B to A I the
 half of the less proportion; and therefore A I is
 greater than A G. Wherefore the proportion of
 A B - A G to A B + A G, by the precedent lemma,
 will be greater than the proportion of A B - A I to
 A B + A I; and therefore also the proportion of
 H G to B H will be greater than that of K I to B K,
 and much greater than the proportion of K I to
 B H, which is greater than B K; for B H is the
 half of B E, as B K is the half of B F, which, by

supposition, is less than B E. Wherefore H G is greater than K I; which was to be proved.

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Coroll. It is manifest from hence, that if any quantity be supposed to be divided into equal parts infinite in number, the difference between the arithmetical and geometrical means will be infinitely little, that is, none at all. And upon this foundation, chiefly, the art of making those numbers, which are called Logarithms, seems to have been built.

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arithmetical
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cal proportions.

29. If any number of quantities be propounded, whether they be unequal, or equal to one another; and there be another quantity, which multiplied by the number of the propounded quantities, is equal to them all; that other quantity is a mean in arithmetical proportion to all those propounded quantities.

CHAP. XIV.

OF STRAIT AND CROOKED, ANGLE AND
FIGURE.

1. The definition and properties of a strait line.—2. The definition and properties of a plane superficies.—3. Several sorts of crooked lines.—4. The definition and properties of a circular line.—5. The properties of a strait line taken in a plane.
6. The definition of tangent lines.—7. The definition of an angle, and the kinds thereof.—8. In concentric circles, arches of the same angle are to one another, as the whole circumferences are.—9. The quantity of an angle, in what it consists.
10. The distinction of angles, simply so called.—11. Of strait lines from the centre of a circle to a tangent of the same.
12. The general definition of parallels, and the properties of strait parallels.—13. The circumferences of circles are to one another, as their diameters are.—14. In triangles, strait lines parallel to the bases are to one another, as the parts of the sides which they cut off from the vertex.—15. By what fraction of a strait line the circumference of a circle is made.
16. That an angle of contingence is quantity, but of a different kind from that of an angle simply so called; and that it can neither add nor take away any thing from the same.
17. That the inclination of planes is angle simply so called.
18. A solid angle what it is.—19. What is the nature of asymptotes.—20. Situation, by what it is determined.—21. What is like situation; what is figure; and what are like figures.

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The definition
and properties
of a strait line.

1. BETWEEN two points given, the shortest line is that, whose extreme points cannot be drawn farther asunder without altering the quantity, that is, without altering the proportion of that line to any other line given. For the magnitude of a line is computed by the greatest distance which may be

between its extreme points ; so that any one line, whether it be extended or bowed, has always one and the same length, because it can have but one greatest distance between its extreme points.

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And seeing the action, by which a strait line is made crooked, or contrarily a crooked line is made strait, is nothing but the bringing of its extreme points nearer to one another, or the setting of them further asunder, a *crooked line* may rightly be defined to be *that, whose extreme points may be understood to be drawn further asunder ;* and a *strait line* to be *that, whose extreme points cannot be drawn further asunder ;* and *comparatively, a more crooked, to be that line whose extreme points are nearer to one another than those of the other, supposing both the lines to be of equal length.* Now, howsoever a line be bowed, it makes always a *sinus* or cavity, sometimes on one side, sometimes on another ; so that the same crooked line may either have its whole cavity on one side only, or it may have it part on one side and part on the other side. Which being well understood, it will be easy to understand the following comparisons of strait and crooked lines.

First, if a strait and a crooked line have their extreme points common, the crooked line is longer than the strait line. For if the extreme points of the crooked line be drawn out to their greatest distance, it will be made a strait line, of which that, which was a strait line from the beginning, will be but a part ; and therefore the strait line was shorter than the crooked line, which had the same extreme points. And for the same reason,

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if two crooked lines have their extreme points common, and both of them have all their cavity on one and the same side, the outermost of the two will be the longest line.

Secondly, a strait line and a perpetually crooked line cannot be coincident, no, not in the least part. For if they should, then not only some strait line would have its extreme points common with some crooked line, but also they would, by reason of their coincidence, be equal to one another; which, as I have newly shown, cannot be.

Thirdly, between two points given, there can be understood but one strait line; because there cannot be more than one least interval or length between the same points. For if there may be two, they will either be coincident, and so both of them will be one strait line; or if they be not coincident, then the application of one to the other by extension will make the extended line have its extreme points at greater distance than the other; and consequently, it was crooked from the beginning.

Fourthly, from this last it follows, that two strait lines cannot include a superficies. For if they have both their extreme points common, they are coincident; and if they have but one or neither of them common, then at one or both ends the extreme points will be disjoined, and include no superficies, but leave all open and undetermined.

Fifthly, every part of a strait line is a strait line. For seeing every part of a strait line is the least that can be drawn between its own extreme points, if all the parts should not constitute a strait

line, they would altogether be longer than the whole line. PART II.
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2. A *plane* or a *plane superficies*, is that which is described by a *strait line* so moved, that all the several points thereof describe several *strait lines*. A *strait line*, therefore, is necessarily all of it in the same plane which it describes. Also the *strait lines*, which are made by the points that describe a plane, are all of them in the same plane. Moreover, if any line whatsoever be moved in a plane, the lines, which are described by it, are all of them in the same plane.

The definition and properties of a plane superficies.

All other superficies, which are not plane, are crooked, that is, are either concave or convex. And the same comparisons, which were made of *strait* and *crooked lines*, may also be made of *plane* and *crooked superficies*.

For, first, if a *plane* and *crooked superficies* be terminated with the same lines, the *crooked superficies* is greater than the *plane superficies*. For if the lines, of which the *crooked superficies* consists, be extended, they will be found to be longer than those of which the *plane superficies* consists, which cannot be extended, because they are *strait*.

Secondly, two superficies, whereof the one is *plane*, and the other continually *crooked*, cannot be coincident, no, not in the least part. For if they were coincident, they would be equal; nay, the same superficies would be both *plane* and *crooked*, which is impossible.

Thirdly, within the same terminating lines there can be no more than one *plane superficies*; because there can be but one least superficies within the same.

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Fourthly, no number of plane superficies can include a solid, unless more than two of them end in a common vertex. For if two planes have both the same terminating lines, they are coincident, that is, they are but one superficies; and if their terminating lines be not the same, they leave one or more sides open.

Fifthly, every part of a plane superficies is a plane superficies. For seeing the whole plane superficies is the least of all those, that have the same terminating lines; and also every part of the same superficies is the least of all those, that are terminated with the same lines; if every part should not constitute a plane superficies, all the parts put together would not be equal to the whole.

Several sorts of crooked lines.

3. Of straitness, whether it be in lines or in superficies, there is but one kind; but of crookedness there are many kinds; for of crooked magnitudes, some are congruous, that is, are coincident when they are applied to one other; others are incongruous. Again, some are *ὁμοιομερεῖς* or uniform, that is, have their parts, howsoever taken, congruous to one another; others are *ἀνομοιομερεῖς* or of several forms. Moreover, of such as are crooked, some are continually crooked, others have parts which are not crooked.

Definition and properties of a circular line.

4. If a strait line be moved in a plane, in such manner, that while one end of it stands still, the whole line be carried round about till it come again into the same place from whence it was first moved, it will describe a plane superficies, which will be terminated every way by that crooked line, which is made by that end of the strait line which

was carried round. Now this superficies is called a **CIRCLE**; and of this circle, the unmoved point is the *centre*; the crooked line which terminates it, the *perimeter*; and every part of that crooked line, a *circumference* or *arch*; the strait line, which generated the *circle*, is the *semidiameter* or *radius*; and any strait line, which passeth through the centre and is terminated on both sides in the circumference, is called the *diameter*. Moreover, every point of the radius, which describes the circle, describes in the same time its own perimeter, terminating its own circle, which is said to be *concentric* to all the other circles, because this and all those have one common centre.

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circular line.

Wherefore in every circle, all strait lines from the centre to the circumference are equal. For they are all coincident with the radius which generates the circle.

Also the diameter divides both the perimeter and the circle itself into two equal parts. For if those two parts be applied to one another, and the semiperimeters be coincident, then, seeing they have one common diameter, they will be equal; and the semicircles will be equal also; for these also will be coincident. But if the semiperimeters be not coincident, then some one strait line, which passes through the centre, which centre is in the diameter, will be cut by them in two points. Wherefore, seeing all the strait lines from the centre to the circumference are equal, a part of the same strait line will be equal to the whole; which is impossible.

For the same reason the perimeter of a circle

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The properties of a strait line taken in a plane.

will be uniform, that is, any one part of it will be coincident with any other equal part of the same.

5. From hence may be collected this property of a strait line, namely, that it is all contained in that plane which contains both its extreme points. For seeing both its extreme points are in the plane, that strait line, which describes the plane, will pass through them both; and if one of them be made a centre, and at the distance between both a circumference be described, whose radius is the strait line which describes the plane, that circumference will pass through the other point. Wherefore between the two propounded points, there is one strait line, by the definition of a circle, contained wholly in the propounded plane; and therefore if another strait line might be drawn between the same points, and yet not be contained in the same plane, it would follow, that between two points two strait lines may be drawn; which has been demonstrated to be impossible.

It may also be collected, that if two planes cut one another, their common section will be a strait line. For the two extreme points of the intersection are in both the intersecting planes; and between those points a strait line may be drawn; but a strait line between any two points is in the same plane, in which the points are; and seeing these are in both the planes, the strait line which connects them will also be in both the same planes, and therefore it is the common section of both. And every other line, that can be drawn between those points, will be either coincident with that line, that is, it will be the same line; or it will not

be coincident, and then it will be in neither, or but in one of those planes.

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As a strait line may be understood to be moved round about whilst one end thereof remains fixed, as the centre; so in like manner it is easy to understand, that a plane may be circumduced about a strait line, whilst the strait line remains still in one and the same place, as the *axis* of that motion. Now from hence it is manifest, that any three points are in some one plane. For as any two points, if they be connected by a strait line, are understood to be in the same plane in which the strait line is; so, if that plane be circumduced about the same strait line, it will in its revolution take in any third point, howsoever it be situate; and then the three points will be all in that plane; and consequently the three strait lines which connect those points, will also be in the same plane.

6. Two lines are said to *touch* one another, which being both drawn to one and the same point, will not cut one another, though they be produced, produced, I say, in the same manner in which they were generated. And therefore if two strait lines touch one another in any one point, they will be contiguous through their whole length. Also two lines continually crooked will do the same, if they be congruous and be applied to one another according to their congruity; otherwise, if they be incongruously applied, they will, as all other crooked lines, touch one another, where they touch, but in one point only. Which is manifest from this, that there can be no congruity between a strait line and a line that is continually crooked; for otherwise the same line might be both strait

Definition of
tangent lines.

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and crooked. Besides, when a strait line touches a crooked line, if the strait line be never so little moved about upon the point of contact, it will cut the crooked line; for seeing it touches it but in one point, if it incline any way, it will do more than touch it; that is, it will either be congruous to it, or it will cut it; but it cannot be congruous to it; and therefore it will cut it.

The definition
of an angle,
and the kinds
thereof.

7. An angle, according to the most general acceptation of the word, may be thus defined; *when two lines, or many superficieses, concur in one sole point, and diverge every where else, the quantity of that divergence is an ANGLE.* And an angle is of two sorts; for, first, it may be made by the concurrence of lines, and then it is a *superficial angle*; or by the concurrence of superficieses, and then it is called a *solid angle*.

Again, from the two ways by which two lines may diverge from one another, superficial angles are divided into two kinds. For two strait lines, which are applied to one another, and are contiguous in their whole length, may be separated or pulled open in such manner, that their concurrence in one point will still remain; and this separation or opening may be either by circular motion, the centre whereof is their point of concurrence, and the lines will still retain their straitness, the quantity of which separation or divergence is an *angle* simply so called; or they may be separated by continual flexion or curvation in every imaginable point; and the quantity of this separation is that, which is called an *angle of contingence*.

Besides, of superficial angles simply so called,

those, which are in a plane superficies, are plane ; and those, which are not plane, are denominated from the superficies in which they are.

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Lastly, those are *strait-lined angles*, which are made by strait lines ; as those which are made by crooked lines are *crooked-lined* ; and those which are made both of strait and crooked lines, are *mixed angles*.

8. Two arches intercepted between two radii of concentric circles, have the same proportion to one another, which their whole perimeters have to one another. For let the point A (in the first figure) be the centre of the two circles B C D and E F G, in which the radii A E B and A F C intercept the arches B C and E F ; I say the proportion of the arch B C to the arch E F is the same with that of the perimeter B C D to the perimeter E F G. For if the radius A F C be understood to be moved about the centre A with circular and uniform motion, that is, with equal swiftness everywhere, the point C will in a certain time describe the perimeter B C D, and in a part of that time the arch B C ; and because the velocities are equal by which both the arch and the whole perimeter are described, the proportion of the magnitude of the perimeter B C D to the magnitude of the arch B C is determined by nothing but the difference of the times in which the perimeter and the arch are described. But both the perimeters are described in one and the same time, and both the arches in one and the same time ; and therefore the proportions of the perimeter B C D to the arch B C, and of the perimeter E F G to the arch E F, are both determined by the same cause. Wherefore

In concentric circles, arches of the same angle are to one another, as the whole circumferences are.

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$BCD \cdot BC :: EFG \cdot EF$ are proportionals (by the 6th art. of the last chapter), and by permutation $BCD \cdot EFG :: BC \cdot EF$ will also be proportionals ; which was to be demonstrated.

The quantity
of an angle,
in what it
consists.

9. Nothing is contributed towards the quantity of an angle, neither by the length, nor by the equality, nor by the inequality of the lines which comprehend it. For the lines AB and AC comprehend the same angle which is comprehended by the lines AE and AF , or AB and AF . Nor is an angle either increased or diminished by the absolute quantity of the arch, which subtends the same ; for both the greater arch BC and the lesser arch EF are subtended to the same angle. But the quantity of an angle is estimated by the quantity of the subtending arch compared with the quantity of the whole perimeter. And therefore the quantity of an angle simply so called may be thus defined : *the quantity of an angle is an arch or circumference of a circle, determined by its proportion to the whole perimeter.* So that when an arch is intercepted between two strait lines drawn from the centre, look how great a portion that arch is of the whole perimeter, so great is the angle. From whence it may be understood, that when the lines which contain an angle are strait lines, the quantity of that angle may be taken at any distance from the centre. But if one or both of the containing lines be crooked, then the quantity of the angle is to be taken in the least distance from the centre, or from their concurrence ; for the least distance is to be considered as a strait line, seeing no crooked line can be imagined so little, but that there may be a less strait line. And

although the least strait line cannot be given, because the least given line may still be divided, yet we may come to a part so small, as is not at all considerable; which we call a point. And this point may be understood to be in a strait line which touches a crooked line; for an angle is generated by separating, by circular motion, one strait line from another which touches it, as has been said above in the 7th article. Wherefore an angle, which two crooked lines make, is the same with that which is made by two strait lines which touch them.

10. From hence it follows, that *vertical angles*, such as are ABC, DBF in the second figure, are equal to one another. For if, from the two semi-perimeters DAC, FDA, which are equal to one another, the common arch DA be taken away, the remaining arches AC, DF will be equal to one another.

The distinction of angles, simply so called.

Another distinction of angles is into *right* and *oblique*. A *right angle* is that, whose quantity is the fourth part of the perimeter. And the lines, which make a right angle, are said to be *perpendicular* to one another. Also, of *oblique angles*, that which is greater than a right, is called an *obtuse angle*; and that which is less, an *acute angle*. From whence it follows, that all the angles that can possibly be made at one and the same point, together taken, are equal to four right angles; because the quantities of them all put together make the whole perimeter. Also, that all the angles, which are made on one side of a strait line, from any one point taken in the same, are equal to two right angles; for if that point be

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made the centre, that strait line will be the diameter of a circle, by whose circumference the quantity of an angle is determined; and that diameter will divide the perimeter into two equal parts.

Of strait lines from the centre of a circle to a tangent of the same.

11. If a tangent be made the diameter of a circle, whose centre is the point of contact, a strait line drawn from the centre of the former circle to the centre of the latter circle, will make two angles with the tangent, that is, with the diameter of the latter circle, equal to two right angles, by the last article. And because, by the 6th article, the tangent has on both sides equal inclination to the circle, each of them will be a right angle; as also the semidiameter will be perpendicular to the same tangent. Moreover, the semidiameter, inasmuch as it is the semidiameter, is the least strait line which can be drawn from the centre to the tangent; and every other strait line, that reaches the tangent, will pass out of the circle, and will therefore be greater than the semidiameter. In like manner, of all the strait lines, which may be drawn from the centre to the tangent, that is the greatest which makes the greatest angle with the perpendicular; which will be manifest, if about the same centre another circle be described, whose semidiameter is a strait line taken nearer to the perpendicular, and there be drawn a perpendicular, that is, a tangent, to the same.

From whence it is also manifest, that if two strait lines, which make equal angles on either side of the perpendicular, be produced to the tangent, they will be equal.

12. There is in Euclid a definition of strait-lined parallels; but I do not find that parallels in general are anywhere defined; and therefore for an universal definition of them, I say that *any two lines whatsoever, strait or crooked, as also any two superficieses, are PARALLEL; when two equal strait lines, wheresoever they fall upon them, make always equal angles with each of them.*

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The general definition of parallels; the properties of strait parallels.

From which definition it follows; first, that any two strait lines, not inclined opposite ways, falling upon two other strait lines, which are parallel, and intercepting equal parts in both of them, are themselves also equal and parallel. As if A B and C D (in the third figure), inclined both the same way, fall upon the parallels A C and B D, and A C and B D be equal, A B and C D will also be equal and parallel. For the perpendiculars B E and D F being drawn, the right angles E B D and F D H will be equal. Wherefore, seeing E F and B D are parallel, the angles E B A and F D C will be equal. Now if D C be not equal to B A, let any other strait line equal to B A be drawn from the point D; which, seeing it cannot fall upon the point C, let it fall upon G. Wherefore A G will be either greater or less than B D; and therefore the angles E B A and F D C are not equal, as was supposed. Wherefore A B and C D are equal; which is the first.

Again, because they make equal angles with the perpendiculars B E and D F; therefore the angle C D H will be equal to the angle A B D, and, by the definition of parallels, A B and C D will be parallel; which is the second.

That plane, which is included both ways within parallel lines, is called a PARALLELOGRAM.

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General defini-
tion of pa-
rallels, &c.

Coroll. I. From this last it follows, that the angles $A B D$ and $C D H$ are equal, that is, that a strait line, as $B H$, falling upon two parallels, as $A B$ and $C D$, makes the internal angle $A B D$ equal to the external and opposite angle $C D H$.

Coroll. II. And from hence again it follows, that a strait line falling upon two parallels, makes the alternate angles equal, that is, the angle $A G F$, in the fourth figure, equal to the angle $G F D$. For seeing $G F D$ is equal to the external opposite angle $E G B$, it will be also equal to its vertical angle $A G F$, which is alternate to $G F D$.

Coroll. III. That the internal angles on the same side of the line $F G$ are equal to two right angles. For the angles at F , namely, $G F C$ and $G F D$, are equal to two right angles. But $G F D$ is equal to its alternate angle $A G F$. Wherefore both the angles $G F C$ and $A G F$, which are internal on the same side of the line $F G$, are equal to two right angles.

Coroll. IV. That the three angles of a strait-lined plain triangle are equal to two right angles; and any side being produced, the external angle will be equal to the two opposite internal angles. For if there be drawn by the vertex of the plain triangle $A B C$ (fig. 5) a parallel to any of the sides, as to $A B$, the angles A and B will be equal to their alternate angles E and F , and the angle C is common. But, by the 10th article, the three angles E , C and F , are equal to two right angles; and therefore the three angles of the triangle are equal to the same; which is the first. Again, the two angles B and D are equal to two right angles, by the 10th article. Wherefore taking

away B, there will remain the angles A and C, equal to the angle D; which is the second. PART II.
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Coroll. v. If the angles A and B be equal, the sides A C and C B will also be equal, because A B and E F are parallel; and, on the contrary, if the sides A C and C B be equal, the angles A and B will also be equal. For if they be not equal, let the angles B and G be equal. Wherefore, seeing G B and E F are parallels, and the angles G and B equal, the sides G C and C B will also be equal; and because C B and A C are equal by supposition, C G and C A will also be equal; which cannot be, by the 11th article. General definition of parallels, &c.

Coroll. vi. From hence it is manifest, that if two radii of a circle be connected by a strait line, the angles they make with that connecting line will be equal to one another; and if there be added that segment of the circle, which is subtended by the same line which connects the radii, then the angles, which those radii make with the circumference, will also be equal to one another. For a strait line, which subtends any arch, makes equal angles with the same; because, if the arch and the subtense be divided in the middle, the two halves of the segment will be congruous to one another, by reason of the uniformity both of the circumference of the circle, and of the strait line.

13. Perimeters of circles are to one another, as their semidiameters are. For let there be any two circles, as, in the first figure, B C D the greater, and E F G the lesser, having their common centre at A; and let their semidiameters be A C and A E. I say, A C has the same proportion to A E, which the perimeter B C D has to the perimeter E F G. The circumferences of circles are to one another as their diameters are.

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For the magnitude of the semidiameters $A C$ and $A E$ is determined by the distance of the points C and E from the centre A ; and the same distances are acquired by the uniform motion of a point from A to C , in such manner, that in equal times the distances acquired be equal. But the perimeters $B C D$ and $E F G$ are also determined by the same distances of the points C and E from the centre A ; and therefore the perimeters $B C D$ and $E F G$, as well as the semidiameters $A C$ and $A E$, have their magnitudes determined by the same cause, which cause makes, in equal times, equal spaces. Wherefore, by the 13th chapter and 6th article, the perimeters of circles and their semidiameters are proportionals; which was to be proved.

In triangles
strait lines pa-
rallel to the
bases are to one
another, as the
parts of the
sides which
they cut off
from the vertex.

14. If two strait lines, which constitute an angle, be cut by strait-lined parallels, the intercepted parallels will be to one another, as the parts which they cut off from the vertex. Let the strait lines $A B$ and $A C$, in the 6th figure, make an angle at A , and be cut by the two strait-lined parallels $B C$ and $D E$, so that the parts cut off from the vertex in either of those lines, as in $A B$, may be $A B$ and $A D$. I say, the parallels $B C$ and $D E$ are to one another, as the parts $A B$ and $A D$. For let $A B$ be divided into any number of equal parts, as into $A F$, $F D$, $D B$; and by the points F and D , let $F G$ and $D E$ be drawn parallel to the base $B C$, and cut $A C$ in G and E ; and again, by the points G and E , let other strait lines be drawn parallel to $A B$, and cut $B C$ in H and I . If now the point A be understood to be moved uniformly over $A B$, and in the same time B be moved to C , and all the

points F, D, and B be moved uniformly and with equal swiftness over F G, D E, and B C ; then shall B pass over B H, equal to F G, in the same time that A passes over A F ; and A F and F G will be to one another, as their velocities are ; and when A is in F, D will be in K ; when A is in D, D will be in E ; and in what manner the point A passes by the points F, D, and B, in the same manner the point B will pass by the points H, I, and C ; and the strait lines F G, D K, K E, B H, H I, and I C, are equal, by reason of their parallelism ; and therefore, as the velocity in A B is to the velocity in B C, so is A D to D E ; but as the velocity in A B is to the velocity in B C, so is A B to B C ; that is to say, all the parallels will be severally to all the parts cut off from the vertex, as A F is to F G. Wherefore, A F. G F :: A D. D E :: A B. B C are proportionals.

The subtenses of equal angles in different circles, as the strait lines B C and F E (in fig. 1), are to one another as the arches which they subtend. For (by art. 8) the arches of equal angles are to one another as their perimeters are ; and (by art. 13) the perimeters as their semidiameters ; but the subtenses B C and F E are parallel to one another by reason of the equality of the angles which they make with the semidiameters ; and therefore the same subtenses, by the last precedent article, will be proportional to the semidiameters, that is, to the perimeters, that is, to the arches which they subtend.

15. If in a circle any number of equal subtenses be placed immediately after one another, and strait lines be drawn from the extreme point of the first

By what fraction of a strait line the circumference of a circle is made.

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By what fraction of a strait line the circumference of a circle is made.

subtense to the extreme points of all the rest, the first subtense being produced will make with the second subtense an external angle double to that, which is made by the same first subtense, and a tangent to the circle touching it in the extreme points thereof; and if a strait line which subtends two of those arches be produced, it will make an external angle with the third subtense, triple to the angle which is made by the tangent with the first subtense; and so continually. For with the radius AB (in fig. 7) let a circle be described, and in it let any number of equal subtenses, BC , CD , and DE , be placed; also let BD and BE be drawn; and by producing BC , BD and BE to any distance in G , H and I , let them make angles with the subtenses which succeed one another, namely, the external angles GCD , and HDE . Lastly, let the tangent KB be drawn, making with the first subtense the angle KBC . I say the angle GCD is double to the angle KBC , and the angle HDE triple to the same angle KBC . For if AC be drawn cutting BD in M , and from the point C there be drawn LC perpendicular to the same AC , then CL and MD will be parallel, by reason of the right angles at C and M ; and therefore the alterne angles LCD and BDC will be equal: as also the angles BDC and CBD will be equal, because of the equality of the strait lines BC and CD . Wherefore the angle GCD is double to either of the angles CBD or CDB ; and therefore also the angle GCD is double to the angle LCD , that is, to the angle KBC . Again, CD is parallel to BE , by reason of the equality of the angles CBE and DEB , and of the strait lines

C B and D E; and therefore the angles G C D and G B E are equal; and consequently G B E, as also D E B is double to the angle K B C. But the external angle H D E is equal to the two internal D E B and D B E; and therefore the angle H D E is triple to the angle K B C, &c.; which was to be proved.

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By what fraction of a strait line the circumference of a circle is made.

Coroll. I. From hence it is manifest, that the angles K B C and C B D, as also, that all the angles that are comprehended by two strait lines meeting in the circumference of a circle and insisting upon equal arches, are equal to one another.

Coroll. II. If the tangent B K be moved in the circumference with uniform motion about the centre B, it will in equal times cut off equal arches; and will pass over the whole perimeter in the same time in which itself describes a semiperimeter about the centre B.

Coroll. III. From hence also we may understand, what it is that determines the bending or curvation of a strait line into the circumference of a circle; namely, that it is fraction continually increasing in the same manner, as numbers, from one upwards, increase by the continual addition of unity. For the indefinite strait line K B being broken in B according to any angle, as that of K B C, and again in C according to a double angle, and in D according to an angle which is triple, and in E according to an angle which is quadruple to the first angle, and so continually, there will be described a figure which will indeed be rectilinear, if the broken parts be considered as having magnitude; but if they be understood to be the least that can be, that is, as so many points, then

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the figure described will not be rectilinear, but a circle, whose circumference will be the broken line.

Coroll. IV. From what has been said in this present article, it may also be demonstrated, that an angle in the centre is double to an angle in the circumference of the same circle, if the intercepted arches be equal. For seeing that strait line, by whose motion an angle is determined, passes over equal arches in equal times, as well from the centre as from the circumference; and while that, which is from the circumference, is passing over half its own perimeter, it passes in the same time over the whole perimeter of that which is from the centre, the arches, which it cuts off in the perimeter whose centre is A, will be double to those, which it makes in its own semiperimeter, whose centre is B. But in equal circles, as arches are to one another, so also are angles.

It may also be demonstrated, that the external angle made by a subtense produced and the next equal subtense is equal to an angle from the centre insisting upon the same arch; as in the last diagram, the angle G C D is equal to the angle C A D; for the external angle G C D is double to the angle C B D; and the angle C A D insisting upon the same arch C D is also double to the same angle C B D or K B C.

That an angle of contingence is quantity, but of a different kind from that of an angle simply so called; and that it can neither add nor

16. An angle of contingence, if it be compared with an angle simply so called, how little soever, has such proportion to it as a point has to a line; that is, no proportion at all, nor any quantity. For first, an angle of contingence is made by continual flexion; so that in the generation of it there is no

circular motion at all, in which consists the nature of an angle simply so called; and therefore it cannot be compared with it according to quantity. Secondly, seeing the external angle made by a subtense produced and the next subtense is equal to an angle from the centre insisting upon the same arch, as in the last figure the angle GCD is equal to the angle CAD , the angle of contingence will be equal to that angle from the centre, which is made by AB and the same AB ; for no part of a tangent can subtend any arch; but as the point of contact is to be taken for the subtense, so the angle of contingence is to be accounted for the external angle, and equal to that angle whose arch is the same point B .

Now, seeing an angle in general is defined to be the opening or divergence of two lines, which concur in one sole point; and seeing one opening is greater than another, it cannot be denied, but that by the very generation of it, an angle of contingence is quantity; for wheresoever there is greater and less, there is also quantity; but this quantity consists in greater and less flexion; for how much the greater a circle is, so much the nearer comes the circumference of it to the nature of a strait line; for the circumference of a circle being made by the curvation of a strait line, the less that strait line is, the greater is the curvation; and therefore, when one strait line is a tangent to many circles, the angle of contingence, which it makes with a less circle, is greater than that which it makes with a greater circle.

Nothing therefore is added to or taken from an angle simply so called, by the addition to it or

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take away
any thing
from the same.

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taking from it of never so many angles of contingence. And as an angle of one sort can never be equal to an angle of the other sort, so they cannot be either greater or less than one another.

From whence it follows, that an angle of a segment, that is, the angle, which any strait line makes with any arch, is equal to the angle which is made by the same strait line, and another which touches the circle in the point of their concurrence; as in the last figure, the angle which is made between G B and B K is equal to that which is made between G B and the arch B C.

That the inclination of planes is angle simply so called.

17. An angle, which is made by two planes, is commonly called the inclination of those planes; and because planes have equal inclination in all their parts, instead of their inclination an angle is taken, which is made by two strait lines, one of which is in one, the other in the other of those planes, but both perpendicular to the common section.

A solid angle what it is.

18. A solid angle may be conceived two ways. First, for the aggregate of all the angles, which are made by the motion of a strait line, while one extreme point thereof remaining fixed, it is carried about any plain figure, in which the fixed point of the strait line is not contained. And in this sense, it seems to be understood by Euclid. Now it is manifest, that the quantity of a solid angle so conceived is no other, than the aggregate of all the angles in a superficies so described, that is, in the superficies of a pyramidal solid. Secondly, when a pyramis or cone has its vertex in the centre of a sphere, a solid angle may be understood to be the proportion of a spherical superficies subtending

that vertex to the whole superficies of the sphere. In which sense, solid angles are to one another as the spherical bases of solids, which have their vertex in the centre of the same sphere.

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19. All the ways, by which two lines respect one another, or all the variety of their position, may be comprehended under four kinds; for any two lines whatsoever are either parallels, or being produced, if need be, or moved one of them to the other parallelly to itself, they make an angle; or else, by the like production and motion, they touch one another; or lastly, they are *asymptotes*. The nature of parallels, angles, and tangents, has been already declared. It remains that I speak briefly of the nature of asymptotes.

What is the
nature of
asymptotes.

Asymptosis depends upon this, that quantity is infinitely divisible. And from hence it follows, that any line being given, and a body supposed to be moved from one extreme thereof towards the other, it is possible, by taking degrees of velocity always less and less, in such proportion as the parts of the line are made less by continual division, that the same body may be always moved forwards in that line, and yet never reach the end of it. For it is manifest, that if any strait line, as A F, (in the 8th figure) be cut anywhere in B, and again B F be cut in C, and C F in D, and D F in E, and so eternally, and there be drawn from the point F, the strait line F F at any angle A F F; and lastly, if the strait lines A F, B F, C F, D F, E F, &c., having the same proportion to one another with the segments of the line A F, be set in order and parallel to the same A F, the crooked line A B C D E, and the strait line F F, will be *asymptotes*, that is, they

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will always come nearer and nearer together, but never touch one another. Now, because any line may be cut eternally according to the proportions which the segments have to one another, therefore the divers kinds of asymptotes are infinite in number, and not necessary to be further spoken of in this place. In the nature of asymptotes in general there is no more, than that they come still nearer and nearer, but never touch. But in special in the asymptosy of hyperbolic lines, it is understood they should approach to a distance less than any given quantity.

Situation, by what it is determined.

20. SITUATION *is the relation of one place to another*; and where there are many places, their situation is determined by four things; by *their distances from one another*; by *several distances from a place assigned*; by *the order of strait lines drawn from a place assigned to the places of them all*; and by *the angles which are made by the lines so drawn*. For if their distances, order, and angles, be given, that is, be certainly known, their several places will also be so certainly known, as that they can be no other.

What is like situation; what is figure; and what are like figures.

21. Points, how many soever they be, have *like situation* with an equal number of other points, when all the strait lines, that are drawn from some one point to all these, have severally the same proportion to those, that are drawn in the same order and at equal angles from some one point to all those. For let there be any number of points as A, B, and C, (in the 9th figure) to which from some one point D let the strait lines D A, D B, and D C be drawn; and let there be an equal number of other points, as E, F, and G, and from some

point H let the strait lines H E, H F, and H G be drawn, so that the angles A D B and B D C be severally and in the same order equal to the angles E H F and F H G, and the strait lines D A, D B, and D C proportional to the strait lines H E, H F, and H G; I say, the three points A, B, and C, have like situation with the three points E, F, and G, or are placed alike. For if H E be understood to be laid upon D A, so that the point H be in D, the point F will be in the strait line D B, by reason of the equality of the angles A D B and E H F; and the point G will be in the strait line D C, by reason of the equality of the angles B D C and F H G; and the strait lines A B and E F, as also B C and F G, will be parallel, because $A D. E H :: B D. F H :: C D. G H$ are proportionals by construction; and therefore the distances between the points A and B, and the points B and C, will be proportional to the distances between the points E and F, and the points F and G. Wherefore, in the situation of the points A, B, and C, and the situation of the points E, F and G, the angles in the same order are equal; so that their situations differ in nothing but the inequality of their distances from one another, and of their distances from the points D and H. Now, in both the orders of points, those inequalities are equal; for $A B. B C :: E F. F G$, which are their distances from one another, as also $D A. D B. D C :: H E. H F. H G$, which are their distances from the assumed points D and H, are proportionals. Their difference, therefore, consists solely in the magnitude of their distances. But, by the definition of *like*, (chapter 1. article 2) those things, which differ only in magnitude, are like. Wherefore the points A, B, and C, have to

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What is like
situation; what
is figure; &c.

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What is like
situation; what
is figure, &c.

one another like situation with the points E, F, and G, or are placed alike; which was to be proved.

FIGURE is quantity, determined by the situation or placing of all its extreme points. Now I call those points extreme, which are contiguous to the place which is without the figure. In lines therefore and superficieses, all points may be called extreme; but in solids only those which are in the superficieses that includes them.

Like figures are those, whose extreme points in one of them are all placed like all the extreme points in the other; for such figures differ in nothing but magnitude.

And like figures are *alike placed*, when in both of them the homologal strait lines, that is, the strait lines which connect the points which answer one another, are parallel, and have their proportional sides inclined the same way.

And seeing every strait line is like every other strait line, and every plane like every other plane, when nothing but planeness is considered; if the lines, which include planes, or the superficieses, which include solids, have their proportions known, it will not be hard to know whether any figure be like or unlike to another propounded figure.

And thus much concerning the *first grounds of philosophy*. The next place belongs to *geometry*; in which the quantities of figures are sought out from the proportions of lines and angles. Wherefore it is necessary for him, that would study geometry, to know first what is the nature of quantity, proportion, angle and figure. Having therefore explained these in the three last chapters, I thought fit to add them to this part; and so pass to the next.

PART III.

PROPORTIONS OF MOTIONS AND MAGNITUDES.

CHAPTER XV.

OF THE NATURE, PROPERTIES, AND DIVERS CONSIDERATIONS OF MOTION AND ENDEAVOUR.

1. Repetition of some principles of the doctrine of motion formerly set down.—2. Other principles added to them.
3. Certain theorems concerning the nature of motion.—4. Divers considerations of motion.—5. The way by which the first endeavour of bodies moved tendeth.—6. In motion which is made by concourse, one of the movents ceasing, the endeavour is made by the way by which the rest tend.—7. All endeavour is propagated in infinitum.—8. How much greater the velocity or magnitude is of a movent, so much the greater is the efficacy thereof upon any other body in its way.

1. THE next things in order to be treated of are **PART III.**
MOTION and MAGNITUDE, which are the most **15.**
common accidents of all bodies. This place there-
fore most properly belongs to the elements of Repetition
of some prin-
ciples of the
doctrine of
motion for-
merly set down.
geometry. But because this part of philosophy,
having been improved by the best wits of all ages,
has afforded greater plenty of matter than can well

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Repetition
of some
principles, &c.

be thrust together within the narrow limits of this discourse, I thought fit to admonish the reader, that before he proceed further, he take into his hands the works of Euclid, Archimedes, Apollonius, and other as well ancient as modern writers. For to what end is it, to do over again that which is already done? The little therefore that I shall say concerning geometry in some of the following chapters, shall be such only as is new, and conducing to natural philosophy.

I have already delivered some of the principles of this doctrine in the eighth and ninth chapters; which I shall briefly put together here, that the reader in going on may have their light nearer at hand.

First, therefore, in chap. VIII. art. 10, *motion* is defined to be *the continual privation of one place, and acquisition of another.*

Secondly, it is there shown, that *whatsoever is moved is moved in time.*

Thirdly, in the same chapter, art. 11, I have defined *rest to be when a body remains for some time in one place.*

Fourthly, it is there shown, that *whatsoever is moved is not in any determined place*; as also that the same *has been moved, is still moved, and will yet be moved*; so that in every part of that space, in which motion is made, we may consider three times, namely, the *past*, the *present*, and the *future time.*

Fifthly, in art. 15 of the same chapter, I have defined *velocity or swiftness to be motion considered as power, namely, that power by which a body moved may in a certain time transmit a*

certain length; which also may more briefly be enunciated thus, *velocity is the quantity of motion determined by time and line.*

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Sixthly, in the same chapter, art. 16, I have shown that *motion is the measure of time.*

Seventhly, in the same chapter, art. 17, I have defined motions to be equally swift, when in equal times equal lengths are transmitted by them.

Eighthly, in art. 18 of the same chapter, *motions are defined to be equal, when the swiftness of one moved body, computed in every part of its magnitude, is equal to the swiftness of another, computed also in every part of its magnitude.* From whence it is to be noted, that *motions equal to one another, and motions equally swift*, do not signify the same thing; for when two horses draw abreast, the motion of both is greater than the motion of either of them singly; but the swiftness of both together is but equal to that of either.

Ninthly, in art. 19 of the same chapter, I have shown, that *whatsoever is at rest will always be at rest, unless there be some other body besides it, which by getting into its place suffers it no longer to remain at rest.* And that *whatsoever is moved, will always be moved, unless there be some other body besides it, which hinders its motion.*

Tenthly, in chap. IX. art. 7, I have demonstrated, that *when any body is moved which was formerly at rest, the immediate efficient cause of that motion is in some other moved and contiguous body.*

Eleventhly, I have shown in the same place, that *whatsoever is moved, will always be moved in the same way, and with the same swiftness, if it be*

PART III. *not hindered by some other moved and contiguous*
 15. *body.*

Other prin-
 ciples added
 to them.

2. To which principles I shall here add those that follow. First, I define ENDEAVOUR *to be motion made in less space and time than can be given; that is, less than can be determined or assigned by exposition or number; that is, motion made through the length of a point, and in an instant or point of time.* For the explaining of which definition it must be remembered, that by a point is not to be understood that which has no quantity, or which cannot by any means be divided; for there is no such thing in nature; but that, whose quantity is not at all considered, that is, whereof neither quantity nor any part is computed in demonstration; so that a point is not to be taken for an indivisible, but for an undivided thing; as also an instant is to be taken for an undivided, and not for an indivisible time.

In like manner, endeavour is to be conceived as motion; but so as that neither the quantity of the time in which, nor of the line in which it is made, may in demonstration be at all brought into comparison with the quantity of that time, or of that line of which it is a part. And yet, as a point may be compared with a point, so one endeavour may be compared with another endeavour, and one may be found to be greater or less than another. For if the vertical points of two angles be compared, they will be equal or unequal in the same proportion which the angles themselves have to one another. Or if a strait line cut many circumferences of concentric circles, the inequality of the points of intersection will be in the same propor-

tion which the perimeters have to one another. And in the same manner, if two motions begin and end both together, their endeavours will be equal or unequal, according to the proportion of their velocities ; as we see a bullet of lead descend with greater endeavour than a ball of wool.

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Other principles added to them.

Secondly, I define **IMPETUS**, or *quickness of motion*, to be the *swiftness or velocity of the body moved, but considered in the several points of that time in which it is moved. In which sense impetus is nothing else but the quantity or velocity of endeavour. But considered with the whole time, it is the whole velocity of the body moved taken together throughout all the time, and equal to the product of a line representing the time, multiplied into a line representing the arithmetically mean impetus or quickness. Which arithmetical mean, what it is, is defined in the 29th article of chapter XIII.*

And because in equal times the ways that are passed are as the velocities, and the *impetus* is the velocity they go withal, reckoned in all the several points of the times, it followeth that during any time whatsoever, howsoever the *impetus* be increased or decreased, the length of the way passed over shall be increased or decreased in the same proportion ; and the same line shall represent both the way of the body moved, and the several *impetus* or degrees of swiftness wherewith the way is passed over.

And if the body moved be not a point, but a strait line moved so as that every point thereof make a several strait line, the plane described by its motion, whether uniform, accelerated, or re-

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ciples added
to them.

tarded, shall be greater or less, the time being the same, in the same proportion with that of the *impetus* reckoned in one motion to the *impetus* reckoned in the other. For the reason is the same in parallelograms and their sides.

For the same cause also, if the body moved be a plane, the solid described shall be still greater or less in the proportions of the several *impetus* or quicknesses reckoned through one line, to the several *impetus* reckoned through another.

This understood, let $ABCD$, (in figure 1, chap. xvii.) be a parallelogram; in which suppose the side AB to be moved parallelly to the opposite side CD , decreasing all the way till it vanish in the point C , and so describing the figure $ABEFC$; the point B , as AB decreaseth, will therefore describe the line $BEFC$; and suppose the time of this motion designed by the line CD ; and in the same time CD , suppose the side AC to be moved parallel and uniformly to BD . From the point O taken at adventure in the line CD , draw OR parallel to BD , cutting the line $BEFC$ in E , and the side AB in R . And again, from the point Q taken also at adventure in the line CD , draw QS parallel to BD , cutting the line $BEFC$ in F , and the side AB in S ; and draw EG and FH parallel to CD , cutting AC in G and H . Lastly, suppose the same construction done in all the points possible of the line $BEFC$. I say, that as the proportions of the swiftness wherewith QF , OE , DB , and all the rest supposed to be drawn parallel to DB and terminated in the line $BEFC$, are to the proportions of their several times designed by the several parallels HF , GE , AB , and all the

rest supposed to be drawn parallel to the line of time CD and terminated in the line $BEFC$, the aggregate to the aggregate, so is the area or plane $DBEFC$ to the area or plane $ACFEB$. For as AB decreasing continually by the line $BEFC$ vanisheth in the time CD into the point C , so in the same time the line DC continually decreasing vanisheth by the same line $CFEB$ into the point B ; and the point D describeth in that decreasing motion the line DB equal to the line AC described by the point A in the decreasing motion of AB ; and their swiftnesses are therefore equal. Again, because in the time GE the point O describeth the line OE , and in the same time the point S describeth the line SE , the line OE shall be to the line SE , as the swiftness wherewith OE is described to the swiftness wherewith SE is described. In like manner, because in the same time HF the point Q describeth the line QF , and the point R the line RF , it shall be as the swiftness by which QF is described to the swiftness by which RF is described, so the line itself QF to the line itself RF ; and so in all the lines that can possibly be drawn parallel to BD in the points where they cut the line $BEFC$. But all the parallels to BD , as SE , RF , AC , and the rest that can possibly be drawn from the line AB to the line $BEFC$, make the area of the plane $ABEFC$; and all the parallels to the same BD , as QF , OE , DB and the rest drawn to the points where they cut the same line $BEFC$, make the area of the plane $BEFCD$. As therefore the aggregate of the swiftnesses wherewith the plane $BEFCD$ is described, is to the aggregate of the swiftnesses wherewith

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Other principles added to them.

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Other principles added to them.

the plane $A C F E B$ is described, so is the plane itself $B E F C D$ to the plane itself $A C F E B$. But the aggregate of the times represented by the parallels $A B$, $G E$, $H F$ and the rest, maketh also the area $A C F E B$. And therefore, as the aggregate of all the lines $Q F$, $O E$, $D B$ and all the rest of the lines parallel to $B D$ and terminated in the line $B E F C$, is to the aggregate of all the lines $H F$, $G E$, $A B$ and all the rest of the lines parallel to $C D$ and terminated in the same line $B E F C$; that is, as the aggregate of the lines of swiftness to the aggregate of the lines of time, or as the whole swiftness in the parallels to $D B$ to the whole time in the parallels to $C D$, so is the plane $B E F C D$ to the plane $A C F E B$. And the proportions of $Q F$ to $F H$, and of $O E$ to $E G$, and of $D B$ to $B A$, and so of all the rest taken together, are the proportions of the plane $D B E F C$ to the plane $A B E F C$. But the lines $Q F$, $O E$, $D B$ and the rest are the lines that design the swiftness; and the lines $H F$, $G E$, $A B$ and the rest are the lines that design the times of the motions; and therefore the proportion of the plane $D B E F C$ to the plane $A B E F C$ is the proportion of all the velocities taken together to all the times taken together. Wherefore, as the proportions of the swiftnesses, &c.; which was to be demonstrated.

The same holds also in the diminution of the circles, whereof the lines of time are the semidiameters, as may easily be conceived by imagining the whole plane $A B C D$ turned round upon the axis $B D$; for the line $B E F C$ will be everywhere in the superficies so made, and the lines $H F$, $G E$, $A B$, which are here parallelograms, will be there

cylinders, the diameters of whose bases are the lines H F, G E, A B, &c. and the altitude a point, that is to say, a quantity less than any quantity that can possibly be named; and the lines Q F, O E, D B, &c. small solids whose lengths and breadths are less than any quantity that can be named.

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But this is to be noted, that unless the proportion of the sum of the swiftesses to the proportion of the sum of the times be determined, the proportion of the figure DBEFC to the figure ABEFC cannot be determined.

Thirdly, I define **RESISTANCE** to be *the endeavour of one moved body either wholly or in part contrary to the endeavour of another moved body, which toucheth the same.* I say, wholly contrary, when the endeavour of two bodies proceeds in the same strait line from the opposite extremes, and contrary in part, when two bodies have their endeavour in two lines, which, proceeding from the extreme points of a strait line, meet without the same.

Fourthly, that I may define what it is to **PRESS**, I say, that *of two moved bodies one presses the other, when with its endeavour it makes either all or part of the other body to go out of its place.*

Fifthly, a body, which is pressed and not wholly removed, is said to **RESTORE** itself, when, *the pressing body being taken away, the parts which were moved do, by reason of the internal constitution of the pressed body, return every one into its own place.* And this we may observe in springs, in blown bladders, and in many other bodies, whose parts yield more or less to the endeavour which the pressing body makes at the

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first arrival; but afterwards, when the pressing body is removed, they do, by some force within them, *restore* themselves, and give their whole body the same figure it had before.

Sixthly, I define FORCE *to be the impetus or quickness of motion multiplied either into itself, or into the magnitude of the movent, by means whereof the said movent works more or less upon the body that resists it.*

Certain theorems concerning the nature of motion.

3. Having premised thus much, I shall now demonstrate, first, that if a point moved come to touch another point which is at rest, how little soever the impetus or quickness of its motion be, it shall move that other point. For if by that impetus it do not at all move it out of its place, neither shall it move it with double the same impetus. For nothing doubled is still nothing; and for the same reason it shall never move it with that impetus, how many times soever it be multiplied, because nothing, however it be multiplied, will for ever be nothing. Wherefore, when a point is at rest, if it do not yield to the least impetus, it will yield to none; and consequently it will be impossible that that, which is at rest, should ever be moved.

Secondly, that when a point moved, how little soever the impetus thereof be, falls upon a point of any body at rest, how hard soever that body be, it will at the first touch make it yield a little. For if it do not yield to the impetus which is in that point, neither will it yield to the impetus of never so many points, which have all their impetus severally equal to the impetus of that point. For seeing all those points together work equally, if any one

of them have no effect, the aggregate of them all together shall have no effect as many times told as there are points in the whole body, that is, still no effect at all; and by consequent there would be some bodies so hard that it would be impossible to break them; that is, a finite hardness, or a finite force, would not yield to that which is infinite; which is absurd.

Coroll. It is therefore manifest, that rest does nothing at all, nor is of any efficacy; and that nothing but motion gives motion to such things as be at rest, and takes it from things moved.

Thirdly, that cessation in the movent does not cause cessation in that which was moved by it. For (by number 11 of art. 1 of this chapter) whatsoever is moved perseveres in the same way and with the same swiftness, as long as it is not hindered by something that is moved against it. Now it is manifest, that cessation is not contrary motion; and therefore it follows that the standing still of the movent does not make it necessary that the thing moved should also stand still.

Coroll. They are therefore deceived, that reckon the taking away of the impediment or resistance for one of the causes of motion.

4. Motion is brought into account for divers respects; first, as in a body *undivided*, that is, considered as a point; or, as in a *divided* body. In an undivided body, when we suppose the way, by which the motion is made, to be a line; and in a divided body, when we compute the motion of the several parts of that body, as of parts.

Divers
considerations
of motions.

Secondly, from the diversity of the regulation of motion, it is in body, considered as undivided.

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Divers
considerations
of motion.

sometimes *uniform* and sometimes *multiform*. *Uniform* is that by which equal lines are always transmitted in equal times ; and *multiform*, when in one time more, in another time less space is transmitted. Again, of multiform motions, there are some in which the degrees of acceleration and retardation proceed in the same proportions, which the spaces transmitted have, whether duplicate, or triplicate, or by whatsoever number multiplied ; and others in which it is otherwise.

Thirdly, from the number of the movents ; that is, one motion is made by one movent only, and another by the concurrence of many movents.

Fourthly, from the position of that line in which a body is moved, in respect of some other line ; and from hence one motion is called *perpendicular*, another *oblique*, another *parallel*.

Fifthly, from the position of the movent in respect of the moved body ; from whence one motion is *pulsion* or driving, another *traction* or drawing. *Pulsion*, when the movent makes the moved body go before it ; and *traction*, when it makes it follow. Again, there are two sorts of *pulsion* ; one, when the motions of the movent and moved body begin both together, which may be called *trusion* or *thrusting* and *vection* ; the other, when the movent is first moved, and afterwards the moved body, which motion is called *percussion* or *stroke*.

Sixthly, motion is considered sometimes from the effect only which the movent works in the moved body, which is usually called *moment*. Now *moment is the excess of motion which the movent has above the motion or endeavour of the resisting body*.

Seventhly, it may be considered from the diversity of the *medium*; as one motion may be made in *vacuity* or *empty place*; another in a *fluid*; another in a *consistent medium*, that is, a *medium* whose parts are by some power so *consistent* and *cohering*, that no part of the same will yield to the movent, unless the whole yield also.

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Eighthly, when a moved body is considered as having parts, there arises another distinction of motion into *simple* and *compound*. *Simple*, when all the several parts describe several equal lines; *compounded*, when the lines described are unequal.

5. All endeavour tends towards that part, that is to say, in that way which is determined by the motion of the movent, if the movent be but one; or, if there be many movents, in that way which their concurrence determines. For example, if a moved body have direct motion, its first endeavour will be in a strait line; if it have circular motion, its first endeavour will be in the circumference of a circle.

The way by which the first endeavour of bodies moved tendeth.

6. And whatsoever the line be, in which a body has its motion from the concurrence of two movents, as soon as in any point thereof the force of one of the movents ceases, there immediately the former endeavour of that body will be changed into an endeavour in the line of the other movent.

In motion, which is made by concurrence, one of the movents ceasing, the endeavour is made by the way by which the rest tend.

Wherefore, when any body is carried on by the concurrence of two winds, one of those winds ceasing, the endeavour and motion of that body will be in that line, in which it would have been carried by that wind alone which blows still. And in the describing of a circle, where that which is moved has its motion determined by a movent in a

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tangent, and by the radius which keeps it in a certain distance from the centre, if the retention of the radius cease, that endeavour, which was in the circumference of the circle, will now be in the tangent, that is, in a strait line. For, seeing endeavour is computed in a less part of the circumference than can be given, that is, in a point, the way by which a body is moved in the circumference is compounded of innumerable strait lines, of which every one is less than can be given; which are therefore called points. Wherefore when any body, which is moved in the circumference of a circle, is freed from the retention of the radius, it will proceed in one of those strait lines, that is, in a tangent.

All endeavour
is propagated
in infinitum.

7. All endeavour, whether strong or weak, is propagated to infinite distance; for it is motion. If therefore the first endeavour of a body be made in space which is empty, it will always proceed with the same velocity; for it cannot be supposed that it can receive any resistance at all from empty space; and therefore, (by art. 7, chap. ix) it will always proceed in the same way and with the same swiftness. And if its endeavour be in space which is filled, yet, seeing endeavour is motion, that which stands next in its way shall be removed, and endeavour further, and again remove that which stands next, and so infinitely. Wherefore the propagation of endeavour, from one part of full space to another, proceeds infinitely. Besides, it reaches in any instant to any distance, how great soever. For in the same instant in which the first part of the full *medium* removes that which is next it, the second also removes that part which is next

to it ; and therefore all endeavour, whether it be in empty or in full space, proceeds not only to any distance, how great soever, but also in any time, how little soever, that is, in an instant. Nor makes it any matter, that endeavour, by proceeding, grows weaker and weaker, till at last it can no longer be perceived by sense ; for motion may be insensible ; and I do not here examine things by sense and experience, but by reason.

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8. When two movents are of equal magnitude, the swifter of them works with greater force than the slower, upon a body that resists their motion. Also, if two movents have equal velocity, the greater of them works with more force than the less. For where the magnitude is equal, the movent of greater velocity makes the greater impression upon that body upon which it falls ; and where the velocity is equal, the movent of greater magnitude falling upon the same point, or an equal part of another body, loses less of its velocity, because the resisting body works only upon that part of the movent which it touches, and therefore abates the impetus of that part only ; whereas in the mean time the parts, which are not touched, proceed, and retain their whole force, till they also come to be touched ; and their force has some effect. Wherefore, for example, in batteries a longer than a shorter piece of timber of the same thickness and velocity, and a thicker than a slenderer piece of the same length and velocity, work a greater effect upon the wall.

How much greater the velocity or magnitude is of a movent, so much the greater is the efficacy thereof upon any other body in its way.

CHAPTER XVI.

OF MOTION ACCELERATED AND UNIFORM, AND
OF MOTION BY CONCOURSE.

1. The velocity of any body, in what time soever it be computed, is that which is made of the multiplication of the impetus, or quickness of its motion into the time.—2-5. In all motion, the lengths which are passed through are to one another, as the products made by the impetus multiplied into the time.—6. If two bodies be moved with uniform motion through two lengths, the proportion of those lengths to one another will be compounded of the proportions of time to time, and impetus to impetus, directly taken.—7. If two bodies pass through two lengths with uniform motion, the proportion of their times to one another will be compounded of the proportions of length to length, and impetus to impetus reciprocally taken; also the proportion of their impetus to one another will be compounded of the proportions of length to length, and time to time reciprocally taken.—8. If a body be carried on with uniform motion by two movents together, which meet in an angle, the line by which it passes will be a strait line, subtending the complement of that angle to two right angles.—9, &c. If a body be carried by two movents together, one of them being moved with uniform, the other with accelerated motion, and the proportion of their lengths to their times being explicable in numbers, how to find out what line that body describes.

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The velocity of any body, in what time soever it be computed, is that which is made of the multiplication of the impetus or quickness

1. THE velocity of any body, in whatsoever time it be moved, has its quantity determined by the sum of all the several quicknesses or impetus, which it hath in the several points of the time of the body's motion. For seeing velocity, (by the definition of it, chap. VIII, art. 15) is that power by which a body can in a certain time pass through a certain length; and quickness of motion or impetus, (by

chap. xv, art. 2, num. 2) is velocity taken in one point of time only, all the impetus, together taken in all the points of time, will be the same thing with the mean impetus multiplied into the whole time, or which is all one, will be the velocity of the whole motion.

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of its motion
into the time.

Coroll. If the impetus be the same in every point, any strait line representing it may be taken for the measure of time : and the quicknesses or impetus applied ordinally to any strait line making an angle with it, and representing the way of the body's motion, will design a parallelogram which shall represent the velocity of the whole motion. But if the impetus or quickness of motion begin from rest and increase uniformly, that is, in the same proportion continually with the times which are passed, the whole velocity of the motion shall be represented by a triangle, one side whereof is the whole time, and the other the greatest impetus acquired in that time ; or else by a parallelogram, one of whose sides is the whole time of motion, and the other, half the greatest impetus ; or lastly, by a parallelogram having for one side a mean proportional between the whole time and the half of that time, and for the other side the half of the greatest impetus. For both these parallelograms are equal to one another, and severally equal to the triangle which is made of the whole line of time, and of the greatest acquired impetus ; as is demonstrated in the elements of geometry.

2. In all uniform motions the lengths which are transmitted are to one another, as the product of the mean impetus multiplied into its time, to the

In all motion,
the lengths
which are pass-
ed through are

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to one another,
as the products
made by the
impetus multi-
plied into time.

product of the mean impetus multiplied also into its time.

For let AB (in fig. 1) be the time, and AC the impetus by which any body passes with uniform motion through the length DE ; and in any part of the time AB , as in the time AF , let another body be moved with uniform motion, first, with the same impetus AC . This body, therefore, in the time AF with the impetus AC will pass through the length AF . Seeing, therefore, when bodies are moved in the same time, and with the same velocity and impetus in every part of their motion, the proportion of one length transmitted to another length transmitted, is the same with that of time to time, it followeth, that the length transmitted in the time AB with the impetus AC will be to the length transmitted in the time AF with the same impetus AC , as AB itself is to AF , that is, as the parallelogram AI is to the parallelogram AH , that is, as the product of the time AB into the mean impetus AC is to the product of the time AF into the same impetus AC . Again, let it be supposed that a body be moved in the time AF , not with the same but with some other uniform impetus, as AL . Seeing therefore, one of the bodies has in all the parts of its motion the impetus AC , and the other in like manner the impetus AL , the length transmitted by the body moved with the impetus AC will be to the length transmitted by the body moved with the impetus AL , as AC itself is to AL , that is, as the parallelogram AH is to the parallelogram FL . Wherefore, by ordinate proportion it will be, as the parallelogram AI to the parallelogram FL , that is, as the pro-

duct of the mean impetus into the time is to the product of the mean impetus into the time, so the length transmitted in the time A B with the impetus A C, to the length transmitted in the time A F with the impetus A L; which was to be demonstrated.

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In all motion,
the lengths, &c.

Coroll. Seeing, therefore, in uniform motion, as has been shown, the lengths transmitted are to one another as the parallelograms which are made by the multiplication of the mean impetus into the times, that is, by reason of the equality of the impetus all the way, as the times themselves, it will also be, by permutation, as time to length, so time to length; and in general, to this place are applicable all the properties and transmutations of analogisms, which I have set down and demonstrated in chapter XIII.

3. In motion begun from rest and uniformly accelerated, that is, where the impetus increaseth continually according to the proportion of the times, it will also be, as one product made by the mean impetus multiplied into the time, to another product made likewise by the mean impetus multiplied into the time, so the length transmitted in the one time to the length transmitted in the other time.

For let A B (in fig. 1) represent a time; in the beginning of which time A, let the impetus be as the point A; but as the time goes on, so let the impetus increase uniformly, till in the last point of that time A B, namely in B, the impetus acquired be B I. Again, let A F represent another time, in whose beginning A, let the impetus be as the point itself A; but as the time proceeds, so let the im-

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In all motion,
the lengths, &c.

petus increase uniformly, till in the last point F of the time A F the impetus acquired be F K; and let D E be the length passed through in the time A B with impetus uniformly increased. I say, the length D E is to the length transmitted in the time A F, as the time A B multiplied into the mean of the impetus increasing through the time A B, is to the time A F multiplied into the mean of the impetus increasing through the time A F.

For seeing the triangle A B I is the whole velocity of the body moved in the time A B, till the impetus acquired be B I; and the triangle A F K the whole velocity of the body moved in the time A F with impetus increasing till there be acquired the impetus F K; the length D E to the length acquired in the time A F with impetus increasing from rest in A till there be acquired the impetus F K, will be as the triangle A B I to the triangle A F K, that is, if the triangles A B I and A F K be like, in duplicate proportion of the time A B to the time A F; but if unlike, in the proportion compounded of the proportions of A B to A F and of B I to F K. Wherefore, as A B I is to A F K, so let D E be to D P; for so, the length transmitted in the time A B with impetus increasing to B I, will be to the length transmitted in the time A F with impetus increasing to F K, as the triangle A B I is to the triangle A F K; but the triangle A B I is made by the multiplication of the time A B into the mean of the impetus increasing to B I; and the triangle A F K is made by the multiplication of the time A F into the mean of the *impetus* increasing to F K; and therefore the length D E which is transmitted in the time A B

with impetus increasing to B I, to the length D P which is transmitted in the time A F with impetus increasing to F K, is as the product which is made of the time A B multiplied into its mean impetus, to the product of the time A F multiplied also into its mean impetus ; which was to be proved.

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In all motion,
the lengths, &c.

Coroll. I. In motion uniformly accelerated, the proportion of the lengths transmitted to that of their times, is compounded of the proportions of their times to their times, and impetus to impetus.

Coroll. II. In motion uniformly accelerated, the lengths transmitted in equal times, taken in continual succession from the beginning of motion, are as the differences of square numbers beginning from unity, namely, as 3, 5, 7, &c. For if in the first time the length transmitted be as 1, in the first and second times the length transmitted will be as 4, which is the square of 2, and in the three first times it will be as 9, which is the square of 3, and in the four first times as 16, and so on. Now the differences of these squares are 3, 5, 7, &c.

Coroll. III. In motion uniformly accelerated from rest, the length transmitted is to another length transmitted uniformly in the same time, but with such impetus as was acquired by the accelerated motion in the last point of that time, as a triangle to a parallelogram, which have their altitude and base common. For seeing the length D E (in fig. 1) is passed through with velocity as the triangle A B I, it is necessary that for the passing through of a length which is double to D E, the velocity be as the parallelogram A I ; for the parallelogram A I is double to the triangle A B I.

4. In motion, which beginning from rest is so ac-

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lerated, that the impetus thereof increases continually in proportion duplicate to the proportion of the times in which it is made, a length transmitted in one time will be to a length transmitted in another time, as the product made by the mean impetus multiplied into the time of one of those motions, to the product of the mean impetus multiplied into the time of the other motion.

For let AB (in fig. 2) represent a time, in whose first instant A let the impetus be as the point A ; but as the time proceeds, so let the impetus increase continually in duplicate proportion to that of the times, till in the last point of time B the impetus acquired be BI ; then taking the point F anywhere in the time AB , let the impetus FK acquired in the time AF be ordinally applied to that point F . Seeing therefore the proportion of FK to BI is supposed to be duplicate to that of AF to AB , the proportion of AF to AB will be subduplicate to that of FK to BI ; and that of AB to AF will be (by chap. XIII. art 16) duplicate to that of BI to FK ; and consequently the point K will be in a parabolical line, whose diameter is AB and base BI ; and for the same reason, to what point soever of the time AB the impetus acquired in that time be ordinally applied, the strait line designing that impetus will be in the same parabolical line AKI . Wherefore the mean impetus multiplied into the whole time AB will be the parabola $AKIB$, equal to the parallelogram AM , which parallelogram has for one side the line of time AB and for the other the line of the impetus AL , which is two-thirds of the impetus BI ; for every parabola is equal to two-

thirds of that parallelogram with which it has its altitude and base common. Wherefore the whole velocity in AB will be the parallelogram AM , as being made by the multiplication of the impetus AL into the time AB . And in like manner, if FN be taken, which is two-thirds of the impetus FK , and the parallelogram FO be completed, FO will be the whole velocity in the time AF , as being made by the uniform impetus AO or FN multiplied into the time AF . Let now the length transmitted in the time AB and with the velocity AM be the strait line DE ; and lastly, let the length transmitted in the time AF with the velocity AN be DP ; I say that as AM is to AN , or as the parabola $AKIB$ to the parabola AKF , so is DE to DP . For as AM is to FL , that is, as AB is to AF , so let DE be to DG . Now the proportion of AM to AN is compounded of the proportions of AM to FL , and of FL to AN . But as AM to FL , so by construction is DE to DG ; and as FL is to AN (seeing the time in both is the same, namely, AF), so is the length DG to the length DP ; for lengths transmitted in the same time are to one another as their velocities are. Wherefore by ordinate proportion, as AM is to AN , that is, as the mean impetus AL multiplied into its time AB , is to the mean impetus AO multiplied into AF , so is DE to DP ; which was to be proved.

Coroll. 1. Lengths transmitted with motion so accelerated, that the impetus increase continually in duplicate proportion to that of their times, if the base represent the impetus, are in triplicate proportion of their impetus acquired in the last

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the lengths, &c.

point of their times. For as the length DE is to the length DP, so is the parallelogram AM to the parallelogram AN, and so the parabola AKIB to the parabola AKF. But the proportion of the parabola AKIB to the parabola AKF is triplicate to the proportion which the base BI has to the base FK. Wherefore also the proportion of DE to DP is triplicate to that of BI to FK.

Coroll. II. Lengths transmitted in equal times succeeding one another from the beginning, by motion so accelerated, that the proportion of the impetus be duplicate to the proportion of the times, are to one another as the differences of cubic numbers beginning at unity, that is as 7, 19, 37, &c. For if in the first time the length transmitted be as 1, the length at the end of the second time will be as 8, at the end of the third time as 27, and at the end of the fourth time as 64, &c.; which are cubic numbers, whose differences are 7, 19, 37, &c.

Coroll. III. In motion so accelerated, as that the length transmitted be always to the length transmitted in duplicate proportion to their times, the length uniformly transmitted in the whole time, and with impetus all the way equal to that which is last acquired, is as a parabola to a parallelogram of the same altitude and base, that is, as 2 to 3. For the parabola AKIB is the impetus increasing in the time AB; and the parallelogram AI is the greatest uniform impetus multiplied into the same time AB. Wherefore the lengths transmitted will be as a parabola to a parallelogram, &c., that is, as 2 to 3.

5. If I should proceed to the explication of such motions as are made by impetus increasing in pro-

portion triplicate, quadruplicate, quintuplicate, &c., to that of their times, it would be a labour infinite and unnecessary. For by the same method by which I have computed such lengths, as are transmitted with impetus increasing in single and duplicate proportion, any man may compute such as are transmitted with impetus increasing in triplicate, quadruplicate, or what other proportion he pleases.

In making which computation he shall find, that where the impetus increase in proportion triplicate to that of the times, there the whole velocity will be designed by the first parabolaster (of which see the next chapter); and the lengths transmitted will be in proportion quadruplicate to that of the times. And in like manner, where the impetus increase in quadruplicate proportion to that of the times, that there the whole velocity will be designed by the second parabolaster, and the lengths transmitted will be in quintuplicate proportion to that of the times; and so on continually.

6. If two bodies with uniform motion transmit two lengths, each with its own impetus and time, the proportion of the lengths transmitted will be compounded of the proportions of time to time, and impetus to impetus, directly taken.

Let two bodies be moved uniformly (as in fig. 3), one in the time A B with the impetus A C, the other in the time A D with the impetus A E. I say the lengths transmitted have their proportion to one another compounded of the proportions of A B to A D, and of A C to A E. For let any length whatsoever, as Z, be transmitted by one of the bodies in the time A B with the impetus A C; and any other length, as X, be transmitted by the

If two bodies be moved with uniform motion through two lengths, the proportion of those lengths to one another, will be compounded of the proportions of time to time, and impetus to impetus, directly taken.

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other body in the time AD with the impetus AE ; and let the parallelograms AF and AG be completed. Seeing now Z is to X (by art. 2) as the impetus AC multiplied into the time AB is to the impetus AE multiplied into the time AD , that is, as AF to AG ; the proportion of Z to X will be compounded of the same proportions, of which the proportion of AF to AG is compounded; but the proportion of AF to AG is compounded of the proportions of the side AB to the side AD , and of the side AC to the side AE (as is evident by the Elements of Euclid), that is, of the proportions of the time AB to the time AD , and of the impetus AC to the impetus AE . Wherefore also the proportion of Z to X is compounded of the same proportions of the time AB to the time AD , and of the impetus AC to the impetus AE ; which was to be demonstrated.

Coroll. I. When two bodies are moved with uniform motion, if the times and impetus be in reciprocal proportion, the lengths transmitted shall be equal. For if it were as AB to AD (in the same fig. 3) so reciprocally AE to AC , the proportion of AF to AG would be compounded of the proportions of AB to AD , and of AC to AE , that is, of the proportions of AB to AD , and of AD to AB . Wherefore, AF would be to AG as AB to AB , that is, equal; and so the two products made by the multiplication of impetus into time would be equal; and by consequent, Z would be equal to X .

Coroll. II. If two bodies be moved in the same time, but with different impetus, the lengths transmitted will be as impetus to impetus. For if the

time of both of them be AD , and their different impetus be AE and AC , the proportion of AG to DC will be compounded of the proportions of AE to AC and of AD to AD , that is, of the proportions of AE to AC and of AC to AC ; and so the proportion of AG to DC , that is, the proportion of length to length, will be as AE to AC , that is, as that of impetus to impetus. In like manner, if two bodies be moved uniformly, and both of them with the same impetus, but in different times, the proportion of the lengths transmitted by them will be as that of their times. For if they have both the same impetus AC , and their different times be AB and AD , the proportion of AF to DC will be compounded of the proportions of AB to AD and of AC to AC ; that is, of the proportions of AB to AD and of AD to AD ; and therefore the proportion of AF to DC , that is, of length to length, will be the same with that of AB to AD , which is the proportion of time to time.

7. If two bodies pass through two lengths with uniform motion, the proportion of the times in which they are moved will be compounded of the proportions of length to length and impetus to impetus reciprocally taken.

If two bodies pass through two lengths with uniform motion, the proportion of their times to one another, will be compounded of the proportions of length to length, and impetus to impetus reciprocally taken; also the proportion of their impetus to one another, will be compounded of the proportions of length to length, and

For let any two lengths be given, as (in the same fig. 3) Z and X , and let one of them be transmitted with the impetus AC , the other with the impetus AE . I say the proportion of the times in which they are transmitted, will be compounded of the proportions of Z to X , and of AE , which is the impetus with which X is transmitted, to AC , the impetus with which Z is transmitted. For seeing AF is the product of the impetus AC multiplied

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time to time
reciprocally
taken.

into the time AB , the time of motion through Z will be a line, which is made by the application of the parallelogram AF to the strait line AC , which line is AB ; and therefore AB is the time of motion through Z . In like manner, seeing AG is the product of the impetus AE multiplied into the time AD , the time of motion through X will be a line which is made by the application of AG to the strait line AD ; but AD is the time of motion through X . Now the proportion of AB to AD is compounded of the proportions of the parallelogram AF to the parallelogram AG , and of the impetus AE to the impetus AC ; which may be demonstrated thus. Put the parallelograms in order AF , AG , DC , and it will be manifest that the proportion of AF to DC is compounded of the proportions of AF to AG and of AG to DC ; but AF is to DC as AB to AD ; wherefore also the proportion of AB to AD is compounded of the proportions of AF to AG and of AG to DC . And because the length Z is to the length X as AF is to AG , and the impetus AE to the impetus AC as AG to DC , therefore the proportion of AB to AD will be compounded of the proportions of the length Z to the length X , and of the impetus AE to the impetus AC ; which was to be demonstrated.

In the same manner it may be proved, that in two uniform motions the proportion of the impetus is compounded of the proportions of length to length and of time to time reciprocally taken.

For if we suppose AC (in the same fig. 3) to be the time, and AB the impetus with which the length Z is passed through; and AE to be the

time, and AD the impetus with which the length X is passed through, the demonstration will proceed as in the last article.

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8. If a body be carried by two movents together, which move with strait and uniform motion, and concur in any given angle, the line by which that body passes will be a strait line.

If a body be carried on with uniform motion by two movents together, which meet in an angle, the line by which it passes will be a strait line, subtending the complement of that angle to 2 right angles.

Let the movent AB (in fig. 4) have strait and uniform motion, and be moved till it come into the place CD ; and let another movent AC , having likewise strait and uniform motion, and making with the movent AB any given angle CAB , be understood to be moved in the same time to DB ; and let the body be placed in the point of their concurrence, A . I say the line which that body describes with its motion is a strait line. For let the parallelogram $ABDC$ be completed, and its diagonal AD be drawn; and in the strait line AB let any point E be taken; and from it let EF be drawn parallel to the strait lines AC and BD , cutting AD in G ; and through the point G let HI be drawn parallel to the strait lines AB and CD ; and lastly, let the measure of the time be AC . Seeing therefore both the motions are made in the same time, when AB is in CD , the body also will be in CD ; and in like manner, when AC is in BD , the body will be in BD . But AB is in CD at the same time when AC is in BD ; and therefore the body will be in CD and BD at the same time; wherefore it will be in the common point D . Again, seeing the motion from AC to BD is uniform, that is, the spaces transmitted by it are in proportion to one another as the times in which they are transmitted, when AC is in EF ,

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the proportion of AB to AE will be the same with that of EF to EG , that is, of the time AC to the time AH . Wherefore AB will be in HI in the same time in which AC is in EF , so that the body will at the same time be in EF and HI , and therefore in their common point G . And in the same manner it will be, wheresoever the point E be taken between A and B . Wherefore the body will always be in the diagonal AD ; which was to be demonstrated.

Coroll. From hence it is manifest, that the body will be carried through the same strait line AD , though the motion be not uniform, provided it have like acceleration; for the proportion of AB to AE will always be the same with that of AC to AH .

If a body be carried by two movents together, one of them being moved with uniform, the other with accelerated motion, and the proportion of their lengths to their times being explicable in numbers, how to find out what line that body describes.

9. If a body be carried by two movents together, which meet in any given angle, and are moved, the one uniformly, the other with motion uniformly accelerated from rest, that is, that the proportion of their impetus be as that of their times, that is, that the proportion of their lengths be duplicate to that of the lines of their times, till the line of greatest impetus acquired by acceleration be equal to that of the line of time of the uniform motion; the line in which the body is carried will be the crooked line of a semiparabola, whose base is the impetus last acquired, and vertex the point of rest.

Let the straight line AB (in fig. 5) be understood to be moved with uniform motion to CD ; and let another movent in the strait line AC be supposed to be moved in the same time to BD , but with motion uniformly accelerated, that is,

with such motion, that the proportion of the spaces which are transmitted be always duplicate to that of the times, till the impetus acquired be BD equal to the strait line AC ; and let the semiparabola $AGDB$ be described. I say that by the concurrence of those two movents, the body will be carried through the semiparabolical crooked line AGD . For let the parallelogram $ABDC$ be completed; and from the point E , taken anywhere in the strait line AB , let EF be drawn parallel to AC and cutting the crooked line in G ; and lastly, through the point G let HI be drawn parallel to the strait lines AB and CD . Seeing therefore the proportion of AB to AE is by supposition duplicate to the proportion of EF to EG , that is, of the time AC to the time AH , at the same time when AC is in EF , AB will be in HI ; and therefore the moved body will be in the common point G . And so it will always be, in what part soever of AB the point E be taken. Wherefore the moved body will always be found in the parabolical line AGD ; which was to be demonstrated.

10. If a body be carried by two movents together, which meet in any given angle, and are moved the one uniformly, the other with impetus increasing from rest, till it be equal to that of the uniform motion, and with such acceleration, that the proportion of the lengths transmitted be every where triplicate to that of the times in which they are transmitted; the line, in which that body is moved, will be the crooked line of the first semiparabolaster of two means, whose base is the impetus last acquired.

Let the strait line AB (in the 6th figure) be moved

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uniformly to CD ; and let another movent AC be moved at the same time to BD with motion so accelerated, that the proportion of the lengths transmitted be everywhere triplicate to the proportion of their times; and let the impetus acquired in the end of that motion be BD , equal to the strait line AC ; and lastly, let AGD be the crooked line of the first semiparabolaster of two means. I say, that by the concurrence of the two movents together, the body will be always in that crooked line AGD . For let the parallelogram $ABDC$ be completed; and from the point E , taken anywhere in the strait line AB , let EF be drawn parallel to AC , and cutting the crooked line in G ; and through the point G let HI be drawn parallel to the strait lines AB and CD . Seeing therefore the proportion of AB to AE is, by supposition, triplicate to the proportion of EF to EG , that is, of the time AC to the time AH , at the same time when AC is in EF , AB will be in HI ; and therefore the moved body will be in the common point G . And so it will always be, in what part soever of AB the point E be taken; and by consequent, the body will always be in the crooked line AGD ; which was to be demonstrated.

11. By the same method it may be shown, what line it is that is made by the motion of a body carried by the concurrence of any two movents, which are moved one of them uniformly, the other with acceleration, but in such proportions of spaces and times as are explicable by numbers, as *duplicate*, *triplicate*, &c., or such as may be designed by any broken number whatsoever. For which this is the rule. Let the two numbers of the length

and time be added together ; and let their sum be the denominator of a fraction, whose numerator must be the number of the length. Seek this fraction in the table of the third article of the xviiith chapter ; and the line sought will be that, which denominates the three-sided figure noted on the left hand ; and the kind of it will be that, which is numbered above over the fraction. For example, let there be a concourse of two movents, whereof one is moved uniformly, the other with motion so accelerated, that the spaces are to the times as 5 to 3. Let a fraction be made whose denominator is the sum of 5 and 3, and the numerator 5, namely the fraction $\frac{5}{8}$. Seek in the table, and you will find $\frac{5}{8}$ to be the third in that row, which belongs to the three-sided figure of four means. Wherefore the line of motion made by the concourse of two such movents, as are last of all described, will be the crooked line of the third parabolaster of four means.

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12. If motion be made by the concourse of two movents, whereof one is moved uniformly, the other beginning from rest in the angle of concourse with any acceleration whatsoever ; the movent, which is moved uniformly, shall put forward the moved body in the several parallel spaces, less than if both the movents had uniform motion ; and still less and less, as the motion of the other movent is more and more accelerated.

Let the body be placed in A, (in the 7th figure) and be moved by two movents, by one with uniform motion from the strait line A B to the strait line C D parallel to it ; and by the other with any acceleration, from the strait line A C to the strait

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line BD parallel to it; and in the parallelogram $ABDC$ let a space be taken between any two parallels EF and GH . I say, that whilst the movent AC passes through the latitude which is between EF and GH , the body is less moved forwards from AB towards CD , than it would have been, if the motion from AC to BD had been uniform.

For suppose that whilst the body is made to descend to the parallel EF by the power of the movent from AC towards BD , the same body in the same time is moved forwards to any point F in the line EF , by the power of the movent from AB towards CD ; and let the strait line AF be drawn and produced indeterminately, cutting GH in H . Seeing therefore, it is as AE to AG , so EF to GH ; if AC should descend towards BD with uniform motion, the body in the time GH , (for I make AC and its parallels the measure of time,) would be found in the point H . But because AC is supposed to be moved towards BD with motion continually accelerated, that is, in greater proportion of space to space, than of time to time, in the time GH the body will be in some parallel beyond it, as between GH and BD . Suppose now that in the end of the time GH it be in the parallel IK , and in IK let IL be taken equal to GH . When therefore the body is in the parallel IK , it will be in the point L . Wherefore when it was in the parallel GH , it was in some point between G and H , as in the point M ; but if both the motions had been uniform, it had been in the point H ; and therefore whilst the movent AC passes over the latitude which is between EF and GH , the body is less moved forwards from AB towards CD , than

it would have been, if both the motions had been uniform ; which was to be demonstrated.

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13. Any length being given, which is passed through in a given time with uniform motion, to find out what length shall be passed through in the same time with motion uniformly accelerated, that is, with such motion that the proportion of the lengths passed through be continually duplicate to that of their times, and that the line of the impetus last acquired be equal to the line of the whole time of the motion.

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Let AB (in the 8th figure) be a length, transmitted with uniform motion in the time AC ; and let it be required to find another length, which shall be transmitted in the same time with motion uniformly accelerated, so that the line of the impetus last acquired be equal to the strait line AC .

Let the parallelogram $ABDC$ be completed; and let BD be divided in the middle at E ; and between BE and BD let BF be a mean proportional; and let AF be drawn and produced till it meet with CD produced in G ; and lastly, let the parallelogram $ACGH$ be completed. I say, AH is the length required.

For as duplicate proportion is to single proportion, so let AH be to AI , that is, let AI be the half of AH ; and let IK be drawn parallel to the strait line AC , and cutting the diagonal AD in K , and the strait line AG in L . Seeing therefore AI is the half of AH , IL will also be the half of BD , that is, equal to BE ; and IK equal to BF ; for BD , that is, GH , BF , and BE , that is, IL , being continual proportionals, AH , AB and AI will also be continual proportionals. But as AB is to

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A I, that is, as A H is to A B, so is B D to I K, and so also is G H, that is, B D to B F; and therefore B F and I K are equal. Now the proportion of A H to A I is duplicate to the proportion of A B to A I, that is, to that of B D to I K, or of G H to I K. Wherefore the point K will be in a parabola, whose diameter is A H, and base G H, which G H is equal to A C. The body therefore proceeding from rest in A, with motion uniformly accelerated in the time A C, when it has passed through the length A H, will acquire the impetus G H equal to the time A C, that is, such impetus, as that with it the body will pass through the length A C in the time A C. Wherefore any length being given, &c., which was propounded to be done.

14. Any length being given, which in a given time is transmitted with uniform motion, to find out what length shall be transmitted in the same time with motion so accelerated, that the lengths transmitted be continually in triplicate proportion to that of their times, and the line of the impetus last of all acquired be equal to the line of time given.

Let the given length A B (in the 9th figure) be transmitted with uniform motion in the time A C; and let it be required to find what length shall be transmitted in the same time with motion so accelerated, that the lengths transmitted be continually in triplicate proportion to that of their times, and the impetus last acquired be equal to the time given.

Let the parallelogram A B D C be completed; and let B D be so divided in E, that B E be a third part of the whole B D; and let B F be a mean pro-

portional between BD and BE ; and let AF be drawn and produced till it meet the strait line CD in G ; and lastly, let the parallelogram $ACGH$ be completed. I say, AH is the length required.

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For as triplicate proportion is to single proportion, so let AH be to another line, AI , that is, make AI a third part of the whole AH ; and let IK be drawn parallel to the strait line AC , cutting the diagonal AD in K , and the strait line AG in L ; then, as AB is to AI , so let AI be to another, AN ; and from the point N let NQ be drawn parallel to AC , cutting AG , AD , and FK produced in P , M , and O ; and last of all, let FO and LM be drawn, which will be equal and parallel to the strait lines BN and IN . By this construction, the lengths transmitted AH , AB , AI , and AN , will be continual proportionals; and, in like manner, the times GH , BF , IL and NP , that is, NQ , NO , NM and NP , will be continual proportionals, and in the same proportion with AH , AB , AI and AN . Wherefore the proportion of AH to AN is the same with that of BD , that is, of NQ to NP ; and the proportion of NQ to NP triplicate to that of NQ to NO , that is, triplicate to that of BD to IK ; wherefore also the length AH is to the length AN in triplicate proportion to that of the time BD , to the time IK ; and therefore the crooked line of the first three-sided figure of two means whose diameter is AH , and base GH equal to AC , shall pass through the point O ; and consequently, AH shall be transmitted in the time AC , and shall have its last acquired impetus GH equal to AC , and the proportions of the lengths acquired in any of the times triplicate to the pro-

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portions of the times themselves. Wherefore AH is the length required to be found out.

By the same method, if a length be given which is transmitted with uniform motion in any given time, another length may be found out which shall be transmitted in the same time with motion so accelerated, that the lengths transmitted shall be to the times in which they are transmitted, in proportion quadruplicate, quintuplicate, and so on infinitely. For if BD be divided in E , so that BD be to BE as 4 to 1; and there be taken between BD and BE a mean proportional FB ; and as AH is to AB , so AB be made to a third, and again so that third to a fourth, and that fourth to a fifth, AN , so that the proportion of AH to AN be quadruplicate to that of AH to AB , and the parallelogram $NBFO$ be completed, the crooked line of the first three-sided figure of three means will pass through the point O ; and consequently, the body moved will acquire the impetus GH equal to AC in the time AC . And so of the rest.

15. Also, if the proportion of the lengths transmitted be to that of their times, as any number to any number, the same method serves for the finding out of the length transmitted with such impetus, and in such time.

For let AC (in the 10th figure) be the time in which a body is transmitted with uniform motion from A to B ; and the parallelogram $ABDC$ being completed, let it be required to find out a length in which that body may be moved in the same time AC from A , with motion so accelerated, that the proportion of the lengths transmitted to that of the times be continually as 3 to 2.

Let BD be so divided in E , that BD be to BE

as 3 to 2; and between B D and B E let B F be a mean proportional; and let A F be drawn and produced till it meet with C D produced in G; and making A M a mean proportional between A H and A B, let it be as A M to A B, so A B to A I; and so the proportion of A H to A I will be to that of A H to A B as 3 to 2; for of the proportions, of which that of A H to A M is one, that of A H to A B is two, and that of A H to A I is three; and consequently, as 3 to 2 to that of G H to B F, and (F K being drawn parallel to B I and cutting A D in K) so likewise to that of G H or B D to I K. Wherefore the proportion of the length A H to A I is to the proportion of the time B D to I K as 3 to 2; and therefore if in the time A C the body be moved with accelerated motion, as was propounded, till it acquire the impetus H G equal to A C, the length transmitted in the same time will be A H.

16. But if the proportion of the lengths to that of the times had been as 4 to 3, there should then have been taken two mean proportionals between A H and A B, and their proportion should have been continued one term further, so that A H to A B might have three of the same proportions, of which A H to A I has four; and all things else should have been done as is already shown. Now the way how to interpose any number of means between two lines given, is not yet found out. Nevertheless this may stand for a general rule; *if there be a time given, and a length be transmitted in that time with uniform motion; as for example, if the time be A C, and the length A B, the strait line A G, which determines the length C G or A H,*

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transmitted in the same time A C with any accelerated motion, shall so cut B D in F, that B F shall be a mean proportional between B D and B E, B E being so taken in B D, that the proportion of length to length be everywhere to the proportion of time to time, as the whole B D is to its part B E.

17. If in a given time two lengths be transmitted, one with uniform motion, the other with motion accelerated in any proportion of the lengths to the times; and again, in part of the same time, parts of the same lengths be transmitted with the same motions, the whole length will exceed the other length in the same proportion in which one part exceeds the other part.

For example, let A B (in the 8th figure) be a length transmitted in the time A C, with uniform motion; and let A H be another length transmitted in the same time with motion uniformly accelerated, so that the impetus last acquired be G H equal to A C; and in A H let any part A I be taken, and transmitted in part of the time A C with uniform motion; and let another part A B be taken and transmitted in the same part of the time A C with motion uniformly accelerated; I say, that as A H is to A B, so will A B be to A I.

Let B D be drawn parallel and equal to H G, and divided in the midst at E, and between B D and B E let a mean proportional be taken as B F; and the strait line A G, by the demonstration of art. 13, shall pass through F. And dividing A H in the midst at I, A B shall be a mean proportional between A H and A I. Again, because A I and A B are described by the same motions, if I K be

drawn parallel and equal to BF or AM , and divided in the midst at N , and between IK and IN be taken the mean proportional IL , the straight line AF will, by the demonstration of the same art. 13, pass through L . And dividing AB in the midst at O , the line AI will be a mean proportional between AB and AO . Where AB is divided in I and O , in like manner as AH is divided in B and I ; and as AH to AB , so is AB to AI . Which was to be proved.

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If a body be carried, &c.

Coroll. Also as AH to AB , so is HB to BI ; and so also BI to IO .

And as this, where one of the motions is uniformly accelerated, is proved out of the demonstration of art. 13; so, when the accelerations are in double proportion to the times, the same may be proved by the demonstration of art. 14; and by the same method in all other accelerations, whose proportions to the times are explicable in numbers.

18. If two sides, which contain an angle in any parallelogram, be moved in the same time to the sides opposite to them, one of them with uniform motion, the other with motion uniformly accelerated; that side, which is moved uniformly, will affect as much with its concurrence through the whole length transmitted, as it would do if the other motion were also uniform, and the length transmitted by it in the same time were a mean proportional between the whole and the half.

Let the side AB of the parallelogram $ABDC$, (in the 11th figure) be understood to be moved with uniform motion till it be coincident with CD ; and let the time of that motion be AC or BD . Also in the same time let the side AC be understood to

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be moved with motion uniformly accelerated, till it be coincident with BD ; then dividing AB in the middle in E , let AF be made a mean proportional between AB and AE ; and drawing FG parallel to AC , let the side AC be understood to be moved in the same time AC with uniform motion till it be coincident with FG . I say, the whole AB confers as much to the velocity of the body placed in A , when the motion of AC is uniformly accelerated till it comes to BD , as the part AF confers to the same, when the side AC is moved uniformly and in the same time to FG .

For seeing AF is a mean proportional between the whole AB and its half AE , BD will (by the 13th article) be the last impetus acquired by AC , with motion uniformly accelerated till it come to the same BD ; and consequently, the strait line FB will be the excess, by which the length, transmitted by AC with motion uniformly accelerated, will exceed the length transmitted by the same AC in the same time with uniform motion, and with impetus every where equal to BD . Wherefore, if the whole AB be moved uniformly to CD in the same time in which AC is moved uniformly to FG , the part FB , seeing it concurs not at all with the motion of the side AC which is supposed to be moved only to FG , will confer nothing to its motion. Again, supposing the side AC to be moved to BD with motion uniformly accelerated, the side AB with its uniform motion to CD will less put forwards the body when it is accelerated in all the parallels, than when it is not at all accelerated; and by how much the greater the acceleration is, by so much the less it will put it for-

wards, as is shown in the 12th article. When therefore AC is in FG with accelerated motion, the body will not be in the side CD at the point G , but at the point D ; so that GD will be the excess, by which the length transmitted with accelerated motion to BD exceeds the length transmitted with uniform motion to FG ; so that the body by its acceleration avoids the action of the part AF , and comes to the side CD in the time AC , and makes the length CD , which is equal to the length AB . Wherefore uniform motion from AB to CD in the time AC , works no more in the whole length AB upon the body uniformly accelerated from AC to BD , than if AC were moved in the same time with uniform motion to FG ; the difference consisting only in this, that when AB works upon the body uniformly moved from AC to FG , that, by which the accelerated motion exceeds the uniform motion, is altogether in FB or GD ; but when the same AB works upon the body accelerated, that, by which the accelerated motion exceeds the uniform motion, is dispersed through the whole length AB or CD , yet, so that if it were collected and put together, it would be equal to the same FB or GD . Wherefore, if two sides which contain an angle, &c.; which was to be demonstrated.

19. If two transmitted lengths have to their times any other proportion explicable by number, and the side AB be so divided in E , that AB be to AE in the same proportion which the lengths transmitted have to the times in which they are transmitted, and between AB and AE there be taken a mean proportional AF ; it may be shown by the same method, that the side, which is moved

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with uniform motion, works as much with its course through the whole length AB , as it would do if the other motion were also uniform, and the length transmitted in the same time AC were that mean proportional AF .

And thus much concerning motion by concourse.

CHAP. XVII.

OF FIGURES DEFICIENT.

1. Definitions of a deficient figure; of a complete figure; of the complement of a deficient figure; and of proportions which are proportional and commensurable to one another.—2. The proportion of a deficient figure to its complement.—3. The proportions of deficient figures to the parallelograms in which they are described, set forth in a table.—4. The description and production of the same figures.—5. The drawing of tangents to them.—6. In what proportion the same figures exceed a strait-lined triangle of the same altitude and base.—7. A table of solid deficient figures described in a cylinder.—8. In what proportion the same figures exceed a cone of the same altitude and base.—9. How a plain deficient figure may be described in a parallelogram, so that it be to a triangle of the same base and altitude, as another deficient figure, plain or solid, twice taken, is to the same deficient figure, together with the complete figure in which it is described.—10. The transferring of certain properties of deficient figures described in a parallelogram to the proportions of the spaces transmitted with several degrees of velocity.—11. Of deficient figures described in a circle.—12. The proposition demonstrated in art. 2 confirmed from the elements of philosophy.—13. An unusual way of reasoning concerning the equality between the superficies of a portion of a sphere and a circle.—14. How from the description of deficient figures in a parallelogram, any number of mean proportionals may be found out between two given strait lines.

Definition of a deficient figure. 1. I CALL those *deficient figures* which may be understood to be generated by the uniform motion

of some quantity, which decreases continually, till at last it have no magnitude at all.

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And I call that a *complete figure*, answering to a deficient figure, which is generated with the same motion and in the same time, by a quantity which retains always its whole magnitude.

Definitions of a complete figure; of the complement of a deficient figure; and of proportions which are proportional & commensurable to one another.

The *complement* of a deficient figure is that which being added to the deficient figure makes it complete.

Four proportions are said to be *proportional*, when the first of them is to the second as the third is to the fourth. For example, if the first proportion be duplicate to the second, and again, the third be duplicate to the fourth, those proportions are said to be *proportional*.

And *commensurable* proportions are those, which are to one another as number to number. As when to a proportion given, one proportion is duplicate, another triplicate, the duplicate proportion will be to the triplicate proportion as 2 to 3; but to the given proportion it will be as 2 to 1; and therefore I call those three proportions *commensurable*.

2. A deficient figure, which is made by a quantity continually decreasing to nothing by proportions everywhere proportional and commensurable, is to its complement, as the proportion of the whole altitude to an altitude diminished in any time is to the proportion of the whole quantity, which describes the figure, to the same quantity diminished in the same time.

The proportion of a deficient figure to its complement.

Let the quantity A B (in fig. 1), by its motion through the altitude A C, describe the complete figure A D; and again, let the same quantity, by

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decreasing continually to nothing in C, describe the deficient figure $ABEFC$, whose complement will be the figure $BDCFE$. Now let AB be supposed to be moved till it lie in GK , so that the altitude diminished be GC , and AB diminished be GE ; and let the proportion of the whole altitude AC to the diminished altitude GC , be, for example, triplicate to the proportion of the whole quantity AB or GK to the diminished quantity GE . And in like manner, let HI be taken equal to GE , and let it be diminished to HF ; and let the proportion of GC to HC be triplicate to that of HI to HF ; and let the same be done in as many parts of the strait line AC as is possible; and a line be drawn through the points B, E, F and C . I say the deficient figure $ABEFC$ is to its complement $BDCFE$ as 3 to 1, or as the proportion of AC to GC is to the proportion of AB , that is, of GK to GE .

For (by art. 2, chapter xv.) the proportion of the complement $BEFCD$ to the deficient figure $ABEFC$ is all the proportions of DB to BA , OE to EG , QF to FH , and of all the lines parallel to DB terminated in the line $BEFC$, to all the parallels to AB terminated in the same points of the line $BEFC$. And seeing the proportions of DB to OE , and of DB to QF &c. are everywhere triplicate of the proportions of AB to GE , and of AB to HF &c. the proportions of HF to AB , and of GE to AB &c. (by art. 16, chap. XIII.), are triplicate of the proportions of QF to DB , and of OE to DB &c. and therefore the deficient figure $ABEFC$, which is the aggre-

gate of all the lines H F, G E, A B; &c. is triple to the complement B E F C D made of all the lines Q F, O E, D B, &c.; which was to be proved.

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of a deficient
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complement.

It follows from hence, that the same complement B E F C D is $\frac{1}{4}$ of the whole parallelogram. And by the same method may be calculated in all other deficient figures, generated as above declared, the proportion of the parallelogram to either of its parts; as that when the parallels increase from a point in the same proportion, the parallelogram will be divided into two equal triangles; when one increase is double to the other, it will be divided into a semiparabola and its complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated thus.

Let the strait line C B be drawn cutting G K in L, and through L let M N be drawn parallel to the strait line A C; wherefore the parallelograms G M and L D will be equal. Then let L K be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of A C to G C, or of G K to G L, hath to the proportion of G K to G E. Therefore L K will be to one of those three parts as the arithmetical proportion between G K and G L is to the arithmetical proportion between G K and the same G K wanting the third part of L K; and K E will be somewhat greater than a third of L K. Seeing now the altitude A G or M L is, by reason of the continual decrease, to be supposed less than any quantity that can be given; L K, which is intercepted between the diagonal B C and the side B D,

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will be also less than any quantity that can be given; and consequently, if G be put so near to A in g , as that the difference between Cg and CA be less than any quantity that can be assigned, the difference also between Cl (removing L to l) and CB , will be less than any quantity that can be assigned; and the line gl being drawn and produced to the line BD in k , cutting the crooked line in e , the proportion of Gk to Gl will still be triplicate to the proportion of Gk to Ge , and the difference between k and e , the third part of kl , will be less than any quantity that can be given; and therefore the parallelogram eD will differ from a third part of the parallelogram Ae by a less difference than any quantity that can be assigned. Again, let HI be drawn parallel and equal to GE , cutting CB in P , the crooked line in F , and OE in I , and the proportion of Cg to CH will be triplicate to the proportion of HF to HP , and IF will be greater than the third part of PI . But again, setting H in h so near to g , as that the difference between Ch and Cg may be but as a point, the point P will also in p be so near to l , as that the difference between Cp and Cl will be but as a point; and drawing hp till it meet with BD in i , cutting the crooked line in f , and having drawn eo parallel to BD , cutting DC in o , the parallelogram fo will differ less from the third part of the parallelogram gf , than by any quantity that can be given. And so it will be in all other spaces generated in the same manner. Wherefore the differences of the arithmetical and geometrical means, which are but as so many points $B, e, f, \&c.$

(seeing the whole figure is made up of so many indivisible spaces) will constitute a certain line, such as is the line $BEFC$, which will divide the complete figure AD into two parts, whereof one, namely, $BEFC$, which I call a deficient figure, is triple to the other, namely, $BDCFE$, which I call the complement thereof. And whereas the proportion of the altitudes to one another is in this case everywhere triplicate to that of the decreasing quantities to one another; in the same manner, if the proportion of the altitudes had been everywhere quadruplicate to that of the decreasing quantities, it might have been demonstrated that the deficient figure had been quadruple to its complement; and so in any other proportion. Wherefore, a deficient figure, which is made, &c. which was to be demonstrated.

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The same rule holdeth also in the diminution of the bases of cylinders, as is demonstrated in the second article of chapter xv.

3. By this proposition, the magnitudes of all deficient figures, when the proportions by which their bases decrease continually are proportional to those by which their altitudes decrease, may be compared with the magnitudes of their complements; and consequently, with the magnitudes of their complete figures. And they will be found to be, as I have set them down in the following tables; in which I compare a parallelogram with three-sided figures; and first, with a strait-lined triangle, made by the base of the parallelogram continually decreasing in such manner, that the altitudes be always in proportion to one another

The proportion of deficient figures to the parallelograms in which they are described, set forth in a table

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of deficient
figures, &c.

as the bases are, and so the triangle will be equal to its complement; or the proportions of the altitudes and bases will be as 1 to 1, and then the triangle will be half the parallelogram. Secondly, with that three-sided figure which is made by the continual decreasing of the bases in subduplicate proportion to that of the altitudes; and so the deficient figure will be double to its complement, and to the parallelogram as 2 to 3. Then, with that where the proportion of the altitudes is triplicate to that of the bases; and then the deficient figure will be triple to its complement, and to the parallelogram as 3 to 4. Also the proportion of the altitudes to that of the bases may be as 3 to 2; and then the deficient figure will be to its complement as 3 to 2, and to the parallelogram as 3 to 5; and so forwards, according as more mean proportionals are taken, or as the proportions are more multiplied, as may be seen in the following table. For example, if the bases decrease so, that the proportion of the altitudes to that of the bases be always as 5 to 2, and it be demanded what proportion the figure made has to the parallelogram, which is supposed to be unity; then, seeing that where the proportion is taken five times, there must be four means; look in the table amongst the three-sided figures of four means, and seeing the proportion was as 5 to 2, look in the uppermost row for the number 2, and descending in the second column till you meet with that three-sided figure, you will find $\frac{4}{7}$; which shows that the deficient figure is to the parallelogram as $\frac{4}{7}$ to 1, or as 5 to 7.

	1	2	3	4	5	6	7
Parallelogram	1	:	:	:	:	:	:
Strait-sided triangle	$\frac{1}{2}$:	:	:	:	:	:
Three-sided figure of 1 mean	$\frac{2}{3}$:	:	:	:	:	:
Three-sided figure of 2 means	$\frac{3}{4}$	$\frac{3}{8}$:	:	:	:	:
Three-sided figure of 3 means	$\frac{4}{5}$	$\frac{4}{9}$	$\frac{4}{7}$:	:	:	:
Three-sided figure of 4 means	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$:	:	:
Three-sided figure of 5 means	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$	$\frac{6}{11}$:	:
Three-sided figure of 6 means	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$:
Three-sided figure of 7 means	$\frac{8}{9}$	$\frac{8}{10}$	$\frac{8}{11}$	$\frac{8}{12}$	$\frac{8}{13}$	$\frac{8}{14}$	$\frac{8}{15}$

4. Now for the better understanding of the nature of these three-sided figures, I will show how they may be described by points; and first, those which are in the first column of the table. Any parallelogram being described, as A B C D (in figure 2) let the diagonal B D be drawn; and the strait-lined triangle B C D will be half the parallelogram; then let any number of lines, as E F, be drawn parallel to the side B C, and cutting the diagonal B D in G; and let it be everywhere, as E F to E G, so E G to another, E H; and through all the points H let the line B H H D be drawn; and the figure B H H D C will be that which I call a three-sided figure of one mean, because in three proportionals, as E F, E G and E H, there is but one mean, namely, E G; and this three-sided figure will be $\frac{2}{3}$ of the parallelogram, and is called a *parabola*. Again, let it be as E G to E H, so E H to another, E I, and let the line B I I D be drawn, making the three-sided figure B I I D C; and this will be $\frac{3}{4}$ of the parallelogram, and is by many called a *cubic parabola*. In like manner, if the

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proportions be further continued in EF , there will be made the rest of the three-sided figures of the first column ; which I thus demonstrate. Let there be drawn strait lines, as HK and GL , parallel to the base DC . Seeing therefore the proportion of EF to EH is duplicate to that of EF to EG , or of BC to BL , that is, of CD to LG , or of KM (producing KH to AD in M) to KH , the proportion of BC to BK will be duplicate to that of KM to KH ; but as BC is to BK , so is DC or KM to KN , and therefore the proportion of KM to KN is duplicate to that of KM to KH ; and so it will be wheresoever the parallel KM be placed. Wherefore the figure $BHHDC$ is double to its complement $BHHDA$, and consequently $\frac{2}{3}$ of the whole parallelogram. In the same manner, if through I be drawn $OPIQ$ parallel and equal to CD , it may be demonstrated that the proportion of OQ to OP , that is, of BC to BO , is triplicate that of OQ to OI , and therefore that the figure $BIIDC$ is triple to its complement $BIIDA$, and consequently $\frac{3}{4}$ of the whole parallelogram, &c.

Secondly, such three-sided figures as are in any of the transverse rows, may be thus described. Let $ABCD$ (in fig. 3) be a parallelogram, whose diagonal is BD . I would describe in it such figures, as in the preceding table I call three-sided figures of three means. Parallel to DC , I draw EF as often as is necessary, cutting BD in G ; and between EF and EG , I take three proportionals EH , EI and EK . If now there be drawn lines through all the points H , I and K , that through all the points H will make the figure $BHDC$, which is the first of those three-sided figures ; and that

through all the points I, will make the figure B I D C, which is the second; and that which is drawn through all the points K, will make the figure B K D C the third of those three-sided figures. The first of these, seeing the proportion of E F to E G is quadruplicate of that E F to E H, will be to its complement as 4 to 1, and to the parallelogram as 4 to 5. The second, seeing the proportion of E F to E G is to that of E F to E I as 4 to 2, will be double to its complement, and $\frac{4}{3}$ or $\frac{2}{3}$ of the parallelogram. The third, seeing the proportion of E F to E G is that of E F to E K as 4 to 3, will be to its complement as 4 to 3, and to the parallelogram as 4 to 7.

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figures.

Any of these figures being described may be produced at pleasure, thus; let A B C D (in fig. 4) be a parallelogram, and in it let the figure B K D C be described, namely, the third three-sided figure of three means. Let B D be produced indefinitely to E, and let E F be made parallel to the base D C, cutting A D produced in G, and B C produced in F; and in G E let the point H be so taken, that the proportion of F E to F G may be quadruplicate to that of F E to F H, which may be done by making F H the greatest of three proportionals between F E and F G; the crooked line B K D produced, will pass through the point H. For if the strait line B H be drawn, cutting C D in I, and H L be drawn parallel to G D, and meeting C D produced in L; it will be as F E to F G, so C L to C I, that is, in quadruplicate proportion to that of F E to F H, or of C D to C I. Wherefore if the line B K D be produced according to its generation, it will fall upon the point H.

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The drawing
of tangents
to them.

5. A strait line may be drawn so as to touch the crooked line of the said figure in any point, in this manner. Let it be required to draw a tangent to the line $BKD H$ (in fig. 4) in the point D . Let the points B and D be connected, and drawing DA equal and parallel to BC , let B and A be connected; and because this figure is by construction the third of three means, let there be taken in AB three points, so, that by them the same AB be divided into four equal parts; of which take three, namely, AM , so that AB may be to AM , as the figure $BKDC$ is to its complement. I say, the strait line MD will touch the figure in the point given D . For let there be drawn anywhere between AB and DC a parallel, as RQ , cutting the strait line BD , the crooked line BKD , the strait line MD , and the strait line AD , in the points P, K, O and Q . RK will therefore, by construction, be the least of three means in geometrical proportion between RQ and RP . Wherefore (by coroll. of art. 28, chapter XIII.) RK will be less than RO ; and therefore MD will fall without the figure. Now if MD be produced to N , FN will be the greatest of three means in arithmetical proportion between FE and FG ; and FH will be the greatest of three means in geometrical proportion between the same FE and FG . Wherefore (by the same coroll. of art. 28, chapter XIII.) FH will be less than FN ; and therefore DN will fall without the figure, and the strait line MN will touch the same figure only in the point D .

In what proportion the same figures exceed

6. The proportion of a deficient figure to its complement being known, it may also be known

what proportion a strait-lined triangle has to the excess of the deficient figure above the same triangle; and these proportions I have set down in the following table; where if you seek, for example, how much the fourth three-sided figure of five means exceeds a triangle of the same altitude and base, you will find in the concourse of the fourth column with the three-sided figures of five means $\frac{1}{5}$; by which is signified, that that three-sided figure exceeds the triangle by two-tenths or by one-fifth part of the same triangle.

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a strait-lined triangle of the same altitude and base.

		1	2	3	4	5	6	7
The excess of	The triangle	1	:	:	:	:	:	:
	A three-sided fig. of 1 mean .	$\frac{1}{3}$:	:	:	:	:	:
	A three-sided fig. of 2 means	$\frac{2}{4}$	$\frac{1}{5}$:	:	:	:	:
	A three-sided fig. of 3 means	$\frac{3}{5}$	$\frac{2}{6}$	$\frac{1}{7}$:	:	:	:
	A three-sided fig. of 4 means	$\frac{4}{6}$	$\frac{3}{7}$	$\frac{2}{8}$	$\frac{1}{9}$:	:	:
	A three-sided fig. of 5 means	$\frac{5}{7}$	$\frac{4}{8}$	$\frac{3}{9}$	$\frac{2}{10}$	$\frac{1}{11}$:	:
	A three-sided fig. of 6 means	$\frac{6}{8}$	$\frac{5}{9}$	$\frac{4}{10}$	$\frac{3}{11}$	$\frac{2}{12}$	$\frac{1}{13}$:
A three-sided fig. of 7 means	$\frac{7}{9}$	$\frac{6}{10}$	$\frac{5}{11}$	$\frac{4}{12}$	$\frac{3}{13}$	$\frac{2}{14}$	$\frac{1}{15}$	

7. In the next table are set down the proportion of a cone and the solids of the said three-sided figures, namely, the proportions between them and a cylinder. As for example, in the concourse of the second column with the three-sided figures of four means, you have $\frac{1}{5}$; which gives you to understand, that the solid of the second three-sided figure of four means is to the cylinder as $\frac{1}{5}$ to 1, or as 5 to 9.

A table of solid deficient figures described in a cylinder.

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		1	2	3	4	5	6	7
The solids of	A cylinder	1	:	:	:	:	:	:
	A cone	$\frac{1}{3}$:	:	:	:	:	:
	A three-sided fig. of 1 mean	$\frac{2}{4}$:	:	:	:	:
	A three-sided fig. of 2 means	$\frac{8}{5}$	$\frac{3}{7}$:	:	:	:
	A three-sided fig. of 3 means	$\frac{4}{6}$	$\frac{4}{8}$	$\frac{4}{10}$:	:	:
	A three-sided fig. of 4 means	$\frac{5}{7}$	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{5}{13}$:	:
	A three-sided fig. of 5 means	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{6}{12}$	$\frac{6}{14}$	$\frac{6}{16}$:
	A three-sided fig. of 6 means	$\frac{7}{9}$	$\frac{7}{11}$	$\frac{7}{13}$	$\frac{7}{15}$	$\frac{7}{17}$	$\frac{7}{19}$	
A three-sided fig. of 7 means	$\frac{8}{10}$	$\frac{8}{12}$	$\frac{8}{14}$	$\frac{8}{16}$	$\frac{8}{18}$	$\frac{8}{20}$	$\frac{8}{22}$	

In what proportion the same figures exceed a cone of the same altitude and base.

8. Lastly, the excess of the solids of the said three-sided figures above a cone of the same altitude and base, are set down in the table which follows :

		1	2	3	4	5	6	7
The excesses of the solids of these three-sided figures above a cone.	The Cone	1	:	:	:	:	:	:
	Of the solid of a three-sided figure of 1 mean	$\frac{6}{12}$:	:	:	:	:	:
	Ditto ditto, 2 means	$\frac{12}{5}$	$\frac{6}{21}$:	:	:	:
	Ditto ditto, 3 means	$\frac{18}{8}$	$\frac{12}{24}$	$\frac{6}{30}$:	:	:
	Ditto ditto, 4 means	$\frac{24}{1}$	$\frac{18}{27}$	$\frac{12}{33}$	$\frac{6}{39}$:	:
	Ditto ditto, 5 means	$\frac{30}{24}$	$\frac{24}{36}$	$\frac{18}{36}$	$\frac{12}{42}$	$\frac{6}{48}$:
	Ditto ditto, 6 means	$\frac{36}{27}$	$\frac{30}{33}$	$\frac{24}{36}$	$\frac{18}{45}$	$\frac{12}{51}$	$\frac{6}{57}$	
	Ditto ditto, 7 means	$\frac{42}{30}$	$\frac{36}{36}$	$\frac{30}{42}$	$\frac{24}{48}$	$\frac{18}{54}$	$\frac{12}{60}$	$\frac{6}{66}$

How a plain deficient figure may be described in a parallelogram, so that it be to a triangle of the same base and altitude, as another deficient figure, plain or solid, twice taken, is to the same deficient

9. If any of these deficient figures, of which I have now spoken, as A B C D (in the 5th figure) be inscribed within the complete figure B E, having A D C E for its complement; and there be made upon C B produced the triangle A B I; and the parallelogram A B I K be completed; and there be drawn parallel to the strait line C I, any number of lines, as M F, cutting every one of them the

crooked line of the deficient figure in D, and the strait lines A C, A B and A I in H, G, and L; and as G F is to G D, so G L be made to another, G N; and through all the points N there be drawn the line A N I: there will be a deficient figure A N I B, whose complement will be A N I K. I say, the figure A N I B is to the triangle A B I, as the deficient figure A B C D twice taken is to the same deficient figure together with the complete figure B E.

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figure, together with the complete figure, in which it is described.

For as the proportion of A B to A G, that is, of G M to G L, is to the proportion of G M to G N, so is the magnitude of the figure A N I B to that of its complement A N I K, by the second article of this chapter.

But, by the same article, as the proportion of A B to A G, that is, of G M to G L, is to the proportion of G F to G D, that is, by construction, of G L to G N, so is the figure A B C D to its complement A D C E.

And by composition, as the proportion of G M to G L, together with that of G L to G N, is to the proportion of G M to G L, so is the complete figure B E to the deficient figure A B C D.

And by conversion, as the proportion of G M to G L is to both the proportions of G M to G L and of G L to G N, that is, to the proportion of G M to G N, which is the proportion compounded of both, so is the deficient figure A B C D to the complete figure B E.

But it was, as the proportion of G M to G L to that of G M to G N, so the figure A N I B to its complement A N I K. And therefore, A B C D. B E :: A N I B. A N I K are proportionals. And by com-

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position, $ABCD + BE$. $ABCD :: BK$. $ANIB$ are proportionals.

And by doubling the consequents, $ABCD + BE$. $2ABCD :: BK$. $2ANIB$ are proportionals.

And by taking the halves of the third and the fourth, $ABCD + BE$. $2ABCD :: ABI$. $ANIB$ are also proportionals; which was to be proved.

10. From what has been said of deficient figures described in a parallelogram, may be found out what proportions spaces, transmitted with accelerated motion in determined times, have to the times themselves, according as the moved body is accelerated in the several times with one or more degrees of velocity.

For let the parallelogram $ABCD$, in the 6th figure, and in it the three-sided figure $DEBC$ be described; and let FG be drawn anywhere parallel to the base, cutting the diagonal BD in H , and the crooked line BED in E ; and let the proportion of BC to BF be, for example, triplicate to that of FG to FE ; whereupon the figure $DEBC$ will be triple to its complement $BEDA$; and in like manner, IE being drawn parallel to BC , the three-sided figure EKB will be triple to its complement $BKEI$. Wherefore the parts of the deficient figure cut off from the vertex by strait lines parallel to the base, namely, $DEBC$ and EKB , will be to one another as the parallelograms AC and IF ; that is, in proportion compounded of the proportions of the altitudes and bases. Seeing therefore the proportion of the altitude BC to the altitude BF is triplicate to the proportion of the base DC to the base FE , the figure $DEBC$ to the

The transferring of certain properties of deficient figures described in a parallelogram to the proportions of spaces transmitted with several degrees of velocity.

figure $E K B F$ will be quadruplicate to the proportion of the same DC to FE . And by the same method, may be found out what proportion any of the said three-sided figures has to any part of the same, cut off from the vertex by a strait line parallel to the base.

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Now as the said figures are understood to be described by the continual decreasing of the base, as of CD , for example, till it end in a point, as in B ; so also they may be understood to be described by the continual increasing of a point, as of B , till it acquire any magnitude, as that of CD .

Suppose now the figure $B E D C$ to be described by the increasing of the point B to the magnitude CD . Seeing therefore the proportion of BC to BF is triplicate to that of CD to FE , the proportion of FE to CD will, by conversion, as I shall presently demonstrate, be triplicate to that BF to BC . Wherefore if the strait line BC be taken for the measure of the time in which the point B is moved, the figure $E K B F$ will represent the sum of all the increasing velocities in the time BF ; and the figure $D E B C$ will in like manner represent the sum of all the increasing velocities in the time BC . Seeing therefore the proportion of the figure $E K B F$ to the figure $D E B C$ is compounded of the proportions of altitude to altitude, and base to base; and seeing the proportion of FE to CD is triplicate to that of BF to BC ; the proportion of the figure $E K B F$ to the figure $D E B C$ will be quadruplicate to that of BF to BC ; that is, the proportion of the sum of the velocities in the time BF , to the sum of the velocities in the time BC , will be quadruplicate to the proportion of BF to

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The transferring of certain properties of deficient figures, &c.

BC. Wherefore if a body be moved from B with velocity so increasing, that the velocity acquired in the time BF be to the velocity acquired in the time BC in triplicate proportion to that of the times themselves BF to BC, and the body be carried to F in the time BF; the same body in the time BC will be carried through a line equal to the fifth proportional from BF in the continual proportion of BF to BC. And by the same manner of working, we may determine what spaces are transmitted by velocities increasing according to any other proportions.

It remains that I demonstrate the proportion of FE to CD to be triplicate to that of BF to BC. Seeing therefore the proportion of CD, that is, of FG to FE is subtriplicate to that of BC to BF; the proportion of FG to FE will also be subtriplicate to that of FG to FH. Wherefore the proportion of FG to FH is triplicate to that of FG, that is, of CD to FE. But in four continual proportionals, of which the least is the first, the proportion of the first to the fourth, (by the 16th article of chapter XIII), is subtriplicate to the proportion of the third to the same fourth. Wherefore the proportion of FH to GF is subtriplicate to that of FE to CD; and therefore the proportion of FE to CD is triplicate to that of FH to FG, that is, of BF to BC; which was to be proved.

It may from hence be collected, that when the velocity of a body is increased in the same proportion with that of the times, the degrees of velocity above one another proceed as numbers do in immediate succession from unity, namely, as 1, 2, 3, 4, &c. And when the velocity is increased in pro-

portion duplicate to that of the times, the degrees proceed as numbers from unity, skipping one, as 1, 3, 5, 7, &c. Lastly, when the proportions of the velocities are triplicate to those of the times, the progression of the degrees is as that of numbers from unity, skipping two in every place, as 1, 4, 7, 10, &c., and so of other proportions. For geometrical proportionals, when they are taken in every point, are the same with arithmetical proportionals.

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11. Moreover, it is to be noted that as in quantities, which are made by any magnitudes decreasing, the proportions of the figures to one another are as the proportions of the altitudes to those of the bases; so also it is in those, which are made with motion decreasing, which motion is nothing else but that power by which the described figures are greater or less. And therefore in the description of *Archimedes' spiral*, which is done by the continual diminution of the semidiameter of a circle in the same proportion in which the circumference is diminished, the space, which is contained within the semidiameter and the spiral line, is a third part of the whole circle. For the semidiameters of circles, inasmuch as circles are understood to be made up of the aggregate of them, are so many sectors; and therefore in the description of a spiral, the sector which describes it is diminished in duplicate proportions to the diminutions of the circumference of the circle in which it is inscribed; so that the complement of the spiral, that is, that space in the circle which is without the spiral line, is double to the space within the spiral line. In the same manner, if

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there be taken a mean proportional everywhere between the semidiameter of the circle, which contains the spiral, and that part of the semidiameter which is within the same, there will be made another figure, which will be half the circle. And to conclude, this rule serves for all such spaces as may be described by a line or superficies decreasing either in magnitude or power; so that if the proportions, in which they decrease, be commensurable to the proportions of the times in which they decrease, the magnitudes of the figures they describe will be known.

The proposition demonstrated in art. 2 confirmed from the elements of philosophy.

12. The truth of that proposition, which I demonstrated in art. 2, which is the foundation of all that has been said concerning deficient figures, may be derived from the elements of philosophy, as having its original in this; *that all equality and inequality between two effects, that is, all proportion, proceeds from, and is determined by, the equal and unequal causes of those effects, or from the proportion which the causes, concurring to one effect, have to the causes which concur to the producing of the other effect*; and that therefore the proportions of quantities are the same with the proportions of their causes. Seeing, therefore, two deficient figures, of which one is the complement of the other, are made, one by motion decreasing in a certain time and proportion, the other by the loss of motion in the same time; the causes, which make and determine the quantities of both the figures, so that they can be no other than they are, differ only in this, that the proportions by which the quantity which generates the figure proceeds in describing of the same, that

is, the proportions of the remainders of all the times and altitudes, may be other proportions than those by which the same generating quantity decreases in making the complement of that figure, that is, the proportions of the quantity which generates the figure continually diminished. Wherefore, as the proportion of the times in which motion is lost, is to that of the decreasing quantities by which the deficient figure is generated, so will the defect or complement be to the figure itself which is generated.

13. There are also other quantities which are determinable from the knowledge of their causes, namely, from the comparison of the motions by which they are made; and that more easily than from the common elements of geometry. For example, that the superficies of any portion of a sphere is equal to that circle, whose radius is a strait line drawn from the pole of the portion to the circumference of its base, I may demonstrate in this manner. Let BAC (in fig. 7) be a portion of a sphere, whose axis is AE , and whose base is BC ; and let AB be the strait line drawn from the pole A to the base in B ; and let AD , equal to AB , touch the great circle BAC in the pole A . It is to be proved that the circle made by the radius AD is equal to the superficies of the portion BAC . Let the plain $AEBD$ be understood to make a revolution about the axis AE ; and it is manifest that by the strait line AD a circle will be described; and by the arch AB the superficies of a portion of a sphere; and lastly, by the subtense AB the superficies of a right cone. Now seeing both the strait line AB and the arch AB make

An unusual way of reasoning concerning the equality between the superficies of a portion of a sphere and a circle.

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ing, &c.

one and the same revolution, and both of them have the same extreme points A and B, the cause why the spherical superficies, which is made by the arch, is greater than the conical superficies, which is made by the subtense, is, that A B the arch is greater than A B the subtense; and the cause why it is greater consists in this, that although they be both drawn from A to B, yet the subtense is drawn strait, but the arch angularly, namely, according to that angle which the arch makes with the subtense, which angle is equal to the angle D A B (for an angle of contingence adds nothing to an angle of a segment, as has been shown in chapter XIV, article 16.) Wherefore the magnitude of the angle D A B is the cause why the superficies of the portion, described by the arch A B, is greater than the superficies of the right cone described by the subtense A B.

Again, the cause why the circle described by the tangent A D is greater than the superficies of the right cone described by the subtense A B (notwithstanding that the tangent and the subtense are equal, and both moved round in the same time) is this, that A D stands at right angles to the axis, but A B obliquely; which obliquity consists in the same angle D A B. Seeing therefore the quantity of the angle D A B is that which makes the excess both of the superficies of the portion, and of the circle made by the radius A D, above the superficies of the right cone described by the subtense A B; it follows, that both the superficies of the portion and that of the circle do equally exceed the superficies of the cone. Wherefore the circle made by A D or A B, and

the spherical superficies made by the arch A B, are equal to one another ; which was to be proved.

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14. If these deficient figures, which I have described in a parallelogram, were capable of exact description, then any number of mean proportionals might be found out between two strait lines given. For example, in the parallelogram A B C D, (in figure 8) let the three-sided figure of two means be described (which many call a *cubical parabola*); and let R and S be two given strait lines ; between which, if it be required to find two mean proportionals, it may be done thus. Let it be as R to S, so B C to B F ; and let F E be drawn parallel to B A, and out the crooked line in E ; then through E let G H be drawn parallel and equal to the strait line A D, and cut the diagonal B D in I ; for thus we have G I the greatest of two means between G H and G E, as appears by the description of the figure in article 4. Wherefore, if it be as G H to G I, so R to another line, T, that T will be the greatest of two means between R and S. And therefore if it be again as R to T, so T to another line, X, that will be done which was required.

How from the description of deficient figures in a parallelogram, any number of mean proportionals may be found out between two given strait lines.

In the same manner, four mean proportionals may be found out, by the description of a three-sided figure of four means ; and so any other number of means, &c.

CHAPTER XVIII.

OF THE EQUATION OF STRAIT LINES WITH THE
CROOKED LINES OF PARABOLAS AND OTHER
FIGURES MADE IN IMITATION OF PARABOLAS.

1. To find the strait line equal to the crooked line of a semiparabola.—2. To find a strait line equal to the crooked line of the first semiparabolaster, or to the crooked line of any other of the deficient figures of the table of the 3d article of the precedent chapter.

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To find a strait
line equal to the
crooked line of
a semiparabola.

1. A PARABOLA being given, to find a strait line equal to the crooked line of the semiparabola.

Let the parabolical line given be ABC (in figure 1), and the diameter found be AD , and the base drawn DC ; and the parallelogram $ADCE$ being completed, draw the strait line AC . Then dividing AD into two equal parts in F , draw FH equal and parallel to DC , cutting AC in K , and the parabolical line in O ; and between FH and FO take a mean proportional FP , and draw AO , AP and PC . I say that the two lines AP and PC , taken together as one line, are equal to the parabolical line $ABOC$.

For the line $ABOC$ being a parabolical line, is generated by the concurrence of two motions, one uniform from A to E , the other in the same time uniformly accelerated from rest in A to D . And because the motion from A to E is uniform, AE may represent the times of both those motions from the beginning to the end. Let therefore AE be the time; and consequently the lines ordi-

nately applied in the semiparabola will design the parts of time wherein the body; that describeth the line $A B O C$, is in every point of the same; so that as at the end of the time $A E$ or $D C$ it is in C , so at the end of the time $F O$ it will be in O . And because the velocity in $A D$ is increased uniformly, that is, in the same proportion with the time, the same lines ordinately applied in the semiparabola will design also the continual augmentation of the impetus, till it be at the greatest, designed by the base $D C$. Therefore supposing uniform motion in the line $A F$, in the time $F K$ the body in A by the concurrence of the two uniform motions in $A F$ and $F K$ will be moved uniformly in the line $A K$; and $K O$ will be the increase of the impetus or swiftness gained in the time $F K$; and the line $A O$ will be uniformly described by the concurrence of the two uniform motions in $A F$ and $F O$ in the time $F O$. From O draw $O L$ parallel to $E C$, cutting $A C$ in L ; and draw $L N$ parallel to $D C$, cutting $E C$ in N , and the parabolical line in M ; and produce it on the other side to $A D$ in I ; and $I N$, $I M$ and $I L$ will be, by the construction of a parabola, in continual proportion, and equal to the three lines $F H$, $F P$ and $F O$; and a strait line parallel to $E C$ passing through M will fall on P ; and therefore $O P$ will be the increase of impetus gained in the time $F O$ or $I L$. Lastly, produce $P M$ to $C D$ in Q ; and $Q C$ or $M N$ or $P H$ will be the increase of impetus proportional to the time $F P$ or $I M$ or $D Q$. Suppose now uniform motion from H to C in the time $P H$. Seeing therefore in the time $F P$ with uniform motion and the impetus increased in proportion to the times, is described

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the straight line AP ; and in the rest of the time and impetus, namely, PH , is described the line CP uniformly; it followeth that the whole line APC is described with the whole impetus, and in the same time wherewith is described the parabolical line ABC ; and therefore the line APC , made of the two strait lines AP and PC , is equal to the parabolical line ABC ; which was to be proved.

To find a strait line equal to the crooked line of the first semiparabolaster or to the crooked line of any other of the deficient figures of the table of art. 3 of the preceding chapter.

2. To find a strait line equal to the crooked line of the first semiparabolaster.

Let ABC be the crooked line of the first semiparabolaster; AD the diameter; DC the base; and let the parallelogram completed be $ADCE$, whose diagonal is AC . Divide the diameter into two equal parts in F , and draw FH equal and parallel to DC , cutting AC in K ; the crooked line in O , and EC in H . Then draw OL parallel to EC , cutting AC in L ; and draw LN parallel to the base DC , cutting the crooked line in M , and the strait line EC in N ; and produce it on the other side to AD in I . Lastly, through the point M draw PMQ parallel and equal to HC , cutting FH in P ; and join CP , AP and AO . I say, the two strait lines AP and PC are equal to the crooked line $ABOC$.

For the line $ABOC$, being the crooked line of the first semiparabolaster, is generated by the concurrence of two motions, one uniform from A to E , the other in the same time accelerated from rest in A to D , so as that the impetus increaseth in proportion perpetually triplicate to that of the increase of the time, or which is all one, the lengths transmitted are in proportion triplicate to

To find a strait
line, &c.

that of the times of their transmission ; for as the impetus or quicknesses increase, so the lengths transmitted increase also. And because the motion from A to E is uniform, the line A E may serve to represent the time, and consequently the lines, ordinately drawn in the semiparabolaster, will design the parts of time wherein the body, beginning from rest in A, describeth by its motion the crooked line A B O C. And because D C, which represents the greatest acquired impetus, is equal to A E, the same ordinate lines will represent the several augmentations of the impetus increasing from rest in A. Therefore, supposing uniform motion from A to F, in the time F K there will be described, by the concourse of the two uniform motions A F and F K, the line A K uniformly, and K O will be the increase of impetus in the time F K ; and by the concourse of the two uniform motions in A F and F O will be described the line A O uniformly. Through the point L draw the strait line L M N parallel to D C, cutting the strait line A D in I, the crooked line A B C in M, and the strait line E C in N ; and through the point M the strait line P M Q parallel and equal to H C, cutting D C in Q and F H in P. By the concourse therefore of the two uniform motions in A F and F P in the time F P will be uniformly described the strait line A P ; and L M or O P will be the increase of impetus to be added for the time F O. And because the proportion of I N to I L is triplicate to the proportion of I N to I M, the proportion of F H to F O will also be triplicate to the proportion of F H to F P ; and the proportional impetus gained in the time F P is P H.

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line, &c.

So that FH being equal to DC, which designed the whole impetus acquired by the acceleration, there is no more increase of impetus to be computed. Now in the time PH suppose an uniform motion from H to C; and by the two uniform motions in CH and HP will be described uniformly the strait line PC. Seeing therefore the two strait lines AP and PC are described in the time AE with the same increase of impetus, wherewith the crooked line ABOC is described in the same time AE, that is, seeing the line APC and the line ABOC are transmitted by the same body in the same time and with equal velocities, the lines themselves are equal; which was to be demonstrated.

By the same method (if any of the semiparabolasters in the table of art. 3 of the precedent chapter be exhibited) may be found a strait line equal to the crooked line thereof, namely, by dividing the diameter into two equal parts, and proceeding as before. Yet no man hitherto hath compared any crooked with any strait line, though many geometricians of every age have endeavoured it. But the cause, why they have not done it, may be this, that there being in Euclid no definition of equality, nor any mark by which to judge of it besides congruity (which is the 8th axiom of the first Book of his Elements) a thing of no use at all in the comparing of strait and crooked; and others after Euclid (except Archimedes and Apollonius, and in our time Bonaventura) thinking the industry of the ancients had reached to all that was to be done in geometry, thought also, that all that could be propounded was either to be

deduced from what they had written, or else that it was not at all to be done : it was therefore disputed by some of those ancients themselves, whether there might be any equality at all between crooked and strait lines ; which question Archimedes, who assumed that some strait line was equal to the circumference of a circle, seems to have despised, as he had reason. And there is a late writer that granteth that between a strait and a crooked line there is equality ; but now, says he, since the fall of Adam, without the special assistance of Divine Grace it is not to be found.

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line, &c.

CHAPTER XIX.

OF ANGLES OF INCIDENCE AND REFLECTION, EQUAL BY SUPPOSITION.

1. If two strait lines falling upon another strait line be parallel, the lines reflected from them shall also be parallel.—2. If two strait lines drawn from one point fall upon another strait line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of incidence.—3. If two strait parallel lines, drawn not oppositely, but from the same parts, fall upon the circumference of a circle, the lines reflected from them, if produced they meet within the circle, will make an angle double to that which is made by two strait lines drawn from the centre to the points of incidence.—4. If two strait lines drawn from the same point without a circle fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to twice that angle, which is made by two strait lines drawn from the centre to the points of incidence, together with the angle which the incident lines themselves make.—5. If two strait lines drawn from one point

fall upon the concave circumference of a circle, and the angle they make be less than twice the angle at the centre, the lines reflected from them and meeting within the circle will make an angle, which being added to the angle of the incident lines will be equal to twice the angle at the centre.—6. If through any one point two unequal chords be drawn cutting one another, and the centre of the circle be not placed between them, and the lines reflected from them concur wheresoever, there cannot through the point, through which the two former lines were drawn, be drawn any other strait line whose reflected line shall pass through the common point of the two former lines reflected.—7. In equal chords the same is not true. 8. Two points being given in the circumference of a circle, to draw two strait lines to them, so that their reflected lines may contain any angle given.—9. If a strait line falling upon the circumference of a circle be produced till it reach the semidiameter, and that part of it, which is intercepted between the circumference and the semidiameter, be equal to that part of the semidiameter which is between the point of concurrence and the centre, the reflected line will be parallel to the semidiameter.—10. If from a point within a circle, two strait lines be drawn to the circumference, and their reflected lines meet in the circumference of the same circle, the angle made by the reflected lines will be a third part of the angle made by the incident lines.

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Angles of
incidence
and reflection.

WHETHER a body falling upon the superficies of another body and being reflected from it, do make equal angles at that superficies, it belongs not to this place to dispute, being a knowledge which depends upon the natural causes of reflection; of which hitherto nothing has been said, but shall be spoken of hereafter.

In this place, therefore, let it be supposed that the angle of incidence is equal to the angle of reflection; that our present search may be applied, not to the finding out of the causes, but some consequences of the same.

I call an *angle of incidence*, that which is made

between a strait line and another line, strait or crooked, upon which it falls, and which I call the *line reflecting*; and an *angle of reflection* equal to it, that which is made at the same point between the strait line which is reflected and the line reflecting.

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1. If two strait lines, which fall upon another strait line, be parallel, their reflected lines shall be also parallel.

If two strait lines falling upon another strait line be parallel, the lines reflected from them shall also be parallel.

Let the two strait lines AB and CD (in fig. 1), which fall upon the strait line EF , at the points B and D , be parallel; and let the lines reflected from them be BG and DH . I say, BG and DH are also parallel.

For the angles ABE and CDE are equal by reason of the parallelism of AB and CD ; and the angles GBF and HDF are equal to them by supposition; for the lines BG and DH are reflected from the lines AB and CD . Wherefore BG and DH are parallel.

2. If two strait lines drawn from the same point fall upon another strait line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle of the incident lines.

If two strait lines drawn from one point fall upon another strait line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of incidence.

From the point A (in fig. 2) let the two strait lines AB and AD be drawn; and let them fall upon the strait line EK at the points B and D ; and let the lines BI and DG be reflected from them. I say, IB and GD do converge, and that if they be produced on the other side of the line EK , they shall meet, as in F ; and that the angle BFD shall be equal to the angle BAD .

For the angle of reflection IBK is equal to the

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angle of incidence ABE ; and to the angle IBK its vertical angle EBF is equal; and therefore the angle ABE is equal to the angle EBF . Again, the angle ADE is equal to the angle of reflection GDK , that is, to its vertical angle EDF ; and therefore the two angles ABD and ADB of the triangle ABD are one by one equal to the two angles FBD and FDB of the triangle FBD ; wherefore also the third angle BAD is equal to the third angle BFD ; which was to be proved.

Coroll. I. If the strait line AF be drawn, it will be perpendicular to the strait line EK . For both the angles at E will be equal, by reason of the equality of the two angles ABE and FBE , and of the two sides AB and FB .

Coroll. II. If upon any point between B and D there fall a strait line, as AC , whose reflected line is CH , this also produced beyond C , will fall upon F ; which is evident by the demonstration above.

3. If from two points taken without a circle, two strait parallel lines, drawn not oppositely, but from the same parts, fall upon the circumference; the lines reflected from them, if produced they meet within the circle, will make an angle double to that which is made by two strait lines drawn from the centre to the points of incidence.

Let the two strait parallels AB and DC (in fig. 3) fall upon the circumference BC at the points B and C ; and let the centre of the circle be E ; and let AB reflected be BF , and DC reflected be CG ; and let the lines FB and GC produced meet within the circle in H ; and let EB and EC

If two strait parallel lines, drawn not oppositely, but from the same parts, fall upon the circumference of a circle, the lines reflected from them, if produced they meet within the circle, will make an angle double to that which is made by two strait lines drawn from the centre to the points of incidence.

be connected. I say the angle FHG is double to the angle BEC .

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For seeing AB and DC are parallels, and EB cuts AB in B , the same EB produced will cut DC somewhere; let it cut it in D ; and let DC be produced howsoever to I , and let the intersection of DC and BF be at K . The angle therefore ICH , being external to the triangle CKH , will be equal to the two opposite angles CKH and CHK . Again, ICE being external to the triangle CDE , is equal to the two angles at D and E . Wherefore the angle ICH , being double to the angle ICE , is equal to the angles at D and E twice taken; and therefore the two angles CKH and CHK are equal to the two angles at D and E twice taken. But the angle CKH is equal to the angles D and ABD , that is, D twice taken; for AB and DC being parallels, the altern angles D and ABD are equal. Wherefore CHK , that is the angle FHG is also equal to the angle at E twice taken; which was to be proved.

If two strait parallel lines, drawn, &c.

Coroll. If from two points taken within a circle two strait parallels fall upon the circumference, the lines reflected from them shall meet in an angle, double to that which is made by two strait lines drawn from the centre to the points of incidence. For the parallels AB and IC falling upon the points B and C , are reflected in the lines BH and CH , and make the angle at H double to the angle at E , as was but now demonstrated.

4. If two strait lines drawn from the same point without a circle fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to

If two strait lines drawn from the same point without a circle fall upon the circumfe-

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rence, and the lines reflected from them being produced meet within the circle, they will make an angle equal to twice that angle, which is made by two strait lines drawn from the centre to the points of incidence, together with the angle which the incident lines themselves make.

twice that angle, which is made by two strait lines drawn from the centre to the points of incidence, together with the angle which the incident lines themselves make.

Let the two strait lines AB and AC (in fig. 4) be drawn from the point A to the circumference of the circle, whose centre is D ; and let the lines reflected from them be BE and CG , and, being produced, make within the circle the angle H ; also let the two strait lines DB and DC be drawn from the centre D to the points of incidence B and C . I say, the angle H is equal to twice the angle at D together with the angle at A .

For let AC be produced howsoever to I . Therefore the angle ICH , which is external to the triangle CKH , will be equal to the two angles CKH and CHK . Again, the angle ICD , which is external to the triangle CLD , will be equal to the two angles CLD and CDL . But the angle ICH is double to the angle ICD , and is therefore equal to the angles CLD and CDL twice taken. Wherefore the angles CKH and CHK are equal to the angles CLD and CDL twice taken. But the angle CLD , being external to the triangle ALB , is equal to the two angles LAB and LBA ; and consequently CLD twice taken is equal to LAB and LBA twice taken. Wherefore CKH and CHK are equal to the angle CDL together with LAB and LBA twice taken. Also the angle CKH is equal to the angle LAB once and ABK , that is, LBA twice taken. Wherefore the angle CHK is equal to the remaining angle CDL , that is, to the angle at D , twice taken, and

the angle $L A B$, that is, the angle at A , once taken ; which was to be proved.

Coroll. If two strait converging lines, as IC and MB , fall upon the concave circumference of a circle, their reflected lines, as CH and BH , will meet in the angle H , equal to twice the angle D , together with the angle at A made by the incident lines produced. Or, if the incident lines be HB and IC , whose reflected lines CH and BM meet in the point N , the angle CNB will be equal to twice the angle D , together with the angle CKH made by the lines of incidence. For the angle CNB is equal to the angle H , that is, to twice the angle D , together with the two angles A , and NBH , that is, KBA . But the angles KBA and A are equal to the angle CKH . Wherefore the angle CNB is equal to twice the angle D , together with the angle CKH made by the lines of incidence IC and HB produced to K .

5. If two strait lines drawn from one point fall upon the concave circumference of a circle, and the angle they make be less than twice the angle at the centre, the lines reflected from them and meeting within the circle will make an angle, which being added to the angle of the incident lines, will be equal to twice the angle at the centre.

Let the two lines AB and AC (in fig. 5), drawn from the point A , fall upon the concave circumference of the circle whose centre is D ; and let their reflected lines BE and CE meet in the point E ; also let the angle A be less than twice the angle D . I say, the angles A and E together taken are equal to twice the angle D .

For let the strait lines AB and EC cut the

If two strait lines drawn from one point fall upon the concave circumference of a circle, and the angle they make be less than twice the angle at the centre, the lines reflected from them and meeting within the circle will make an angle, which being added to the angle of the incident lines will be equal to twice the angle at the centre.

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If two strait
lines drawn
from one, &c.

strait lines DC and DB in the points G and H ; and the angle BHC will be equal to the two angles EBH and E ; also the same angle BHC will be equal to the two angles D and DCH ; and in like manner the angle BGC will be equal to the two angles ACD and A , and the same angle BGC will be also equal to the two angles DBG and D . Wherefore the four angles EBH , E , ACD and A , are equal to the four angles D , DCH , DBG and D . If, therefore, equals be taken away on both sides, namely, on one side ACD and EBH , and on the other side DCH and DBG , (for the angle EBH is equal to the angle DBG , and the angle ACD equal to the angle DCH), the remainders on both sides will be equal, namely, on one side the angles A and E , and on the other the angle D twice taken. Wherefore the angles A and E are equal to twice the angle D .

Coroll. If the angle A be greater than twice the angle D , their reflected lines will diverge. For, by the corollary of the third proposition, if the angle A be equal to twice the angle D , the reflected lines BE and CE will be parallel; and if it be less, they will concur, as has now been demonstrated. And therefore, if it be greater, the reflected lines BE and CE will diverge, and consequently, if they be produced the other way, they will concur and make an angle equal to the excess of the angle A above twice the angle D ; as is evident by art. 4.

If through any
one point two
unequal chords
be drawn cut-
ting one ano-
ther, and the
centre of the
circle be not

6. If through any one point two unequal chords be drawn cutting one another, either within the circle, or, if they be produced, without it, and the centre of the circle be not placed between them, and the lines reflected from them concur where-

soever ; there cannot, through the point through which the former lines were drawn, be drawn another strait line, whose reflected line shall pass through the point where the two former reflected lines concur.

Let any two unequal chords, as BK and CH (in fig. 6), be drawn through the point A in the circle BC ; and let their reflected lines BD and CE meet in F ; and let the centre not be between AB and AC ; and from the point A let any other strait line, as AG , be drawn to the circumference between B and C . I say, GN , which passes through the point F , where the reflected lines BD and CE meet, will not be the reflected line of AG .

For let the arch BL be taken equal to the arch BG , and the strait line BM equal to the strait line BA ; and LM being drawn, let it be produced to the circumference in O . Seeing therefore BA and BM are equal, and the arch BL equal to the arch BG , and the angle MBL equal to the angle ABG , AG and ML will also be equal, and, producing GA to the circumference in I , the whole lines LO and GI will in like manner be equal. But LO is greater than GFN , as shall presently be demonstrated; and therefore also GI is greater than GN . Wherefore the angles NGC and IGB are not equal. Wherefore the line GFN is not reflected from the line of incidence AG , and consequently no other strait line, besides AB and AC , which is drawn through the point A , and falls upon the circumference BC , can be reflected to the point F ; which was to be demonstrated.

It remains that I prove LO to be greater than GN ; which I shall do in this manner. LO and

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placed between them, and the lines reflected from them concur wheresoever, there cannot through the point, through which the two former lines were drawn, be drawn any other strait line whose reflected line shall pass through the common point of the two former lines reflected.

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GN cut one another in P ; and PL is greater than PG . Seeing now $LP.PG::PN.PO$ are proportionals, therefore the two extremes LP and PO together taken, that is LO , are greater than PG and PN together taken, that is, GN ; which remained to be proved.

In equal chords
the same is not
true.

7. But if two equal chords be drawn through one point within a circle, and the lines reflected from them meet in another point, then another strait line may be drawn between them through the former point, whose reflected line shall pass through the latter point.

Let the two equal chords BC and ED (in the 7th figure) cut one another in the point A within the circle BCD ; and let their reflected lines CH and DI meet in the point F . Then dividing the arch CD equally in G , let the two chords GK and GL be drawn through the points A and F . I say, GL will be the line reflected from the chord KG . For the four chords BC , CH , ED and DI are by supposition all equal to one another; and therefore the arch BCH is equal to the arch EDI ; as also the angle BCH to the angle EDI ; and the angle AMC to its verticle angle FMD ; and the strait line DM to the strait line GM ; and, in like manner, the strait line AC to the strait line FD ; and the chords CG and GD being drawn, will also be equal; and also the angles FDG and ACG , in the equal segments GDI and GCB . Wherefore the strait lines FG and AG are equal; and, therefore, the angle FGD is equal to the angle AGC , that is, the angle of incidence equal to the angle of reflection. Wherefore the line GL is reflected from the incident line CG ; which was to be proved.

Coroll. By the very sight of the figure it is manifest, that if G be not the middle point between C and D , the reflected line GL will not pass through the point F .

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8. Two points in the circumference of a circle being given to draw two strait lines to them, so as that their reflected lines may be parallel, or contain any angle given.

Two points being given in the circumference of a circle, to draw two strait lines to them so as that their reflected lines may contain any angle given.

In the circumference of the circle, whose centre is A , (in the 8th figure) let the two points B and C be given; and let it be required to draw to them from two points taken without the circle two incident lines, so that their reflected lines may, first, be parallel.

Let AB and AC be drawn; as also any incident line DC , with its reflected line CF ; and let the angle ECD be made double to the angle A ; and let HB be drawn parallel to EC , and produced till it meet with DC produced in I . Lastly, producing AB indefinitely to K , let GB be drawn so that the angle GBK may be equal to the angle HBK , and then GB will be the reflected line of the incident line HB . I say, DC and HB are two incident lines, whose reflected lines CF and BG are parallel.

For seeing the angle ECD is double to the angle BAC , the angle HIC is also, by reason of the parallels EC and HI , double to the same BAC ; therefore also FC and GB , namely, the lines reflected from the incident lines DC and HB , are parallel. Wherefore the first thing required is done.

Secondly, let it be required to draw to the points B and C two strait lines of incidence, so that the

PART III. lines reflected from them may contain the given
 19. angle Z.

Two points being given in the circumference of a circle, &c.

To the angle ECD made at the point C , let there be added on one side the angle DCL equal to half Z , and on the other side the angle ECM equal to the angle DCL ; and let the strait line BN be drawn parallel to the strait line CM ; and let the angle KBO be made equal to the angle NBK ; which being done, BO will be the line of reflection from the line of incidence NB . Lastly, from the incident line LC , let the reflected line CO be drawn, cutting BO at O , and making the angle COB . I say, the angle COB is equal to the angle Z .

Let NB be produced till it meet with the strait line LC produced in P . Seeing, therefore, the angle LCM is, by construction, equal to twice the angle BAC , together with the angle Z ; the angle NPL , which is equal to LCM by reason of the parallels NP and MC , will also be equal to twice the same angle BAC , together with the angle Z . And seeing the two strait lines OC and OB fall from the point O upon the points C and B ; and their reflected lines LC and NB meet in the point P ; the angle NPL will be equal to twice the angle BAC together with the angle COB . But I have already proved the angle NPL to be equal to twice the angle BAC together with the angle Z . Therefore the angle COB is equal to the angle Z ; wherefore, two points in the circumference of a circle being given, I have drawn, &c.; which was to be done.

But if it be required to draw the incident lines from a point within the circle, so that the lines reflected from them may contain an angle equal to

the angle Z , the same method is to be used, saving that in this case the angle Z is not to be added to twice the angle BAC , but to be taken from it.

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9. If a strait line, falling upon the circumference of a circle, be produced till it reach the semidiameter, and that part of it which is intercepted between the circumference and the semidiameter be equal to that part of the semidiameter which is between the point of concourse and the centre, the reflected line will be parallel to the semidiameter.

If a strait line falling upon the circumference of a circle be produced till it reach the semidiameter, and that part of it, which is intercepted between the circumference and the semidiameter, be equal to that part of the semidiameter which is between the point of concourse and the centre, the reflected line will be parallel to the semidiameter.

Let any line AB (in the 9th figure) be the semidiameter of the circle whose centre is A ; and upon the circumference BD let the strait line CD fall, and be produced till it cut AB in E , so that ED and EA may be equal; and from the incident line CD let the line DF be reflected. I say, AB and DF will be parallel.

Let AG be drawn through the point D . Seeing, therefore, ED and EA are equal, the angles EDA and EAD will also be equal. But the angles FDG and EDA are equal; for each of them is half the angle EDH or FDC . Wherefore the angles FDG and EAD are equal; and consequently DF and AB are parallel; which was to be proved.

Coroll. If EA be greater than ED , then DF and AB being produced will concur; but if EA be less than ED , then BA and DH being produced will concur.

10. If from a point within a circle two strait lines be drawn to the circumference, and their reflected lines meet in the circumference of the same circle, the angle made by the lines of reflection will be a third part of the angle made by the lines of incidence.

If from a point within a circle two strait lines be drawn to the circumference, and their reflected lines meet in the cir-

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cumference of
the same circle,
the angle made
by the reflected
lines will be a
third part of the
angle made by
the incident
lines.

From the point B (in the 10th figure) taken within the circle whose centre is A , let the two straight lines BC and BD be drawn to the circumference; and let their reflected lines CE and DE meet in the circumference of the same circle at the point E . I say, the angle CED will be a third part of the angle CBD .

Let AC and AD be drawn. Seeing, therefore, the angles CED and CBD together taken are equal to twice the angle CAD (as has been demonstrated in the 5th article); and the angle CAD twice taken is quadruple to the angle CED ; the angles CED and CBD together taken will also be equal to the angle CED four times taken; and therefore if the angle CED be taken away on both sides, there will remain the angle CBD on one side, equal to the angle CED thrice taken on the other side; which was to be demonstrated.

Coroll. Therefore a point being given within a circle, there may be drawn two lines from it to the circumference, so as their reflected lines may meet in the circumference. For it is but trisecting the angle CBD , which how it may be done shall be shown in the following chapter.

CHAPTER XX.

OF THE DIMENSION OF A CIRCLE, AND THE
DIVISION OF ANGLES OR ARCHES.

1. The dimension of a circle never determined in numbers by Archimedes and others.—2. The first attempt for the finding out of the dimension of a circle by lines.—3. The second attempt for the finding out of the dimension of a circle from the consideration of the nature of crookedness.—4. The third attempt; and some things propounded to be further searched into.—5. The equation of the spiral of Archimedes with a strait line.—6. Of the analysis of geometricians by the powers of lines.

1. IN the comparing of an arch of a circle with a strait line, many and great geometricians, even from the most ancient times, have exercised their wits; and more had done the same, if they had not seen their pains, though undertaken for the common good, if not brought to perfection, vilified by those that envy the praises of other men. Amongst those ancient writers whose works are come to our hands, Archimedes was the first that brought the length of the perimeter of a circle within the limits of numbers very little differing from the truth; demonstrating the same to be less than three diameters and a seventh part, but greater than three diameters and ten seventy-one parts of the diameter. So that supposing the radius to consist of 10,000,000 equal parts, the arch of a quadrant will be between 15,714,285 and 15,704,225 of the same parts. In our times, Ludovicus Van Cullen and Willebrordus Snellius,

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The dimension of a circle never determined in numbers by Archimedes and others.

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The dimension
of a circle never
determined in
numbers by
Archimedes
and others.

with joint endeavour, have come yet nearer to the truth ; and pronounced from true principles, that the arch of a quadrant, putting, as before, 10,000,000 for radius, differs not one whole unity from the number 15,707,963 ; which, if they had exhibited their arithmetical operations, and no man had discovered any error in that long work of theirs, had been demonstrated by them. This is the furthest progress that has been made by the way of numbers ; and they that have proceeded thus far deserve the praise of industry. Nevertheless, if we consider the benefit, which is the scope at which all speculation should aim, the improvement they have made has been little or none. For any ordinary man may much sooner and more accurately find a strait line equal to the perimeter of a circle, and consequently square the circle, by winding a small thread about a given cylinder, than any geometrician shall do the same by dividing the radius into 10,000,000 equal parts. But though the length of the circumference were exactly set out, either by numbers, or mechanically, or only by chance, yet this would contribute no help at all towards the section of angles, unless happily these two problems, *to divide a given angle according to any proportion assigned, and to find a strait line equal to the arch of a circle,* were reciprocal, and followed one another. Seeing therefore the benefit proceeding from the knowledge of the length of the arch of a quadrant consists in this, that we may thereby divide an angle according to any proportion, either accurately, or at least accurately enough for common use ; and seeing this cannot be done by arithmetic, I

thought fit to attempt the same by geometry, and in this chapter to make trial whether it might not be performed by the drawing of strait and circular lines.

2. Let the square $ABCD$ (in the first figure) be described; and with the radii AB , BC , and DC , the three arches BD , CA , and AC ; of which let the two BD and CA cut one another in E , and the two BD and AC in F . The diagonals therefore BD and AC being drawn will cut one another in the centre of the square G , and the two arches BD and CA in two equal parts in H and Y ; and the arch BHD will be trisected in F and E . Through the centre G let the two strait lines KGL and MGN be drawn parallel and equal to the sides of the square AB and AD , cutting the four sides of the same square in the points K , L , M , and N ; which being done, KL will pass through F , and MN through E . Then let OP be drawn parallel and equal to the side BC , cutting the arch BFD in F , and the sides AB and DC in O and P . Therefore OF will be the sine of the arch BF , which is an arch of 30 degrees; and the same OF will be equal to half the radius. Lastly, dividing the arch BF in the middle in Q , let RQ , the sine of the arch BQ , be drawn and produced to S , so that QS be equal to RQ , and consequently RS be equal to the chord of the arch BF ; and let FS be drawn and produced to T in the side BC . I say, the strait line BT is equal to the arch BF ; and consequently that BV , the triple of BT , is equal to the arch of the quadrant $B FED$.

The first attempt for the finding out of the dimension of a circle by lines.

Let TF be produced till it meet the side BA produced in X ; and dividing OF in the middle

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The first
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the finding
out of the
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circle by lines.

in Z , let QZ be drawn and produced till it meet with the side BA produced. Seeing therefore the strait lines RS and OF are parallel, and divided in the midst in Q and Z , QZ produced will fall upon X , and XZQ produced to the side BC will cut BT in the midst in a .

Upon the strait line FZ , the fourth part of the radius AB , let the equilateral triangle aZF be constituted; and upon the centre a , with the radius aZ , let the arch ZF be drawn; which arch ZF will therefore be equal to the arch QF , the half of the arch BF . Again, let the strait line ZO be cut in the midst in b , and the strait line bO in the midst in c ; and let the bisection be continued in this manner till the last part Oc be the least that can possibly be taken; and upon it, and all the rest of the parts equal to it into which the strait line OF may be cut, let so many equilateral triangles be understood to be constituted; of which let the last be dOc . If, therefore, upon the centre d , with the radius dO , be drawn the arch Oc , and upon the rest of the equal parts of the strait line OF be drawn in like manner so many equal arches, all those arches together taken will be equal to the whole arch BF , and the half of them, namely, those that are comprehended between O and Z , or between Z and F , will be equal to the arch BQ or QF , and in sum, what part soever the strait line Oc be of the strait line OF , the same part will the arch Oc be of the arch BF , though both the arch and the chord be infinitely bisected. Now seeing the arch Oc is more crooked than that part of the arch BF which is equal to it; and seeing also

that the more the strait line Xc is produced, the more it diverges from the strait line XO , if the points O and c be understood to be moved forwards with strait motion in XO and Xc , the arch Oc will thereby be extended by little and little, till at the last it come somewhere to have the same crookedness with that part of the arch BF which is equal to it. In like manner, if the strait line Xb be drawn, and the point b be understood to be moved forwards at the same time, the arch cb will also by little and little be extended, till its crookedness come to be equal to the crookedness of that part of the arch BF which is equal to it. And the same will happen in all those small equal arches which are described upon so many equal parts of the strait line OF . It is also manifest, that by strait motion in XO and XZ all those small arches will lie in the arch BF , in the points B , Q and F . And though the same small equal arches should not be coincident with the equal parts of the arch BF in all the other points thereof, yet certainly they will constitute two crooked lines, not only equal to the two arches BQ and QF , and equally crooked, but also having their cavity towards the same parts; which how it should be, unless all those small arches should be coincident with the arch BF in all its points, is not imaginable. They are therefore coincident, and all the strait lines drawn from X , and passing through the points of division of the strait line OF , will also divide the arch BF into the same proportions into which OF is divided.

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The first attempt for the finding out of the dimension of a circle by lines.

Now seeing Xb cuts off from the point B the

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fourth part of the arch B F, let that fourth part be $B e$; and let the sine thereof, $f e$, be produced to F T in g , for so $f e$ will be the fourth part of the strait line $f g$, because as $O b$ is to O F, so is $f e$ to $f g$. But B T is greater than $f g$; and therefore the same B T is greater than four sines of the fourth part of the arch B F. And in like manner, if the arch B F be subdivided into any number of equal parts whatsoever, it may be proved that the strait line B T is greater than the sine of one of those small arches, so many times taken as there be parts made of the whole arch B F. Wherefore the strait line B T is not less than the arch B F. But neither can it be greater, because if any strait line whatsoever, less than B T, be drawn below B T, parallel to it, and terminated in the strait lines X B and X T, it would cut the arch B F; and so the sine of some one of the parts of the arch B F, taken so often as that small arch is found in the whole arch B F, would be greater than so many of the same arches; which is absurd. Wherefore the strait line B T is equal to the arch B F; and the strait line B V equal to the arch of the quadrant B F D; and B V four times taken, equal to the perimeter of the circle described with the radius A B. Also the arch B F and the strait line B T are everywhere divided into the same proportions; and consequently any given angle, whether greater or less than B A F, may be divided into any proportion given.

But the strait line B V, though its magnitude fall within the terms assigned by Archimedes, is found, if computed by the canon of signs, to be somewhat greater than that which is exhibited by

the Rudolphine numbers. Nevertheless, if in the place of BT , another strait line, though never so little less, be substituted, the division of angles is immediately lost, as may by any man be demonstrated by this very scheme.

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The first attempt for the finding out of the dimension of a circle by lines.

Howsoever, if any man think this my strait line BV to be too great, yet, seeing the arch and all the parallels are everywhere so exactly divided, and BV comes so near to the truth, I desire he would search out the reason, why, granting BV to be precisely true, the arches cut off should not be equal.

But some man may yet ask the reason why the strait lines, drawn from X through the equal parts of the arch BF , should cut off in the tangent BV so many strait lines equal to them, seeing the connected straight line XV passes not through the point D , but cuts the strait line AD produced in l ; and consequently require some determination of this problem. Concerning which, I will say what I think to be the reason, namely, that whilst the magnitude of the arch doth not exceed the magnitude of the radius, that is, the magnitude of the tangent BC , both the arch and the tangent are cut alike by the strait lines drawn from X ; otherwise not. For AV being connected, cutting the arch BHD in I , if XC being drawn should cut the same arch in the same point I , it would be as true that the arch BI is equal to the radius BC , as it is true that the arch BF is equal to the strait line BT ; and drawing XK it would cut the arch BI in the midst in i ; also drawing Ai and producing it to the tangent BC in k , the strait line Bk will be the

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tangent of the arch $B i$; (which arch is equal to half the radius) and the same strait line $B k$ will be equal to the strait line $k I$. I say all this is true, if the preceding demonstration be true; and consequently the proportional section of the arch and its tangent proceeds hitherto. But it is manifest by the golden rule, that taking $B h$ double to $B T$, the line $X h$ shall not cut off the arch $B E$, which is double to the arch $B F$, but a much greater. For the magnitude of the straight lines $X M$, $X B$, and $M E$, being known (in numbers), the magnitude of the strait line cut off in the tangent by the strait line $X E$ produced to the tangent, may also be known; and it will be found to be less than $B h$; Wherefore the strait line $X h$ being drawn, will cut off a part of the arch of the quadrant greater than the arch $B E$. But I shall speak more fully in the next article concerning the magnitude of the arch $B I$.

And let this be the first attempt for the finding out of the dimension of a circle by the section of the arch $B F$.

The second attempt for the finding out of the dimension of a circle from the consideration of the nature of crookedness.

3. I shall now attempt the same by arguments drawn from the nature of the crookedness of the circle itself; but I shall first set down some premises necessary for this speculation; and

First, if a strait line be bowed into an arch of a circle equal to it, as when a stretched thread, which toucheth a right cylinder, is so bowed in every point, that it be everywhere coincident with the perimeter of the base of the cylinder, the flexion of that line will be equal, in all its points; and consequently the crookedness of the arch of a

circle is everywhere uniform ; which needs no other demonstration than this, that the perimeter of a circle is an uniform line.

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The second attempt for the finding, &c.

Secondly, and consequently : if two unequal arches of the same circle be made by the bowing of two strait lines equal to them, the flexion of the longer line, whilst it is bowed into the greater arch, is greater than the flexion of the shorter line, whilst it is bowed into the lesser arch, according to the proportion of the arches themselves ; and consequently, the crookedness of the greater arch is to the crookedness of the lesser arch, as the greater arch is to the lesser arch.

Thirdly : if two unequal circles and a strait line touch one another in the same point, the crookedness of any arch taken in the lesser circle, will be greater than the crookedness of an arch equal to it taken in the greater circle, in reciprocal proportion to that of the radii with which the circles are described ; or, which is all one, any strait line being drawn from the point of contact till it cut both the circumferences, as the part of that strait line cut off by the circumference of the greater circle to that part which is cut off by the circumference of the lesser circle.

For let A B and A C (in the second figure) be two circles, touching one another, and the strait line A D in the point A ; and let their centres be E and F ; and let it be supposed, that as A E is to A F, so is the arch A B to the arch A H. I say the crookedness of the arch A C is to the crookedness of the arch A H, as A E is to A F. For let the strait line A D be supposed to be equal to the arch A B, and the strait line A G to the arch A C ; and

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The second
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let $A D$, for example, be double to $A G$. Therefore, by reason of the likeness of the arches $A B$ and $A C$, the strait line $A B$ will be double to the strait line $A C$, and the radius $A E$ double to the radius $A F$, and the arch $A B$ double to the arch $A H$. And because the strait line $A D$ is so bowed to be coincident with the arch $A B$ equal to it, as the strait line $A G$ is bowed to be coincident with the arch $A C$ equal also to it, the flexion of the strait line $A G$ into the crooked line $A C$ will be equal to the flexion of the strait line $A D$ into the crooked line $A B$. But the flexion of the strait line $A D$ into the crooked line $A B$ is double to the flexion of the strait line $A G$ into the crooked line $A H$; and therefore the flexion of the strait line $A G$ into the crooked line $A C$ is double to the flexion of the same strait line $A G$ into the crooked line $A H$. Wherefore, as the arch $A B$ is to the arch $A C$ or $A H$; or as the radius $A E$ is to the radius $A F$; or as the chord $A B$ is to the chord $A C$; so reciprocally is the flexion or uniform crookedness of the arch $A C$, to the flexion or uniform crookedness of the arch $A H$, namely, here double. And this may by the same method be demonstrated in circles whose perimeters are to one another triple, quadruple, or in whatsoever given proportion. The crookedness therefore of two equal arches taken in several circles are in proportion reciprocal to that of their radii, or like arches, or like chords; which was to be demonstrated.

Let the square $A B C D$ be again described (in the third figure), and in it the quadrants $A B D$, $B C A$ and $D A C$; and dividing each side of the square $A B C D$ in the midst in E , F , G and H , let

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EG and FH be connected, which will cut one another in the centre of the square at I, and divide the arch of the quadrant ABD into three equal parts in K and L. Also the diagonals AC and BD being drawn will cut one another in I, and divide the arches BKD and CLA into two equal parts in M and N. Then with the radius BF let the arch FE be drawn, cutting the diagonal BD in O; and dividing the arch BM in the midst in P, let the strait line E*a* equal to the chord BP be set off from the point E in the arch EF, and let the arch *ab* be taken equal to the arch *Oa*, and let Ba and B*b* be drawn and produced to the arch AN in *c* and *d*; and lastly, let the strait line A*d* be drawn. I say the strait line A*d* is equal to the arch AN or BM.

I have proved in the preceding article, that the arch EO is twice as crooked as the arch BP, that is to say, that the arch EO is so much more crooked than the arch BP, as the arch BP is more crooked than the strait line E*a*. The crookedness therefore of the chord E*a*, of the arch BP, and of the arch EO, are as 0, 1, 2. Also the difference between the arches EO and EO, the difference between the arches EO and E*a*, and the difference between the arches EO and E*b*, are as 0, 1, 2. So also the difference between the arches AN and AN, the difference between the arches AN and A*c*, and the difference between the arches AN and A*d*, are as 0, 1, 2; and the strait line A*c* is double to the chord BP or E*a*, and the strait line A*d* double to the chord E*b*.

Again, let the strait line BF be divided in the midst in Q, and the arch BP in the midst in R;

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and describing the quadrant BQS (whose arch QS is a fourth part of the arch of the quadrant BMD , as the arch BR is a fourth part of the arch BM , which is the arch of the semiquadrant ABM) let the chord Se equal to the chord BR be set off from the point S in the arch SQ ; and let Be be drawn and produced to the arch AN in f ; which being done, the strait line Af will be quadruple to the chord BR or Se . And seeing the crookedness of the arch Se , or of the arch Ac , is double to the crookedness of the arch BR , the excess of the crookedness of the arch Af above the crookedness of the arch Ac will be subduple to the excess of the crookedness of the arch Ac above the crookedness of the arch AN ; and therefore the arch Nc will be double to the arch cf . Wherefore the arch cd is divided in the midst in f , and the arch Nf is $\frac{2}{4}$ of the arch Nd . And in like manner if the arch BR be bisected in V , and the strait line BQ in X , and the quadrant BXY be described, and the strait line Yg equal to the chord BV be set off from the point Y in the arch YX , it may be demonstrated that the strait line Bg being drawn and produced to the arch AN , will cut the arch fd into two equal parts, and that a strait line drawn from A to the point of that section, will be equal to eight chords of the arch BV , and so on perpetually; and consequently, that the strait line Ad is equal to so many equal chords of equal parts of the arch BM , as may be made by infinite bisections. Wherefore the strait line Ad is equal to the arch BM or AN , that is, to half the arch of the quadrant ABD or BCA .

Coroll. An arch being given not greater than

the arch of a quadrant (for being made greater, it comes again towards the radius BA produced, from which it receded before) if a strait line double to the chord of half the given arch be adapted from the beginning of the arch, and by how much the arch that is subtended by it is greater than the given arch, by so much a greater arch be subtended by another strait line, this strait line shall be equal to the first given arch.

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Supposing the strait line BV (in fig. 1) be equal to the arch of the quadrant BHD, and AV be connected cutting the arch BHD in I, it may be asked what proportion the arch BI has to the arch ID. Let therefore the arch AY be divided in the midst in o, and in the strait line AD let Ap be taken equal, and Aq double to the drawn chord Ao. Then upon the centre A, with the radius Aq, let an arch of a circle be drawn cutting the arch AY in r, and let the arch Yr be doubled at t; which being done, the drawn strait line At (by what has been last demonstrated) will be equal to the arch AY. Again, upon the centre A with the radius At let the arch tu be drawn cutting AD in u; and the strait line Au will be equal to the arch AY. From the point u let the strait line us be drawn equal and parallel to the strait line AB, cutting MN in x, and bisected by MN in the same point x. Therefore the strait line Ax being drawn and produced till it meet with BC produced in V, it will cut off BV double to Bs, that is, equal to the arch BHD. Now let the point, where the strait line AV cuts the arch BHD, be I; and let the arch DI be divided in the midst in y; and in the strait line DC, let Dz

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be taken equal, and $D\delta$ double to the drawn chord $D\gamma$; and upon the centre D with the radius $D\delta$ let an arch of a circle be drawn cutting the arch BHD in the point n ; and let the arch nm be taken equal to the arch In ; which being done, the strait line Dm will (by the last foregoing corollary) be equal to the arch DI . If now the strait lines Dm and CV be equal, the arch BI will be equal to the radius AB or BC ; and consequently XC being drawn, will pass through the point I . Moreover, if the semicircle $BHD\epsilon$ being completed, the strait lines ϵI and BI be drawn, making a right angle (in the semicircle) at I , and the arch BI be divided in the midst at i , it will follow that Ai being connected will be parallel to the strait line ϵI , and being produced to BC in k , will cut off the strait line Bk equal to the strait line kI , and equal also to the strait line $A\gamma$ cut off in AD by the strait line ϵI . All which is manifest, supposing the arch BI and the radius BC to be equal.

But that the arch BI and the radius BC are precisely equal, cannot (how true soever it be) be demonstrated, unless that be first proved which is contained in art. 1, namely, that the strait lines drawn from X through the equal parts of OF (produced to a certain length) cut off so many parts also in the tangent BC severally equal to the several arches cut off; which they do most exactly as far as BC in the tangent, and BI in the arch BE ; insomuch that no inequality between the arch BI and the radius BC can be discovered either by the hand or by ratiocination. It is therefore to be further enquired, whether the

strait line $A V$ cut the arch of the quadrant in I in the same proportion as the point C divides the strait line $B V$, which is equal to the arch of the quadrant. But however this be, it has been demonstrated that the strait line $B V$ is equal to the arch $B H D$.

4. I shall now attempt the same dimension of a circle another way, assuming the two following lemmas. The third attempt; and some things propounded to be further searched into.

Lemma I. If to the arch of a quadrant, and the radius, there be taken in continual proportion a third line Z ; then the arch of the semiquadrant, half the chord of the quadrant, and Z , will also be in continual proportion.

For seeing the radius is a mean proportional between the chord of a quadrant and its semichord, and the same radius a mean proportional between the arch of the quadrant and Z , the square of the radius will be equal as well to the rectangle made of the chord and semichord of the quadrant, as to the rectangle made of the arch of the quadrant and Z ; and these two rectangles will be equal to one another. Wherefore, as the arch of a quadrant is to its chord, so reciprocally is half the chord of the quadrant to Z . But as the arch of the quadrant is to its chord, so is half the arch of the quadrant to half the chord of the quadrant. Wherefore, as half the arch of the quadrant is to half the chord of the quadrant (or to the sine of 45 degrees), so is half the chord of the quadrant to Z ; which was to be proved.

Lemma II. The radius, the arch of the semiquadrant, the sine of 45 degrees, and the semiradius, are proportional.

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For seeing the sine of 45 degrees is a mean proportional between the radius and the semiradius; and the same sine of 45 degrees is also a mean proportional (by the precedent lemma) between the arch of 45 degrees and Z ; the square of the sine of 45 degrees will be equal as well to the rectangle made of the radius and semiradius, as to the rectangle made of the arch of 45 degrees and Z . Wherefore, as the radius is to the arch of 45 degrees, so reciprocally is Z to the semiradius; which was to be demonstrated.

Let now $ABCD$ (in fig. 4) be a square; and with the radii AB , BC and DA , let the three quadrants ABD , BCA and DAC , be described; and let the strait lines EF and GH , drawn parallel to the sides BC and AB , divide the square $ABCD$ into four equal squares. They will therefore cut the arch of the quadrant ABD into three equal parts in I and K , and the arch of the quadrant BCA into three equal parts in K and L . Also let the diagonals AC and BD be drawn, cutting the arches BID and ALC in M and N . Then upon the centre H with the radius HF equal to half the chord of the arch BMD , or to the sine of 45 degrees, let the arch FO be drawn cutting the arch CK in O ; and let AO be drawn and produced till it meet with BC produced in P ; also let it cut the arch BMD in Q , and the strait line DC in R . If now the strait line HQ be equal to the strait line DR , and being produced to DC in S , cut off DS equal to half the strait line BP ; I say then the strait line BP will be equal to the arch BMD .

For seeing PBA and ADR are like triangles,

it will be as P B to the radius B A or A D, so A D to D R ; and therefore as well P B, A D and D R, as P B, A D (or A Q) and Q H are in continual proportion ; and producing H O to D C in T, D T will be equal to the sine of 45 degrees, as shall by and by be demonstrated. Now D S, D T and D R are in continual proportion by the first lemma ; and by the second lemma D C. D S :: D R. D F are proportionals. And thus it will be, whether B P be equal or not equal to the arch of the quadrant B M D. But if they be equal, it will then be, as that part of the arch B M D which is equal to the radius, is to the remainder of the same arch B M D ; so A Q to H Q, or so B C to C P. And then will B P and the arch B M D be equal. But it is not demonstrated that the strait lines H Q and D R are equal ; though if from the point B there be drawn (by the construction of fig. 1) a strait line equal to the arch B M D, then D R to H Q, and also the half of the strait line B P to D S, will always be so equal, that no inequality can be discovered between them. I will therefore leave this to be further searched into. For though it be almost out of doubt, that the strait line B P and the arch B M D are equal, yet that may not be received without demonstration ; and means of demonstration the circular line admitteth none that is not grounded upon the nature of flexion, or of angles. But by that way I have already exhibited a strait line equal to the arch of a quadrant in the first and second aggression.

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It remains that I prove D T to be equal to the sine of 45 degrees.

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In BA produced let AV be taken equal to the sine of 45 degrees; and drawing and producing VH , it will cut the arch of the quadrant CNA in the midst in N , and the same arch again in O , and the strait line DC in T , so that DT will be equal to the sine of 45 degrees, or to the strait line AV ; also the strait line VH will be equal to the strait line HI , or the sine of 60 degrees.

For the square of AV is equal to two squares of the semiradius; and consequently the square of VH is equal to three squares of the semiradius. But HI is a mean proportional between the semiradius and three semiradii; and, therefore, the square of HI is equal to three squares of the semiradius. Wherefore HI is equal to HV . But because AD is cut in the midst in H , therefore VH and HT are equal; and, therefore, also DT is equal to the sine of 45 degrees. In the radius BA let BX be taken equal to the sine of 45 degrees; for so VX will be equal to the radius; and it will be as VA to AH the semiradius, so VX the radius to XN the sine of 45 degrees. Wherefore VH produced passes through N . Lastly, upon the centre V with the radius VA let the arch of a circle be drawn cutting VH in Y ; which being done, VY will be equal to HO (for HO is, by construction, equal to the sine of 45 degrees) and YH will be equal to OT ; and, therefore, VT passes through O . All which was to be demonstrated.

I will here add certain problems, of which if any analyst can make the construction, he will thereby be able to judge clearly of what I have now said concerning the dimension of a circle. Now

these problems are nothing else (at least to sense) but certain symptoms accompanying the construction of the first and third figure of this chapter.

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Describing, therefore, again, the square $A B C D$ (in fig. 5) and the three quadrants $A B D$, $B C A$ and $D A C$, let the diagonals $A C$ and $B D$ be drawn, cutting the arches $B H D$ and $C I A$ in the middle in H and I ; and the strait lines $E F$ and $G L$, dividing the square $A B C D$ into four equal squares, and trisecting the arches $B H D$ and $C I A$, namely, $B H D$ in K and M , and $C I A$ in M and O . Then dividing the arch $B K$ in the midst in P , let $Q P$ the sine of the arch $B P$, be drawn and produced to R , so that $Q R$ be double to $Q P$; and, connecting $K R$, let it be produced one way to $B C$ in S , and the other way to $B A$ produced in T . Also let $B V$ be made triple to $B S$, and consequently, (by the second article of this chapter) equal to the arch $B D$. This construction is the same with that of the first figure, which I thought fit to renew discharged of all lines but such as are necessary for my present purpose.

In the first place, therefore, if $A V$ be drawn, cutting the arch $B H D$ in X , and the side $D C$ in Z , I desire some analyst would, if he can, give a reason why the strait lines $T E$ and $T C$ should cut the arch $B D$, the one in Y , the other in X , so as to make the arch $B Y$ equal to the arch $Y X$; or if they be not equal, that he would determine their difference.

Secondly, if in the side $D A$, the strait line $D a$ be taken equal to $D Z$, and $V a$ be drawn; why $V a$ and $V B$ should be equal; or if they be not equal, what is the difference.

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Thirdly, drawing Zb parallel and equal to the side CB , cutting the arch BHD in c , and drawing the strait line Ac , and producing it to BV in d ; why Ad should be equal and parallel to the strait line aV , and consequently equal also to the arch BD .

Fourthly, drawing eK the sine of the arch BK , and taking (in eA produced) ef equal to the diagonal AC , and connecting fC ; why fC should pass through a (which point being given, the length of the arch BHD is also given) and c ; and why fe and fc should be equal; or if not, why unequal.

Fifthly, drawing fZ , I desire he would show, why it is equal to BV , or to the arch BD ; or if they be not equal, what is their difference.

Sixthly, granting fZ to be equal to the arch BD , I desire he would determine whether it fall all without the arch BCA , or cut the same, or touch it, and in what point.

Seventhly, the semicircle BDg being completed, why gI being drawn and produced, should pass through X , by which point X the length of the arch BD is determined. And the same gI being yet further produced to DC in h , why Ad , which is equal to the arch BD , should pass through that point h .

Eighthly, upon the centre of the square $ABCD$, which let be k , the arch of the quadrant EiL being drawn, cutting eK produced in i , why the drawn strait line iX should be parallel to the side CD .

Ninthly, in the sides BA and BC taking gl and Bm severally equal to half BV , or to the arch BH , and drawing mn parallel and equal to the

side B A, cutting the arch B D in *o*, why the strait line which connects V *l* should pass through the point *o*.

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Tenthly, I would know of him why the strait line which connects *a* H should be equal to B *m* ; or if not, how much it differs from it.

The analyst that can solve these problems without knowing first the length of the arch B D, or using any other known method than that which proceeds by perpetual bisection of an angle, or is drawn from the consideration of the nature of flexion, shall do more than ordinary geometry is able to perform. But if the dimension of a circle cannot be found by any other method, then I have either found it, or it is not at all to be found.

From the known length of the arch of a quadrant, and from the proportional division of the arch and of the tangent B C, may be deduced the section of an angle into any given proportion ; as also the squaring of the circle, the squaring of a given sector, and many the like propositions, which it is not necessary here to demonstrate. I will, therefore, only exhibit a strait line equal to the spiral of Archimides, and so dismiss this speculation.

5. The length of the perimeter of a circle being found, that strait line is also found, which touches a spiral at the end of its first conversion. For upon the centre A (in fig. 6) let the circle B C D E be described ; and in it let Archimedes' spiral A F G H B be drawn, beginning at A and ending at B. Through the centre A let the strait line C E be drawn, cutting the diameter B D at right angles ; and let it be produced to I, so that A I be equal to the perimeter B C D E B. Therefore I B being drawn will touch

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the spiral $A F G H B$ in B ; which is demonstrated by Archimedes in his book *De Spiralibus*.

And for a strait line equal to the given spiral $A F G H B$, it may be found thus.

Let the strait line $A I$, which is equal to the perimeter $B C D E$, be bisected in K ; and taking $K L$ equal to the radius $A B$, let the rectangle $I L$ be completed. Let $M L$ be understood to be the axis, and $K L$ the base of a parabola, and let $M K$ be the crooked line thereof. Now if the point M be conceived to be so moved by the concurrence of two movents, the one from $I M$ to $K L$ with velocity encreasing continually in the same proportion with the times, the other from $M L$ to $I K$ uniformly, that both those motions begin together in M and end in K ; Galilæus has demonstrated that by such motion of the point M , the crooked line of a parabola will be described. Again, if the point A be conceived to be moved uniformly in the strait line $A B$, and in the same time to be carried round upon the centre A by the circular motion of all the points between A and B ; Archimedes has demonstrated that by such motion will be described a spiral line. And seeing the circles of all these motions are concentric in A ; and the interior circle is always less than the exterior in the proportion of the times in which $A B$ is passed over with uniform motion; the velocity also of the circular motion of the point A will continually increase proportionally to the times. And thus far the generations of the parabolical line $M K$, and of the spiral line $A F G H B$, are like. But the uniform motion in $A B$ concurring with circular motion in the perimeters of all the concentric circles, describes that

circle, whose centre is A, and perimeter B C D E; and, therefore, that circle is (by the coroll. of art. 1, chap. XVI) the aggregate of all the velocities together taken of the point A whilst it describes the spiral A F G H B. Also the rectangle I K L M is the aggregate of all the velocities together taken of the point M, whilst it describes the crooked line M K. And, therefore the whole velocity by which the parabolical line M K is described, is to the whole velocity with which the spiral line A F G H B is described in the same time, as the rectangle I K L M is to the circle B C D E, that is to the triangle A I B. But because A I is bisected in K, and the strait lines I M and A B are equal, therefore the rectangle I K L M and the triangle A I B are also equal. Wherefore the spiral line A F G H B, and the parabolical line M K, being described with equal velocity and in equal times, are equal to one another. Now, in the first article of chap. XVIII, a strait line is found out equal to any parabolical line. Wherefore also a strait line is found out equal to a given spiral line of the first revolution described by Archimedes; which was to be done.

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The equation of the spiral of Archimedes with a strait line

6. In the sixth chapter, which is of *Method*, that which I should there have spoken of the analytics of geometricians I thought fit to defer, because I could not there have been understood, as not having then so much as named *lines, superficies, solids, equal and unequal, &c.* Wherefore I will in this place set down my thoughts concerning it.

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Analysis is continual reasoning from the definitions of the terms of a proposition we suppose true, and again from the definitions of the terms of

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those definitions, and so on, till we come to some things known, the composition whereof is the demonstration of the truth or falsity of the first supposition; and this composition or demonstration is that we call *Synthesis*. *Analytica*, therefore, is that art, by which our reason proceeds from something supposed, to principles, that is, to prime propositions, or to such as are known by these, till we have so many known propositions as are sufficient for the demonstration of the truth or falsity of the thing supposed. *Synthetica* is the art itself of demonstration. *Synthesis*, therefore, and *analysis*, differ in nothing, but in proceeding forwards or backwards; and *Logistica* comprehends both. So that in the analysis or synthesis of any question, that is to say, of any problem, the terms of all the propositions ought to be convertible; or if they be enunciated hypothetically, the truth of the consequent ought not only to follow out of the truth of its antecedent, but contrarily also the truth of the antecedent must necessarily be inferred from the truth of the consequent. For otherwise, when by resolution we are arrived at principles, we cannot by composition return directly back to the thing sought for. For those terms which are the first in analysis, will be the last in synthesis; as for example, when in *resolving*, we say, these two rectangles are equal, and therefore their sides are reciprocally proportional, we must necessarily in *compounding* say, the sides of these rectangles are reciprocally proportional, and therefore the rectangles themselves are equal; which we could not say, unless *rectangles have their sides recipro-*

cally proportional, and rectangles are equal, were terms convertible.

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Now in every analysis, that which is sought is the proportion of two quantities; by which proportion, a figure being described, the quantity sought for may be exposed to sense. And this *exposition* is the end and solution of the question, or the construction of the problem.

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And seeing analysis is reasoning from something supposed, till we come to principles, that is, to definitions, or to theorems formerly known; and seeing the same reasoning tends in the last place to some equation, we can therefore make no end of resolving, till we come at last to the causes themselves of equality and inequality, or to theorems formerly demonstrated from those causes; and so have a sufficient number of those theorems for the demonstration of the thing sought for.

And seeing also, that the end of the analytics is either the construction of such a problem as is possible, or the detection of the impossibility thereof; whensoever the problem may be solved, the analyst must not stay, till he come to those things which contain the efficient cause of that whereof he is to make construction. But he must of necessity stay, when he comes to prime propositions; and these are definitions. These definitions therefore must contain the efficient cause of his construction; I say of his construction, not of the conclusion which he demonstrates; for the cause of the conclusion is contained in the premised propositions; that is to say, the truth of the proposition he proves is drawn from the propositions which prove the same.

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But the cause of his construction is in the things themselves, and consists in motion, or in the course of motions. Wherefore those propositions, in which analysis ends, are definitions, but such as signify in what manner the construction or generation of the thing proceeds. For otherwise, when he goes back by synthesis to the proof of his problem, he will come to no demonstration at all; there being no true demonstration but such as is scientific; and no demonstration is scientific, but that which proceeds from the knowledge of the causes from which the construction of the problem is drawn. To collect therefore what has been said into few words; ANALYSIS *is ratiocination from the supposed construction or generation of a thing to the efficient cause or coefficient causes of that which is constructed or generated.* And SYNTHESIS *is ratiocination from the first causes of the construction, continued through all the middle causes till we come to the thing itself which is constructed or generated.*

But because there are many means by which the same thing may be generated, or the same problem be constructed, therefore neither do all geometers, nor doth the same geometrician always, use one and the same method. For, if to a certain quantity given, it be required to construct another quantity equal, there may be some that will inquire whether this may not be done by means of some motion. For there are quantities, whose equality and inequality may be argued from motion and time, as well as from congruence; and there is motion, by which two quantities, whether lines or superficies, though one of them be crooked, the

other strait, may be made congruous or coincident. And this method Archimedes made use of in his book *De Spiralibus*. Also the equality or inequality of two quantities may be found out and demonstrated from the consideration of weight, as the same Archimedes did in his quadrature of the parabola. Besides, equality and inequality are found out often by the division of the two quantities into parts which are considered as indivisible; as Cavallerius Bonaventura has done in our time, and Archimedes often. Lastly, the same is performed by the consideration of the powers of lines, or the roots of those powers, and by the multiplication, division, addition, and subtraction, as also by the extraction of the roots of those powers, or by finding where strait lines of the same proportion terminate. For example, when any number of strait lines, how many soever, are drawn from a strait line and pass all through the same point, look what proportion they have, and if their parts continued from the point retain everywhere the same proportion, they shall all terminate in a strait line. And the same happens if the point be taken between two circles. So that the places of all their points of termination make either strait lines, or circumferences of circles, and are called *plane places*. So also when strait parallel lines are applied to one strait line, if the parts of the strait line to which they are applied be to one another in proportion duplicate to that of the contiguous applied lines, they will all terminate in a conical section; which section, being the place of their termination, is called a *solid place*, because it serves for the finding out of the quantity of any

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equation which consists of three dimensions. There are therefore three ways of finding out the cause of equality or inequality between two given quantities; namely, first, by the computation of *motions*; for by equal motion, and equal time, equal spaces are described; and ponderation is motion. Secondly, by *indivisibles*: because all the parts together taken are equal to the whole. And thirdly, by the *powers*: for when they are equal, their roots also are equal; and contrarily, the powers are equal, when their roots are equal. But if the question be much complicated, there cannot by any of these ways be constituted a certain rule, from the supposition of which of the unknown quantities the analysis may best begin; nor out of the variety of equations, that at first appear, which we were best to choose; but the success will depend upon dexterity, upon formerly acquired science, and many times upon fortune.

For no man can ever be a good analyst without being first a good geometrician; nor do the rules of analysis make a geometrician, as synthesis doth; which begins at the very elements, and proceeds by a logical use of the same. For the true teaching of geometry is by synthesis, according to Euclid's method; and he that hath Euclid for his master, may be a geometrician without Vieta, though Vieta was a most admirable geometrician; but he that has Vieta for his master, not so, without Euclid.

And as for that part of analysis which works by the powers, though it be esteemed by some geometricians, not the chiefest, to be the best way of solving all problems, yet it is a thing of no great extent; it being all contained in the doctrine of

rectangles, and rectangled solids. So that although they come to an equation which determines the quantity sought, yet they cannot sometimes by art exhibit that quantity in a plane, but in some conic section; that is, as geometricians say, not geometrically, but mechanically. Now such problems as these, they call *solid*; and when they cannot exhibit the quantity sought for with the help of a conic section, they call it a *lineary* problem. And therefore in the quantities of angles, and of the arches of circles, there is no use at all of the analytics which proceed by the powers; so that the ancients pronounced it impossible to exhibit in a plane the division of angles, except bisection, and the bisection of the bisected parts, otherwise than mechanically. For Pappus, (before the 31st proposition of his fourth book) distinguishing and defining the several kinds of problems, says that "some are *plane*, others *solid*, and others *lineary*. Those, therefore, which may be solved by strait lines and the circumferences of circles, (that is, which may be described with the rule and compass, without any other instrument), are fitly called *plane*; for the lines, by which such problems are found out, have their generation in a plane. But those which are solved by the using of some one or more conic sections in their construction, are called *solid*, because their construction cannot be made without using the superficies of solid figures, namely, of cones. There remains the third kind, which is called *lineary*, because other lines besides those already mentioned are made use of in their construction, &c." And a

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little after he says, “of this kind are the *spiral* lines, the *quadratrices*, the *conchoeides*, and the *cissoeides*. And geometricians think it no small fault, when for the finding out of a plane problem any man makes use of conics, or new lines.” Now he ranks the trisection of an angle among solid problems, and the quinquesection among lineary. But what! are the ancient geometricians to be blamed, who made use of the quadratrix for the finding out of a strait line equal to the arch of a circle? And Pappus himself, was he faulty, when he found out the trisection of an angle by the help of an hyperbole? Or am I in the wrong, who think I have found out the construction of both these problems by the rule and compass only? Neither they, nor I. For the ancients made use of this analysis which proceeds by the powers; and with them it was a fault to do that by a more remote power, which might be done by a nearer; as being an argument that they did not sufficiently understand the nature of the thing.

The virtue of this kind of analysis consists in the changing and turning and tossing of rectangles and analogisms; and the skill of analysts is mere logic, by which they are able methodically to find out whatsoever lies hid either in the subject or predicate of the conclusion sought for. But this doth not properly belong to algebra, or the analytics specious, symbolical, or cossick; which are, as I may say, the *brachygraphy* of the analytics, and an art neither of teaching nor learning geometry, but of registering with brevity and celerity the inventions of geometricians. For though it be easy to discourse

by symbols of very remote propositions; yet whether such discourse deserve to be thought very profitable, when it is made without any ideas of the things themselves, I know not.

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CHAPTER XXI.

OF CIRCULAR MOTION.

1. In simple motion, every strait line taken in the body moved is so carried, that it is always parallel to the places in which it formerly was.—2. If circular motion be made about a resting centre, and in that circle there be an epicycle, whose revolution is made the contrary way, in such manner that in equal times it make equal angles, every strait line taken in that epicycle will be so carried, that it will always be parallel to the places in which it formerly was.—3. The properties of simple motion.—4. If a fluid be moved with simple circular motion, all the points taken in it will describe their circles in times proportional to the distances from the centre.—5. Simple motion dissipates heterogeneous and congregates homogeneous bodies.—6. If a circle made by a movent moved with simple motion be commensurable to another circle made by a point which is carried about by the same movent, all the points of both the circles will at some time return to the same situation. 7. If a sphere have simple motion, its motion will more dissipate heterogeneous bodies by how much it is more remote from the poles.—8. If the simple circular motion of a fluid body be hindered by a body which is not fluid, the fluid body will spread itself upon the superficies of that body.—9. Circular motion about a fixed centre casteth off by the tangent such things as lie upon the circumference and stick not to it. 10. Such things, as are moved with simple circular motion, beget simple circular motion.—11. If that which is so moved have one side hard and the other side fluid, its motion will not be perfectly circular.

1. I HAVE already defined *simple motion* to be In simple motion, &c. that, in which the several points taken in a moved

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In simple motion, every strait line taken in the body moved is so carried, that it is always parallel to the places in which it formerly was.

body do in several equal times describe several equal arches. And therefore in simple circular motion it is necessary that every strait line taken in the moved body be always carried parallel to itself; which I thus demonstrate.

First, let AB (in the first figure) be any strait line taken in any solid body; and let AD be any arch drawn upon any centre C and radius CA . Let the point B be understood to describe towards the same parts the arch BE , like and equal to the arch AD . Now in the same time in which the point A transmits the arch AD , the point B , which by reason of its simple motion is supposed to be carried with a velocity equal to that of A , will transmit the arch BE ; and at the end of the same time the whole AB will be in DE ; and therefore AB and DE are equal. And seeing the arches AD and BE are like and equal, their subtending strait lines AD and BE will also be equal; and therefore the four-sided figure $ABDE$ will be a parallelogram. Wherefore AB is carried parallel to itself. And the same may be proved by the same method, if any other strait line be taken in the same moved body in which the strait line AB was taken. So that all strait lines, taken in a body moved with simple circular motion, will be carried parallel to themselves.

Coroll. I. It is manifest that the same will also happen in any body which hath simple motion, though not circular. For all the points of any strait line whatsoever will describe lines, though not circular, yet equal; so that though the crooked lines AD and BE were not arches of circles, but of parabolas, ellipses, or of any other figures,

yet both they, and their subtenses, and the strait lines which join them, would be equal and parallel.

Coroll. II. It is also manifest, that the radii of the equal circles AD and BE , or the axis of a sphere, will be so carried, as to be always parallel to the places in which they formerly were. For the strait line BF drawn to the centre of the arch BE being equal to the radius AC , will also be equal to the strait line FE or CD ; and the angle BFE will be equal to the angle ACD . Now the intersection of the strait lines CA and BE being at G , the angle CGE (seeing BE and AD are parallel) will be equal to the angle DAC . But the angle EBF is equal to the same angle DAC ; and therefore the angles CGE and EBF are also equal. Wherefore AC and BF are parallel; which was to be demonstrated.

2. Let there be a circle given (in the second figure) whose centre is A , and radius AB ; and upon the centre B and any radius BC let the epicycle CDE be described. Let the centre B be understood to be carried about the centre A , and the whole epicycle with it till it be coincident with the circle FGH , whose centre is I ; and let BAI be any angle given. But in the time that the centre B is moved to I , let the epicycle CDE have a contrary revolution upon its own centre, namely from E by D to C , according to the same proportions; that is, in such manner, that in both the circles, equal angles be made in equal times. I say EC , the axis of the epicycle, will be always carried parallel to itself. Let the angle FIG be made equal to the angle BAI ; IF and AB will

If circular motion be made about a resting centre, and in that circle there be an epicycle whose revolution is made the contrary way, in such manner that in equal times it make equal angles, every strait line taken in that epicycle will be so carried, that it will always be parallel to the places in which it formerly was.

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If circular
motion, &c.

therefore be parallel; and how much the axis AG has departed from its former place AC (the measure of which progression is the angle CAG , or CBD , which I suppose equal to it) so much in the same time has the axis IG , the same with BC , departed from its own former situation. Wherefore, in what time BC comes to IG by the motion from B to I upon the centre A , in the same time G will come to F by the contrary motion of the epicycle; that is, it will be turned backwards to F , and IG will lie in IF . But the angles FIG and GAC are equal; and therefore AC , that is, BC , and IF , (that is the axis, though in different places) will be parallel. Wherefore, the axis of the epicycle EDC will be carried always parallel to itself; which was to be proved.

Coroll. From hence it is manifest, that those two annual motions which Copernicus ascribes to the earth, are reducible to this one circular simple motion, by which all the points of the moved body are carried always with equal velocity, that is, in equal times they make equal revolutions uniformly.

This, as it is the most simple, so it is the most frequent of all circular motions; being the same which is used by all men when they turn anything round with their arms, as they do in grinding or sifting. For all the points of the thing moved describe lines which are like and equal to one another. So that if a man had a ruler, in which many pens' points of equal length were fastened, he might with this one motion write many lines at once.

3. Having shown what simple motion is, I will here also set down some properties of the same.

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First, when a body is moved with simple motion in a fluid medium which hath no vacuity, it changes the situation of all the parts of the fluid ambient which resist its motion; I say there are no parts so small of the fluid ambient, how far soever it be continued, but do change their situation in such manner, as that they leave their places continually to other small parts that come into the same.

Properties of
simple motion:

For (in the same second figure) let any body, as $KLMN$, be understood to be moved with simple circular motion; and let the circle, which every point thereof describes, have any determined quantity, suppose that of the same $KLMN$. Wherefore the centre A and every other point, and consequently the moved body itself, will be carried sometimes towards the side where is K , and sometimes towards the other side where is M . When therefore it is carried to K , the parts of the fluid medium on that side will go back; and, supposing all space to be full, others on the other side will succeed. And so it will be when the body is carried to the side M , and to N , and every way. Now when the nearest parts of the fluid medium go back, it is necessary that the parts next to those nearest parts go back also; and supposing still all space to be full, other parts will come into their places with succession perpetual and infinite. Wherefore all, even the least parts of the fluid medium, change their places, &c. Which was to be proved.

It is evident from hence, that simple motion, whether circular or not circular, of bodies which

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make perpetual returns to their former places, hath greater or less force to dissipate the parts of resisting bodies, as it is more or less swift, and as the lines described have greater or less magnitude. Now the greatest velocity that can be, may be understood to be in the least circuit, and the least in the greatest; and may be so supposed, when there is need.

If a fluid be moved with simple circular motion, all the points taken in it will describe their circles in times proportional to the distances from the centre.

4. Secondly, supposing the same simple motion in the air, water, or other fluid medium; the parts of the medium, which adhere to the moved body, will be carried about with the same motion and velocity, so that in what time soever any point of the movent finishes its circle, in the same time every part of the medium, which adheres to the movent, shall also describe such a part of its circle, as is equal to the whole circle of the movent; I say, it shall describe a part, and not the whole circle, because all its parts receive their motion from an interior concentric movent, and of concentric circles the exterior are always greater than the interior; nor can the motion imprinted by any movent be of greater velocity than that of the movent itself. From whence it follows, that the more remote parts of the fluid ambient shall finish their circles in times, which have to one another the same proportion with their distances from the movent. For every point of the fluid ambient, as long as it toucheth the body which carries it about, is carried about with it, and would make the same circle, but that it is left behind so much as the exterior circle exceeds the interior. So that if we suppose some thing, which is not fluid, to float in that part of the fluid ambient which is

nearest to the movent, it will together with the movent be carried about. Now that part of the fluid ambient, which is not the nearest but almost the nearest, receiving its degree of velocity from the nearest, which degree cannot be greater than it was in the giver, doth therefore in the same time make a circular line, not a whole circle, yet equal to the whole circle of the nearest. Therefore in the same time that the movent describes its circle, that which doth not touch it shall not describe its circle; yet it shall describe such a part of it, as is equal to the whole circle of the movent. And after the same manner, the more remote parts of the ambient will describe in the same time such parts of their circles, as shall be severally equal to the whole circle of the movent; and, by consequent, they shall finish their whole circles in times proportional to their distances from the movent; which was to be proved.

5. Thirdly, the same simple motion of a body placed in a fluid medium, congregates or gathers into one place such things as naturally float in that medium, if they be homogeneous; and if they be heterogeneous, it separates and dissipates them. But if such things as be heterogeneous do not float, but settle, then the same motion stirs and mingles them disorderly together. For seeing bodies, which are unlike to one another, that is, heterogeneous bodies, are not unlike in that they are bodies; for bodies, as bodies, have no difference; but only from some special cause, that is, from some internal motion, or motions of their smallest parts (for I have shown in chap. ix, art. 9, that all mutation is such motion), it remains that

Simple motion
dissipates hetero-
geneous and
congregates
homogeneous
bodies.

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Simple motion
dissipates hetero-
geneous, &c.

heterogeneous bodies have their unlikeness or difference from one another from their internal or specific motions. Now bodies which have such difference receive unlike and different motions from the same external common movent; and therefore they will not be moved together, that is to say, they will be dissipated. And being dissipated they will necessarily at some time or other meet with bodies like themselves, and be moved alike and together with them; and afterwards meeting with more bodies like themselves, they will unite and become greater bodies. Wherefore homogeneous bodies are congregated, and heterogeneous dissipated by simple motion in a medium where they naturally float. Again, such as being in a fluid medium do not float, but sink, if the motion of the fluid medium be strong enough, will be stirred up and carried away by that motion, and consequently they will be hindered from returning to that place to which they sink naturally, and in which only they would unite, and out of which they are promiscuously carried; that is, they are disorderly mingled.

Now this motion, by which homogeneous bodies are congregated and heterogeneous are scattered, is that which is commonly called *fermentation*, from the Latin *fervere*; as the Greeks have their Ζύμη, which signifies the same, from Ζέω *ferveo*. For seething makes all the parts of the water change their places; and the parts of any thing, that is thrown into it, will go several ways according to their several natures. And yet all *fervour* or seething is not caused by fire; for new wine and many other things have also their fer-

mentation and fervour, to which fire contributes little, and sometimes nothing. But when in fermentation we find heat, it is made by the fermentation.

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6. Fourthly, in what time soever the movent, whose centre is A (in fig. 2) moved in K L N, shall, by any number of revolutions, that is, when the perimeters B I and K L N be commensurable, have described a line equal to the circle which passes through the points B and I; in the same time all the points of the floating body, whose centre is B, shall return to have the same situation in respect of the movent, from which they departed. For seeing it is as the distance B A, that is, as the radius of the circle which passes through B I is to the perimeter itself B I, so the radius of the circle K L N is to the perimeter K L N; and seeing the velocities of the points B and K are equal, the time also of the revolution in I B to the time of one revolution in K L N, will be as the perimeter B I to the perimeter K L N; and therefore so many revolutions in K L N, as together taken are equal to the perimeter B I, will be finished in the same time in which the whole perimeter B I is finished; and therefore also the points L, N, F and H, or any of the rest, will in the same time return to the same situation from which they departed; and this may be demonstrated, whatsoever be the points considered. Wherefore all the points shall in that time return to the same situation; which was to be proved.

If a circle made by a movent moved with simple motion, be commensurable to another circle made by a point which is carried about by the same movent, all the points of both the circles will at some time return to the same situation.

From hence it follows, that if the perimeters B I and L K N be not commensurable, then all the

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If a sphere have simple motion, its motion will more dissipate heterogeneous bodies by how much it is more remote from the poles.

points will never return to have the same situation or configuration in respect of one another.

7. In simple motion, if the body moved be of a spherical figure, it hath less force towards its poles than towards its middle to dissipate heterogeneous, or to congregate homogeneous bodies.

Let there be a sphere (as in the third figure) whose centre is A and diameter B C; and let it be conceived to be moved with simple circular motion; of which motion let the axis be the strait line D E, cutting the diameter B C at right angles in A. Let now the circle, which is described by any point B of the sphere, have B F for its diameter; and taking F G equal to B C, and dividing it in the middle in H, the centre of the sphere A will, when half a revolution is finished, lie in H. And seeing H F and A B are equal, a circle described upon the centre H with the radius H F or H G, will be equal to the circle whose centre is A and radius A B. And if the same motion be continued, the point B will at the end of another half revolution return to the place from whence it began to be moved; and therefore at the end of half a revolution, the point B will be carried to F, and the whole hemisphere D B E into that hemisphere in which are the points L, K and F. Wherefore that part of the fluid medium, which is contiguous to the point F, will in the same time go back the length of the strait line B F; and in the return of the point F to B, that is, of G to C, the fluid medium will go back as much in a strait line from the point C. And this is the effect of simple motion in the middle of the sphere, where the distance from the poles is greatest. Let now the point I be taken in the same sphere nearer to

the pole E, and through it let the strait line I K be drawn parallel to the strait line B F, cutting the arch F L in K, and the axis H L in M; then connecting H K, upon H F let the perpendicular K N be drawn. In the same time therefore that B comes to F the point I will come to K, B F and I K being equal and described with the same velocity. Now the motion in I K to the fluid medium upon which it works, namely, to that part of the medium which is contiguous to the point K, is oblique, whereas if it proceeded in the strait line H K it would be perpendicular; and therefore the motion which proceeds in I K has less power than that which proceeds in H K with the same velocity. But the motions in H K and H F do equally thrust back the medium; and therefore the part of the sphere at K moves the medium less than the part at F, namely, so much less as K N is less than H F. Wherefore also the same motion hath less power to disperse heterogeneous, and to congregate homogeneous bodies, when it is nearer, than when it is more remote from the poles; which was to be proved.

Coroll. It is also necessary, that in planes which are perpendicular to the axis, and more remote than the pole itself from the middle of the sphere, this simple motion have no effect. For the axis D E with simple motion describes the superficies of a cylinder; and towards the bases of the cylinder there is in this motion no endeavour at all.

8. If in a fluid medium moved about, as hath been said, with simple motion, there be conceived to float some other spherical body which is not fluid, the parts of the medium, which are stopped by that

If a simple circular motion of a fluid body be hindered by a body which is not fluid, the

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fluid body will spread itself upon the superficies of that body.

body, will endeavour to spread themselves every way upon the superficies of it. And this is manifest enough by experience, namely, by the spreading of water poured out upon a pavement. But the reason of it may be this. Seeing the sphere A (in fig. 3) is moved towards B, the medium also in which it is moved will have the same motion. But because in this motion it falls upon a body not liquid, as G, so that it cannot go on; and seeing the small parts of the medium cannot go forwards, nor can they go directly backwards against the force of the movent; it remains, therefore, that they diffuse themselves upon the superficies of that body, as towards O and P; which was to be proved.

Circular motion about a fixed centre casteth off by the tangent such things as lie upon the circumference & stick not to it.

9. Compounded circular motion, in which all the parts of the moved body do at once describe circumferences, some greater, others less, according to the proportion of their several distances from the common centre, carries about with it such bodies, as being not fluid, adhere to the body so moved; and such as do not adhere, it casteth forwards in a strait line which is a tangent to the point from which they are cast off.

For let there be a circle whose radius is A B (in fig. 4); and let a body be placed in the circumference in B, which if it be fixed there, will necessarily be carried about with it, as is manifest of itself. But whilst the motion proceeds, let us suppose that body to be unfixed in B. I say, the body will continue its motion in the tangent B C. For let both the radius A B and the sphere B be conceived to consist of hard matter; and let us suppose the radius A B to be stricken in the point B

by some other body which falls upon it in the tangent D B. Now, therefore, there will be a motion made by the concurrence of two things, the one, endeavour towards C in the strait line D B produced, in which the body B would proceed, if it were not retained by the radius A B; the other, the retention itself. But the retention alone causeth no endeavour towards the centre; and, therefore, the retention being taken away, which is done by the unfixing of B, there will remain but one endeavour in B, namely, that in the tangent B C. Wherefore the motion of the body B unfix'd will proceed in the tangent B C; which was to be proved.

By this demonstration it is manifest, that circular motion about an unmoved axis shakes off and puts further from the centre of its motion such things as touch, but do not stick fast to its superficies; and the more, by how much the distance is greater from the poles of the circular motion; and so much the more also, by how much the things, that are shaken off, are less driven towards the centre by the fluid ambient, for other causes.

10. If in a fluid medium a spherical body be moved with simple circular motion, and in the same medium there float another sphere whose matter is not fluid, this sphere also shall be moved with simple circular motion.

Such things as are moved with simple circular motion, beget simple circular motion.

Let B C D (in fig. 5) be a circle, whose centre is A, and in whose circumference there is a sphere so moved, that it describes with simple motion the the perimeter B C D. Let also E F G be another sphere of consistent matter, whose semidiameter is E H, and centre H; and with the radius A H let the circle H I be described. I say, the sphere

PART III. EFG will, by the motion of the body in BCD, be moved in the circumference HI with simple motion.

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Such things as
are moved, &c.

For seeing the motion in BCD (by art. 4 of this chapter) makes all the points of the fluid medium describe in the same time circular lines equal to one another, the points E, H and G of the strait line EHG will in the same time describe with equal radii equal circles. Let EB be drawn equal and parallel to the strait line AH; and let AB be connected, which will therefore be equal and parallel to EH; and therefore also, if upon the centre B and radius BE the arch EK be drawn equal to the arch HI, and the strait lines AI, BK and IK be drawn, BK and AI will be equal; and they will also be parallel, because the two arches EK and HI, that is, the two angles KBE and IAH are equal; and, consequently, the strait lines AB and KI, which connect them, will also be equal and parallel. Wherefore KI and EH are parallel. Seeing, therefore, E and H are carried in the same time to K and I, the whole strait line IK will be parallel to EH, from whence it departed. And, therefore, seeing the sphere EFG is supposed to be of consistent matter, so as all its points keep always the same situation, it is necessary that every other strait line, taken in the same sphere, be carried always parallel to the places in which it formerly was. Wherefore the sphere EFG is moved with simple circular motion; which was to be demonstrated.

If that which is
so moved have
one side hard
and the other

11. If in a fluid medium, whose parts are stirred by a body moved with simple motion, there float another body, which hath its superficies either

wholly hard, or wholly fluid, the parts of this body shall approach the centre equally on all sides; that is to say, the motion of the body shall be circular, and concentric with the motion of the movent. But if it have one side hard, and the other side fluid, then both those motions shall not have the same centre, nor shall the floating body be moved in the circumference of a perfect circle.

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side fluid, its motion will not be perfectly circular.

Let a body be moved in the circumference of the circle *KLMN* (in fig 2.) whose centre is *A*. And let there be another body at *I*, whose superficies is either all hard or all fluid. Also let the medium, in which both these bodies are placed, be fluid. I say, the body at *I* will be moved in the circle *IB* about the centre *A*. For this has been demonstrated in the last article.

Wherefore let the superficies of the body at *I* be fluid on one side, and hard on the other. And first, let the fluid side be towards the centre. Seeing, therefore, the motion of the medium is such, as that its parts do continually change their places, (as hath been shown in art 5); if this change of place be considered in those parts of the medium which are contiguous to the fluid superficies, it must needs be that the small parts of that superficies enter into the places of the small parts of the medium which are contiguous to them; and the like change of place will be made with the next contiguous parts towards *A*. And if the fluid parts of the body at *I* have any degree at all of tenacity (for there are degrees of tenacity, as in the air and water) the whole fluid side will be lifted up a little, but so much the less, as its parts have less tenacity; whereas the hard part of the superficies.

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If that which is
so moved, &c.

which is contiguous to the fluid part, has no cause at all of elevation, that is to say, no endeavour towards A.

Secondly, let the hard superficies of the body at I be towards A. By reason, therefore, of the said change of place of the parts which are contiguous to it, the hard superficies must, of necessity, seeing by supposition there is no empty space, either come nearer to A, or else its smallest parts must supply the contiguous places of the medium, which otherwise would be empty. But this cannot be, by reason of the supposed hardness; and, therefore, the other must needs be, namely, that the body come nearer to A. Wherefore the body at I has greater endeavour towards the centre A, when its hard side is next it, than when it is averted from it. But the body in I, while it is moving in the circumference of the circle I B, has sometimes one side, sometimes another, turned towards the centre; and, therefore, it is sometimes nearer, sometimes further off from the centre A. Wherefore the body at I is not carried in the circumference of a perfect circle; which was to be demonstrated.

CHAPTER XXII.

OF OTHER VARIETY OF MOTION.

1. Endeavour and pressure how they differ.—2. Two kinds of mediums in which bodies are moved.—3. Propagation of motion, what it is.—4. What motion bodies have, when they press one another.—5. Fluid bodies, when they are pressed together, penetrate one another.—6. When one body presseth another and doth not penetrate it, the action of the pressing body is perpendicular to the superficies of the body pressed.—7. When a hard body, pressing another body, penetrates the same, it doth not penetrate it perpendicularly, unless it fall perpendicularly upon it.—8. Motion sometimes opposite to that of the movent.—9. In a full medium, motion is propagated to any distance.—10. Dilatation and contraction what they are. 11. Dilatation and contraction suppose mutation of the smallest parts in respect of their situation.—12. All traction is pulsion. 13. Such things as being pressed or bent restore themselves, have motion in their internal parts.—14. Though that which carrieth another be stopped, the body carried will proceed. 15, 16. The effects of percussion not to be compared with those of weight.—17, 18. Motion cannot begin first in the internal parts of a body.—19. Action and reaction proceed in the same line.—20. Habit, what it is.

1. I HAVE already (chapter xv. art. 2) defined endeavour to be motion through some length, though not considered as length, but as a point. Whether, therefore, there be resistance or no resistance, the endeavour will still be the same. For simply to endeavour is to go. But when two bodies, having opposite endeavours, press one another, then the endeavour of either of them is that which we call pressure, and is mutual when their pressures are opposite.

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Endeavour and pressure how they differ.

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Two kinds of
mediums in
which bodies
are moved.

2. Bodies moved, and also the mediums in which they are moved, are of two kinds. For either they have their parts coherent in such manner, as no part of the moved body will easily yield to the movent, except the whole body yield also, and such are the things we call *hard*: or else their parts, while the whole remains unmoved, will easily yield to the movent, and these we call *fluid* or *soft* bodies. For the words *fluid*, *soft*, *tough*, and *hard*, in the same manner as *great* and *little*, are used only comparatively; and are not different kinds, but different degrees of quality.

Propagation
of motion,
what it is.

3. To *do*, and to *suffer*, is to move and to be moved; and nothing is moved but by that which toucheth it and is also moved, as has been formerly shown. And how great soever the distance be, we say the first movent moveth the last moved body, but mediately; namely so, as that the first moveth the second, the second the third, and so on, till the last of all be touched. When therefore one body, having opposite endeavour to another body, moveth the same, and that moveth a third, and so on, I call that action *propagation of motion*.

What motion
bodies have
when they press
one another.

4. When two fluid bodies, which are in a free and open space, press one another, their parts will endeavour, or be moved, towards the sides; not only those parts which are there where the mutual contact is, but all the other parts. For in the first contact, the parts, which are pressed by both the endeavouring bodies, have no place either forwards or backwards in which they can be moved; and therefore they are pressed out towards the sides. And this expressure, when the forces are equal, is

in a line perpendicular to the bodies pressing. But whensoever the foremost parts of both the bodies are pressed, the hindermost also must be pressed at the same time; for the motion of the hindermost parts cannot in an instant be stopped by the resistance of the foremost parts, but proceeds for some time; and therefore, seeing they must have some place in which they may be moved, and that there is no place at all for them forwards, it is necessary that they be moved into the places which are towards the sides every way. And this effect follows of necessity, not only in fluid, but in consistent and hard bodies, though it be not always manifest to sense. For though from the compression of two stones we cannot with our eyes discern any swelling outwards towards the sides, as we perceive in two bodies of wax; yet we know well enough by reason, that some tumour must needs be there, though it be but little.

5. But when the space is enclosed, and both the bodies be fluid, they will, if they be pressed together, penetrate one another, though differently, according to their different endeavours. For suppose a hollow cylinder of hard matter, well stopped at both ends, but filled first, below with some heavy fluid body, as quicksilver, and above with water or air. If now the bottom of the cylinder be turned upwards, the heaviest fluid body, which is now at the top, having the greatest endeavour downwards, and being by the hard sides of the vessel hindered from extending itself sideways, must of necessity either be received by the lighter body, that it may sink through it, or else it must open a passage through itself, by

Fluid bodies, when they are pressed together, penetrate one another.

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Fluid bodies,
when they are
pressed together,
penetrate
one another.

which the lighter body may ascend. For of the two bodies, that, whose parts are most easily separated, will be the first divided; which being done, it is not necessary that the parts of the other suffer any separation at all. And therefore when two liquors, which are enclosed in the same vessel, change their places, there is no need that their smallest parts should be mingled with one another; for a way being opened through one of them, the parts of the other need not be separated.

Now if a fluid body, which is not enclosed, press a hard body, its endeavour will indeed be towards the internal parts of that hard body; but being excluded by the resistance of it, the parts of the fluid body will be moved every way according to the superficies of the hard body, and that equally, if the pressure be perpendicular; for when all the parts of the cause are equal, the effects will be equal also. But if the pressure be not perpendicular, then the angles of the incidence being unequal, the expansion also will be unequal, namely, greater on that side where the angle is greater, because that motion is most direct which proceeds by the directest line.

When one body
presseth another
and doth not
penetrate it,
the action of
the pressing
body is perpendicular
to the superficies
of the body
pressed.

6. If a body, pressing another body, do not penetrate it, it will nevertheless give to the part it presseth an endeavour to yield, and recede in a strait line perpendicular to its superficies in that point in which it is pressed.

Let A B C D (in fig. 1) be a hard body, and let another body, falling upon it in the strait line E A, with any inclination or without inclination, press it in the point A. I say the body so pressing, and not penetrating it, will give to the part A an

endeavour to yield or recede in a strait line perpendicular to the line A D. PART III.
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For let A B be perpendicular to A D, and let B A be produced to F. If therefore A F be coincident with A E, it is of itself manifest that the motion in E A will make A to endeavour in the line A B. Let now E A be oblique to A D, and from the point E let the strait line E C be drawn, cutting A D at right angles in D, and let the rectangles A B C D and A D E F be completed. I have shown (in the 8th article of chapter xvi) that the body will be carried from E to A by the concurrence of two uniform motions, the one in E F and its parallels, the other in E D and its parallels. But the motion in E F and its parallels, whereof D A is one, contributes nothing to the body in A to make it endeavour or press towards B; and therefore the whole endeavour, which the body hath in the inclined line E A to pass or press the strait line A D, it hath it all from the perpendicular motion or endeavour in F A. Wherefore the body E, after it is in A, will have only that perpendicular endeavour which proceeds from the motion in F A, that is, in A B; which was to be proved.

When one body presseth, &c.

7. If a hard body falling upon or pressing another body penetrate the same, its endeavour after its first penetration will be neither in the inclined line produced, nor in the perpendicular, but sometimes betwixt both, sometimes without them. When a hard body, pressing another body, penetrates the same, it doth not penetrate it perpendicularly, unless it fall perpendicularly upon it.

Let E A G (in the same fig. 1) be the inclined line produced; and first, let the passage through the medium, in which E A is, be easier than the passage through the medium in which A G is. As

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When a hard
body, &c.

soon therefore as the body is within the medium in which is A G, it will find greater resistance to its motion in D A and its parallels, than it did whilst it was above A D ; and therefore below A D it will proceed with slower motion in the parallels of D A, than above it. Wherefore the motion which is compounded of the two motions in E F and E D will be slower below A D than above it ; and therefore also, the body will not proceed from A in E A produced, but below it. Seeing, therefore, the endeavour in A B is generated by the endeavour in F A ; if to the endeavour in F A there be added the endeavour in D A, which is not all taken away by the immersion of the point A into the lower medium, the body will not proceed from A in the perpendicular A B, but beyond it ; namely, in some strait line between A B and A G, as in the line A H.

Secondly, let the passage through the medium E A be less easy than that through A G. The motion, therefore, which is made by the concurrence of the motions in E F and F B, is slower above A D than below it ; and consequently, the endeavour will not proceed from A in E A produced, but beyond it, as in A I. Wherefore, if a hard body falling, &c. ; which was to be proved.

This divergency of the strait line A H from the strait line A G is that which, the writers of optics commonly called *refraction*, which, when the passage is easier in the first than in the second medium, is made by diverging from the line of inclination towards the perpendicular ; and contrarily, when the passage is not so easy in the

first medium, by departing further from the perpendicular.

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8. By the 6th theorem it is manifest, that the force of the movent may be so placed, as that the body moved by it may proceed in a way almost directly contrary to that of the movent, as we see in the motion of ships.

Motion sometimes opposite to that of the movent.

For let AB (in fig. 2) represent a ship, whose length from the prow to the poop is AB , and let the wind lie upon it in the strait parallel lines CB , DE and FG ; and let DE and FG be cut in E and G by a strait line drawn from B perpendicular to AB ; also let BE and EG be equal, and the angle ABC any angle how small soever. Then between BC and BA let the strait line BI be drawn; and let the sail be conceived to be spread in the same line BI , and the wind to fall upon it in the points L , M and B ; from which points, perpendicular to BI , let BK , MQ and LP be drawn. Lastly, let EN and GO be drawn perpendicular to BG , and cutting BK in H and K ; and let HN and KO be made equal to one another, and severally equal to BA . I say, the ship BA , by the wind falling upon it in CB , DE , FG , and other lines parallel to them, will be carried forwards almost opposite to the wind, that is to say, in a way almost contrary to the way of the movent.

For the wind that blows in the line CB will (as hath been shown in art. 6) give to the point B an endeavour to proceed in a strait line perpendicular to the strait line BI , that is, in the strait line BK ; and to the points M and L an endeavour to proceed in the strait lines MQ and LP , which are parallel to BK . Let now the measure of the time

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Motion some-
times opposite
to that of the
movent.

be $B G$, which is divided in the middle in E ; and let the point B be carried to H in the time $B E$. In the same time, therefore, by the wind blowing in $D M$ and $F L$, and as many other lines as may be drawn parallel to them, the whole ship will be applied to the strait line $H N$. Also at the end of the second time $E G$, it will be applied to the strait line $K O$. Wherefore the ship will always go forward; and the angle it makes with the wind will be equal to the angle $A B C$, how small soever that angle be; and the way it makes will in every time be equal to the strait line $E H$. I say, thus it would be, if the ship might be moved with as great celerity sideways from $B A$ towards $K O$, as it may be moved forwards in the line $B A$. But this is impossible, by reason of the resistance made by the great quantity of water which presseth the side, much exceeding the resistance made by the much smaller quantity which presseth the prow of the ship; so that the way the ship makes sideways is scarce sensible; and, therefore, the point B will proceed almost in the very line $B A$, making with the wind the angle $A B C$, how acute soever; that is to say, it will proceed almost in the strait line $B C$, that is, in a way almost contrary to the way of the movent; which was to be demonstrated.

But the sail in $B I$ must be so stretched as that there be left in it no bosom at all; for otherwise the strait lines $L P$, $M Q$ and $B K$ will not be perpendicular to the plane of the sail, but falling below P , Q and K , will drive the ship backwards. But by making use of a small board for a sail, a little waggon with wheels for the ship, and of a smooth pavement for the sea, I have by experience found

this to be so true, that I could scarce oppose the board to the wind in any obliquity, though never so small, but the waggon was carried forwards by it.

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Motion sometimes opposite to that of the movent.

By the same 6th theorem it may be found, how much a stroke, which falls obliquely, is weaker than a stroke falling perpendicularly, they being like and equal in all other respects.

Let a stroke fall upon the wall AB obliquely, as for example, in the strait line CA (in fig. 3.) Let CE be drawn parallel to AB , and DA perpendicular to the same AB and equal to CA ; and let both the velocity and time of the motion in CA be equal to the velocity and time of the motion in DA . I say, the stroke in CA will be weaker than that in DA , in the proportion of EA to DA . For producing DA howsoever to F , the endeavour of both the strokes will (by art. 6) proceed from A in the perpendicular AF . But the stroke in CA is made by the concurrence of two motions in CE and EA , of which that in CE contributes nothing to the stroke in A , because CE and BA are parallels; and, therefore, the stroke in CA is made by the motion which is in EA only. But the velocity or force of the perpendicular stroke in EA , to the velocity or force of the stroke in DA , is as EA to DA . Wherefore the oblique stroke in CA is weaker than the perpendicular stroke in DA , in the proportion of EA to DA or CA ; which was to be proved.

9. In a full medium, all endeavour proceeds as far as the medium itself reacheth; that is to say, if the medium be infinite, the endeavour will proceed infinitely.

In a full medium, motion is propagated to any distance.

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In a full medium, motion is propagated to any distance.

For whatsoever endeavoureth is moved, and therefore whatsoever standeth in its way it maketh it yield, at least a little, namely, so far as the movent itself is moved forwards. But that which yieldeth is also moved, and consequently maketh that to yield which is in its way, and so on successively as long as the medium is full; that is to say, infinitely, if the full medium be infinite; which was to be proved.

Now although endeavour thus perpetually propagated do not always appear to the senses as motion, yet it appears as action, or as the efficient cause of some mutation. For if there be placed before our eyes some very little object, as for example, a small grain of sand, which at a certain distance is visible; it is manifest that it may be removed to such a distance as not to be any longer seen, though by its action it still work upon the organs of sight, as is manifest from that which was last proved, that all endeavour proceeds infinitely. Let it be conceived therefore to be removed from our eyes to any distance how great soever, and a sufficient number of other grains of sand of the same bigness added to it; it is evident that the aggregate of all those sands will be visible; and though none of them can be seen when it is single and severed from the rest, yet the whole heap or hill which they make will manifestly appear to the sight; which would be impossible, if some action did not proceed from each several part of the whole heap.

Dilatation & contraction what they are.

10 Between the degrees of hard and soft are those things which we call *tough*, tough being that which may be bent without being altered from

what it was; and the bending of a line is either the adduction or diduction of the extreme parts, that is, a motion from straitness to crookedness, or contrarily, whilst the line remains still the same it was; for by drawing out the extreme points of a line to their greatest distance, the line is made strait, which otherwise is crooked. So also the bending of a superficies is the diduction or adduction of its extreme lines, that is, their dilatation and contraction.

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11. *Dilatation* and *contraction*, as also all *flexion*, supposes necessarily that the internal parts of the body bowed do either come nearer to the external parts, or go further from them. For though *flexion* be considered only in the length of a body, yet when that body is bowed, the line which is made on one side will be convex, and the line on the other side will be concave; of which the concave, being the interior line, will, unless something be taken from it and added to the convex line, be the more crooked, that is, the greater of the two. But they are equal; and, therefore, in flexion there is an accession made from the interior to the exterior parts; and, on the contrary, in tension, from the exterior to the interior parts. And as for those things which do not easily suffer such transposition of their parts, they are called brittle; and the great force they require to make them yield, makes them also with sudden motion to leap asunder, and break in pieces.

Dilatation & contraction suppose mutation of the smallest parts in respect of their situation.

12. Also motion is distinguished into *pulsion* and *traction*. And *pulsion*, as I have already defined it, is when that which is moved goes before that which moveth it. But contrarily, in *traction*

All traction is pulsion.

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the movent goes before that which is moved. Nevertheless, considering it with greater attention, it seemeth to be the same with pulsion. For of two parts of a hard body, when that which is foremost drives before it the medium in which the motion is made, at the same time that which is thrust forwards thrusteth the next, and this again the next, and so on successively. In which action, if we suppose that there is no place void, it must needs be, that by continual pulsion, namely, when that action has gone round, the movent will be behind that part, which at the first seemed not to be thrust forwards, but to be drawn; so that now the body, which was drawn, goes before the body which gives it motion; and its motion is no longer traction, but pulsion.

Such things as being pressed or bent restore themselves, have motion in their internal parts.

13. Such things as are removed from their places by forcible compression or extension, and, as soon as the force is taken away, do presently return and restore themselves to their former situation, have the beginning of their restitution within themselves, namely, a certain motion in their internal parts, which was there, when, before the taking away of the force, they were compressed, or extended. For that restitution is motion, and that which is at rest cannot be moved, but by a moved and a contiguous movent. Nor doth the cause of their restitution proceed from the taking away of the force by which they were compressed or extended; for the removing of impediments hath not the efficacy of a cause, as has been shown at the end of the 3rd article of chap. xv. The cause therefore of their restitution is some motion either of the parts of the ambient, or of the parts of the

body compressed or extended. But the parts of the ambient have no endeavour which contributes to their compression or extension, nor to the setting of them at liberty, or restitution. It remains therefore that from the time of their compression or extension there be left some endeavour or motion, by which, the impediment being removed, every part resumes its former place; that is to say, the whole restores itself.

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14. In the carriage of bodies, if that body, which carries another, hit upon any obstacle, or be by any means suddenly stopped, and that which is carried be not stopped, it will go on, till its motion be by some external impediment taken away.

Though that which carrieth another be stopped, the body carried will proceed.

For I have demonstrated (chap. VIII, art. 19) that motion, unless it be hindered by some external resistance, will be continued eternally with the same celerity; and in the 7th article of chap. IX, that the action of an external agent is of no effect without contact. When therefore that, which carrieth another thing, is stopped, that stop doth not presently take away the motion of that which is carried. It will therefore proceed, till its motion be by little and little extinguished by some external resistance: which was to be proved; though experience alone had been sufficient to prove this.

In like manner, if that body which carrieth another be put from rest into sudden motion, that which is carried will not be moved forwards together with it, but will be left behind. For the contiguous part of the body carried hath almost the same motion with the body which carries it; and the remote parts will receive different velocities according to their different distances from the body

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that carries them ; namely, the more remote the parts are, the less will be their degrees of velocity. It is necessary, therefore, that the body, which is carried, be left accordingly more or less behind. And this also is manifest by experience, when at the starting forward of the horse the rider falleth backwards.

The effects of percussion not to be compared with those of weight.

15. In *percussion*, therefore, when one hard body is in some small part of it stricken by another with great force, it is not necessary that the whole body should yield to the stroke with the same celerity with which the stricken part yields. For the rest of the parts receive their motion from the motion of the part stricken and yielding, which motion is less propagated every way towards the sides, than it is directly forwards. And hence it is, that sometimes very hard bodies, which being erected can hardly be made to stand, are more easily broken than thrown down by a violent stroke ; when, nevertheless, if all their parts together were by any weak motion thrust forwards, they would easily be cast down.

16. Though the difference between *trusion* and *percussion* consist only in this, that in *trusion* the motion both of the movent and moved body begin both together in their very contact ; and in *percussion* the striking body is first moved, and afterwards the body stricken ; yet their effects are so different, that it seems scarce possible to compare their forces with one another. I say, any effect of *percussion* being propounded, as for example, the stroke of a beetle of any weight assigned, by which a pile of any given length is to be driven

into earth of any tenacity given, it seems to me very hard, if not impossible, to define with what weight, or with what stroke, and in what time, the same pile may be driven to a depth assigned into the same earth. The cause of which difficulty is this, that the velocity of the percutient is to be compared with the magnitude of the ponderant. Now velocity, seeing it is computed by the length of space transmitted, is to be accounted but as one dimension; but weight is as a solid thing, being measured by the dimension of the whole body. And there is no comparison to be made of a solid body with a length, that is, with a line.

17. If the internal parts of a body be at rest, or retain the same situation with one another for any time how little soever, there cannot in those parts be generated any new motion or endeavour, whereof the efficient cause is not without the body of which they are parts. For if any small part, which is comprehended within the superficies of the whole body, be supposed to be now at rest, and by and by to be moved, that part must of necessity receive its motion from some moved and contiguous body. But by supposition, there is no such moved and contiguous part within the body. Wherefore, if there be any endeavour or motion or change of situation in the internal parts of that body, it must needs arise from some efficient cause that is without the body which contains them; which was to be proved.

Motion cannot begin first in the internal parts of a body.

18. In hard bodies, therefore, which are compressed or extended, if that which compresseth or extendeth them being taken away, they restore

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themselves to their former place or situation, it must needs be that that endeavour or motion of their internal parts, by which they were able to recover their former places or situations, was not extinguished when the force by which they were compressed or extended was taken away. Therefore, when the lath of a cross-bow bent doth, as soon as it is at liberty, restore itself, though to him, that judges by sense, both it and all its parts seem to be at rest; yet he, that judging by reason doth not account the taking away of impediment for an efficient cause, nor conceives that without an efficient cause any thing can pass from rest to motion, will conclude that the parts were already in motion before they began to restore themselves.

Action and reaction proceed in the same line.

19. *Action* and *reaction* proceed in the same line, but from opposite terms. For seeing reaction is nothing but endeavour in the patient to restore itself to that situation from which it was forced by the agent; the endeavour or motion both of the agent and patient or reagent will be propagated between the same terms; yet so, as that in action the term, *from which*, is in reaction the term *to which*. And seeing all action proceeds in this manner, not only between the opposite terms of the whole line in which it is propagated, but also in all the parts of that line, the terms *from which* and *to which*, both of the action and reaction, will be in the same line. Wherefore action and reaction proceed in the same line, &c.

Habit, what it is.

20. To what has been said of motion, I will add what I have to say concerning habit. *Habit*, therefore, is a generation of motion, not of motion

simply, but an easy conducting of the moved body in a certain and designed way. And seeing it is attained by the weakening of such endeavours as divert its motion, therefore such endeavours are to be weakened by little and little. But this cannot be done but by the long continuance of action, or by actions often repeated; and therefore custom begets that facility, which is commonly and rightly called *habit*; and it may be defined thus: *HABIT is motion made more easy and ready by custom; that is to say, by perpetual endeavour, or by iterated endeavours in a way differing from that in which the motion proceeded from the beginning, and opposing such endeavours as resist.* And to make this more perspicuous by example, we may observe, that when one that has no skill in music first puts his hand to an instrument, he cannot after the first stroke carry his hand to the place where he would make the second stroke, without taking it back by a new endeavour, and, as it were beginning again, pass from the first to the second. Nor will he be able to go on to the third place without another new endeavour; but he will be forced to draw back his hand again, and so successively, by renewing his endeavour at every stroke; till at the last, by doing this often, and by compounding many interrupted motions or endeavours into one equal endeavour, he be able to make his hand go readily on from stroke to stroke in that order and way which was at the first designed. Nor are habits to be observed in living creatures only, but also in bodies inanimate. For we find that when the lath of a cross-bow is strongly bent,

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what it is.

and would if the impediment were removed return again with great force ; if it remain a long time bent, it will get such a habit, that when it is loosed and left to its own freedom, it will not only not restore itself, but will require as much force for the bringing of it back to its first posture, as it did for the bending of it at the first.

CHAP. XXIII.

OF THE CENTRE OF EQUIPONDERATION ; OF BODIES PRESSING DOWNWARDS IN STRAIT PARALLEL LINES.

1. Definitions and suppositions.—2. Two planes of equiponderation are not parallel.—3. The centre of equiponderation is in every plane of equiponderation.—4. The moments of equal ponderants are to one another as their distances from the centre of the scale.—5, 6. The moments of unequal ponderants have their proportion to one another compounded of the proportions of their weights and distances from the centre of the scale.—7. If two ponderants have their weights and distances from the centre of the scale in reciprocal proportion, they are equally poised ; and contrarily.—8. If the parts of any ponderant press the beams of the scale every where equally, all the parts cut off, reckoned from the centre of the scale, will have their moments in the same proportion with that of the parts of a triangle cut off from the vertex by strait lines parallel to the base.—9. The diameter of equiponderation of figures, which are deficient according to commensurable proportions of their altitudes and bases, divides the axis, so that the part taken next the vertex is to the other part of the complete figure to the deficient figure.—10. The diameter of equiponderation of the complement of the half of any of the said deficient figures, divides that line which is drawn through the vertex parallel to the base, so that the part next the vertex is to the other part as the complete figure to the

complement.—11. The centre of equiponderation of the half of any of the deficient figures in the first row of the table of art. 3, chap. XVII, may be found out by the numbers of the second row.—12. The centre of equiponderation of the half of any of the figures of the second row of the same table, may be found out by the numbers of the fourth row.—13. The centre of equiponderation of the half of any of the figures in the same table being known, the centre of the excess of the same figure above a triangle of the same altitude and base is also known.—14. The centre of equiponderation of a solid sector is in the axis so divided, that the part next the vertex be to the whole axis, wanting half the axis of the portion of the sphere, as 3 to 4.

DEFINITIONS.

I. A *scale* is a strait line, whose middle point is immovable, all the rest of its points being at liberty; and that part of the scale, which reaches from the centre to either of the weights, is called the *beam*. Definitions.

II. *Equiponderation* is when the endeavour of one body, which presses one of the beams, resists the endeavour of another body pressing the other beam, so that neither of them is moved; and the bodies, when neither of them is moved, are said to be *equally poised*.

III. *Weight* is the aggregate of all the endeavours, by which all the points of that body, which presses the beam, tend downwards in lines parallel to one another; and the body which presses is called the *ponderant*.

IV. *Moment* is the power which the ponderant has to move the beam, by reason of a determined situation.

V. *The plane of equiponderation* is that by which the ponderant is so divided, that the moments on both sides remain equal.

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Definitions.

VI. *The diameter of equiponderation* is the common section of the two planes of equiponderation, and is in the strait line by which the weight is hanged.

VII. *The centre of equiponderation* is the common point of the two diameters of equiponderation.

SUPPOSITIONS.

Suppositions.

I. When two bodies are equally poised, if weight be added to one of them and not to the other, their equiponderation ceases.

II. When two ponderants of equal magnitude, and of the same species or matter, press the beam on both sides at equal distances from the centre of the scale, their moments are equal. Also when two bodies endeavour at equal distances from the centre of the scale, if they be of equal magnitude and of the same species, their moments are equal.

Two planes
of equiponde-
ration are
not parallel.

2. No two planes of equiponderation are parallel.

Let $A B C D$ (in fig. 1) be any ponderant whatsoever; and in it let $E F$ be a plane of equiponderation; parallel to which, let any other plane be drawn, as $G H$. I say, $G H$ is not a plane of equiponderation. For seeing the parts $A E F D$ and $E B C F$ of the ponderant $A B C D$ are equally poised; and the weight $E G H F$ is added to the part $A E F D$, and nothing is added to the part $E B C F$, but the weight $E G H F$ is taken from it; therefore, by the first supposition, the parts $A G H D$ and $G B C H$ will not be equally poised; and consequently $G H$ is not a plane of equiponderation. Wherefore, no two planes of equiponderation are parallel; which was to be proved.

3. The centre of equiponderation is in every plane of equiponderation. PART III.
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For if another plane of equiponderation be taken, it will not, by the last article, be parallel to the former plane; and therefore both those planes will cut one another. Now that section (by the 6th definition) is the diameter of equiponderation. Again, if another diameter of equiponderation be taken, it will cut that former diameter; and in that section (by the 7th definition) is the centre of equiponderation. Wherefore the centre of equiponderation is in that diameter which lies in the said plane of equiponderation.

4. The moment of any ponderant applied to one point of the beam, to the moment of the same or an equal ponderant applied to any other point of the beam, is as the distance of the former point from the centre of the scale, to the distance of the latter point from the same centre. Or thus, those moments are to one another, as the arches of circles which are made upon the centre of the scale through those points, in the same time. Or lastly thus, they are as the parallel bases of two triangles, which have a common angle at the centre of the scale.

Let A (in fig. 2) be the centre of the scale; and let the equal ponderants D and E press the beam A B in the points B and C; also let the strait lines B D and C E be diameters of equiponderation; and the points D and E in the ponderants D and E be their centres of equiponderation. Let A G F be drawn howsoever, cutting D B produced in F, and E C in G; and lastly, upon the common centre A, let the two arches B H and C I be described, cut-

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ting A G F in H and I. I say, the moment of the ponderant D to the moment of the ponderant E is as AB to AC, or as BH to CI, or as BF to CG. For the effect of the ponderant D, in the point B, is circular motion in the arch BH; and the effect of the ponderant E, in the point C, circular motion in the arch CI; and by reason of the equality of the ponderants D and E, these motions are to one another as the quicknesses or velocities with which the points B and C describe the arches BH and CI, that is, as the arches themselves BH and CI, or as the strait parallels BF and CG, or as the parts of the beam AB and AC; for $AB \cdot AC :: BF \cdot CG :: BH \cdot CI$. are proportionals; and therefore the effects, that is, by the 4th definition, the moments of the equal ponderants applied to several points of the beam, are to one another as AB and AC; or as the distances of those points from the centre of the scale; or as the parallel bases of the triangles which have a common angle at A; or as the concentric arches BH and CI; which was to be demonstrated.

The moments of unequal ponderants have their proportion to one another compounded of the proportions of their weights and distances from the centre of the scale.

5. Unequal ponderants, when they are applied to several points of the beam, and hang at liberty, that is, so as the line by which they hang be the diameter of equiponderation, whatsoever be the figure of the ponderant, have their moments to one another in proportion compounded of the proportions of their distances from the centre of the scale, and of their weights.

Let A (in fig. 3) be the centre of the scale, and AB the beam; to which let the two ponderants C and D be applied at the points B and E. I say, the proportion of the moment of the ponderant C

to the moment of the ponderant D, is compounded of the proportions of A B to A E, and of the weight C to the weight D ; or, if C and D be of the same species, of the magnitude C to the magnitude D.

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Let either of them, as C, be supposed to be bigger than the other, D. If, therefore, by the addition of F, F and D together be as one body equal to C, the moment of C to the moment of F+D will be (by the last article) as BG is to EH. Now as F+D is to D, so let EH be to another EI; and the moment of F+D, that is of C, to the moment of D, will be as BG to EI. But the proportion of BG to EI is compounded of the proportions of BG to EH, that is, of AB to AE, and of EH to EI, that is, of the weight C to the weight D. Wherefore unequal ponderants, when they are applied, &c. Which was to be proved.

6. The same figure remaining, if IK be drawn parallel to the beam AB, and cutting AG in K; and KL be drawn parallel to BG, cutting AB in L, the distances AB and AL from the centre will be proportional to the moments of C and D. For the moment of C is BG, and the moment of D is EI, to which KL is equal. But as the distance AB from the centre is to the distance AL from the centre, so is BG, the moment of the ponderant C, to LK, or EI the moment of the ponderant D.

7. If two ponderants have their weights and distances from the centre in reciprocal proportion, and the centre of the scale be between the points to which the ponderants are applied, they will be equally poised. And contrarily, if they be equally poised, their weights and distances from the centre of the scale will be in reciprocal proportion.

If two ponderants have their weights and distances from the centre of the scale in reciprocal proportion, they are equally poised; and contrarily.

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Let the centre of the scale (in the same third figure) be A, the beam A B; and let any ponderant C, having B G for its moment, be applied to the point B; also let any other ponderant D, whose moment is E I, be applied to the point E. Through the point I let I K be drawn parallel to the beam A B, cutting A G in K; also let K L be drawn parallel to B G, K L will then be the moment of the ponderant D; and by the last article, it will be as B G, the moment of the ponderant C in the point B, to L K the moment of the ponderant D in the point E, so A B to A L. On the other side of the centre of the scale, let A N be taken equal to A L; and to the point N let there be applied the ponderant O, having to the ponderant C the proportion of A B to A N. I say, the ponderants in B and N will be equally poised. For the proportion of the moment of the ponderant O, in the point N, to the moment of the ponderant C in the point B, is by the 5th article, compounded of the proportions of the weight O to the weight C, and of the distance from the centre of the scale A N or A L to the distance from the centre of the scale A B. But seeing we have supposed, that the distance A B to the distance A N is in reciprocal proportion of the weight O to the weight C, the proportion of the moment of the ponderant O, in the point N, to the moment of the ponderant C, in the point B, will be compounded of the proportions of A B to A N, and of A N to A B. Wherefore, setting in order A B, A N, A B, the moment of O to the moment of C will be as the first to the last, that is, as A B to A B. Their moments therefore are equal; and consequently the plane which passes through

A will (by the fifth definition) be a plane of equiponderation. Wherefore they will be equally poised; as was to be proved.

Now the converse of this is manifest. For if there be equiponderation and the proportion of the weights and distances be not reciprocal, then both the weights will always have the same moments, although one of them have more weight added to it or its distance changed.

Coroll. When ponderants are of the same species, and their moments be equal; their magnitudes and distances from the centre of the scale will be reciprocally proportional. For in homogeneous bodies, it is as weight to weight, so magnitude to magnitude.

8. If to the whole length of the beam there be applied a parallelogram, or a parallelopipedum, or a prisma, or a cylinder, or the superficies of a cylinder, or of a sphere, or of any portion of a sphere or prisma; the parts of any of them cut off with planes parallel to the base will have their moments in the same proportion with the parts of a triangle, which has its vertex in the centre of the scale, and for one of its sides the beam itself, which parts are cut off by planes parallel to the base.

If the parts of any ponderant press the beams of the scale every where equally, all the parts cut off, reckoned from centre of the scale, will have their moments in the same proportion with that of the parts of a triangle cut off from the vertex by strait lines parallel to the base.

First, let the rectangled parallelogram $A B C D$ (in figure 4) be applied to the whole length of the beam $A B$; and producing $C B$ howsoever to E , let the triangle $A B E$ be described. Let now any part of the parallelogram, as $A F$, be cut off by the plane $F G$, parallel to the base $C B$; and let $F G$ be produced to $A E$ in the point H . I say, the moment of the whole $A B C D$ to the moment of its

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part A F, is as the triangle A B E to the triangle A G H, that is, in proportion duplicate to that of the distances from the centre of the scale.

For, the parallelogram A B C D being divided into equal parts, infinite in number, by strait lines drawn parallel to the base; and supposing the moment of the strait line C B to be B E, the moment of the strait line F G will (by the 7th article) be G H; and the moments of all the strait lines of that parallelogram will be so many strait lines in the triangle A B E drawn parallel to the base B E; all which parallels together taken are the moment of the whole parallelogram A B C D; and the same parallels do also constitute the superficies of the triangle A B E. Wherefore the moment of the parallelogram A B C D is the triangle A B E; and for the same reason, the moment of the parallelogram A F is the triangle A G H; and therefore the moment of the whole parallelogram to the moment of a parallelogram which is part of the same, is as the triangle A B E to the triangle A G H, or in proportion duplicate to that of the beams to which they are applied. And what is here demonstrated in the case of a parallelogram may be understood to serve for that of a cylinder, and of a prisma, and their superficies; as also for the superficies of a sphere, of an hemisphere, or any portion of a sphere. For the parts of the superficies of a sphere have the same proportion with that of the parts of the axis cut off by the same parallels, by which the parts of the superficies are cut off, as Archimedes has demonstrated; and therefore when the parts of any of these figures are equal and at equal

distances from the centre of the scale, their moments also are equal, in the same manner as they are in parallelograms.

Secondly, let the parallelogram $AKIB$ not be rectangled; the strait line IB will nevertheless press the point B perpendicularly in the strait line BE ; and the strait line LG will press the point G perpendicularly in the strait line GH ; and all the rest of the strait lines which are parallel to IB will do the like. Whatsoever therefore the moment be which is assigned to the strait line IB , as here, for example, it is supposed to be BE , if AE be drawn, the moment of the whole parallelogram AI will be the triangle ABE ; and the moment of the part AL will be the triangle AGH . Wherefore the moment of any ponderant, which has its sides equally applied to the beam, whether they be applied perpendicularly or obliquely, will be always to the moment of a part of the same in such proportion as the whole triangle has to a part of the same cut off by a plane which is parallel to the base.

9. The centre of equiponderation of any figure, which is deficient according to commensurable proportions of the altitude and base diminished, and whose complete figure is either a parallelogram or a cylinder, or a parallelopipedum, divides the axis, so, that the part next the vertex, to the other part, is as the complete figure to the deficient figure.

The diameter of equiponderation of figures which are deficient according to commensurable proportions of their altitudes and bases, divides the axis, so that the part taken next the vertex is to the other part as the complete figure to the deficient figure.

For let $CIAPE$ (in fig. 5) be a deficient figure, whose axis is AB , and whose complete figure is $CDFE$; and let the axis AB be so divided in Z , that AZ be to ZB as $CDFE$ is to $CIAPE$. I

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say, the centre of equiponderation of the figure CIAPE will be in the point Z.

First, that the centre of equiponderation of the figure CIAPE is somewhere in the axis AB is manifest of itself; and therefore AB is a diameter of equiponderation. Let AE be drawn, and let BE be put for the moment of the strait line CE; the triangle ABE will therefore (by the third article) be the moment of the complete figure CDFE. Let the axis AB be equally divided in L, and let GLH be drawn parallel and equal to the strait line CE, cutting the crooked line CIAPE in I and P, and the strait lines AC and AE in K and M. Moreover, let ZO be drawn parallel to the same CE; and let it be, as LG to LI, so LM to another, LN; and let the same be done in all the rest of the strait lines possible, parallel to the base; and through all the points N, let the line ANE be drawn; the three-sided figure ANEB will therefore be the moment of the figure CIAPE. Now the triangle ABE is (by the 9th article of chapter xvii) to the three-sided figure ANEB, as ABCD+AICB is to AICB twice taken, that is, as CDFE+CIAPE is to CIAPE twice taken. But as CIAPE is to CDFE, that is, as the weight of the deficient figure is to the weight of the complete figure, so is CIAPE twice taken to CDFE twice taken. Wherefore, setting in order CDFE+CIAPE. 2 CIAPE. 2 CDFE; the proportion of CDFE+CIAPE to CDFE twice taken will be compounded of the proportion of CDFE+CIAPE to CIAPE twice taken, that is, of the proportion

of the triangle ABE to the three-sided figure $ANEB$, that is, of the moment of the complete figure to the moment of the deficient figure, and of the proportion of $CIAPE$ twice taken to $CDFE$ twice taken, that is, to the proportion reciprocally taken of the weight of the deficient figure to the weight of the complete figure.

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Again, seeing by supposition $AZ.ZB :: CDFE.CIAPE$ are proportionals; $AB.AZ :: CDFE + CIAPE.CDFE$ will also, by compounding, be proportionals. And seeing AL is the half of AB , $AL.AZ :: CDFE + CIAPE.2CDFE$ will also be proportionals. But the proportion of $CDFE + CIAPE$ to $2CDFE$ is compounded, as was but now shown, of the proportions of moment to moment, &c., and therefore the proportion of AL to AZ is compounded of the proportion of the moment of the complete figure $CDFE$ to the moment of the deficient figure $CIAPE$, and of the proportion of the weight of the deficient figure $CIAPE$ to the weight of the complete figure $CDFE$; but the proportion of AL to AZ is compounded of the proportions of AL to BZ and of BZ to AZ . Now the proportion of BZ to AZ is the proportion of the weights reciprocally taken, that is to say, of the weight $CIAPE$ to the weight $CDFE$. Therefore the remaining proportion of AL to BZ , that is, of LB to BZ , is the proportion of the moment of the weight $CDFE$ to the moment of the weight $CIAPE$. But the proportion of AL to BZ is compounded of the proportions of AL to AZ and of AZ to ZB ; of which proportions that of AZ to ZB is the proportion of the weight $CDFE$ to the weight $CIAPE$. Wherefore (by art. 5 of this

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chapter) the remaining proportion of AL to AZ is the proportion of the distances of the points Z and L from the centre of the scale, which is A. And, therefore, (by art. 6) the weight CIAPE shall hang from O in the strait line OZ. So that OZ is one diameter of equiponderation of the weight CIAPE. But the strait line AB is the other diameter of equiponderation of the same weight CIAPE. Wherefore (by the 7th definition) the point Z is the centre of the same equiponderation; which point, by construction, divides the axis so, that the part AZ, which is the part next the vertex, is to the other part ZB, as the complete figure CDFE is to the deficient figure CIAPE; which is that which was to be demonstrated.

Coroll. I. The centre of equiponderation of any of those plane three-sided figures, which are compared with their complete figures in the table of art. 3, chap. xvii, is to be found in the same table, by taking the denominator of the fraction for the part of the axis cut off next the vertex, and the numerator for the other part next the base. For example, if it be required to find the centre of equiponderation of the second three-sided figure of four means, there is in the concourse of the second column with the row of three-sided figures of four means this fraction $\frac{5}{7}$, which signifies that that figure is to its parallelogram or complete figure as $\frac{5}{7}$ to unity, that is, as $\frac{5}{7}$ to 7, or as 5 to 7; and, therefore the centre of equiponderation of that figure divides the axis, so that the part next the vertex is to the other part as 7 to 5.

Coroll. II. The centre of equiponderation of any of the solids of those figures, which are contained

in the table of art. 7 of the same chap. xvii, is exhibited in the same table. For example, if the centre of equiponderation of a cone be sought for, the cone will be found to be $\frac{1}{3}$ of its cylinder; and, therefore, the centre of its equiponderation will so divide the axis, that the part next the vertex to the other part will be as 3 to 1. Also the solid of a three-sided figure of one mean, that is, a parabolical solid, seeing it is $\frac{1}{2}$, that is $\frac{1}{2}$ of its cylinder, will have its centre of equiponderation in that point, which divides the axis, so that the part towards the vertex be double to the part towards the base.

10. The diameter of equiponderation of the complement of the half of any of those figures which are contained in the table of art. 3, chap. xvii, divides that line which is drawn through the vertex parallel and equal to the base, so that the part next the vertex will be to the other part, as the complete figure to the complement.

For let A I C B (in the same fig. 5) be the half of a parabola, or of any other of those three-sided figures which are in the table of art. 3, chap. xvii, whose axis is A B, and base B C, having A D drawn from the vertex, equal and parallel to the base B C, and whose complete figure is the parallelogram A B C D. Let I Q be drawn at any distance from the side C D, but parallel to it; and let A D be the altitude of the complement A I C D, and Q I a line ordinately applied in it. Wherefore the altitude A L in the deficient figure A I C B is equal to Q I the line ordinately applied in its complement; and contrarily, L I the line ordinately applied in the figure A I C B is equal to the altitude

The diameter of equiponderation of the complement of the half of any of the said deficient figures, divides that line which is drawn through the vertex parallel to the base, so that the part next the vertex is to the other part as the complete figure to the complement.

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A Q in its complement ; and so in all the rest of the ordinate lines and altitudes the mutation is such, that that line, which is ordinately applied in the figure, is the altitude of its complement. And, therefore, the proportion of the altitudes decreasing to that of the ordinate lines decreasing, being multiply according to any number in the deficient figure, is submultiply according to the same number in its complement. For example, if A I C B be a parabola, seeing the proportion of A B to A L is duplicate to that of B C to L I, the proportion of A D to A Q in the complement A I C D, which is the same with that of B C to L I, will be subduplicate to that of C D to Q I, which is the same with that of A B to A L ; and consequently, in a parabola, the complement will be to the parallelogram as 1 to 3 ; in a three-sided figure of two means, as 1 to 4 ; in a three-sided figure of three means, as 1 to 5, &c. But all the ordinate lines together in A I C D are its moment ; and all the ordinate lines in A I C B are its moment. Wherefore the moments of the complements of the halves of deficient figures in the table of art. 3 of chap. xvii, being compared, are as the deficient figures themselves ; and, therefore, the diameter of equiponderation will divide the strait line A D in such proportion, that the part next the vertex be to the other part, as the complete figure A B C D is to the complement A I C D.

Coroll. The diameter of equiponderation of these halves may be found by the table of art. 3 of chap. xvii, in this manner. Let there be propounded any deficient figure, namely, the second three-sided figure of two means. This figure is to the com-

plete figure as $\frac{3}{5}$ to 1, that is 3 to 5. Wherefore the complement to the same complete figure is as 2 to 5; and, therefore, the diameter of equiponderation of this complement will cut the strait line drawn from the vertex parallel to the base, so that the part next the vertex will be to the other part as 5 to 2. And, in like manner, any other of the said three-sided figures being propounded, if the numerator of its fraction found out in the table be taken from the denominator, the strait line drawn from the vertex is to be divided, so that the part next the vertex be to the other part, as the denominator is to the remainder which that subtraction leaves.

11. The centre of equiponderation of the half of any of those crooked-lined figures, which are in the first row of the table of art. 3 of chap. xvii, is in that strait line which, being parallel to the axis, divides the base according to the numbers of the fraction next below it in the second row, so that the numerator be answerable to that part which is towards the axis.

The centre of equiponderation of the half of any of the deficient figures in the first row of the table of art. 3, chapter xvii, may be found out by the numbers of the second row.

For example, let the first figure of three means be taken, whose half is A B C D (in fig. 6), and let the rectangle A B E D be completed. The complement therefore will be B C D E. And seeing A B E D is to the figure A B C D (by the table) as 5 to 4, the same A B E D will be to the complement B C D E as 5 to 1. Wherefore, if F G be drawn parallel to the base D A, cutting the axis so that A G be to G B as 4 to 5, the centre of equiponderation of the figure A B C D will, by the precedent article, be somewhere in the same F G. Again, seeing, by the same article, the complete

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figure $A B E D$, is to the complement $B C D E$ as 5 to 1, therefore if BE and AD be divided in I and H as 5 to 1, the centre of equiponderation of the complement $B C D E$ will be somewhere in the strait line which connects H and I . Let now the strait line $L K$ be drawn through M the centre of the complete figure, parallel to the base; and the strait line $N O$ through the same centre M , perpendicular to it; and let the strait lines $L K$ and $F G$ cut the strait line $H I$ in P and Q . Let $P R$ be taken quadruple to $P Q$; and let $R M$ be drawn and produced to $F G$ in S . $R M$ therefore will be to $M S$ as 4 to 1, that is, as the figure $A B C D$ to its complement $B C D E$. Wherefore, seeing M is the centre of the complete figure $A B E D$, and the distances of R and S from the centre M be in proportion reciprocal to that of the weight of the complement $B C D E$ to the weight of the figure $A B C D$, R and S will either be the centres of equiponderation of their own figures, or those centres will be in some other points of the diameters of equiponderation $H I$ and $F G$. But this last is impossible. For no other strait line can be drawn through the point M terminating in the strait lines $H I$ and $F G$, and retaining the proportion of $M R$ to $M S$, that is, of the figure $A B C D$ to its complement $B C D E$. The centre, therefore, of equiponderation of the figure $A B C D$ is in the point S . Now, seeing $P M$ hath the same proportion to $Q S$ which $R P$ hath to $R Q$, $Q S$ will be 5 of those parts of which $P M$ is four, that is, of which $I N$ is four. But $I N$ or $P M$ is 2 of those parts of which $E B$ or $F G$ is 6; and, therefore, if it be as 4 to 5, so 2 to a fourth, that fourth

will be $2\frac{1}{2}$. Wherefore QS is $2\frac{1}{2}$ of those parts of which FG is 6. But FQ is 1; and, therefore, FS is $3\frac{1}{2}$. Wherefore the remaining part GS is $2\frac{1}{2}$. So that FG is so divided in S, that the part towards the axis is in proportion to the other part, as $2\frac{1}{2}$ to $3\frac{1}{2}$, that is as 5 to 7; which answereth to the fraction $\frac{5}{7}$ in the second row, next under the fraction $\frac{1}{2}$ in the first row. Wherefore drawing ST parallel to the axis, the base will be divided in like manner.

By this method it is manifest, that the base of a semiparabola will be divided into 3 and 5; and the base of the first three-sided figure of two means, into 4 and 6; and of the first three-sided figure of four means, into 6 and 8. The fractions, therefore, of the second row denote the proportions, into which the bases of the figures of the first row are divided by the diameters of equiponderation. But the first row begins one place higher than the second row.

12. The centre of equiponderation of the half of any of the figures in the second row of the same table of art. 3, chap. XVII, is in a strait line parallel to the axis, and dividing the base according to the numbers of the fraction in the fourth row, two places lower, so as that the numerator be answerable to that part which is next the axis.

The centre of equiponderation of the half of any of the figures of the second row of the same table may be found out by the numbers of the fourth row.

Let the half of the second three-sided figure of two means be taken; and let it be ABCD (in fig. 7); whose complement is BCDE, and the rectangle completed ABED. Let this rectangle be divided by the two strait lines LK and NO, cutting one another in the centre M at right angles; and because ABED is to ABCD as 5 to

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3, let AB be divided in G , so that AG to BG be as 3 to 5; and let FG be drawn parallel to the base. Also because $ABED$ is (by art. 9) to $BCDE$ as 5 to 2, let BE be divided in the point I , so that BI be to IE as 5 to 2; and let IH be drawn parallel to the axis, cutting LK and FG in P and Q . Let now PR be so taken, that it be to PQ as 3 to 2, and let RM be drawn and produced to FG in S . Seeing, therefore, RP is to PQ , that is, RM to MS , as $ABCD$ is to its complement $BCDE$, and the centres of equiponderation of $ABCD$ and $BCDE$ are in the strait lines FG and HI , and the centre of equiponderation of them both together in the point M ; R will be the centre of the complement $BCDE$, and S the centre of the figure $ABCD$. And seeing PM , that is IN , is to QS , as RP is to RQ ; and IN or PM is 3 of those parts, of which BE , that is FG , is 14; therefore QS is 5 of the same parts; and EI , that is FQ , 4; and FS , 9; and GS , 5. Wherefore the strait line ST being drawn parallel to the axis, will divide the base AD into 5 and 9. But the fraction $\frac{5}{14}$ is found in the fourth row of the table, two places below the fraction $\frac{3}{14}$ in the second row.

By the same method, if in the same second row there be taken the second three-sided figure of three means, the centre of equiponderation of the half of it will be found to be in a strait line parallel to the axis, dividing the base according to the numbers of the fraction $\frac{1}{10}$, two places below in the fourth row. And the same way serves for all the rest of the figures in the second row. In like manner, the centre of equiponderation of the third three-sided figure of three means will be found to

be in a strait line parallel to the axis, dividing the base, so that the part next the axis be to the other part as 7 to 13, &c.

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Coroll. The centres of equiponderation of the halves of the said figures are known, seeing they are in the intersection of the strait lines ST and FG , which are both known.

13. The centre of equiponderation of the half of any of the figures, which (in the table of art. 3, chap. xvii) are compared with their parallelograms, being known; the centre of equiponderation of the excess of the same figure above its triangle is also known.

The centre of equiponderation of the half of any of the figures in the same table being known, the centre of the excess of the same figure above a triangle of the same altitude and base is also known.

For example, let the semiparabola $ABCD$ (in fig. 8) be taken, whose axis is AB ; whose complete figure is $ABED$; and whose excess above its triangle is $BCDB$. Its centre of equiponderation may be found out in this manner. Let FG be drawn parallel to the base, so that AF be a third part of the axis; and let HI be drawn parallel to the axis, so that AH be a third part of the base. This being done, the centre of equiponderation of the triangle ABD will be I . Again, let KL be drawn parallel to the base, so that AK be to AB as 2 to 5; and MN parallel to the axis, so that AM be to AD as 3 to 8; and let MN terminate in the strait line KL . The centre, therefore, of equiponderation of the parabola $ABCD$ is N ; and therefore we have the centres of equiponderation of the semiparabola $ABCD$, and of its part the triangle ABD . That we may now find the centre of equiponderation of the remaining part $BCDB$, let IN be drawn and produced to O , so that NO be triple to IN ; and

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O will be the centre sought for. For seeing the weight of A B D to the weight of B C D B is in proportion reciprocal to that of the strait line N O to the strait line I N; and N is the centre of the whole, and I the centre of the triangle A B D; O will be the centre of the remaining part, namely, of the figure B D C B; which was to be found.

Coroll. The centre of equiponderation of the figure B D C B is in the concourse of two strait lines, whereof one is parallel to the base, and divides the axis, so that the part next the base be $\frac{1}{3}$ or $\frac{1}{15}$ of the whole axis; the other is parallel to the axis, and so divides the base, that the part towards the axis be $\frac{1}{3}$, or $\frac{1}{15}$ of the whole base. For drawing O P parallel to the base, it will be as I N to N O, so F K to K P, that is, so 1 to 3, or 5 to 15. But A F is $\frac{1}{15}$, or $\frac{1}{3}$ of the whole A B; and A K is $\frac{1}{15}$, or $\frac{1}{3}$; and F K $\frac{1}{15}$; and K P $\frac{1}{15}$; and therefore A P is $\frac{1}{15}$ of the axis A B. Also A H is $\frac{1}{3}$, or $\frac{2}{15}$; and A M $\frac{2}{3}$, or $\frac{10}{15}$ of the whole base; and therefore O Q being drawn parallel to the axis, M Q, which is triple to H M, will be $\frac{2}{15}$. Wherefore A Q is $\frac{1}{15}$, or $\frac{1}{3}$ of the base A D.

The excesses of the rest of the three-sided figures in the first row of the table of art. 3, chap. xvii, have their centres of equiponderation in two strait lines, which divide the axis and base according to those fractions, which add 4 to the numerators of the fractions of a parabola $\frac{2}{15}$ and $\frac{1}{15}$; and 6 to the denominators, in this manner:—

In a parabola,	the axis $\frac{1}{15}$, the base $\frac{2}{15}$.
In the first three-sided figure,	the axis $\frac{1}{15}$, the base $\frac{1}{15}$.
In the second three-sided figure,	the axis $\frac{2}{15}$, the base $\frac{1}{15}$, &c.

And by the same method, any man, if it be

worth the pains, may find out the centres of equiponderation of the excesses above their triangles of the rest of the figures in the second and third row, &c.

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14. The centre of equiponderation of the sector of a sphere, that is, of a figure compounded of a right cone, whose vertex is the centre of the sphere, and the portion of the sphere whose base is the same with that of the cone, divides the straight line which is made of the axis of the cone and half the axis of the portion together taken, so that the part next the vertex be triple to the other part, or to the whole straight line as 3 to 4.

The centre of equiponderation of a solid sector is in the axis so divided, that the part next the vertex be to the whole axis, wanting half the axis of the portion of the sphere, as 3 to 4.

For let ABC (in fig. 9) be the sector of a sphere, whose vertex is the centre of the sphere A ; whose axis is AD ; and the circle upon BC is the common base of the portion of the sphere and of the cone whose vertex is A ; the axis of which portion is ED , and the half thereof FD ; and the axis of the cone, AE . Lastly, let AG be $\frac{3}{4}$ of the straight line AF . I say, G is the centre of equiponderation of the sector ABC .

Let the straight line FH be drawn of any length, making right angles with AF at F ; and drawing the straight line AH , let the triangle $A FH$ be made. Then upon the same centre A let any arch IK be drawn, cutting AD in L ; and its chord, cutting AD in M ; and dividing ML equally in N , let NO be drawn parallel to the straight line FH , and meeting with the straight line AH in O .

Seeing now BDC is the spherical superficies of the portion cut off with a plane passing through BC , and cutting the axis at right angles; and seeing FH divides ED , the axis of the portion;

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into two equal parts in F ; the centre of equiponderation of the superficies BDC will be in F (by art. 8); and for the same reason the centre of equiponderation of the superficies ILK , K being in the strait line AC , will be in N . And in like manner, if there were drawn, between the centre of the sphere A and the outermost spherical superficies of the sector, arches infinite in number, the centres of equiponderation of the spherical superficies, in which those arches are, would be found to be in that part of the axis, which is intercepted between the superficies itself and a plane passing along by the chord of the arch, and cutting the axis in the middle at right angles.

Let it now be supposed that the moment of the outermost spherical superficies BDC is FH . Seeing therefore the superficies BDC is to the superficies ILK in proportion duplicate to that of the arch BDC to the arch ILK , that is, of BE to IM , that is, of FH to NO ; let it be as FH to NO , so NO to another NP ; and again, as NO to NP , so NP to another NQ ; and let this be done in all the strait lines parallel to the base FH that that can possibly be drawn between the base and the vertex of the triangle $A FH$. If then through all the points Q there be drawn the crooked line AQH , the figure $A FHQA$ will be the complement of the first three-sided figure of two means; and the same will also be the moment of all the spherical superficies, of which the solid sector $ABCD$ is compounded; and by consequent, the moment of the sector itself. Let now FH be understood to be the semidiameter of the base of a right cone, whose side is AH , and axis AF

Wherefore, seeing the bases of the cones, which pass through F and N and the rest of the points of the axis, are in proportion duplicate to that of the strait lines FH and NO, &c., the moment of all the bases together, that is, of the whole cone, will be the figure itself AFHQ A; and therefore the centre of equiponderation of the cone AFH is the same with that of the solid sector. Wherefore, seeing AG is $\frac{1}{4}$ of the axis AF, the centre of equiponderation of the cone AFH is in G; and therefore the centre of the solid sector is in G also, and divides the part AF of the axis so that AG is triple to GF; that is, AG is to AF as 3 to 4; which was to be demonstrated.

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Note, that when the sector is a hemisphere, the axis of the cone vanisheth into that point which is the centre of the sphere; and therefore it addeth nothing to half the axis of the portion. Wherefore, if in the axis of the hemisphere there be taken from the centre $\frac{1}{4}$ of half the axis, that is, $\frac{3}{8}$ of the semidiameter of the sphere, there will be the centre of equiponderation of the hemisphere.

CHAPTER XXIV.

OF REFRACTION AND REFLECTION.

1. Definitions.—2. In perpendicular motion there is no refraction.—3. Things thrown out of a thinner into a thicker medium are so refracted that the angle refracted is greater than the angle of inclination.—4. Endeavour, which from one point tendeth every way, will be so refracted, as that the sine of the angle refracted will be to the sine of the angle of inclination, as the density of the first medium is to the density of the second medium, reciprocally taken.—5. The sine of the refracted angle in one inclination is to the sine of the refracted angle in another inclination, as the sine of the angle of that inclination is to the sine of the angle of this inclination.—6. If two lines of incidence, having equal inclination, be the one in a thinner, the other in a thicker medium, the sine of the angle of inclination will be a mean proportional between the two sines of the refracted angles.—7. If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of their densities be the same with that of the diagonal to the side of a square, and the separating superficies be plane, the refracted line will be in the separating superficies.—8. If a body be carried in a strait line upon another body, and do not penetrate the same, but be reflected from it, the angle of reflection will be equal to the angle of incidence.—9. The same happens in the generation of motion in the line of incidence.

DEFINITIONS.

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Definitions.

I. REFRACTION is the *breaking* of that strait line, in which a body is moved or its action would proceed in one and the same medium, into two strait lines, by reason of the different natures of the two mediums.

II. The former of these is called the *line of incidence*; the latter the *refracted line*.

III. The *point of refraction* is the common point of the line of incidence, and of the refracted line.

IV. The *refracting superficies*, which also is the *separating superficies* of the two mediums, is that in which is the point of refraction.

V. The *angle refracted* is that, which the refracted line makes in the point of refraction with that line, which from the same point is drawn perpendicular to the separating superficies in a different medium.

VI. The *angle of refraction* is that which the refracted line makes with the line of incidence produced.

VII. The *angle of inclination* is that which the line of incidence makes with that line, which from the point of refraction is drawn perpendicular to the separating superficies.

VIII. The *angle of incidence* is the complement to a right angle of the angle of inclination.

And so, (in fig. 1) the refraction is made in A B F. The refracted line is B F. The line of incidence is A B. The point of incidence and of refraction is B. The refracting or separating superficies is D B E. The line of incidence produced directly is A B C. The perpendicular to the separating superficies is B H. The angle of refraction is C B F. The angle refracted is H B F. The angle of inclination is A B G or H B C. The angle of incidence is A B D.

IX. Moreover the *thinner medium* is understood to be that in which there is less resistance to motion, or to the generation of motion; and the thicker that wherein there is greater resistance.

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x. And that medium in which there is equal resistance everywhere, is a *homogeneous medium*. All other mediums are *heterogeneous*.

In perpendicular motion there is no refraction.

2. If a body pass, or there be generation of motion from one medium to another of different density, in a line perpendicular to the separating superficies, there will be no refraction.

For seeing on every side of the perpendicular all things in the mediums are supposed to be like and equal, if the motion itself be supposed to be perpendicular, the inclinations also will be equal, or rather none at all; and therefore there can be no cause from which refraction may be inferred to be on one side of the perpendicular, which will not conclude the same refraction to be on the other side. Which being so, refraction on one side will destroy refraction on the other side; and consequently either the refracted line will be everywhere, which is absurd, or there will be no refracted line at all; which was to be demonstrated.

Coroll. It is manifest from hence, that the cause of refraction consisteth only in the obliquity of the line of incidence, whether the incident body penetrate both the mediums, or without penetrating, propagate motion by pressure only.

Things thrown out of a thinner into a thicker medium are so refracted that the angle refracted is greater than the angle of inclination.

3. If a body, without any change of situation of its internal parts, as a stone, be moved obliquely out of the thinner medium, and proceed penetrating the thicker medium, and the thicker medium be such, as that its internal parts being moved restore themselves to their former situation; the angle refracted will be greater than the angle of inclination.

For let DBE (in the same first figure) be the separating superficies of two mediums; and let a body, as a stone thrown, be understood to be moved as is supposed in the strait line ABC ; and let AB be in the thinner medium, as in the air; and BC in the thicker, as in the water. I say the stone, which being thrown, is moved in the line AB , will not proceed in the line BC , but in some other line, namely, that, with which the perpendicular BH makes the refracted angle HBF greater than the angle of inclination HBC .

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Things thrown out of a thinner into a thicker medium, &c.

For seeing the stone coming from A , and falling upon B , makes that which is at B proceed towards H , and that the like is done in all the strait lines which are parallel to BH ; and seeing the parts moved restore themselves by contrary motion in the same line; there will be contrary motion generated in HB , and in all the strait lines which are parallel to it. Wherefore, the motion of the stone will be made by the concurrence of the motions in AG , that is, in DB , and in GB , that is, in BH , and lastly, in HB , that is, by the concurrence of three motions. But by the concurrence of the motions in AG and BH , the stone will be carried to C ; and therefore by adding the motion in HB , it will be carried higher in some other line, as in BF , and make the angle HBF greater than the angle HBC .

And from hence may be derived the cause, why bodies which are thrown in a very oblique line, if either they be any thing flat, or be thrown with great force, will, when they fall upon the water, be cast up again from the water into the air.

For let AB (in fig. 2) be the superficies of the

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water ; into which, from the point C, let a stone be thrown in the strait line CA, making with the line BA produced a very little angle CAD ; and producing BA indefinitely to D, let CD be drawn perpendicular to it, and AE parallel to CD. The stone therefore will be moved in CA by the concurrence of two motions in CD and DA, whose velocities are as the lines themselves CD and DA. And from the motion in CD and all its parallels downwards, as soon as the stone falls upon A, there will be reaction upwards, because the water restores itself to its former situation. If now the stone be thrown with sufficient obliquity, that is, if the strait line CD be short enough, that is, if the endeavour of the stone downwards be less than the reaction of the water upwards, that is, less than the endeavour it hath from its own gravity (for that may be), the stone will by reason of the excess of the endeavour which the water hath to restore itself, above that which the stone hath downwards, be raised again above the superficies AB, and be carried higher, being reflected in a line which goes higher, as the line AG.

Endeavour, which from one point tendeth every way, will be so refracted, as that the sine of the angle refracted will be to the sine of the angle of inclination, as the density of the first medium is to the density of the second medium, reciprocally taken.

4. If from a point, whatsoever the medium be, endeavour be propagated every way into all the parts of that medium ; and to the same endeavour there be obliquely opposed another medium of a different nature, that is, either thinner or thicker ; that endeavour will be so refracted, that the sine of the angle refracted, to the sine of the angle of inclination, will be as the density of the first medium to the density of the second medium, reciprocally taken.

First, let a body be in the thinner medium in A

Endeavour,
which, &c.

(fig. 3), and let it be understood to have endeavour every way, and consequently, that its endeavour proceed in the lines AB and $A b$; to which let $B b$ the superficies of the thicker medium be obliquely opposed in B and b , so that AB and $A b$ be equal; and let the strait line $B b$ be produced both ways. From the points B and b , let the perpendiculars BC and bc be drawn; and upon the centres B and b , and at the equal distances BA and bA , let the circles AC and $A c$ be described, cutting BC and bc in C and c , and the same CB and cb produced in D and d , as also AB and $A b$ produced in E and e . Then from the point A to the strait lines BC and bc let the perpendiculars AF and Af be drawn. AF therefore will be the sine of the angle of inclination of the strait line AB , and Af the sine of the angle of inclination of the strait line $A b$, which two inclinations are by construction made equal. I say, as the density of the medium in which are BC and bc is to the density of the medium in which are BD and bd , so is the sine of the angle refracted, to the sine of the angle of inclination.

Let the strait line FG be drawn parallel to the strait line AB , meeting with the strait line bB produced in G .

Seeing therefore AF and BG are also parallels, they will be equal; and consequently, the endeavour in AF is propagated in the same time, in which the endeavour in BG would be propagated if the medium were of the same density. But because BG is in a thicker medium, that is, in a medium which resists the endeavour more than the medium in which AF is, the endeavour will be

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which, &c.

propagated less in BG than in AF , according to the proportion which the density of the medium, in which AF is, hath to the density of the medium in which BG is. Let therefore the density of the medium, in which BG is, be to the density of the medium, in which AF is, as BG is to BH ; and let the measure of the time be the radius of the circle. Let HI be drawn parallel to BD , meeting with the circumference in I ; and from the point I let IK be drawn perpendicular to BD ; which being done, BH and IK will be equal; and IK will be to AF , as the density of the medium in which is AF is to the density of the medium in which is IK . Seeing therefore in the time AB , which is the radius of the circle, the endeavour is propagated in AF in the thinner medium, it will be propagated in the same time, that is, in the time BI in the thicker medium from K to I . Therefore, BI is the refracted line of the line of incidence AB ; and IK is the sine of the angle refracted; and AF the sine of the angle of inclination. Wherefore, seeing IK is to AF , as the density of the medium in which is AF to the density of the medium in which is IK ; it will be as the density of the medium in which is AF or BC to the density of the medium in which is IK or BD , so the sine of the angle refracted to the sine of the angle of inclination. And by the same reason it may be shown, that as the density of the thinner medium is to the density of the thicker medium, so will KI the sine of the angle refracted be to AF the sine of the angle of inclination.

Secondly, let the body, which endeavoureth every

way, be in the thicker medium at I. If, therefore, both the mediums were of the same density, the endeavour of the body in IB would tend directly to L; and the sine of the angle of inclination LM would be equal to IK or BH. But because the density of the medium, in which is IK, to the density of the medium, in which is LM, is as BH to BG, that is, to AF, the endeavour will be propagated further in the medium in which LM is, than in the medium in which IK is, in the proportion of density to density, that is, of ML to AF. Wherefore, BA being drawn, the angle refracted will be CBA, and its sine AF. But LM is the sine of the angle of inclination; and therefore again, as the density of one medium is to the density of the different medium, so reciprocally is the sine of the angle refracted to the sine of the angle of inclination; which was to be demonstrated.

In this demonstration, I have made the separating superficies B*b* plane by construction. But though it were concave or convex, the theorem would nevertheless be true. For the refraction being made in the point B of the plane separating superficies, if a crooked line, as PQ, be drawn, touching the separating line in the point B; neither the refracted line BI, nor the perpendicular BD, will be altered; and the refracted angle KBI, as also its sine KI, will be still the same they were.

5. The sine of the angle refracted in one inclination is to the sine of the angle refracted in another inclination, as the sine of the angle of that inclination to the sine of the angle of this inclination.

The sine of the refracted angle in one inclination is to the sine of the refracted angle in another incli-

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nation, as the sine of the angle of that inclination is to the sine of the angle of this inclination.

For seeing the sine of the refracted angle is to the sine of the angle of inclination, whatsoever that inclination be, as the density of one medium to the density of the other medium; the proportion of the sine of the refracted angle, to the sine of the angle of inclination, will be compounded of the proportions of density to density, and of the sine of the angle of one inclination to the sine of the angle of the other inclination. But the proportions of the densities in the same homogeneous body are supposed to be the same. Wherefore refracted angles in different inclinations are as the sines of the angles of those inclinations; which was to be demonstrated.

If two lines of incidence, having equal inclination, be one in a thinner the other in a thicker medium, the sine of the angle of inclination will be a mean proportional between the two sines of the refracted angles.

6. If two lines of incidence, having equal inclination, be the one in a thinner, the other in a thicker medium, the sine of the angle of their inclination will be a mean proportional between the two sines of their angles refracted.

For let the strait line AB (in fig. 3) have its inclination in the thinner medium, and be refracted in the thicker medium in BI ; and let EB have as much inclination in the thicker medium, and be refracted in the thinner medium in BS ; and let RS , the sine of the angle refracted, be drawn. I say, the strait lines RS , AF , and IK are in continual proportion. For it is, as the density of the thicker medium to the density of the thinner medium, so RS to AF . But it is also as the density of the same thicker medium to that of the same thinner medium, so AF to IK . Wherefore RS , AF , and IK are in continual proportion; that is, RS , AF , and IK are in continual proportion, and AF is the mean proportional; which was to be proved.

7. If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of the densities be as that of a diagonal to the side of its square, and the separating superficies be plain, the refracted line will be in that separating superficies.

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If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of their densities be the same with that of the diagonal to the side of a square, and the separating superficies be plain, the refracted line will be in the separating superficies.

For in the circle A C (fig. 4) let the angle of inclination A B C be an angle of 45 degrees. Let C B be produced to the circumference in D; and let C E, the sine of the angle E B C, be drawn, to which let B F be taken equal in the separating line B G. B C E F will therefore be a parallelogram, and F E and B C, that is F E and B G equal. Let A G be drawn, namely the diagonal of the square whose side is B G, and it will be, as A G to E F so B G to B F; and so, by supposition, the density of the medium, in which C is, to the density of the medium in which D is; and so also the sine of the angle refracted to the sine of the angle of inclination. Drawing therefore F D, and from D the line D H perpendicular to A B produced, D H will be the sine of the angle of inclination. And seeing the sine of the angle refracted is to the sine of the angle of inclination, as the density of the medium, in which is C, is to the density of the medium in which is D, that is, by supposition, as A G is to F E, that is as B G is to D H; and seeing D H is the sine of the angle of inclination, B G will therefore be the sine of the angle refracted. Wherefore B G will be the refracted line, and lye in the plain separating superficies; which was to be demonstrated.

Coroll. It is therefore manifest, that when the inclination is greater than 45 degrees, as also

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when it is less, provided the density be greater, it may happen that the refraction will not enter the thinner medium at all.

If a body be carried in a strait line upon another body, and do not penetrate it, but be reflected from it, the angle of reflection will be equal to the angle of incidence.

8. If a body fall in a strait line upon another body, and do not penetrate it, but be reflected from it, the angle of reflection will be equal to the angle of incidence.

Let there be a body at A (in fig. 5), which falling with strait motion in the line A C upon another body at C, passeth no further, but is reflected; and let the angle of incidence be any angle, as A C D. Let the strait line C E be drawn, making with D C produced the angle E C F equal to the angle A C D; and let A D be drawn perpendicular to the strait line D F. Also in the same strait line D F let C G be taken equal to C D; and let the perpendicular G E be raised, cutting C E in E. This being done, the triangles A C D and E C G will be equal and like. Let C H be drawn equal and parallel to the strait line A D; and let H C be produced indefinitely to I. Lastly let E A be drawn, which will pass through H, and be parallel and equal to G D. I say the motion from A to C, in the strait line of incidence A C, will be reflected in the strait line C E.

For the motion from A to C is made by two coefficient or concurrent motions, the one in A H parallel to D G, the other in A D perpendicular to the same D G; of which two motions that in A H works nothing upon the body A after it has been moved as far as C, because, by supposition, it doth not pass the strait line D G; whereas the endeavour in A D, that is in H C, worketh further towards I. But seeing it doth only press and not

penetrate, there will be reaction in H, which causeth motion from C towards H; and in the meantime the motion in HE remains the same it was in AH; and therefore the body will now be moved by the concurrence of two motions in CH and HE, which are equal to the two motions it had formerly in AH and HC. Wherefore it will be carried on in CE. The angle therefore of reflection will be ECG, equal, by construction, to the angle ACD; which was to be demonstrated.

Now when the body is considered but as a point, it is all one whether the superficies or line in which the reflection is made be strait or crooked; for the point of incidence and reflection C is as well in the crooked line which toucheth DG in C, as in DG itself.

9. But if we suppose that not a body be moved, but some endeavour only be propagated from A to C, the demonstration will nevertheless be the same. For all endeavour is motion; and when it hath reached the solid body in C, it presseth it, and endeavoureth further in CI. Wherefore the reaction will proceed in CH; and the endeavour in CH concurring with the endeavour in HE, will generate the endeavour in CE, in the same manner as in the repercussion of bodies moved.

The same happens in the generation of motion in the line of incidence.

If therefore endeavour be propagated from any point to the concave superficies of a spherical body, the reflected line with the circumference of a great circle in the same sphere will make an angle equal to the angle of incidence.

For if endeavour be propagated from A (in fig. 6) to the circumference in B, and the centre of the sphere be C, and the line CB be drawn, as

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also the tangent DBE ; and lastly if the angle FBD be made equal to the angle ABE , the reflection will be made in the line BF , as hath been newly shown. Wherefore the angles, which the strait lines AB and FB make with the circumference, will also be equal. But it is here to be noted, that if CB be produced howsoever to G , the endeavour in the line $GB C$ will proceed only from the perpendicular reaction in GB ; and that therefore there will be no other endeavour in the point B towards the parts which are within the sphere, besides that which tends towards the centre.

And here I put an end to the third part of this discourse; in which I have considered motion and magnitude by themselves in the abstract. The fourth and last part, concerning the *phenomena of nature*, that is to say, concerning the motions and magnitudes of the bodies which are parts of the world, real and existent, is that which follows.

PART IV.

PHYSICS, OR THE PHENOMENA OF NATURE.

CHAPTER XXV.

OF SENSE AND ANIMAL MOTION.

1. The connexion of what hath been said with that which followeth.—2. The investigation of the nature of sense, and the definition of sense.—3. The subject and object of sense.
4. The organs of sense.—5. All bodies are not indued with sense.—6. But one phantasm at one and the same time.
7. Imagination the remains of past sense, which also is memory. Of sleep.—8. How phantasms succeed one another.—9. Dreams, whence they proceed.—10. Of the senses, their kinds, their organs, and phantasms proper and common.—11. The magnitude of images, how and by what it is determined.
12. Pleasure, pain, appetite and aversion, what they are.
13. Deliberation and will, what.

1. I HAVE, in the first chapter, defined philosophy to be *knowledge of effects acquired by true ratiocination, from knowledge first had of their causes and generation; and of such causes or generations as may be, from former knowledge of their effects or appearances.* There are, therefore, two methods of philosophy; one, from the generation of things to their possible effects; and the

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other, from their effects or appearances to some possible generation of the same. In the former of these the truth of the first principles of our ratiocination, namely definitions, is made and constituted by ourselves, whilst we consent and agree about the appellations of things. And this part I have finished in the foregoing chapters; in which, if I am not deceived, I have affirmed nothing, saving the definitions themselves, which hath not good coherence with the definitions I have given; that is to say, which is not sufficiently demonstrated to all those, that agree with me in the use of words and appellations; for whose sake only I have written the same. I now enter upon the other part; which is the finding out by the appearances or effects of nature, which we know by sense, some ways and means by which they may be, I do not say they are, generated. The principles, therefore, upon which the following discourse depends, are not such as we ourselves make and pronounce in general terms, as definitions; but such, as being placed in the things themselves by the Author of Nature, are by us observed in them; and we make use of them in single and particular, not universal propositions. Nor do they impose upon us any necessity of constituting theorems; their use being only, though not without such general propositions as have been already demonstrated, to show us the possibility of some production or generation. Seeing, therefore, the science, which is here taught, hath its principles in the appearances of nature, and endeth in the attaining of some knowledge of natural causes, I have given to this part the title of PHYSICS, or the *Phœnomena of Nature*. Now

such things as appear, or are shown to us by nature, we call phenomena or appearances.

Of all the phenomena or appearances which are near us, the most admirable is apparition itself, τὸ φαίνεσθαι; namely, that some natural bodies have in themselves the patterns almost of all things, and others of none at all. So that if the appearances be the principles by which we know all other things, we must needs acknowledge sense to be the principle by which we know those principles, and that all the knowledge we have is derived from it. And as for the causes of sense, we cannot begin our search of them from any other phenomenon than that of sense itself. But you will say, by what sense shall we take notice of sense? I answer, by sense itself, namely, by the memory which for some time remains in us of things sensible, though they themselves pass away. For he that perceives that he hath perceived, remembers.

In the first place, therefore, the causes of our perception, that is, the causes of those ideas and phantasms which are perpetually generated within us whilst we make use of our senses, are to be enquired into; and in what manner their generation proceeds. To help which inquisition, we may observe first of all, that our phantasms or ideas are not always the same; but that new ones appear to us, and old ones vanish, according as we apply our organs of sense, now to one object, now to another. Wherefore they are generated, and perish. And from hence it is manifest, that they are some change or mutation in the sentient.

2. Now that all mutation or alteration is motion or endeavour (and endeavour also is motion)

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in the internal parts of the thing that is altered, hath been proved (in art. 9, chap. VIII) from this, that whilst even the least parts of any body remain in the same situation in respect of one another, it cannot be said that any alteration, unless perhaps that the whole body together hath been moved, hath happened to it; but that it both appeareth and is the same it appeared and was before. Sense, therefore, in the sentient, can be nothing else but motion in some of the internal parts of the sentient; and the parts so moved are parts of the organs of sense. For the parts of our body, by which we perceive any thing, are those we commonly call the organs of sense. And so we find what is the subject of our sense, namely, that in which are the phantasms; and partly also we have discovered the nature of sense, namely, that it is some internal motion in the sentient.

I have shown besides (in chap. IX, art. 7) that no motion is generated but by a body contiguous and moved: from whence it is manifest, that the immediate cause of sense or perception consists in this, that the first organ of sense is touched and pressed. For when the uttermost part of the organ is pressed, it no sooner yields, but the part next within it is pressed also; and, in this manner, the pressure or motion is propagated through all the parts of the organ to the innermost. And thus also the pressure of the uttermost part proceeds from the pressure of some more remote body, and so continually, till we come to that from which, as from its fountain, we derive the phantasm or idea that is made in us by our sense. And this, whatsoever it be, is that we commonly call *the object*,

Sense, therefore, is some internal motion in the sentient, generated by some internal motion of the parts of the object, and propagated through all the media to the innermost part of the organ. By which words I have almost defined what sense is.

Moreover, I have shown (art. 2, chap. xv) that all resistance is endeavour opposite to another endeavour, that is to say, reaction. Seeing, therefore, there is in the whole organ, by reason of its own internal natural motion, some resistance or reaction against the motion which is propagated from the object to the innermost part of the organ, there is also in the same organ an endeavour opposite to the endeavour which proceeds from the object; so that when that endeavour inwards is the last action in the act of sense, then from the reaction, how little soever the duration of it be, a phantasm or idea hath its being; which, by reason that the endeavour is now outwards, doth always appear as something situate without the organ. So that now I shall give you the whole definition of sense, as it is drawn from the explication of the causes thereof and the order of its generation, thus: *SENSE is a phantasm, made by the reaction and endeavour outwards in the organ of sense, caused by an endeavour inwards from the object, remaining for some time more or less.*

3. The *subject* of sense is the *sentient* itself, The subject and object of sense. namely, some living creature; and we speak more correctly, when we say a living creature seeth, than when we say the eye seeth. The object is the thing received; and it is more accurately said, that we see the sun, than that we see the light. For light and colour, and heat and sound, and

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The subject
and object of
sense.

other qualities which are commonly called sensible, are not objects, but phantasms in the sentient.

For a phantasm is the act of sense, and differs no otherwise from sense than *fieri*, that is, being a doing, differs from *factum esse*, that is, being done; which difference, in things that are done in an instant, is none at all; and a phantasm is made in an instant. For in all motion which proceeds by perpetual propagation, the first part being moved moves the second, the second the third, and so on to the last, and that to any distance, how great soever. And in what point of time the first or foremost part proceeded to the place of the second, which is thrust on, in the same point of time the last save one proceeded into the place of the last yielding part; which by reaction, in the same instant, if the reaction be strong enough, makes a phantasm; and a phantasm being made, perception is made together with it.

The organs of
sense.

4. The *organs* of sense, which are in the sentient, are such parts thereof, that if they be hurt, the very generation of phantasms is thereby destroyed, though all the rest of the parts remain entire. Now these parts in the most of living creatures are found to be certain spirits and membranes, which, proceeding from the *pia mater*, involve the brain and all the nerves; also the brain itself, and the arteries which are in the brain; and such other parts, as being stirred, the heart also, which is the fountain of all sense, is stirred together with them. For whensoever the action of the object reacheth the body of the sentient, that action is by some nerve propagated to the brain; and if the nerve leading thither be

so hurt or obstructed, that the motion can be propagated no further, no sense follows. Also if the motion be intercepted between the brain and the heart by the defect of the organ by which the action is propagated, there will be no perception of the object.

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5. But though all sense, as I have said, be made by reaction, nevertheless it is not necessary that every thing that reacteth should have sense. I know there have been philosophers, and those learned men, who have maintained that all bodies are endued with sense. Nor do I see how they can be refuted, if the nature of sense be placed in reaction only. And, though by the reaction of bodies inanimate a phantasm might be made, it would nevertheless cease, as soon as ever the object were removed. For unless those bodies had organs, as living creatures have, fit for the retaining of such motion as is made in them, their sense would be such, as that they should never remember the same. And therefore this hath nothing to do with that sense which is the subject of my discourse. For by sense, we commonly understand the judgment we make of objects by their phantasms; namely, by comparing and distinguishing those phantasms; which we could never do, if that motion in the organ, by which the phantasm is made, did not remain there for some time, and make the same phantasm return. Wherefore sense, as I here understand it, and which is commonly so called, hath necessarily some memory adhering to it, by which former and later phantasms may be compared together, and distinguished from one another.

All bodies are
not endued
with sense.

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All bodies are
not endued
with sense.

Sense, therefore, properly so called, must necessarily have in it a perpetual variety of phantasms, that they may be discerned one from another. For if we should suppose a man to be made with clear eyes, and all the rest of his organs of sight well disposed, but endued with no other sense; and that he should look only upon one thing, which is always of the same colour and figure, without the least appearance of variety, he would seem to me, whatsoever others may say, to see, no more than I seem to myself to feel the bones of my own limbs by my organs of feeling; and yet those bones are always and on all sides touched by a most sensible membrane. I might perhaps say he were astonished, and looked upon it; but I should not say he saw it; it being almost all one for a man to be always sensible of one and the same thing, and not to be sensible at all of any thing.

But one phantasm at one and the same time.

6. And yet such is the nature of sense, that it does not permit a man to discern many things at once. For seeing the nature of sense consists in motion; as long as the organs are employed about one object, they cannot be so moved by another at the same time, as to make by both their motions one sincere phantasm of each of them at once. And therefore two several phantasms will not be made by two objects working together, but only one phantasm compounded from the action of both.

Besides, as when we divide a body, we divide its place; and when we reckon many bodies, we must necessarily reckon as many places; and contrarily, as I have shown in the seventh chapter; so what number soever we say there be of times, we

must understand the same number of motions also; and as oft as we count many motions, so oft we reckon many times. For though the object we look upon be of divers colours, yet with those divers colours it is but one varied object, and not variety of objects.

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But one phantasm at one and the same time.

Moreover, whilst those organs which are common to all the senses, such as are those parts of every organ which proceed in men from the root of the nerves to the heart, are vehemently stirred by a strong action from some one object, they are, by reason of the contumacy which the motion, they have already, gives them against the reception of all other motion, made the less fit to receive any other impression from whatsoever other objects, to what sense soever those objects belong. And hence it is, that an earnest studying of one object, takes away the sense of all other objects for the present. For *study* is nothing else but a possession of the mind, that is to say, a vehement motion made by some one object in the organs of sense, which are stupid to all other motions as long as this lasteth; according to what was said by Terence, "*Populus studio stupidus in funambulo animum occuparat.*" For what is *stupor* but that which the Greeks call *ἀνασθησία*, that is, a cessation from the sense of other things? Wherefore at one and the same time, we cannot by sense perceive more than one single object; as in reading, we see the letters successively one by one, and not all together, though the whole page be presented to our eye; and though every several letter be distinctly written there, yet when we look upon the whole page at once, we read nothing.

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From hence it is manifest, that every endeavour of the organ outwards, is not to be called sense, but that only, which at several times is by vehemence made stronger and more predominant than the rest; which deprives us of the sense of other phantasms, no otherwise than the sun deprives the rest of the stars of light, not by hindering their action, but by obscuring and hiding them with his excess of brightness.

Imagination,
the remains of
past sense;
which also
is memory.
Of sleep.

7. But the motion of the organ, by which a phantasm is made, is not commonly called sense, except the object be present. And the phantasm remaining after the object is removed or past by, is called *fancy*, and in Latin *imaginatio*; which word, because all phantasms are not images, doth not fully answer the signification of the word *fancy* in its general acceptation. Nevertheless I may use it safely enough, by understanding it for the Greek *Φαντασία*.

IMAGINATION therefore is nothing else but *sense decaying*, or *weakened*, by the absence of the object. But what may be the cause of this decay or weakening? Is the motion the weaker, because the object is taken away? If it were, then phantasms would always and necessarily be less clear in the imagination, than they are in sense; which is not true. For in dreams, which are the imaginations of those that sleep, they are no less clear than in sense itself. But the reason why in men waking the phantasms of things past are more obscure than those of things present, is this, that their organs being at the same time moved by other present objects, those phantasms are the less predominant. Whereas in sleep, the passages

being shut up, external action doth not at all disturb or hinder internal motion.

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If this be true, the next thing to be considered, will be, whether any cause may be found out, from the supposition whereof it will follow, that the passage is shut up from the external objects of sense to the internal organ. I suppose, therefore, that by the continual action of objects, to which a reaction of the organ, and more especially of the spirits, is necessarily consequent, the organ is wearied, that is, its parts are no longer moved by the spirits without some pain; and consequently the nerves being abandoned and grown slack, they retire to their fountain, which is the cavity either of the brain or of the heart; by which means the action which proceeded by the nerves is necessarily intercepted. For action upon a patient, that retires from it, makes but little impression at the first; and at last, when the nerves are by little and little slackened, none at all. And therefore there is no more reaction, that is, no more sense, till the organ being refreshed by rest, and by a supply of new spirits recovering strength and motion, the sentient awaketh. And thus it seems to be always, unless some other preternatural cause intervene; as heat in the internal parts from lassitude, or from some disease stirring the spirits and other parts of the organ in some extraordinary manner.

8. Now it is not without cause, nor so casual a thing as many perhaps think it, that phantasms in this their great variety proceed from one another; and that the same phantasms sometimes bring into the mind other phantasms like themselves, and at

How phantasms succeed one another.

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How phantasms succeed one another.

other times extremely unlike. For in the motion of any continued body, one part follows another by cohesion ; and therefore, whilst we turn our eyes and other organs successively to many objects, the motion which was made by every one of them remaining, the phantasms are renewed as often as any one of those motions comes to be predominant above the rest ; and they become predominant in the same order in which at any time formerly they were generated by sense. So that when by length of time very many phantasms have been generated within us by sense, then almost any thought may arise from any other thought ; insomuch that it may seem to be a thing indifferent and casual, which thought shall follow which. But for the most part this is not so uncertain a thing to waking as to sleeping men. For the thought or phantasm of the desired end brings in all the phantasms, that are means conducing to that end, and that in order backwards from the last to the first, and again forwards from the beginning to the end. But this supposes both appetite, and judgment to discern what means conduce to the end, which is gotten by experience ; and experience is store of phantasms, arising from the sense of very many things. For *φανταζεσθαι* and *meminisse*, *fancy* and *memory*, differ only in this, that memory supposeth the time past, which fancy doth not. In memory, the phantasms we consider are as if they were worn out with time ; but in our fancy we consider them as they are ; which distinction is not of the things themselves, but of the considerations of the sentient. For there is in memory something like that which happens in looking upon things at a great

distance ; in which as the small parts of the object are not discerned, by reason of their remoteness ; so in memory, many accidents and places and parts of things, which were formerly perceived by sense, are by length of time decayed and lost.

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The perpetual arising of phantasms, both in sense and imagination, is that which we commonly call discourse of the mind, and is common to men with other living creatures. For he that thinketh, compareth the phantasms that pass, that is, taketh notice of their likeness or unlikeness to one another. And as he that observes readily the likenesses of things of different natures, or that are very remote from one another, is said to have a good fancy ; so he is said to have a good judgment, that finds out the unlikenesses or differences of things that are like one another. Now this observation of differences is not perception made by a common organ of sense, distinct from sense or perception properly so called, but is memory of the differences of particular phantasms remaining for some time ; as the distinction between hot and lucid, is nothing else but the memory both of a heating, and of an enlightening object.

9. The phantasms of men that sleep, are *dreams*. Dreams, whence they proceed. Concerning which we are taught by experience these five things. First, that for the most part there is neither order nor coherence in them. Secondly, that we dream of nothing but what is compounded and made up of the phantasms of sense past. Thirdly, that sometimes they proceed, as in those that are drowsy, from the interruption of their phantasms by little and little, broken and altered through sleepiness ; and sometimes also .

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whence they
proceed.

they begin in the midst of sleep. Fourthly, that they are clearer than the imaginations of waking men, except such as are made by sense itself, to which they are equal in clearness. Fifthly, that when we dream, we admire neither the places nor the looks of the things that appear to us. Now from what hath been said, it is not hard to show what may be the causes of these phenomena. For as for the first, seeing all order and coherence proceeds from frequent looking back to the end, that is, from consultation; it must needs be, that seeing in sleep we lose all thought of the end, our phantasms succeed one another, not in that order which tends to any end, but as it happeneth, and in such manner, as objects present themselves to our eyes when we look indifferently upon all things before us, and see them, not because we would see them, but because we do not shut our eyes; for then they appear to us without any order at all. The second proceeds from this, that in the silence of sense there is no new motion from the objects, and therefore no new phantasm, unless we call that new, which is compounded of old ones, as a chimaera, a golden mountain, and the like. As for the third, why a dream is sometimes as it were the continuation of sense, made up of broken phantasms, as in men distempered with sickness, the reason is manifestly this, that in some of the organs sense remains, and in others it faileth. But how some phantasms may be revived, when all the exterior organs are benumbed with sleep, is not so easily shown. Nevertheless that, which hath already been said, contains the reason of this also. For whatsoever strikes the *pia mater*, reviveth

some of those phantasms that are still in motion in the brain ; and when any internal motion of the heart reacheth that membrane, then the predominant motion in the brain makes the phantasm. Now the motions of the heart are appetites and aversions, of which I shall presently speak further. And as appetites and aversions are generated by phantasms, so reciprocally phantasms are generated by appetites and aversions. For example, heat in the heart proceeds from anger and fighting ; and again, from heat in the heart, whatsoever be the cause of it, is generated anger and the image of an enemy, in sleep. And as love and beauty stir up heat in certain organs ; so heat in the same organs, from whatsoever it proceeds, often causeth desire and the image of an unresisting beauty. Lastly, cold doth in the same manner generate fear in those that sleep, and causeth them to dream of ghosts, and to have phantasms of horror and danger ; as fear also causeth cold in those that wake. So reciprocal are the motions of the heart and brain. The fourth, namely, that the things we seem to see and feel in sleep, are as clear as in sense itself, proceeds from two causes ; one, that having then no sense of things without us, that internal motion which makes the phantasm, in the absence of all other impressions, is predominant ; and the other, that the parts of our phantasms which are decayed and worn out by time, are made up with other fictitious parts. To conclude, when we dream, we do not wonder at strange places and the appearances of things unknown to us, because admiration requires that the things appearing be new and unusual, which can

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happen to none but those that remember former appearances; whereas in sleep, all things appear as present.

But it is here to be observed, that certain dreams, especially such as some men have when they are between sleeping and waking, and such as happen to those that have no knowledge of the nature of dreams and are withal superstitious, were not heretofore nor are now accounted dreams. For the apparitions men thought they saw, and the voices they thought they heard in sleep, were not believed to be phantasms, but things subsisting of themselves, and objects without those that dreamed. For to some men, as well sleeping as waking, but especially to guilty men, and in the night, and in hallowed places, fear alone, helped a little with the stories of such apparitions, hath raised in their minds terrible phantasms, which have been and are still deceitfully received for things really true, under the names of *ghosts* and *incorporeal substances*.

Of the senses,
their kinds,
their organs
and phantasms,
proper and
common.

10. In most living creatures there are observed five kinds of senses, which are distinguished by their organs, and by their different kinds of phantasms; namely, *sight*, *hearing*, *smell*, *taste*, and *touch*; and these have their organs partly peculiar to each of them severally, and partly common to them all. The organ of sight is partly animate, and partly inanimate. The inanimate parts are the three humours; namely, the watery humour, which by the interposition of the membrane called uvea, the perforation whereof is called the apple of the eye, is contained on one side by the first concave superficies of the eye, and on the other side by the

ciliary processes, and the coat of the crystalline humour; the crystalline, which, hanging in the midst between the ciliary processes, and being almost of spherical figure, and of a thick consistence, is enclosed on all sides with its own transparent coat; and the vitreous or glassy humour, which filleth all the rest of the cavity of the eye, and is somewhat thicker than the watery humour, but thinner than the crystalline. The animate part of the organ is, first, the membrane *chorooides*, which is a part of the *pia mater*, saving that it is covered with a coat derived from the marrow of the optic nerve, which is called the *retina*; and this *chorooides*, seeing it is part of the *pia mater*, is continued to the beginning of the *medulla spinalis* within the skull, in which all the nerves which are within the head have their roots. Wherefore all the animal spirits that the nerves receive, enter into them there; for it is not imaginable that they can enter into them anywhere else. Seeing therefore sense is nothing else but the action of objects propagated to the furthest part of the organ; and seeing also that animal spirits are nothing but vital spirits purified by the heart, and carried from it by the arteries; it follows necessarily, that the action is derived from the heart by some of the arteries to the roots of the nerves which are in the head, whether those arteries be the *plexus retiformis*, or whether they be other arteries which are inserted into the substance of the brain. And, therefore, those arteries are the complement or the remaining part of the whole organ of sight. And this last part is a common organ to all the senses; whereas, that which reacheth from the eye to the

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roots of the nerves is proper only to sight. The proper organ of hearing is the tympanum of the ear and its own nerve; from which to the heart the organ is common. So the proper organs of smell and taste are nervous membranes, in the palate and tongue for the taste, and in the nostrils for the smell; and from the roots of those nerves to the heart all is common. Lastly, the proper organ of touch are nerves and membranes dispersed through the whole body; which membranes are derived from the root of the nerves. And all things else belonging alike to all the senses seem to be administered by the arteries, and not by the nerves.

The proper phantasm of sight is light; and under this name of light, colour also, which is nothing but perturbed light, is comprehended. Wherefore the phantasm of a lucid body is light; and of a coloured body, colour. But the object of sight, properly so called, is neither light nor colour, but the body itself which is lucid, or enlightened, or coloured. For light and colour, being phantasms of the sentient, cannot be accidents of the object. Which is manifest enough from this, that visible things appear oftentimes in places in which we know assuredly they are not, and that in different places they are of different colours, and may at one and the same time appear in divers places. Motion, rest, magnitude, and figure, are common both to the sight and touch; and the whole appearance together of figure, and light or colour, is by the Greeks commonly called *εἶδος*, and *εἶδωλον*, and *ἰδέα*; and by the Latins, *species* and

imago; all which names signify no more but appearance. PART IV.
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The phantasm, which is made by hearing, is sound; by smell, odour; by taste, savour; and by touch, hardness and softness, heat and cold, wetness, oiliness, and many more, which are easier to be distinguished by sense than words. Smoothness, roughness, rarity, and density, refer to figure, and are therefore common both to touch and sight. And as for the objects of hearing, smell, taste, and touch, they are not sound, odour, savour, hardness, &c., but the bodies themselves from which sound, odour, savour, hardness, &c. proceed; of the causes of which, and of the manner how they are produced, I shall speak hereafter.

But these phantasms, though they be effects in the sentient, as subject, produced by objects working upon the organs; yet there are also other effects besides these, produced by the same objects in the same organs; namely certain motions proceeding from sense, which are called *animal motions*. For seeing in all sense of external things there is mutual action and reaction, that is, two endeavours opposing one another, it is manifest that the motion of both of them together will be continued every way, especially to the confines of both the bodies. And when this happens in the internal organ, the endeavour outwards will proceed in a solid angle, which will be greater, and consequently the idea greater, than it would have been if the impression had been weaker.

11. From hence the natural cause is manifest, first, why those things seem to be greater, which, *cæteris paribus*, are seen in a greater angle: The magnitude of images, how and by what it is determined.

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secondly, why in a serene cold night, when the moon doth not shine, more of the fixed stars appear than at another time. For their action is less hindered by the serenity of the air, and not obscured by the greater light of the moon, which is then absent; and the cold, making the air more pressing, helpeth or strengtheneth the action of the stars upon our eyes; in so much as stars may then be seen which are seen at no other time. And this may suffice to be said in general concerning sense made by the reaction of the organ. For, as for the place of the image, the deceptions of sight, and other things of which we have experience in ourselves by sense, seeing they depend for the most part upon the fabric itself of the eye of man, I shall speak of them then when I come to speak of man.

Pleasure, pain, appetite, and aversion, what they are.

12. But there is another kind of sense, of which I will say something in this place, namely, the sense of pleasure and pain, proceeding not from the reaction of the heart outwards, but from continual action from the outermost part of the organ towards the heart. For the original of life being in the heart, that motion in the sentient, which is propagated to the heart, must necessarily make some alteration or diversion of vital motion, namely, by quickening or slackening, helping or hindering the same. Now when it helpeth, it is pleasure; and when it hindereth, it is pain, trouble, grief, &c. And as phantasms seem to be without, by reason of the endeavour outwards, so pleasure and pain, by reason of the endeavour of the organ inwards, seem to be within; namely, there where the first cause of the pleasure or pain is; as when the

pain proceeds from a wound, we think the pain and the wound are both in the same place.

Now vital motion is the motion of the blood, perpetually circulating (as hath been shown from many infallible signs and marks by Doctor Harvey, the first observer of it) in the veins and arteries. Which motion, when it is hindered by some other motion made by the action of sensible objects, may be restored again either by bending or setting strait the parts of the body; which is done when the spirits are carried now into these, now into other nerves, till the pain, as far as is possible, be quite taken away. But if vital motion be helped by motion made by sense, then the parts of the organ will be disposed to guide the spirits in such manner as conduceth most to the preservation and augmentation of that motion, by the help of the nerves. And in animal motion this is the very first endeavour, and found even in the embryo; which while it is in the womb, moveth its limbs with voluntary motion, for the avoiding of whatsoever troubleth it, or for the pursuing of what pleaseth it. And this first endeavour, when it tends towards such things as are known by experience to be pleasant, is called *appetite*, that is, an approaching; and when it shuns what is troublesome, *aversion*, or flying from it. And little infants, at the beginning and as soon as they are born, have appetite to very few things, as also they avoid very few, by reason of their want of experience and memory; and therefore they have not so great a variety of animal motion as we see in those that are more grown. For it is not possible, with-

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out such knowledge as is derived from sense, that is, without experience and memory, to know what will prove pleasant or hurtful ; only there is some place for conjecture from the looks or aspects of things. And hence it is, that though they do not know what may do them good or harm, yet sometimes they approach and sometimes retire from the same thing, as their doubt prompts them. But afterwards, by accustoming themselves by little and little, they come to know readily what is to be pursued and what to be avoided ; and also to have a ready use of their nerves and other organs, in the pursuing and avoiding of good and bad. Wherefore appetite and aversion are the first endeavours of animal motion.

Consequent to this first endeavour, is the impulsion into the nerves and retraction again of animal spirits, of which it is necessary there be some receptacle or place near the original of the nerves ; and this motion or endeavour is followed by a swelling and relaxation of the muscles ; and lastly, these are followed by contraction and extension of the limbs, which is animal motion.

Deliberation
and will, what.

13. The considerations of appetites and aversions are divers. For seeing living creatures have sometimes appetite and sometimes aversion to the same thing, as they think it will either be for their good or their hurt ; while that vicissitude of appetites and aversions remains in them, they have that series of thoughts which is called *deliberation* ; which lasteth as long as they have it in their power to obtain that which pleaseth, or to avoid that which displeaseth them. Appetite, therefore, and aversion are simply so called as long as they follow

not deliberation. But if deliberation have gone before, then the last act of it, if it be appetite, is called *will*; if aversion, *unwillingness*. So that the same thing is called both will and appetite; but the consideration of them, namely, before and after deliberation, is divers. Nor is that which is done within a man whilst he willeth any thing, different from that which is done in other living creatures, whilst, deliberation having preceded, they have appetite.

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Neither is the freedom of willing or not willing, greater in man, than in other living creatures. For where there is appetite, the entire cause of appetite hath preceded; and, consequently, the act of appetite could not choose but follow, that is, hath of necessity followed (as is shown in chapter IX, article 5). And therefore such a liberty as is free from necessity, is not to be found in the will either of men or beasts. But if by liberty we understand the faculty or power, not of willing, but of doing what they will, then certainly that liberty is to be allowed to both, and both may equally have it, whensoever it is to be had.

Again, when appetite and aversion do with celerity succeed one another, the whole series made by them hath its name sometimes from one, sometimes from the other. For the same deliberation, whilst it inclines sometimes to one, sometimes to the other, is from appetite called *hope*, and from aversion, *fear*. For where there is no hope, it is not to be called fear, but *hate*; and where no fear, not hope, but *desire*. To conclude, all the passions, called passions of the mind, consist of appetite and aversion, except pure pleasure and pain, which are

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a certain fruition of good or evil ; as anger is aversion from some imminent evil, but such as is joined with appetite of avoiding that evil by force. But because the passions and perturbations of the mind are innumerable, and many of them not to be discerned in any creatures besides men ; I will speak of them more at large in that section which is concerning *man*. As for those objects, if there be any such, which do not at all stir the mind, we are said to contemn them.

And thus much of sense in general. In the next place I shall speak of sensible objects.

CHAPTER XXVI.

OF THE WORLD AND OF THE STARS.

1. The magnitude and duration of the world, inscrutable.—2. No place in the world empty.—3. The arguments of Lucretius for vacuum, invalid.—4. Other arguments for the establishing of vacuum, invalid.—5. Six suppositions for the salving of the phenomena of nature.—6. Possible causes of the motions annual and diurnal ; and of the apparent direction, station, and retrogradation of the planets.—7. The supposition of simple motion, why likely.—8. The cause of the eccentricity of the annual motion of the earth.—9. The cause why the moon hath always one and the same face turned towards the earth.
10. The cause of the tides of the ocean.—11. The cause of the precession of the equinoxes.

The magnitude
and duration of
the world, in-
scrutable.

1. CONSEQUENT to the contemplation of sense is the contemplation of bodies, which are the efficient causes or objects of sense. Now every object is either a part of the whole world, or an aggregate of parts. The greatest of all bodies, or sensible objects, is the world itself ; which we behold when,

we look round about us from this point of the same which we call the earth. Concerning the world, as it is one aggregate of many parts, the things that fall under inquiry are but few; and those we can determine, none. Of the whole world we may inquire what is its magnitude, what its duration, and how many there be, but nothing else. For as for place and time, that is to say, magnitude and duration, they are only our own fancy of a body simply so called, that is to say, of a body indefinitely taken, as I have shown before in chapter VII. All other phantasms are of bodies or objects, as they are distinguished from one another; as colour, the phantasm of coloured bodies; sound, of bodies that move the sense of hearing, &c. The questions concerning the magnitude of the world are whether it be finite or infinite, full or not full; concerning its duration, whether it had a beginning, or be eternal; and concerning the number, whether there be one or many; though as concerning the number, if it were of infinite magnitude, there could be no controversy at all. Also if it had a beginning, then by what cause and of what matter it was made; and again, from whence that cause and that matter had their being, will be new questions; till at last we come to one or many eternal cause or causes. And the determination of all these things belongeth to him that professeth the universal doctrine of philosophy, in case as much could be known as can be sought. But the knowledge of what is infinite can never be attained by a finite inquirer. Whatsoever we know that are men, we learn it from our phantasms; and of infinite, whether magnitude or time, there is no phantasm

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at all; so that it is impossible either for a man or any other creature to have any conception of infinite. And though a man may from some effect proceed to the immediate cause thereof, and from that to a more remote cause, and so ascend continually by right ratiocination from cause to cause; yet he will not be able to proceed eternally, but wearied will at last give over, without knowing whether it were possible for him to proceed to an end or not. But whether we suppose the world to be finite or infinite, no absurdity will follow. For the same things which now appear, might appear, whether the Creator had pleased it should be finite or infinite. Besides, though from this, that nothing can move itself, it may rightly be inferred that there was some first eternal movent; yet it can never be inferred, though some used to make such inference, that that movent was eternally immovable, but rather eternally moved. For as it is true, that nothing is moved by itself; so it is true also that nothing is moved but by that which is already moved. The questions therefore about the magnitude and beginning of the world, are not to be determined by philosophers, but by those that are lawfully authorized to order the worship of God. For as Almighty God, when he had brought his people into Judæa, allowed the priests the first fruits reserved to himself; so when he had delivered up the world to the disputations of men, it was his pleasure that all opinions concerning the nature of infinite and eternal, known only to himself, should, as the first fruits of wisdom, be judged by those whose ministry he meant to use in the ordering of religion. I cannot therefore commend those that

boast they have demonstrated, by reasons drawn from natural things, that the world had a beginning. They are contemned by idiots, because they understand them not; and by the learned, because they understand them; by both deservedly. For who can commend him that demonstrates thus? "If the world be eternal, then an infinite number of days, or other measures of time, preceded the birth of Abraham. But the birth of Abraham preceded the birth of Isaac; and therefore one infinite is greater than another infinite, or one eternal than another eternal; which," he says, "is absurd." This demonstration is like his, who from this, that the number of even numbers is infinite, would conclude that there are as many even numbers as there are numbers simply, that is to say, the even numbers are as many as all the even and odd together. They, which in this manner take away eternity from the world, do they not by the same means take away eternity from the Creator of the world? From this absurdity therefore they run into another, being forced to call eternity *nunc stans*, a standing still of the present time, or an abiding now; and, which is much more absurd, to give to the infinite number of numbers the name of unity. But why should eternity be called an abiding now, rather than an abiding then? Wherefore there must either be many eternities, or *now* and *then* must signify the same. With such demonstrators as these, that speak in another language, it is impossible to enter into disputation. And the men, that reason thus absurdly, are not idiots, but, which makes the absurdity unpardonable, geometricians, and such as take upon them to be judges,

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impertinent, but severe judges of other men's demonstrations. The reason is this, that as soon as they are entangled in the words *infinite* and *eternal*, of which we have in our mind no idea, but that of our own insufficiency to comprehend them, they are forced either to speak something absurd, or, which they love worse, to hold their peace. For geometry hath in it somewhat like wine, which, when new, is windy; but afterwards though less pleasant, yet more wholesome. Whatsoever therefore is true, young geometricians think demonstrable; but elder not. Wherefore I purposely pass over the questions of infinite and eternal; contenting myself with that doctrine concerning the beginning and magnitude of the world, which I have been persuaded to by the holy Scriptures and fame of the miracles which confirm them; and by the custom of my country, and reverence due to the laws. And so I pass on to such things as it is not unlawful to dispute of.

No place in the world empty.

2. Concerning the world it is further questioned, whether the parts thereof be contiguous to one another, in such manner as not to admit of the least empty space between; and the disputation both for and against it is carried on with probability enough. For the taking away of vacuum, I will instance in only one experiment, a common one, but I think unanswerable.

Let A B (in fig. 1) represent a vessel, such as gardeners use to water their gardens withal; whose bottom B is full of little holes; and whose mouth A may be stopped with one's finger, when there shall be need. If now this vessel be filled with water, the hole at the top A being stopped, the

water will not flow out at any of the holes in the bottom B. But if the finger be removed to let in the air above, it will run out at them all; and as soon as the finger is applied to it again, the water will suddenly and totally be stayed again from running out. The cause whereof seems to be no other but this, that the water cannot by its natural endeavour to descend drive down the air below it, because there is no place for it to go into, unless either by thrusting away the next contiguous air, it proceed by continual endeavour to the hole A, where it may enter and succeed into the place of the water that floweth out, or else, by resisting the endeavour of the water downwards, penetrate the same and pass up through it. By the first of these ways, while the hole at A remains stopped, there is no possible passage; nor by the second, unless the holes be so great that the water, flowing out at them, can by its own weight force the air at the same time to ascend into the vessel by the same holes: as we see it does in a vessel whose mouth is wide enough, when we turn suddenly the bottom upwards to pour out the water; for then the air being forced by the weight of the water, enters, as is evident by the sobbing and resistance of the water, at the sides or circumference of the orifice. And this I take for a sign that all space is full; for without this, the natural motion of the water, which is a heavy body, downwards, would not be hindered.

3. On the contrary, for the establishing of vacuum, many and specious arguments and experiments have been brought. Nevertheless there seems to be something wanting in all of them to

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conclude it firmly. These arguments for vacuum are partly made by the followers of the doctrine of Epicurus; who taught that the world consists of very small spaces not filled by any body, and of very small bodies that have within them no empty space, which by reason of their hardness he calls *atoms*; and that these small bodies and spaces are every where intermingled. Their arguments are thus delivered by Lucretius.

And first he says, that unless it were so, there could be no motion. For the office and property of bodies is to withstand and hinder motion. If, therefore, the universe were filled with body, motion would everywhere be hindered, so as to have no beginning anywhere; and consequently there would be no motion at all. It is true that in whatsoever is full and at rest in all its parts, it is not possible motion should have beginning. But nothing is drawn from hence for the proving of vacuum. For though it should be granted that there is vacuum, yet if the bodies which are intermingled with it, should all at once and together be at rest, they would never be moved again. For it has been demonstrated above, in chap. ix, art. 7, that nothing can be moved but by that which is contiguous and already moved. But supposing that all things are at rest together, there can be nothing contiguous and moved, and therefore no beginning of motion. Now the denying of the beginning of motion, doth not take away present motion, unless beginning be taken away from body also. For motion may be either co-eternal, or concreated with body. Nor doth it seem more necessary that bodies were first at rest,

and afterwards moved, than that they were first moved, and rested, if ever they rested at all, afterwards. Neither doth there appear any cause, why the matter of the world should, for the admission of motion, be intermingled with empty spaces rather than full; I say full, but withal fluid. Nor, lastly, is there any reason why those hard atoms may not also, by the motion of intermingled fluid matter, be congregated and brought together into compounded bodies of such bigness as we see. Wherefore nothing can by this argument be concluded, but that motion was either coeternal, or of the same duration with that which is moved; neither of which conclusions consisteth with the doctrine of Epicurus, who allows neither to the world nor to motion any beginning at all. The necessity, therefore, of vacuum is not hitherto demonstrated. And the cause, as far as I understand from them that have discoursed with me of vacuum, is this, that whilst they contemplate the nature of fluid, they conceive it to consist, as it were, of small grains of hard matter, in such manner as meal is fluid, made so by grinding of the corn; when nevertheless it is possible to conceive fluid to be of its own nature as homogeneous as either an atom, or as vacuum itself.

The second of their arguments is taken from weight, and is contained in these verses of Lucretius:

Corporis officium est quoniam premere omnia deorsum;
 Contra autem natura manet sine pondere inanis;
 Ergo, quod magnum est æque, leviusque videtur,
 Nimirum plus esse sibi declarat inanis.—I. 363-66.

That is to say, *seeing the office and property of*

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body is to press all things downwards; and on the contrary, seeing the nature of vacuum is to have no weight at all; therefore when of two bodies of equal magnitude, one is lighter than the other, it is manifest that the lighter body hath in it more vacuum than the other.

To say nothing of the assumption concerning the endeavour of bodies downwards, which is not rightly assumed, because the world hath nothing to do with downwards, which is a mere fiction of ours; nor of this, that if all things tended to the same lowest part of the world, either there would be no coalescence at all of bodies, or they would all be gathered together into the same place: this only is sufficient to take away the force of the argument, that air, intermingled with those his atoms, had served as well for his purpose as his intermingled vacuum.

The third argument is drawn from this, that lightning, sound, heat and cold, do penetrate all bodies, except atoms, how solid soever they be. But this reason, except it be first demonstrated that the same things cannot happen without vacuum by perpetual generation of motion, is altogether invalid. But that all the same things may so happen, shall in due place be demonstrated.

Lastly, the fourth argument is set down by the same Lucretius in these verses:

Duo de concursu corpora lata
Si cita dissiliant, nempe aer omne necesse est,
Inter corpora quod fuerat, possidat inane.
Is porro quamvis circum celerantibus auris
Confluat, haud poterit tamen uno tempore totum
Compleri spatium; nam primum quemque necesse est
Occupet ille locum, deinde omnia possideantur.—I. 385-91.

That is, *if two flat bodies be suddenly pulled asunder, of necessity the air must come between them to fill up the space they left empty. But with what celerity soever the air flow in, yet it cannot in one instant of time fill the whole space, but first one part of it, then successively all.*

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Which nevertheless is more repugnant to the opinion of Epicurus, than of those that deny vacuum. For though it be true, that if two bodies were of infinite hardness, and were joined together by their superficies which were most exactly plane, it would be impossible to pull them asunder, in regard it could not be done but by motion in an instant ; yet, if as the greatest of all magnitudes cannot be given, nor the swiftest of all motions, so neither the hardest of all bodies ; it might be, that by the application of very great force, there might be place made for a successive flowing in of the air, namely, by separating the parts of the joined bodies by succession, beginning at the outermost and ending at the innermost part. He ought, therefore, first to have proved, that there are some bodies extremely hard, not relatively as compared with softer bodies, but absolutely, that is to say, infinitely hard ; which is not true. But if we suppose, as Epicurus doth, that atoms are indivisible, and yet have small superficies of their own ; then if two bodies should be joined together by many, or but one only small superficies of either of them, then I say this argument of Lucretius would be a firm demonstration, that no two bodies made up of atoms, as he supposes, could ever, possibly be pulled asunder by any force whatsoever. But this is repugnant to daily experience.

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4. And thus much of the arguments of Lucretius. Let us now consider the arguments which are drawn from the experiments of later writers.

I. The first experiment is this : that if a hollow vessel be thrust into water with the bottom upwards, the water will ascend into it ; which they say it could not do, unless the air within were thrust together into a narrower place ; and that this were also impossible, except there were little empty places in the air. Also, that when the air is compressed to a certain degree, it can receive no further compression, its small particles not suffering themselves to be pent into less room. This reason, if the air could not pass through the water as it ascends within the vessel, might seem valid. But it is sufficiently known, that air will penetrate water by the application of a force equal to the gravity of the water. If therefore the force, by which the vessel is thrust down, be greater or equal to the endeavour by which the water naturally tendeth downwards, the air will go out that way where the resistance is made, namely, towards the edges of the vessel. For, by how much the deeper is the water which is to be penetrated, so much greater must be the depressing force. But after the vessel is quite under water, the force by which it is depressed, that is to say, the force by which the water riseth up, is no longer increased. There is therefore such an equilibration between them, as that the natural endeavour of the water downwards is equal to the endeavour by which the same water is to be penetrated to the increased depth.

II. The second experiment is, that if a concave

cylinder of sufficient length, made of glass, that the experiment may be the better seen, having one end open and the other close shut, be filled with quicksilver, and the open end being stopped with one's finger, be together with the finger dipped into a dish or other vessel, in which also there is quicksilver, and the cylinder be set upright, we shall, the finger being taken away to make way for the descent of the quicksilver, see it descend into the vessel under it, till there be only so much remaining within the cylinder as may fill about twenty-six inches of the same; and thus it will always happen whatsoever be the cylinder, provided that the length be not less than twenty-six inches. From whence they conclude that the cavity of the cylinder above the quicksilver remains empty of all body. But in this experiment I find no necessity at all of vacuum. For when the quicksilver which is in the cylinder descends, the vessel under it must needs be filled to a greater height, and consequently so much of the contiguous air must be thrust away as may make place for the quicksilver which is descended. Now if it be asked whither that air goes, what can be answered but this, that it thrusteth away the next air, and that the next, and so successively, till there be a return to the place where the propulsion first began. And there, the last air thus thrust on will press the quicksilver in the vessel with the same force with which the first air was thrust away; and if the force with which the quicksilver descends be great enough, which is greater or less as it descends from a place of greater or less height, it will make the air penetrate the quicksilver in the

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vessel, and go up into the cylinder to fill the place which they thought was left empty. But because the quicksilver hath not in every degree of height force enough to cause such penetration, therefore in descending it must of necessity stay somewhere, namely, there, where its endeavour downwards, and the resistance of the same to the penetration of the air, come to an equilibrium. And by this experiment it is manifest, that this equilibrium will be at the height of twenty-six inches, or thereabouts.

III. The third experiment is, that when a vessel hath as much air in it as it can naturally contain, there may nevertheless be forced into it as much water as will fill three quarters of the same vessel. And the experiment is made in this manner. Into the glass bottle, represented (in figure 2) by the sphere FG, whose centre is A, let the pipe BAC be so fitted, that it may precisely fill the mouth of the bottle; and let the end B be so near the bottom, that there may be only space enough left for the free passage of the water which is thrust in above. Let the upper end of this pipe have a cover at D, with a spout at E, by which the water, when it ascends in the pipe, may run out. Also let HC be a cock, for the opening or shutting of the passage of the water between B and D, as there shall be occasion. Let the cover DE be taken off, and the cock HC being opened, let a syringe full of water be forced in; and before the syringe be taken away, let the cock be turned to hinder the going out of the air. And in this manner let the injection of water be repeated as often as it shall be requisite, till the water rise within the bottle; for example, to GF. Lastly, the cover being

fastened on again, and the cock H C opened, the water will run swiftly out at E, and sink by little and little from G F to the bottom of the pipe B.

From this phenomenon, they argue for the necessity of vacuum in this manner. The bottle, from the beginning, was full of air; which air could neither go out by penetrating so great a length of water as was injected by the pipe, nor by any other way. Of necessity, therefore, all the water as high as F G, as also all the air that was in the bottle before the water was forced in, must now be in the same place, which at first was filled by the air alone; which were impossible, if all the space within the bottle were formerly filled with air precisely, that is, without any vacuum. Besides, though some man perhaps may think the air, being a thin body, may pass through the body of the water contained in the pipe, yet from that other phenomenon, namely, that all the water which is in the space B F G is cast out again by the spout at E, for which it seems impossible that any other reason can be given besides the force by which the air frees itself from compression, it follows, that either there was in the bottle some space empty, or that many bodies may be together in the same place. But this last is absurd; and therefore the former is true, namely, that there was vacuum.

This argument is infirm in two places. For first, that is assumed which is not to be granted; and in the second place, an experiment is brought, which I think is repugnant to vacuum. That which is assumed is, that the air can have no passage out through the pipe. Nevertheless, we see daily that air easily ascends from the bottom to the

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superficies of a river, as is manifest by the bubbles that rise; nor doth it need any other cause to give it this motion, than the natural endeavour downwards of the water. Why, therefore, may not the endeavour upwards of the same water, acquired by the injection, which endeavour upwards is greater than the natural endeavour of the water downwards, cause the air in the bottle to penetrate in like manner the water that presseth it downwards; especially, seeing the water, as it riseth in the bottle, doth so press the air that is above it, as that it generateth in every part thereof an endeavour towards the external superficies of the pipe, and consequently maketh all the parts of the enclosed air to tend directly towards the passage at B? I say, this is no less manifest, than that the air which riseth up from the bottom of a river should penetrate the water, how deep soever it be. Wherefore I do not yet see any cause why the force, by which the water is injected, should not at the same time eject the air.

And as for their arguing the necessity of vacuum from the rejection of the water; in the first place, supposing there is vacuum, I demand by what principle of motion that ejection is made. Certainly, seeing this motion is from within outwards, it must needs be caused by some agent within the bottle; that is to say, by the air itself. Now the motion of that air, being caused by the rising of the water, begins at the bottom, and tends upwards; whereas the motion by which it ejecteth the water ought to begin above, and tend downwards. From whence therefore hath the enclosed air this endeavour towards the bottom? To this question I know not

what answer can be given, unless it be said, that the air descends of its own accord to expel the water. Which, because it is absurd, and that the air, after the water is forced in, hath as much room as its magnitude requires, there will remain no cause at all why the water should be forced out. Wherefore the assertion of vacuum is repugnant to the very experiment which is here brought to establish it.

Many other phenomena are usually brought for vacuum, as those of *weather-glasses*, *æolipyles*, *wind-guns*, &c. which would all be very hard to be salved, unless water be penetrable by air, without the intermixture of empty space. But now, seeing air may with no great endeavour pass through not only water, but any other fluid body though never so stubborn, as quicksilver, these phenomena prove nothing. Nevertheless, it might in reason be expected, that he that would take away vacuum, should without vacuum show us such causes of these phenomena, as should be at least of equal, if not greater probability. This therefore shall be done in the following discourse, when I come to speak of these phenomena in their proper places. But first, the most general hypotheses of natural philosophy are to be premised.

And seeing that suppositions are put for the true causes of apparent effects, every supposition, except such as be absurd, must of necessity consist of some supposed possible motion; for rest can never be the efficient cause of anything; and motion supposeth bodies moveable; of which there are three kinds, *fluid*, *consistent*, and *mixed of both*. *Fluid* are those, whose parts may by very weak endeavour

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be separated from one another; and *consistent* those for the separation of whose parts greater force is to be applied. There are therefore degrees of consistency; which degrees, by comparison with more or less consistent, have the names of *hardness* or *softness*. Wherefore a fluid body is always divisible into bodies equally fluid, as quantity into quantities; and soft bodies, of whatsoever degree of softness, into soft bodies of the same degree. And though many men seem to conceive no other difference of *fluidity*, but such as ariseth from the different magnitudes of the parts, in which sense dust, though of diamonds, may be called fluid; yet I understand by *fluidity*, that which is made such by nature equally in every part of the fluid body; not as dust is fluid, for so a house which is falling in pieces may be called fluid; but in such manner as water seems fluid, and to divide itself into parts perpetually fluid. And this being well understood, I come to my suppositions.

Six suppositions for the salving of the phenomena of nature.

5. First, therefore, I suppose that the immense space, which we call the world, is the aggregate of all bodies which are either consistent and visible, as the earth and the stars; or invisible, as the small atoms which are disseminated through the whole space between the earth and the stars; and lastly, that most fluid ether, which so fills all the rest of the universe, as that it leaves in it no empty place at all.

Secondly, I suppose with Copernicus, that the greater bodies of the world, which are both consistent and permanent, have such order amongst themselves, as that the sun hath the first place,

Mercury the second, Venus the third, the Earth with the moon going about it the fourth, Mars the fifth, Jupiter with his attendants the sixth, Saturn the seventh; and after these, the fixed stars have their several distances from the sun.

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Thirdly, I suppose that in the sun and the rest of the planets there is and always has been a simple circular motion.

Fourthly, I suppose that in the body of the air there are certain other bodies intermingled, which are not fluid; but withal that they are so small, that they are not perceptible by sense; and that these also have their proper simple motion, and are some of them more, some less hard or consistent.

Fifthly, I suppose with Kepler that as the distance between the sun and the earth is to the distance between the moon and the earth, so the distance between the moon and the earth is to the semidiameter of the earth.

As for the magnitude of the circles, and the times in which they are described by the bodies which are in them, I will suppose them to be such as shall seem most agreeable to the phenomena in question.

6. The causes of the different seasons of the year, and of the several variations of days and nights in all the parts of the superficies of the earth, have been demonstrated, first by Copernicus, and since by Kepler, Galileus, and others, from the supposition of the earth's diurnal revolution about its own axis, together with its annual motion about the sun in the ecliptic according to the order of the signs; and thirdly, by the annual

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revolution of the same earth about its own centre, contrary to the order of the signs. I suppose with Copernicus, that the diurnal revolution is from the motion of the earth, by which the equinoctial circle is described about it. And as for the other two annual motions, they are the efficient cause of the earth's being carried about in the ecliptic in such manner, as that its axis is always kept parallel to itself. Which parallelism was for this reason introduced, lest by the earth's annual revolution its poles should seem to be necessarily carried about the sun, contrary to experience. I have, in art. 10, chap. XXI, demonstrated, from the supposition of simple circular motion in the sun, that the earth is so carried about the sun, as that its axis is thereby kept always parallel to itself. Wherefore, from these two supposed motions in the sun, the one simple circular motion, the other circular motion about its own centre, it may be demonstrated that the year hath both the same variations of days and nights, as have been demonstrated by Copernicus.

For if the circle $abcd$ (in fig. 3) be the ecliptic, whose centre is e , and diameter aec ; and the earth be placed in a , and the sun be moved in the little circle $fghi$, namely, according to the order f, g, h , and i , it hath been demonstrated, that a body placed in a will be moved in the same order through the points of the ecliptic a, b, c , and d , and will always keep its axis parallel to itself.

But if, as I have supposed, the earth also be moved with simple circular motion in a plane that passeth through a , cutting the plane of the ecliptic so as that the common section of both the planes

be in ac , thus also the axis of the earth will be kept always parallel to itself. For let the centre of the earth be moved about in the circumference of the epicycle, whose diameter is lak , which is a part of the strait line lac ; therefore lak , the diameter of the epicycle, passing through the centre of the earth, will be in the plane of the ecliptic. Wherefore seeing that by reason of the earth's simple motion both in the ecliptic and in its epicycle, the strait line lak is kept always parallel to itself, every other strait line also taken in the body of the earth, and consequently its axis, will in like manner be kept always parallel to itself; so that in what part soever of the ecliptic the centre of the epicycle be found, and in what part soever of the epicycle the centre of the earth be found at the same time, the axis of the earth will be parallel to the place where the same axis would have been, if the centre of the earth had never gone out of the ecliptic.

Now as I have demonstrated the simple annual motion of the earth from the supposition of simple motion in the sun; so from the supposition of simple motion in the earth may be demonstrated the monthly simple motion of the moon. For if the names be but changed, the demonstration will be the same, and therefore need not be repeated.

7. That which makes this supposition of the sun's simple motion in the epicycle $fghi$ probable, is first, that the periods of all the planets are not only described about the sun, but so described, as that they are all contained within the zodiac, that is to say, within the latitude of about sixteen degrees; for the cause of this seems to

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depend upon some power in the sun, especially in that part of the sun which respects the zodiac. Secondly, that in the whole compass of the heavens there appears no other body from which the cause of this phenomenon can in probability be derived. Besides, I could not imagine that so many and such various motions of the planets should have no dependance at all upon one another. But, by supposing motive power in the sun, we suppose motion also ; for power to move without motion is no power at all. I have therefore supposed that there is in the sun for the governing of the primary planets, and in the earth for the governing of the moon, such motion, as being received by the primary planets and by the moon, makes them necessarily appear to us in such manner as we see them. Whereas, that circular motion, which is commonly attributed to them, about a fixed axis, which is called conversion, being a motion of their parts only, and not of their whole bodies, is insufficient to salve their appearances. For seeing whatsoever is so moved, hath no endeavour at all towards those parts which are without the circle, they have no power to propagate any endeavour to such bodies as are placed without it. And as for them that suppose this may be done by magnetical virtue, or by incorporeal and immaterial species, they suppose no natural cause ; nay, no cause at all. For there is no such thing as an incorporeal movent, and magnetical virtue is a thing altogether unknown ; and whensoever it shall be known, it will be found to be a motion of body. It remains, therefore, that if the primary planets be carried about by the sun, and the moon by the earth, they

have the simple circular motions of the sun and the earth for the causes of their circulations. Otherwise, if they be not carried about by the sun and the earth, but that every planet hath been moved, as it is now moved, ever since it was made, there will be of their motions no cause natural. For either these motions were concreated with their bodies, and their cause is supernatural; or they are coeternal with them, and so they have no cause at all. For whatsoever is eternal was never generated.

I may add besides, to confirm the probability of this simple motion, that as almost all learned men are now of the same opinion with Copernicus concerning the parallelism of the axis of the earth, it seemed to me to be more agreeable to truth, or at least more handsome, that it should be caused by simple circular motion alone, than by two motions, one in the ecliptic, and the other about the earth's own axis the contrary way, neither of them simple, nor either of them such as might be produced by any motion of the sun. I thought best therefore to retain this hypothesis of simple motion, and from it to derive the causes of as many of the phenomena as I could, and to let such alone as I could not deduce from thence.

It will perhaps be objected, that although by this supposition the reason may be given of the parallelism of the axis of the earth, and of many other appearances, nevertheless, seeing it is done by placing the body of the sun in the centre of that orb which the earth describes with its annual motion, the supposition itself is false; because this annual orb is eccentric to the sun. In the first

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place, therefore, let us examine what that eccentricity is, and whence it proceeds.

8. Let the annual circle of the earth $abcd$ (in fig. 3) be divided into four equal parts by the strait lines ac and bd , cutting one another in the centre e ; and let a be the beginning of Libra, b of Capricorn, c of Aries and d of Cancer; and let the whole orb $abcd$ be understood, according to Copernicus, to have every way so great distance from the zodiac of the fixed stars, that it be in comparison with it but as a point. Let the earth be now supposed to be in the beginning of Libra at a . The sun, therefore, will appear in the beginning of Aries at c . Wherefore, if the earth be moved from a to b , the apparent motion of the sun will be from c to the beginning of Cancer in d ; and the earth being moved forwards from b to c , the sun also will appear to be moved forwards to the beginning of Libra in a ; wherefore cda will be the summer arch, and the winter arch will be abc . Now, in the time of the sun's apparent motion in the summer arch, there are numbered $186\frac{1}{2}$ days; and, consequently, the earth makes in the same time the same number of diurnal conversions in the arch abc ; and, therefore, the earth in its motion through the arch cda will make only $178\frac{1}{2}$ diurnal conversions. Wherefore the arch abc ought to be greater than the arch cda by $8\frac{1}{2}$ days, that is to say, by almost so many degrees. Let the arch ar , as also cs , be each of them an arch of two degrees and $\frac{1}{8}$. Wherefore the arch rbs will be greater than the semicircle abc by $4\frac{1}{2}$ degrees, and greater than the arch sdr by $8\frac{1}{2}$ degrees. The equinoxes, therefore, will be

in the points r and s ; and therefore also, when the earth is in r , the sun will appear in s . Wherefore the true place of the sun will be in t , that is to say, without the centre of the earth's annual motion by the quantity of the sine of the arch $a r$, or the sine of two degrees and 16 minutes. Now this sine, putting 100,000 for the radius, will be near 3580 parts thereof. And so much is the eccentricity of the earth's annual motion, provided that that motion be in a perfect circle; and s and r are the equinoctial parts. And the strait lines $s r$ and $c a$, produced both ways till they reach the zodiac of the fixed stars, will fall still upon the same fixed stars; because the whole orb $a b c d$ is supposed to have no magnitude at all in respect of the great distance of the fixed stars.

Supposing now the sun to be in c , it remains that I show the cause why the earth is nearer to the sun, when in its annual motion it is found to be in d , than when it is in b . And I take the cause to be this. When the earth is in the beginning of Capricorn at b , the sun appears in the beginning of Cancer at d ; and then is the midst of summer. But in the midst of summer, the northern parts of the earth are towards the sun, which is almost all dry land, containing all Europe and much the greatest part of Asia and America. But when the earth is in the beginning of Cancer at d , it is the midst of winter, and that part of the earth is towards the sun, which contains those great seas called the South Sea and the Indian Sea, which are of far greater extent than all the dry land in that hemisphere. Wherefore by the last article of chapter XXI, when the earth is in d , it will come nearer to

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its first movent, that is, to the sun which is in t ; that is to say, the earth is nearer to the sun in the midst of winter when it is in d , than in the midst of summer when it is in b ; and, therefore, during the winter the sun is in its *Perigæum*, and in its *Apogæum* during the summer. And thus I have shown a possible cause of the eccentricity of the earth; which was to be done.

I am, therefore, of Kepler's opinion in this, that he attributes the eccentricity of the earth to the difference of the parts thereof, and supposes one part to be affected, and another disaffected to the sun. And I dissent from him in this, that he thinks it to be by magnetic virtue, and that this magnetic virtue or attraction and thrusting back of the earth is wrought by immateriate species; which cannot be, because nothing can give motion but a body moved and contiguous. For if those bodies be not moved which are contiguous to a body unmoved, how this body should begin to be moved is not imaginable; as has been demonstrated in art. 7, chap. IX, and often inculcated in other places, to the end that philosophers might at last abstain from the use of such unconceivable connexions of words. I dissent also from him in this, that he says the similitude of bodies is the cause of their mutual attraction. For if it were so, I see no reason why one egg should not be attracted by another. If, therefore, one part of the earth be more affected by the sun than another part, it proceeds from this, that one part hath more water, the other more dry land. And from hence it is, as I showed above, that the earth comes nearer to the sun when it shines upon that part where there is more water,

than when it shines upon that where there is more dry land.

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9. This eccentricity of the earth is the cause why the way of its annual motion is not a perfect circle, but either an elliptical, or almost an elliptical line; as also why the axis of the earth is not kept exactly parallel to itself in all places, but only in the equinoctial points.

The cause why the moon hath always one and the same face turned towards the earth.

Now seeing I have said that the moon is carried about by the earth, in the same manner that the earth is by the sun; and that the earth goeth about the sun in such manner as that it shows sometimes one hemisphere, sometimes the other to the sun; it remains to be enquired, why the moon has always one and the same face turned towards the earth.

Suppose, therefore, the sun to be moved with simple motion in the little circle $fg hi$, (in fig. 4) whose centre is t ; and let $v \ominus \triangle w$ be the annual circle of the earth; and a the beginning of Libra. About the point a let the little circle lk be described; and in it let the centre of the earth be understood to be moved with simple motion; and both the sun and the earth to be moved according to the order of the signs. Upon the centre a let the way of the moon $mno p$ be described; and let $q r$ be the diameter of a circle cutting the globe of the moon into two hemispheres, whereof one is seen by us when the moon is at the full, and the other is turned from us.

The diameter therefore of the moon $q o r$ will be perpendicular to the strait line ta . Wherefore the moon is carried, by reason of the motion of the earth, from o towards p . But by reason of the

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motion of the sun, if it were in p it would at the same time be carried from p towards o ; and by these two contrary movents the strait line qr will be turned about; and, in a quadrant of the circle $mno p$, it will be turned so much as makes the fourth part of its whole conversion. Wherefore when the moon is in p , qr will be parallel to the strait line mo . Secondly, when the moon is in m , the strait line qr will, by reason of the motion of the earth, be in mo . But by the working of the sun's motion upon it in the quadrant pm , the same qr will be turned so much as makes another quarter of its whole conversion. When, therefore, the moon is in m , qr will be perpendicular to the strait line om . By the same reason, when the moon is in n , qr will be parallel to the strait line mo ; and, the moon returning to o , the same qr will return to its first place; and the body of the moon will in one entire period make also one entire conversion upon her own axis. In the making of which, it is manifest, that one and the same face of the moon is always turned towards the earth. And if any diameter were taken in that little circle, in which the moon were supposed to be carried about with simple motion, the same effect would follow; for if there were no action from the sun, every diameter of the moon would be carried about always parallel to itself. Wherefore I have given a possible cause why one and the same face of the moon is always turned towards the earth.

But it is to be noted, that when the moon is without the ecliptic, we do not always see the same face precisely. For we see only that part which is illuminated. But when the moon is without the

ecliptic, that part which is towards us is not exactly the same with that which is illuminated.

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10. To these three simple motions, one of the sun, another of the moon, and the third of the earth, in their own little circles $f g h i$, $l k$, and $q r$, together with the diurnal conversion of the earth, by which conversion all things that adhere to its superficies are necessarily carried about with it, may be referred the three phenomena concerning the tides of the ocean. Whereof the first is the alternate elevation and depression of the water at the shores, twice in the space of twenty-four hours and near upon fifty-two minutes; for so it has constantly continued in all ages. The second, that at the new and full moons, the elevations of the water are greater than at other times between. And the third, that when the sun is in the equinoctial, they are yet greater than at any other time. For the salving of which phenomena, we have already the four above-mentioned motions; to which I assume also this, that the part of the earth which is called America, being higher than the water, and extended almost the space of a whole semicircle from north to south, gives a stop to the motion of the water.

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This being granted, in the same 4th figure, where $l b k c$ is supposed to be in the plane of the moon's monthly motion, let the little circle $l d k e$ be described about the same centre a in the plane of the equinoctial. This circle therefore will decline from the circle $l b k c$ in an angle of almost $28\frac{1}{2}$ degrees; for the greatest declination of the ecliptic is $23\frac{1}{2}$, to which adding 5 for the greatest declination of the moon from the ecliptic, the sum will be $28\frac{1}{2}$

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degrees. Seeing now the waters, which are under the circle of the moon's course, are by reason of the earth's simple motion in the plane of the same circle moved together with the earth, that is to say, together with their own bottoms, neither outgoing nor outgone; if we add the diurnal motion, by which the other waters which are under the equinoctial are moved in the same order, and consider withal that the circles of the moon and of the equinoctial intersect one another; it will be manifest, that both those waters, which are under the circle of the moon, and under the equinoctial, will run together under the equinoctial; and consequently, that their motion will not only be swifter than the ground that carries them; but also that the waters themselves will have greater elevation whensoever the earth is in the equinoctial. Wherefore, whatsoever the cause of the tides may be, this may be the cause of their augmentation at that time.

Again, seeing I have supposed the moon to be carried about by the simple motion of the earth in the little circle $l b k c$; and demonstrated, at the 4th article of chapter XXI, that whatsoever is moved by a movent that hath simple motion, will be moved always with the same velocity; it follows, that the centre of the earth will be carried in the circumference $l b k c$ with the same velocity with which the moon is carried in the circumference $m n o p$. Wherefore the time, in which the moon is carried about in $m n o p$, is to the time, in which the earth is carried about in $l b k c$, as one circumference to the other, that is, as $a o$ to $a k$. But $a o$ is observed to be to the semidiameter of the

earth as 59 to 1 ; and therefore the earth, if *ak* be put for its semidiameter, will make fifty-nine revolutions in *lbkc* in the time that the moon makes one monthly circuit in *mno**p*. But the moon makes her monthly circuit in little more than twenty-nine days. Wherefore the earth shall make its circuit in the circumference *lbkc* in twelve hours and a little more, namely, about twenty-six minutes more ; that is to say, it shall make two circuits in twenty-four hours and almost fifty-two minutes ; which is observed to be the time between the high-water of one day and the high-water of the day following. Now the course of the waters being hindered by the southern part of America, their motion will be interrupted there ; and consequently, they will be elevated in those places, and sink down again by their own weight, twice in the space of twenty-four hours and fifty-two minutes. And thus I have given a possible cause of the diurnal reciprocation of the ocean.

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Now from this swelling of the ocean in those parts of the earth, proceed the flowings and ebbings in the Atlantic, Spanish, British, and German seas ; which though they have their set times, yet upon several shores they happen at several hours of the day. And they receive some augmentation from the north, by reason that the shores of China and Tartary, hindering the general course of the waters, make them swell there, and discharge themselves in part through the strait of Anian into the Northern Ocean, and so into the German Sea.

As for the spring tides which happen at the time of the new and full moons, they are caused by that simple motion, which at the beginning I

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supposed to be always in the moon. For as, when I showed the cause of the eccentricity of the earth, I derived the elevation of the waters from the simple motion of the sun; so the same may here be derived from the simple motion of the moon. For though from the generation of clouds, there appear in the sun a more manifest power of elevating the waters than in the moon; yet the power of increasing moisture in vegetables and living creatures appears more manifestly in the moon than in the sun; which may perhaps proceed from this, that the sun raiseth up greater, and the moon lesser drops of water. Nevertheless, it is more likely, and more agreeable to common observation, that rain is raised not only by the sun, but also by the moon; for almost all men expect change of weather at the time of the conjunctions of the sun and moon with one another and with the earth, more than in the time of their quarters.

In the last place, the cause why the spring tides are greater at the time of the equinoxes hath been already sufficiently declared in this article, where I have demonstrated, that the two motions of the earth, namely, its simple motion in the little circle *lbkc*, and its diurnal motion in *ldke*, cause necessarily a greater elevation of waters when the sun is about the equinoxes, than when he is in other places. I have therefore given possible causes of the phenomenon of the flowing and ebbing of the ocean.

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precession of
the equinoxes.

11. As for the explication of the yearly *precession of the equinoctial points*, we must remember that, as I have already shown, the annual motion of the earth is not in the circumference of a circle,

but of an ellipsis, or a line not considerably different from that of an ellipsis. In the first place, therefore, this elliptical line is to be described.

Let the ecliptic $\triangle \varpi \gamma \ominus$ (in fig. 5) be divided into four equal parts by the two strait lines ab and $\varpi \ominus$, cutting one another at right angles in the centre c . And taking the arch bd of two degrees and sixteen minutes, let the strait line de be drawn parallel to ab , and cutting $\varpi \ominus$ in f ; which being done, the eccentricity of the earth will be cf . Seeing therefore the annual motion of the earth is in the circumference of an ellipsis, of which $\varpi \ominus$ is the greater axis, ab cannot be the lesser axis; for ab and $\varpi \ominus$ are equal. Wherefore the earth passing through a and b , will either pass above ϖ , as through g , or passing through ϖ , will fall between c and a ; it is no matter which. Let it pass therefore through g ; and let gl be taken equal to the strait line $\varpi \ominus$; and dividing gl equally in i , gi will be equal to ϖf , and il equal to $f \ominus$; and consequently the point i will cut the eccentricity cf into two equal parts; and taking ih equal to if , hi will be the whole eccentricity. If now a strait line, namely, the line $\triangle i \gamma$, be drawn through i parallel to the strait lines ab and ed , the way of the sun in summer, namely, the arch $\triangle g \gamma$, will be greater than his way in winter, by $8\frac{1}{4}$ degrees. Wherefore the true equinoxes will be in the strait line $\triangle i \gamma$; and therefore the ellipsis of the earth's annual motion will not pass through a, g, b , and l ; but through $\triangle g, \gamma$ and l . Wherefore the annual motion of the earth is in the ellipsis $\triangle g \gamma l$; and cannot be, the eccentricity being salved, in any

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other line. And this perhaps is the reason, why Kepler, against the opinion of all the astronomers of former time, thought fit to bisect the eccentricity of the earth, or, according to the ancients, of the sun, not by diminishing the quantity of the same eccentricity, (because the true measure of that quantity is the difference by which the summer arch exceeds the winter arch), but by taking for the centre of the ecliptic of the great orb the point c nearer to f , and so placing the whole great orb as much nearer to the ecliptic of the fixed stars towards ε , as is the distance between c and i . For seeing the whole great orb is but as a point in respect of the immense distance of the fixed stars, the two strait lines $\triangle r$ and ab , being produced both ways to the beginnings of Aries and Libra, will fall upon the same points of the sphere of the fixed stars. Let therefore the diameter of the earth mn be in the plane of the earth's annual motion. If now the earth be moved by the sun's simple motion in the circumference of the ecliptic about the centre i , this diameter will be kept always parallel to itself and to the strait line gl . But seeing the earth is moved in the circumference of an ellipsis without the ecliptic, the point n , whilst it passeth through $\triangle v r$, will go in a lesser circumference than the point m ; and consequently, as soon as ever it begins to be moved, it will lose its parallelism with the strait line $vs \varepsilon$; so that mn produced will at last cut the strait line gl produced. And contrarily, as soon as mn is past ε , the earth making its way in the internal elliptical line $r l \triangle$, the same mn produced towards m , will cut lg produced. And when the

earth hath almost finished its whole circumference, the same $m n$ shall again make a right angle with a line drawn from the centre i , a little short of the point from which the earth began its motion. And there the next year shall be one of the equinoctial points, namely, near the end of m ; the other shall be opposite to it near the end of x . And thus the points in which the days and nights are made equal do every year fall back; but with so slow a motion, that, in a whole year, it makes but 51 first minutes. And this relapse being contrary to the order of the signs, is commonly called the *precession of the equinoxes*. Of which I have from my former suppositions deduced a possible cause; which was to be done.

Cause of the
precession of
the equinoxes.

According to what I have said concerning the cause of the eccentricity of the earth; and according to Kepler, who for the cause thereof supposeth one part of the earth to be affected to the sun, the other part to be disaffected; the apogæum and perigæum of the sun should be moved every year in the same order, and with the same velocity, with which the equinoctial points are moved; and their distance from them should always be the quadrant of a circle; which seems to be otherwise. For astronomers say, that the equinoxes are now, the one about 28 degrees gone back from the first star of Aries, the other as much from the beginning of Libra; so that the apogæum of the sun or the aphelium of the earth ought to be about the 28th degree of Cancer. But it is reckoned to be in the 7th degree. Seeing, therefore, we have not sufficient evidence of the $\acute{o}r\acute{i}$ (that so it is,) it is in vain to seek for the $\delta\acute{i}o\acute{r}\acute{i}$ (why it is so.) Wherefore, as

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long as the motion of the apogæum is not observable by reason of the slowness thereof, and as long as it remains doubtful whether their distance from the equinoctial points be more or less than a quadrant precisely; so long it may be lawful for me to think they proceed both of them with equal velocity.

Also, I do not at all meddle with the causes of the eccentricities of Saturn, Jupiter, Mars, and Mercury. Nevertheless, seeing the eccentricity of the earth may, as I have shewn, be caused by the unlike constitution of the several parts of the earth which are alternately turned towards the sun, it is credible also, that like effects may be produced in these other planets from their having their superficies of unlike parts.

And this is all I shall say concerning Sidereal Philosophy. And, though the causes I have here supposed be not the true causes of these phenomena, yet I have demonstrated that they are sufficient to produce them, according to what I at first propounded.

CHAPTER XXVII.

OF LIGHT, HEAT, AND OF COLOURS.

1. Of the immense magnitude of some bodies, and the unspeakable littleness of others.—2. Of the cause of the light of the sun.—3. How light heateth.—4. The generation of fire from the sun.—5. The generation of fire from collision.—6. The cause of light in glow-worms, rotten wood, and the Bolognan stone.—6. The cause of light in the concussion of sea water.—8. The cause of flame, sparks, and colliquation.—9. The cause why wet hay sometimes burns of its own accord; also the cause of lightning.—10. The cause of the force of gunpowder; and what is to be ascribed to the coals, what to the brimstone, and what to the nitre.—11. How heat is caused by attrition.—12. The distinction of light into first, second, &c.—13. The causes of the colours we see in looking through a prisma of glass, namely, of red, yellow, blue, and violet colour.
14. Why the moon and the stars appear redder in the horizon than in the midst of the heaven.—15. The cause of whiteness.—16. The cause of blackness.

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1. BESIDES the stars, of which I have spoken in the last chapter, whatsoever other bodies there be in the world, they may be all comprehended under the name of intersidereal bodies. And these I have already supposed to be either the most fluid æther, or such bodies whose parts have some degree of cohesion. Now, these differ from one another in their several *consistencies, magnitudes, motions, and figures*. In consistency, I suppose some bodies to be harder, others softer through all the several degrees of *tenacity*. In magnitude, some to be greater, others less, and many unspeakably little. For we must remember that, by the understanding,

Of the immense magnitude of some bodies, and the unspeakable littleness of others.

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magnitude, &c.

quantity is divisible into divisibles perpetually. And, therefore, if a man could do as much with his hands as he can with his understanding, he would be able to take from any given magnitude a part which should be less than any other magnitude given. But the Omnipotent Creator of the world can actually from a part of any thing take another part, as far as we by our understanding can conceive the same to be divisible. Wherefore there is no impossible smallness of bodies. And what hinders but that we may think this likely? For we know there are some living creatures so small that we can scarce see their whole bodies. Yet even these have their young ones; their little veins and other vessels, and their eyes so small as that no microscope can make them visible. So that we cannot suppose any magnitude so little, but that our very supposition is actually exceeded by nature. Besides, there are now such microscopes commonly made, that the things we see with them appear a hundred thousand times bigger than they would do if we looked upon them with our bare eyes. Nor is there any doubt but that by augmenting the power of these microscopes (for it may be augmented as long as neither matter nor the hands of workmen are wanting) every one of those hundred thousandth parts might yet appear a hundred thousand times greater than they did before. Neither is the smallness of some bodies to be more admired than the vast greatness of others. For it belongs to the same Infinite Power, as well to augment infinitely as infinitely to diminish. To make the great orb, namely, that whose radius reacheth from the earth to the sun, but as a

point in respect of the distance between the sun and the fixed stars; and, on the contrary, to make a body so little, as to be in the same proportion less than any other visible body, proceeds equally from one and the same Author of Nature. But this of the immense distance of the fixed stars, which for a long time was accounted an incredible thing, is now believed by almost all the learned. Why then should not that other, of the smallness of some bodies, become credible at some time or other? For the Majesty of God appears no less in small things than in great; and as it exceedeth human sense in the immense greatness of the universe, so also it doth in the smallness of the parts thereof. Nor are the first elements of compositions, nor the first beginnings of actions, nor the first moments of times more credible, than that which is now believed of the vast distance of the fixed stars.

Some things are acknowledged by mortal men to be very great, though finite, as seeing them to be such. They acknowledge also that some things, which they do not see, may be of infinite magnitude. But they are not presently nor without great study persuaded, that there is any mean between infinite and the greatest of those things which either they see or imagine. Nevertheless, when after meditation and contemplation many things which we wondered at before are now grown more familiar to us, we then believe them, and transfer our admiration from the creatures to the Creator. But how little soever some bodies may be, yet I will not suppose their quantity to be less than is requisite for the salving of the phenomena. And in like manner I shall suppose their motion, namely,

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their velocity and slowness, and the variety of their figures, to be only such as the explication of their natural causes requires. And lastly, I suppose, that the parts of the pure æther, as if it were the first matter, have no motion at all but what they receive from bodies which float in them, and are not themselves fluid.

Of the cause
of the light
of the sun.

2. Having laid these grounds, let us come to speak of causes ; and in the first place let us inquire what may be the cause of the light of the sun. Seeing, therefore, the body of the sun doth by its simple circular motion thrust away the ambient ethereal substance sometimes one way sometimes another, so that those parts, which are next the sun, being moved by it, do propagate that motion to the next remote parts, and these to the next, and so on continually ; it must needs be that, notwithstanding any distance, the foremost part of the eye will at last be pressed ; and by the pressure of that part, the motion will be propagated to the innermost part of the organ of sight, namely, to the heart ; and from the reaction of the heart, there will proceed an endeavour back by the same way, ending in the endeavour outwards of the coat of the eye, called the *retina*. But this endeavour outwards, as has been defined in chapter xxv, is the thing which is called light, or the phantasm of a lucid body. For it is by reason of this phantasm that an object is called lucid. Wherefore we have a possible cause of the light of the sun ; which I undertook to find.

How light
heateth.

3. The generation of the light of the sun is accompanied with the generation of heat. Now every man knows what heat is in himself, by feeling

it when he grows hot ; but what it is in other things, he knows only by ratiocination. For it is one thing to grow hot, and another thing to heat or make hot. And therefore though we perceive that the fire or the sun heateth, yet we do not perceive that it is itself hot. That other living creatures, whilst they make other things hot, are hot themselves, we infer by reasoning from the like sense in ourselves. But this is not a necessary inference. For though it may truly be said of living creatures, that *they heat, therefore they are themselves hot* ; yet it cannot from hence be truly inferred that *fire heateth, therefore it is itself hot* ; no more than this, *fire causeth pain, therefore it is itself in pain*. Wherefore, that is only and properly called hot, which when we feel we are necessarily hot.

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heateth.

Now when we grow hot, we find that our spirits and blood, and whatsoever is fluid within us, is called out from the internal to the external parts of our bodies, more or less, according to the degree of the heat ; and that our skin swelleth. He, therefore, that can give a possible cause of this evocation and swelling, and such as agrees with the rest of the phenomena of heat, may be thought to have given the cause of the heat of the sun.

It hath been shown, in the 5th article of chapter XXI, that the fluid medium, which we call the air, is so moved by the simple circular motion of the sun, as that all its parts, even the least, do perpetually change places with one another ; which change of places is that which there I called fermentation. From this fermentation of the air, I have, in the 8th article of the last chapter, demon-

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strated that the water may be drawn up into the clouds.

And I shall now show that the fluid parts may, in like manner, by the same fermentation, be drawn out from the internal to the external parts of our bodies. For seeing that wheresoever the fluid medium is contiguous to the body of any living creature, there the parts of that medium are, by perpetual change of place, separated from one another; the contiguous parts of the living creature must, of necessity, endeavour to enter into the spaces of the separated parts. For otherwise those parts, supposing there is no vacuum, would have no place to go into. And therefore that, which is most fluid and separable in the parts of the living creature which are contiguous to the medium, will go first out; and into the place thereof will succeed such other parts as can most easily transpire through the pores of the skin. And from hence it is necessary that the rest of the parts, which are not separated, must altogether be moved outwards, for the keeping of all places full. But this motion outwards of all parts together must, of necessity, press those parts of the ambient air which are ready to leave their places; and therefore all the parts of the body, endeavouring at once that way, make the body swell. Wherefore a possible cause is given of heat from the sun; which was to be done.

The generation
of fire from the
sun.

4. We have now seen how light and heat are generated; heat by the simple motion of the medium, making the parts perpetually change places with one another; and light by this, that by the same simple motion action is propagated in a

strait line. But when a body hath its parts so moved, that it sensibly both heats and shines at the same time, then it is that we say fire is generated.

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Now by fire I do not understand a body distinct from matter combustibile or glowing, as wood or iron, but the matter itself, not simply and always, but then only when it shineth and heateth. He, therefore, that renders a cause possible and agreeable to the rest of the phenomena, namely, whence, and from what action, both the *shining* and *heating* proceed, may be thought to have given a possible cause of the generation of *fire*.

Let, therefore, A B C (in the first figure) be a sphere, or the portion of a sphere, whose centre is D; and let it be transparent and homogeneous, as crystal, glass, or water, and objected to the sun. Wherefore, the foremost part A B C will, by the simple motion of the sun, by which it thrusts forwards the medium, be wrought upon by the sunbeams in the strait lines E A, F B, and G C; which strait lines may, in respect of the great distance of the sun, be taken for parallels. And seeing the medium within the sphere is thicker than the medium without it, those beams will be refracted towards the perpendiculars. Let the strait lines E A and G C be produced till they cut the sphere in H and I; and drawing the perpendiculars A D and C D, the refracted beams E A and G C will of necessity fall, the one between A H and A D, the other between C I and C D. Let those refracted beams be A K and C L. And again, let the lines D K M and D L N be drawn perpendicular to the sphere; and let A K and C L be

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produced till they meet with the strait line BD produced in O . Seeing, therefore, the medium within the sphere is thicker than that without it, the refracted line AK will recede further from the perpendicular KM than KO will recede from the same. Wherefore KO will fall between the refracted line and the perpendicular. Let, therefore, the refracted line be KP , cutting FO in P ; and for the same reason the strait line LP will be the refracted line of the strait line CL . Wherefore, seeing the beams are nothing else but the ways in which the motion is propagated, the motion about P will be so much more vehement than the motion about ABC , by how much the base of the portion ABC is greater than the base of a like portion in the sphere, whose centre is P , and whose magnitude is equal to that of the little circle about P , which comprehendeth all the beams that are propagated from ABC ; and this sphere being much less than the sphere ABC , the parts of the medium, that is, of the air about P , will change places with one another with much greater celerity than those about ABC . If, therefore, any matter combustible, that is to say, such as may be easily dissipated, be placed in P , the parts of that matter, if the proportion be great enough between AC and a like portion of the little circle about P , will be freed from their mutual cohesion, and being separated will acquire simple motion. But vehement simple motion generates in the beholder a phantasm of lucid and hot, as I have before demonstrated of the simple motion of the sun; and therefore the combustible matter which is placed in P will be made lucid and hot, that is to say, will be fire.

Wherefore I have rendered a possible cause of fire ; which was to be done. PART IV.
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5. From the manner by which the sun generateth fire, it is easy to explain the manner by which fire may be generated by the collision of two flints. For by that collision some of those particles of which the stone is compacted, are violently separated and thrown off ; and being withal swiftly turned round, the eye is moved by them, as it is in the generation of light by the sun. Wherefore they shine ; and falling upon matter which is already half dissipated, such as is tinder, they thoroughly dissipate the parts thereof, and make them turn round. From whence, as I have newly shown, light and heat, that is to say fire, is generated.

6. The shining of glow-worms, some kinds of rotten wood, and of a kind of stone made at Bologna, may have one common cause, namely, the exposing of them to the hot sun. We find by experience that the Bologna stone shines not, unless it be so exposed ; and after it has been exposed it shines but for a little time, namely, as long as it retains a certain degree of heat. And the cause may be that the parts, of which it is made, may together with heat have simple motion imprinted in them by the sun. Which if it be so, it is necessary that it shine in the dark, as long as there is sufficient heat in it ; but this ceasing, it will shine no longer. Also we find by experience that in the glow-worm there is a certain thick humour, like the crystalline humour of the eye ; which if it be taken out and held long enough in one's fingers, and then be carried into the dark, it will shine by reason of the warmth it received from the fingers ; but as soon

The generation of fire from collision.

The cause of light in glow-worms, rotten wood, and the Bologna stone

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as it is cold it will cease shining. From whence, therefore, can these creatures have their light, but from lying all day in the sunshine in the hottest time of summer? In the same manner, rotten wood, except it grow rotten in the sunshine, or be afterwards long enough exposed to the sun, will not shine. That this doth not happen in every worm, nor in all kinds of rotten wood, nor in all calcined stones, the cause may be that the parts, of which the bodies are made, are different both in motion and figure from the parts of bodies of other kinds.

The cause of light in the concussion of sea water.

7. Also the sea water shineth when it is either dashed with the strokes of oars, or when a ship in its course breaks strongly through it; but more or less, according as the wind blows from different points. The cause whereof may be this, that the particles of salt, though they never shine in the salt-pits, where they are but slowly drawn up by the sun, being here beaten up into the air in greater quantities and with more force, are withal made to turn round, and consequently to shine, though weakly. I have, therefore, given a possible cause of this phenomenon.

The cause of flame, sparks, & colliquation.

8. If such matter as is compounded of hard little bodies be set on fire, it must needs be, that, as they fly out in greater or less quantities, the flame which is made by them will be greater or less. And if the ethereal or fluid part of that matter fly out together with them, their motion will be the swifter, as it is in wood and other things which flame with a manifest mixture of wind. When, therefore, these hard particles by their flying out move the eye strongly, they shine bright; and a

great quantity of them flying out together, they make a great shining body. For flame being nothing but an aggregate of shining particles, the greater the aggregate is, the greater and more manifest will be the flame. I have, therefore, shown a possible cause of flame. And from hence the cause appears evidently, why glass is so easily and quickly melted by the small flame of a candle blown, which will not be melted without blowing but by a very strong fire.

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Now, if from the same matter there be a part broken off, namely, such a part as consisteth of many of the small particles, of this is made a spark. For from the breaking off it hath a violent turning round, and from hence it shines. But though from this matter there fly neither flame nor sparks, yet some of the smallest parts of it may be carried out as far as to the superficies, and remain there as ashes; the parts whereof are so extremely small, that it cannot any longer be doubted how far nature may proceed in dividing.

Lastly, though by the application of fire to this matter there fly little or nothing from it, yet there will be in the parts an endeavour to simple motion; by which the whole body will either be melted, or, which is a degree of melting, softened. For all motion has some effect upon all matter whatsoever, as has been shown at art. 3, chap. xv. Now if it be softened to such a degree, as that the stubbornness of the parts be exceeded by their gravity, then we say it is melted; otherwise, softened and made pliant and ductile.

Again, the matter having in it some particles hard, others ethereal or watery; if, by the appli-

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cation of fire, these latter be called out, the former will thereby come to a more full contact with one another; and, consequently, will not be so easily separated; that is to say, the whole body will be made harder. And this may be the cause why the same fire makes some things soft, others hard.

The cause why wet hay sometimes burns of its own accord; also the cause of lightning.

9. It is known by experience that if hay be laid wet together in a heap, it will after a time begin to smoke, and then burn as it were of itself. The cause whereof seems to be this, that in the air, which is enclosed within the hay, there are those little bodies, which, as I have supposed, are moved freely with simple motion. But this motion being by degrees hindered more and more by the descending moisture, which at the last fills and stops all the passages, the thinner parts of the air ascend by penetrating the water; and those hard little bodies, being so thrust together that they touch and press one another, acquire stronger motion; till at last by the increased strength of this motion the watery parts are first driven outwards, from whence appears vapour; and by the continued increase of this motion, the smallest particles of the dried hay are forced out, and recovering their natural simple motion, they grow hot and shine, that is to say, they are set on fire.

The same also may be the cause of lightning, which happens in the hottest time of the year, when the water is raised up in greatest quantity and carried highest. For after the first clouds are raised, others after others follow them; and being congealed above, they happen, whilst some of them ascend and others descend, to fall one upon another in such manner, as that in some places all their parts

are joined together, in others they leave hollow spaces between them ; and into these spaces, the ethereal parts being forced out by the compressure of the clouds, many of the harder little bodies are so pent together, as they have not the liberty of such motion as is natural to the air. Wherefore their endeavour grows more vehement, till at last they force their way through the clouds, sometimes in one place, sometimes in another ; and, breaking through with great noise, they move the air violently, and striking our eyes, generate light, that is to say, they shine. And this shining is that we call lightning.

10. The most common phenomenon proceeding from fire, and yet the most admirable of all others, is the force of gunpowder fired ; which being compounded of nitre, brimstone and coals, beaten small, hath from the coals its first taking fire ; from the brimstone its nourishment and flame, that is to say, light and motion, and from the nitre the vehemence of both. Now if a piece of nitre, before it is beaten, be laid upon a burning coal, first it melts, and, like water, quencheth that part of the coal it toucheth. Then vapour or air, flying out where the coal and nitre join, bloweth the coal with great swiftness and vehemence on all sides. And from hence it comes to pass, that by two contrary motions, the one, of the particles which go out of the burning coal, the other, of those of the ethereal and watery substance of the nitre, is generated that vehement motion and inflammation. And, lastly, when there is no more action from the nitre, that is to say, when the volatile parts of the nitre are flown out, there is found about the sides a cer-

The cause of the force of gunpowder ; and what is to be ascribed to the coals, what to the brimstone, and what to the nitre.

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the force of
gunpowder, &c.

tain white substance, which being thrown again into the fire, will grow red-hot again, but will not be dissipated, at least unless the fire be augmented. If now a possible cause of this be found out, the same will also be a possible cause why a grain of gunpowder set on fire doth expand itself with such vehement motion, and shine. And it may be caused in this manner.

Let the particles, of which nitre consisteth, be supposed to be some of them hard, others watery, and the rest ethereal. Also let the hard particles be supposed to be spherically hollow, like small bubbles, so that many of them growing together may constitute a body, whose little caverns are filled with a substance which is either watery, or ethereal, or both. As soon, therefore, as the hard particles are dissipated, the watery and ethereal particles will necessarily fly out; and as they fly, of necessity blow strongly the burning coals and brimstone which are mingled together; whereupon there will follow a great expansion of light, with vehement flame, and a violent dissipation of the particles of the nitre, the brimstone and the coals. Wherefore I have given a possible cause of the force of fired gunpowder.

It is manifest from hence, that for the rendering of the cause why a bullet of lead or iron, shot from a piece of ordnance, flies with so great velocity, there is no necessity to introduce such rarefaction, as, by the common definition of it, makes the same matter to have sometimes more, sometimes less quantity; which is inconceivable. For every thing is said to be greater or less, as it hath more or less quantity. The violence with which a bullet

is thrust out of a gun, proceeds from the swiftness of the small particles of the fired powder ; at least it may proceed from that cause without the supposition of any empty space.

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11. Besides, by the attrition or rubbing of one body against another, as of wood against wood, we find that not only a certain degree of heat, but fire itself is sometimes generated. For such motion is the reciprocation of pressure, sometimes one way, sometimes the other ; and by this reciprocation whatsoever is fluid in both the pieces of wood is forced hither and thither ; and consequently, to an endeavour of getting out ; and at last by breaking out makes fire.

How heat is caused by attrition.

12. Now light is distinguished into, first, second, third, and so on infinitely. And we call that first light, which is in the first lucid body ; as the sun, fire, &c. : second, that which is in such bodies, as being not transparent are illuminated by the sun ; as the moon, a wall, &c. : and third, that which is in bodies not transparent, but illuminated by second light, &c.

The distinction of light into first, second, &c.

13. *Colour* is light, but troubled light, namely, such as is generated by perturbed motion ; as shall be made manifest by the red, yellow, blue and purple, which are generated by the interposition of a diaphanous prisma, whose opposite bases are triangular, between the light and that which is enlightened.

The causes of the colours we see in looking through a prisma of glass, namely, of red, yellow, blue, & violet colour.

For let there be a prisma of glass, or of any other transparent matter which is of greater density than air ; and let the triangle A B C be the base of this prisma. Also let the strait line D E be the diameter of the sun's body, having oblique position to

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the strait line $A B$; and let the sunbeams pass in the lines $D A$ and $E B C$. And lastly, let the strait lines $D A$ and $E C$ be produced indefinitely to F and G . Seeing therefore the strait line $D A$, by reason of the density of the glass, is refracted towards the perpendicular; let the line refracted at the point A be the strait line $A H$. And again, seeing the medium below $A C$ is thinner than that above it, the other refraction, which will be made there, will diverge from the perpendicular. Let therefore this second refracted line be $A I$. Also let the same be done at the point C , by making the first refracted line to be $C K$, and the second $C L$. Seeing therefore the cause of the refraction in the point A of the strait line of $A B$ is the excess of the resistance of the medium in $A B$ above the resistance of the air, there must of necessity be reaction from the point A towards the point B ; and consequently the medium at A within the triangle $A B C$ will have its motion troubled, that is to say, the strait motion in $A F$ and $A H$ will be mixed with the transverse motion between the same $A F$ and $A H$, represented by the short transverse lines in the triangle $A F H$. Again, seeing at the point A of the strait line $A C$ there is a second refraction from $A H$ in $A I$, the motion of the medium will again be perturbed by reason of the transverse reaction from A towards C , represented likewise by the short transverse lines in the triangle $A H I$. And in the same manner there is a double perturbation represented by the transverse lines in the triangles $C G K$ and $C K L$. But as for the light between $A I$ and $C G$, it will not be perturbed; because, if there were in all the points of the strait

lines A B and A C the same action which is in the points A and C, then the plane of the triangle C G K would be everywhere coincident with the plane of the triangle A F H ; by which means all would appear alike between A and C. Besides, it is to be observed, that all the reaction at A tends towards the illuminated parts which are between A and C, and consequently perturbeth the first light. And on the contrary, that all the reaction at C tends towards the parts without the triangle or without the prisma A B C, where there is none but second light ; and that the triangle A F H shows that perturbation of light which is made in the glass itself ; as the triangle A H I shows that perturbation of light which is made below the glass. In like manner, that C G K shows the perturbation of light within the glass ; and C K L that which is below the glass. From whence there are four divers motions, or four different illuminations or colours, whose differences appear most manifestly to the sense in a prisma, whose base is an equilateral triangle, when the sunbeams that pass through it are received upon a white paper. For the triangle A F H appears red to the sense ; the triangle A H I yellow ; the triangle C G K green, and approaching to blue ; and lastly, the triangle C K L appears purple. It is therefore evident that when weak but first light passeth through a more resisting diaphanous body, as glass, the beams, which fall upon it transversely, make redness ; and when the same first light is stronger, as it is in the thinner medium below the strait line A C, the transverse beams make yellowness. Also when second light is strong, as it is in the triangle C G K, which is nearest to the first

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Why the moon
and the stars
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than in the
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light, the transverse beams make greenness; and when the same second light is weaker, as in the triangle C K L, they make a purple colour.

14. From hence may be deduced a cause, why the moon and stars appear bigger and redder near the horizon than in the mid-heaven. For between the eye and the apparent horizon there is more impure air, such as is mingled with watery and earthy little bodies, than is between the same eye and the more elevated part of heaven. But vision is made by beams which constitute a cone, whose base, if we look upon the moon, is the moon's face, and whose vertex is in the eye; and therefore, many beams from the moon must needs fall upon little bodies that are without the visual cone, and be by them reflected to the eye. But these reflected beams tend all in lines which are transverse to the visual cone, and make at the eye an angle which is greater than the angle of the cone. Wherefore, the moon appears greater in the horizon, than when she is more elevated. And because those reflected beams go transversely, there will be generated, by the last article, redness. A possible cause therefore is shown, why the moon as also the stars appear greater and redder in the horizon, than in the midst of heaven. The same also may be the cause, why the sun appears in the horizon greater and of a colour more degenerating to yellow, than when he is higher elevated. For the reflection from the little bodies between, and the transverse motion of the medium, are still the same. But the light of the sun is much stronger than that of the moon; and therefore, by the last article, his splendour

must needs by this perturbation degenerate into yellowness.

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But for the generation of these four colours, it is not necessary that the figure of the glass be a prisma; for if it were spherical it would do the same. For in a sphere the sunbeams are twice refracted and twice reflected. And this being observed by Des Cartes, and withal that a rainbow never appears but when it rains; as also, that the drops of rain have their figures almost spherical; he hath shown from thence the cause of the colours in the rainbow; which therefore need not be repeated.

15. *Whiteness* is light, but light perturbed by the reflections of many beams of light coming to the eye together within a little space. For if glass or any other diaphanous body be reduced to very small parts by contusion or concussion, every one of those parts, if the beams of a lucid body be from any one point of the same reflected to the eye, will represent to the beholder an idea or image of the whole lucid body, that is to say, a phantasm of white. For the strongest light is the most white; and therefore many such parts will make many such images. Wherefore, if those parts lie thick and close together, those many images will appear confusedly, and will by reason of the confused light represent a white colour. So that from hence may be deduced a possible cause, why glass beaten, that is, reduced to powder, looks white. Also why water and snow are white; they being nothing but a heap of very small diaphanous bodies, namely, of little bubbles, from whose several convex superficies there are by reflection made several confused phan-

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tasms of the whole lucid body, that is to say, whiteness. For the same reason, salt and nitre are white, as consisting of small bubbles which contain within them water and air; as is manifest in nitre, from this, that being thrown into the fire it violently blows the same; which salt also doth, but with less violence. But if a white body be exposed, not to the light of the day, but to that of the fire or of a candle, it will not at the first sight be easily judged whether it be white or yellow; the cause whereof may be this, that the light of those things, which burn and flame, is almost of a middle colour between whiteness and yellowness.

The cause of
blackness.

16. As whiteness is light, so *blackness* is the privation of light, or darkness. And, from hence it is, first, that all holes, from which no light can be reflected to the eye, appear black. Secondly, that when a body hath little eminent particles erected straight up from the superficies, so that the beams of light which fall upon them are reflected not to the eye but to the body itself, that superficies appears black; in the same manner as the sea appears black when ruffled by the wind. Thirdly, that any combustible matter is by the fire made to look black before it shines. For the endeavour of the fire being to dissipate the smallest parts of such bodies as are thrown into it, it must first raise and erect those parts before it can work their dissipation. If, therefore, the fire be put out before the parts are totally dissipated, the coal will appear black; for the parts being only erected, the beams of light falling upon them will not be reflected to the eye, but to the coal itself. Fourthly, that burning glasses do more easily burn black things than

white. For in a white superficies the eminent parts are convex, like little bubbles; and therefore the beams of light, which fall upon them, are reflected every way from the reflecting body. But in a black superficies, where the eminent particles are more erected, the beams of light falling upon them are all necessarily reflected towards the body itself; and, therefore, bodies that are black are more easily set on fire by the sun beams, than those that are white. Fifthly, that all colours that are made of the mixture of white and black proceed from the different position of the particles that rise above the superficies, and their different forms of asperity. For, according to these differences, more or fewer beams of light are reflected from several bodies to the eye. But in regard those differences are innumerable, and the bodies themselves so small that we cannot perceive them; the explication and precise determination of the causes of all colours is a thing of so great difficulty, that I dare not undertake it.

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CHAPTER XXVIII.

OF COLD, WIND, HARD, ICE, RESTITUTION OF BODIES BENT, DIAPHANOUS, LIGHTNING AND THUNDER; AND OF THE HEADS OF RIVERS.

1. Why breath from the same mouth sometimes heats and sometimes cools.—2. Wind, and the inconstancy of winds, whence.
3. Why there is a constant, though not a great wind, from east to west, near the equator.—4. What is the effect of air pent in between the clouds.—5. No change from soft to hard, but by motion.—6. What is the cause of cold near the poles.
7. The cause of ice; and why the cold is more remiss in rainy than in clear weather. Why water doth not freeze in deep wells as it doth near the superficies of the earth. Why ice is not so heavy as water; and why wine is not so easily frozen as water.—8. Another cause of hardness from the fuller contact of atoms; also, how hard things are broken.—9. A third cause of hardness from heat.—10. A fourth cause of hardness from the motion of atoms enclosed in a narrow space.—11. How hard things are softened.—12. Whence proceed the spontaneous restitution of things bent.—13. Diaphanous and opacous, what they are, and whence.—14. The cause of lightning and thunder.—15. Whence it proceeds that clouds can fall again after they are once elevated and frozen.—16. How it could be that the moon was eclipsed, when she was not diametrically opposite to the sun.—17. By what means many suns may appear at once.—18. Of the heads of rivers.

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Why breath from the same mouth sometimes heats and sometimes cools.

1. As, when the motion of the ambient ethereal substance makes the spirits and fluid parts of our bodies tend outwards, we acknowledge heat; so, by the endeavour inwards of the same spirits and humours, we feel cold. So that to cool is to make the exterior parts of the body endeavour inwards,

by a motion contrary to that of calefaction, by which the internal parts are called outwards. He, therefore, that would know the cause of cold, must find by what motion or motions the exterior parts of any body endeavour to retire inwards. To begin with those phenomena which are the most familiar. There is almost no man but knows, that breath blown strongly, and which comes from the mouth with violence, that is to say, the passage being strait, will cool the hand; and that the same breath blown gently, that is to say, through a greater aperture, will warm the same. The cause of which phenomenon may be this, the breath going out hath two motions; the one, of the whole and direct, by which the foremost parts of the hand are driven inwards; the other, simple motion of the small particles of the same breath, which, (as I have shown in the 3rd article of the last chapter, causeth heat. According, therefore, as either of these motions is predominant, so there is the sense sometimes of cold, sometimes of heat. Wherefore, when the breath is softly breathed out at a large passage, that simple motion which causeth heat prevaieth, and consequently heat is felt; and when, by compressing the lips, the breath is more strongly blown out, then is the direct motion prevalent, which makes us feel cold. For, the direct motion of the breath or air is wind; and all wind cools or diminisheth former heat.

2. And seeing not only great wind, but almost
 any ventilation and stirring of the air, doth refrigerate; the reason of many experiments concerning cold cannot well be given without finding first what are the causes of wind. Now, wind is

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 winds, whence.

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winds, whence.

nothing else but the direct motion of the air thrust forwards; which, nevertheless, when many winds concur, may be circular or otherwise indirect, as it is in whirlwinds. Wherefore, in the first place we are to enquire into the causes of winds. Wind is air moved in a considerable quantity, and that either in the manner of waves, which is both forwards and also up and down, or else forwards only.

Supposing, therefore, the air both clear and calm for any time how little soever, yet, the greater bodies of the world being so disposed and ordered as has been said, it will be necessary that a wind presently arise somewhere. For, seeing that motion of the parts of the air, which is made by the simple motion of the sun in his own epicycle, causeth an exhalation of the particles of water from the seas and all other moist bodies, and those particles make clouds; it must needs follow, that, whilst the particles of water pass upwards, the particles of air, for the keeping of all spaces full, be jostled out on every side, and urge the next particles, and these the next; till having made their circuit, there comes continually so much air to the hinder parts of the earth as there went water from before it. Wherefore, the ascending vapours move the air towards the sides every way; and all direct motion of the air being wind, they make a wind. And if this wind meet often with other vapours which arise in other places, it is manifest that the force thereof will be augmented, and the way or course of it changed. Besides, according as the earth, by its diurnal motion, turns sometimes the drier, sometimes the moister part towards the sun,

so sometimes a greater, sometimes a less, quantity of vapours will be raised ; that is to say, sometimes there will be a less, sometimes a greater wind. Wherefore, I have rendered a possible cause of such winds as are generated by vapours ; and also of their inconstancy.

From hence it follows that these winds cannot be made in any place, which is higher than that to which vapours may ascend. Nor is that incredible which is reported of the highest mountains, as the Peak of Teneriffe and the Andes of Peru, namely, that they are not at all troubled with these inconstant winds. And if it were certain that neither rain nor snow were ever seen in the highest tops of those mountains, it could not be doubted but that they are higher than any place to which vapours use to ascend.

3. Nevertheless, there may be wind there, though not that which is made by the ascent of vapours, yet a less and more constant wind, like the continued blast of a pair of bellows, blowing from the east. And this may have a double cause ; the one, the diurnal motion of the earth ; the other, its simple motion in its own epicycle. For these mountains being, by reason of their height, more eminent than all the rest of the parts of the earth, do by both these motions drive the air from the west eastwards. To which, though the diurnal motion contribute but little, yet seeing I have supposed that the simple motion of the earth, in its own epicycle, makes two revolutions in the same time in which the diurnal motion makes but one, and that the semidiameter of the epicycle is double to the semidiameter of the diurnal conversion, the

Why there is a constant, though not a great wind, from east to west, near the equator.

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motion of every point of the earth in its own epicycle will have its velocity quadruple to that of the diurnal motion; so that by both these motions together, the tops of those hills will sensibly be moved against the air; and consequently a wind will be felt. For whether the air strike the sentient, or the sentient the air, the perception of motion will be the same. But this wind, seeing it is not caused by the ascent of vapours, must necessarily be very constant.

What is the effect of air pent in between the clouds.

4. When one cloud is already ascended into the air, if another cloud ascend towards it, that part of the air, which is intercepted between them both, must of necessity be pressed out every way. Also when both of them, whilst the one ascends and the other either stays or descends, come to be joined in such manner as that the ethereal substance be shut within them on every side, it will by this compression also go out by penetrating the water. But in the meantime, the hard particles, which are mingled with the air and are agitated, as I have supposed, with simple motion, will not pass through the water of the clouds, but be more straitly compressed within their cavities. And this I have demonstrated at the 4th and 5th articles of chapter XXII. Besides, seeing the globe of the earth floateth in the air which is agitated by the sun's motion, the parts of the air resisted by the earth will spread themselves every way upon the earth's superficies; as I have shown at the 8th article of chapter XXI.

No change from soft to hard, but by motion.

5. We perceive a body to be hard, from this, that, when touching it, we would thrust forwards that part of the same which we touch, we cannot

do it otherwise than by thrusting forwards the whole body. We may indeed easily and sensibly thrust forwards any particle of the air or water which we touch, whilst yet the rest of its parts remain to sense unmoved. But we cannot do so to any part of a stone. Wherefore I define a *hard* body to be that whereof no part can be sensibly moved, unless the whole be moved. Whatsoever therefore is soft or fluid, the same can never be made hard but by such motion as makes many of the parts together stop the motion of some one part, by resisting the same.

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6. Those things premised, I shall show a possible cause why there is greater cold near the poles of the earth, than further from them. The motion of the sun between the tropics, driving the air towards that part of the earth's superficies which is perpendicularly under it, makes it spread itself every way; and the velocity of this expansion of the air grows greater and greater, as the superficies of the earth comes to be more and more straitened, that is to say, as the circles which are parallel to the equator come to be less and less. Wherefore this expansive motion of the air drives before it the parts of the air, which are in its way, continually towards the poles more and more strongly, as its force comes to be more and more united, that is to say, as the circles which are parallel to the equator are less and less; that is, so much the more, by how much they are nearer to the poles of the earth. In those places, therefore, which are nearer to the poles, there is greater cold than in those which are more remote from them. Now this expansion of the air upon the superficies of the

What is the
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earth, from east to west, doth, by reason of the sun's perpetual accession to the places which are successively under it, make it cold at the time of the sun's rising and setting ; but as the sun comes to be continually more and more perpendicular to those cooled places, so by the heat, which is generated by the supervening simple motion of the sun, that cold is again remitted ; and can never be great, because the action by which it was generated is not permanent. Wherefore I have rendered a possible cause of cold in those places that are near the poles, or where the obliquity of the sun is great.

The cause of ice ; and why the cold is more remiss in rainy than in clear weather. Why water doth not freeze in deep wells, as it doth near the superficies of the earth. Why ice is not so heavy as water ; and why wine is not so easily frozen as water.

7. How water may be congealed by cold, may be explained in this manner. Let A (in figure 1) represent the sun, and B the earth. A will therefore be much greater than B. Let EF be in the plane of the equinoctial ; to which let GH, IK, and LC be parallel. Lastly, let C and D be the poles of the earth. The air, therefore, by its action in those parallels, will rake the superficies of the earth ; and that with motion so much the stronger, by how much the parallel circles towards the poles grow less and less. From whence must arise a wind, which will force together the uppermost parts of the water, and withal raise them a little, weakening their endeavour towards the centre of the earth. And from their endeavour towards the centre of the earth, joined with the endeavour of the said wind, the uppermost parts of the water will be pressed together and coagulated, that is to say, the top of the water will be skinned over and hardened. And so again, the water next the top will be hardened in the same manner, till at length the ice be thick. And this

ice, being now compacted of little hard bodies, must also contain many particles of air received into it.

As rivers and seas, so also in the same manner may the clouds be frozen. For when, by the ascending and descending of several clouds at the same time, the air intercepted between them is by compression forced out, it rakes, and by little and little hardens them. And though those small drops, which usually make clouds, be not yet united into greater bodies, yet the same wind will be made; and by it, as water is congealed into ice, so will vapours in the same manner be congealed into snow. From the same cause it is that ice may be made by art, and that not far from the fire. For it is done by the mingling of snow and salt together, and by burying in it a small vessel full of water. Now while the snow and salt, which have in them a great deal of air, are melting, the air, which is pressed out every way in wind, rakes the sides of the vessel; and as the wind by its motion rakes the vessel, so the vessel by the same motion and action congeals the water within it.

We find by experience, that cold is always more remiss in places where it rains, or where the weather is cloudy, things being alike in all other respects, than where the air is clear. And this agreeth very well with what I have said before. For in clear weather, the course of the wind which, as I said even now, rakes the superficies of the earth, as it is free from all interruption, so also it is very strong. But when small drops of water are either rising or falling, that wind is repelled,

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broken, and dissipated by them ; and the less the wind is, the less is the cold.

We find also by experience, that in deep wells the water freezeth not so much as it doth upon the superficies of the earth. For the wind, by which ice is made, entering into the earth by reason of the laxity of its parts, more or less, loseth some of its force, though not much. So that if the well be not deep, it will freeze ; whereas if it be so deep, as that the wind which causeth cold cannot reach it, it will not freeze.

We find moreover by experience, that ice is lighter than water. The cause whereof is manifest from that which I have already shown, namely, that air is received in and mingled with the particles of the water whilst it is congealing.

Lastly, wine is not so easily congealed as water, because in wine there are particles, which, being not fluid, are moved very swiftly, and by their motion congelation is retarded. But if the cold prevail against this motion, then the outermost parts of the wine will be first frozen, and afterwards the inner parts ; whereof this is a sign, that the wine which remains unfrozen in the midst will be very strong.

Another cause of hardness from the fuller contact of atoms. Also how hard things are broken.

8. We have seen one way of making things hard, namely, by congelation. Another way is thus. Having already supposed that innumerable atoms, some harder than others and that have several simple motions of their own, are intermingled with the ethereal substance ; it follows necessarily from hence, that by reason of the fermentation of the whole air, of which I have spoken in chapter XXI, some of those atoms meeting with others will

cleave together, by applying themselves to one another in such manner as is agreeable to their motions and mutual contacts ; and, seeing there is no vacuum, cannot be pulled asunder, but by so much force as is sufficient to overcome their hardness.

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Another cause
of hardness, &c.

Now there are innumerable degrees of hardness. As for example, there is a degree of it in water, as is manifest from this, that upon a plane it may be drawn any way at pleasure by one's finger. There is a greater degree of it in clammy liquors, which, when they are poured out, do in falling downwards dispose themselves into one continued thread ; which thread, before it be broken, will by little and little diminish its thickness, till at last it be so small, as that it seems to break only in a point ; and in their separation the external parts break first from one another, and then the more internal parts successively one after another. In wax there is yet a greater degree of hardness. For when we would pull one part of it from another, we first make the whole mass slenderer, before we can pull it asunder. And how much the harder anything is which we would break, so much the more force we must apply to it. Wherefore, if we go on to harder things, as ropes, wood, metals, stones, &c., reason prompteth us to believe that the same, though not always sensibly, will necessarily happen ; and that even the hardest things are broken asunder in the same manner, namely, by solution of their continuity begun in the outermost superficies, and proceeding successively to the innermost parts. In like manner, when the parts of bodies are to be separated, not by pulling them

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asunder, but by breaking them, the first separation will necessarily be in the convex superficies of the bowed part of the body, and afterwards in the concave superficies. For in all bowing there is in the convex superficies an endeavour in the parts to go one from another, and in the concave superficies to penetrate one another.

This being well understood, a reason may be given how two bodies, which are contiguous in one common superficies, may by force be separated without the introduction of vacuum; though Lucretius thought otherwise, believing that such separation was a strong establishment of vacuum. For a marble pillar being made to hang by one of its bases, if it be long enough, it will by its own weight be broken asunder; and yet it will not necessarily follow that there should be vacuum, seeing the solution of its continuity may begin in the circumference, and proceed successively to the midst thereof.

A third cause of hardness, from heat.

9. Another cause of hardness in some things may be in this manner. If a soft body consist of many hard particles, which by the intermixture of many other fluid particles cohere but loosely together, those fluid parts, as hath been shown in the last article of chapter XXI, will be exhaled; by which means each hard particle will apply itself to the next to it according to a greater superficies, and consequently they will cohere more closely to one another, that is to say, the whole mass will be made harder.

A fourth cause of hardness, from the motion of atoms

10. Again, in some things hardness may be made to a certain degree in this manner. When any fluid substance hath in it certain very small bodies

intermingled, which, being moved with simple motion of their own, contribute like motion to the parts of the fluid substance, and this be done in a small enclosed space, as in the hollow of a little sphere, or a very slender pipe, if the motion be vehement and there be a great number of these small enclosed bodies, two things will happen; the one, that the fluid substance will have an endeavour of dilating itself at once every way; the other, that if those small bodies can nowhere get out, then from their reflection it will follow, that the motion of the parts of the enclosed fluid substance, which was vehement before, will now be much more vehement. Wherefore, if any one particle of that fluid substance should be touched and pressed by some external movent, it could not yield but by the application of very sensible force. Wherefore the fluid substance, which is enclosed and so moved, hath some degree of hardness. Now, greater and less degree of hardness depends upon the quantity and velocity of those small bodies, and upon the narrowness of the place both together.

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enclosed in a
narrow space.

11. Such things as are made hard by sudden heat, namely such as are hardened by fire, are commonly reduced to their former soft form by maceration. For fire hardens by evaporation, and therefore if the evaporated moisture be restored again, the former nature and form is restored together with it. And such things as are frozen with cold, if the wind by which they were frozen change into the opposite quarter, they will be unfrozen again, unless they have gotten a habit of new motion or endeavour by long continuance in

How hard
things are
softened.

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that hardness. Nor is it enough to cause thawing, that there be a cessation of the freezing wind ; for the taking away of the cause doth not destroy a produced effect ; but the thawing also must have its proper cause, namely, a contrary wind, or at least a wind opposite in some degree. And this we find to be true by experience. For, if ice be laid in a place so well enclosed that the motion of the air cannot get to it, that ice will remain unchanged, though the place be not sensibly cold.

Whence proceeds the spontaneous restitution of things bent.

12. Of hard bodies, some may manifestly be bowed ; others not, but are broken in the very first moment of their bending. And of such bodies as may manifestly be bended, some being bent, do, as soon as ever they are set at liberty, restore themselves to their former posture ; others remain still bent. Now if the cause of this restitution be asked, I say, it may be in this manner, namely, that the particles of the bended body, whilst it is held bent, do nevertheless retain their motion ; and by this motion they restore it as soon as the force is removed by which it was bent. For when any thing is bent, as a plate of steel, and, as soon as the force is removed, restores itself again, it is evident that the cause of its restitution cannot be referred to the ambient air ; nor can it be referred to the removal of the force by which it was bent ; for in things that are at rest the taking away of impediments is not a sufficient cause of their future motion ; there being no other cause of motion, but motion. The cause therefore of such restitution is in the parts of the steel itself. Wherefore, whilst it remains bent, there is in the parts, of which it consisteth, some motion though invisible ; that is to

say, some endeavour at least that way by which the restitution is to be made; and therefore this endeavour of all the parts together is the first beginning of restitution; so that the impediment being removed, that is to say, the force by which it was held bent, it will be restored again. Now the motion of the parts, by which this done, is that which I called simple motion, or motion returning into itself. When therefore in the bending of a plate the ends are drawn together, there is on one side a mutual compression of the parts; which compression is one endeavour opposite to another endeavour: and on the other side a divulsion of the parts. The endeavour therefore of the parts on one side tends to the restitution of the plate from the middle towards the ends; and on the other side, from the ends towards the middle. Wherefore the impediment being taken away, this endeavour, which is the beginning of restitution, will restore the plate to its former posture. And thus I have given a possible cause why some bodies, when they are bent, restore themselves again; which was to be done.

As for stones, seeing they are made by the accretion of many very hard particles within the earth; which particles have no great coherence, that is to say, touch one another in small latitude, and consequently admit many particles of air; it must needs be that, in bending of them, their internal parts will not easily be compressed, by reason of their hardness. And because their coherence is not firm, as soon as the external hard particles are disjoined, the ethereal parts will

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Whence proceeds the spontaneous restitution of things bent.

PART IV. necessarily break out, and so the body will suddenly be broken.
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Diaphanous
and opacous,
what they are,
and whence.

13. Those bodies are called *diaphanous*, upon which, whilst the beams of a lucid body do work, the action of every one of those beams is propagated in them in such manner, as that they still retain the same order amongst themselves, or the inversion of that order; and therefore bodies, which are perfectly diaphanous, are also perfectly homogeneous. On the contrary, an *opacous* body is that, which, by reason of its heterogeneous nature, doth by innumerable reflections and refractions in particles of different figures and unequal hardness, weaken the beams that fall upon it before they reach the eye. And of diaphanous bodies, some are made such by nature from the beginning; as the substance of the air, and of the water, and perhaps also some parts of stones, unless these also be water that has been long congealed. Others are made so by the power of heat, which congregates homogeneous bodies. But such, as are made diaphanous in this manner, consist of parts which were formerly diaphanous.

The cause of
lightning and
thunder.

14. In what manner clouds are made by the motion of the sun, elevating the particles of water from the sea and other moist places, hath been explained in chapter xxvi. Also how clouds come to be frozen, hath been shown above at the 7th article. Now from this, that air may be enclosed as it were in caverns, and pent together more and more by the meeting of ascending and descending clouds, may be deduced a possible cause of *thunder* and *lightning*. For seeing the air consists of two

parts, the one ethereal, which has no proper motion of its own, as being a thing divisible into the least parts; the other hard, namely, consisting of many hard atoms, which have every one of them a very swift simple motion of its own: whilst the clouds by their meeting do more and more straiten such cavities as they intercept, the ethereal parts will penetrate and pass through their watery substance; but the hard parts will in the meantime be the more thrust together, and press one another; and consequently, by reason of their vehement motions, they will have an endeavour to rebound from each other. Whensoever, therefore, the compression is great enough, and the concave parts of the clouds are, for the cause I have already given, congealed into ice, the cloud will necessarily be broken; and this breaking of the cloud produceth the first clap of thunder. Afterwards the air, which was pent in, having now broken through, makes a concussion of the air without, and from hence proceeds the roaring and murmur which follows; and both the first clap and the murmur that follows it make that noise which is called *thunder*. Also, from the same air breaking through the clouds and with concussion falling upon the eye, proceeds that action upon our eye, which causeth in us a perception of that light, which we call *lightning*. Wherefore I have given a possible cause of thunder and lightning.

15. But if the vapours, which are raised into clouds, do run together again into water or be congealed into ice, from whence is it, seeing both ice and water are heavy, that they are sustained in the air? Or rather, what may the cause be, that

Whence it proceeds that clouds can fall again, after they are once elevated and frozen.

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Whence it
proceeds that
clouds, &c.

being once elevated, they fall down again? For there is no doubt but the same force which could carry up that water, could also sustain it there. Why therefore being once carried up, doth it fall again? I say it proceeds from the same simple motion of the sun, both that vapours are forced to ascend, and that water gathered into clouds is forced to descend. For in chapter XXI, article 11, I have shown how vapours are elevated; and in the same chapter, article 5, I have also shown how by the same motion homogeneous bodies are congregated, and heterogeneous dissipated; that is to say, how such things, as have a like nature to that of the earth, are driven towards the earth; that is to say, what is the cause of the descent of heavy bodies. Now if the action of the sun be hindered in the raising of vapours, and be not at all hindered in the casting of them down, the water will descend. But a cloud cannot hinder the action of the sun in making things of an earthly nature descend to the earth, though it may hinder it in making vapours ascend. For the lower part of a thick cloud is so covered by its upper part, as that it cannot receive that action of the sun by which vapours are carried up; because vapours are raised by the perpetual fermentation of the air, or by the separating of its smallest parts from one another, which is much weaker when a thick cloud is interposed, than when the sky is clear. And therefore, whensoever a cloud is made thick enough, the water, which would not descend before, will then descend, unless it be kept up by the agitation of the wind. Wherefore I have rendered a possible cause, both why the clouds may

be sustained in the air, and also why they may fall down again to the earth ; which was propounded to be done.

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16. Granting that the clouds may be frozen, it is no wonder if the moon have been seen eclipsed at such time as she hath been almost two degrees above the horizon, the sun at the same time appearing in the horizon ; for such an eclipse was observed by Mæstlin, at Tübingen, in the year 1590. For it might happen that a frozen cloud was then interposed between the sun and the eye of the observer. And if it were so, the sun, which was then almost two degrees below the horizon, might appear to be in it, by reason of the passing of his beams through the ice. And it is to be noted that those, that attribute such refractions to the atmosphere, cannot attribute to it so great a refraction as this. Wherefore not the atmosphere, but either water in a continued body, or else ice, must be the cause of that refraction.

How it could be that the moon was eclipsed, when she was not diametrically opposite to the sun.

17. Again, granting that there may be ice in the clouds, it will be no longer a wonder that many suns have sometimes appeared at once. For looking-glasses may be so placed, as by reflections to show the same object in many places. And may not so many frozen clouds serve for so many looking-glasses ? And may they not be fitly disposed for that purpose ? Besides, the number of appearances may be increased by refractions also ; and therefore it would be a greater wonder to me, if such phenomena as these should never happen.

By what means many suns may appear at once.

And were it not for that one phenomenon of the new star which was seen in Cassiopea, I should think comets were made in the same manner,

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namely, by vapours drawn not only from the earth but from the rest of the planets also, and congealed into one continued body. For I could very well from hence give a reason both of their hair, and of their motions. But seeing that star remained sixteen whole months in the same place amongst the fixed stars, I cannot believe the matter of it was ice. Wherefore I leave to others the disquisition of the cause of comets; concerning which nothing that hath hitherto been published, besides the bare histories of them, is worth considering.

Of the heads
of rivers.

18. The heads of rivers may be deduced from rain-water, or from melted snows, very easily; but from other causes, very hardly, or not at all. For both rain-water and melted snows run down the descents of mountains; and if they descend only by the outward superficies, the showers or snows themselves may be accounted the springs or fountains; but if they enter the earth and descend within it, then, wheresoever they break out, there are their springs. And as these springs make small streams, so, many small streams running together make rivers. Now, there was never any spring found, but where the water which flowed to it, was either further, or at least as far from the centre of the earth, as the spring itself. And whereas it has been objected by a great philosopher, that in the top of Mount Cenis, which parts Savoy from Piedmont, there springs a river which runs down by Susa; it is not true. For there are above that river, for two miles length, very high hills on both sides, which are almost perpetually covered with snow; from which innumerable little streams running down do manifestly supply that river with water sufficient for its magnitude.

CHAP. XXIX.

OF SOUND, ODOUR, SAVOUR, AND TOUCH.

1. The definition of sound, and the distinctions of sounds.
2. The cause of the degrees of sounds.—3. The difference between sounds acute and grave.—4. The difference between clear and hoarse sounds, whence.—5. The sound of thunder and of a gun, whence it proceeds.—6. Whence it is that pipes, by blowing into them, have a clear sound.—7. Of reflected sound.—8. From whence it is that sound is uniform and lasting.—9. How sound may be helped and hindered by the wind.
10. Not only air, but other bodies how hard soever they be, convey sound.—11. The causes of grave and acute sounds, and of concert.—12. Phenomena for smelling.—13. The first organ and the generation of smelling.—14. How it is helped by heat and by wind.—15. Why such bodies are least smelt, which have least intermixture of air in them.—16. Why odorous things become more odorous by being bruised:—17. The first organ of tasting; and why some savours cause nauseousness.—18. The first organ of feeling; and how we come to the knowledge of such objects as are common to the touch and other senses.

1. SOUND *is sense generated by the action of the medium, when its motion reacheth the ear and the rest of the organs of sense.* Now, the motion of the medium is not the sound itself, but the cause of it. For the phantasm which is made in us, that is to say, the reaction of the organ, is properly that which we call *sound*.

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The definition of sound, and the distinction of sounds.

The principal distinctions of sounds are these; first, that one sound is stronger, another weaker. Secondly, that one is more grave, another more acute. Thirdly, that one is clear, another hoarse. Fourthly, that one is primary, another derivative.

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Fifthly, that one is uniform, another not. Sixthly, that one is more durable, another less durable. Of all which distinctions the members may be subdistinguished into parts distinguishable almost infinitely. For the variety of sounds seems to be not much less than that of colours.

As vision, so hearing is generated by the motion of the medium, but not in the same manner. For sight is from pressure, that is, from an endeavour ; in which there is no perceptible progression of any of the parts of the medium ; but one part urging or thrusting on another propagateth that action successively to any distance whatsoever ; whereas the motion of the medium, by which sound is made, is a stroke. For when we hear, the drum of the ear, which is the first organ of hearing, is stricken ; and the drum being stricken, the *pia mater* is also shaken, and with it the arteries which are inserted into it ; by which the action being propagated to the heart itself, by the reaction of the heart a phantasm is made which we call sound ; and because the reaction tendeth outwards, we think it is without.

The cause of
the degrees of
sounds.

2. And seeing the effects produced by motion are greater or less, not only when the velocity is greater or less, but also when the body hath greater or less magnitude though the velocity be the same ; a sound may be greater or less both these ways. And because neither the greatest nor the least magnitude or velocity can be given, it may happen that either the motion may be of so small velocity, or the body itself of so small magnitude, as to produce no sound at all ; or either of them may be so

great, as to take away the faculty of sense by hurting the organ. PART IV.
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From hence may be deduced possible causes of the strength and weakness of sounds in the following phenomena. The cause of the degrees of sounds.

The first whereof is this, that if a man speak through a trunk which hath one end applied to the mouth of the speaker, and the other to the ear of the hearer, the sound will come stronger than it would do through the open air. And the cause, not only the possible, but the certain and manifest cause is this, that the air which is moved by the first breath and carried forwards in the trunk, is not diffused as it would be in the open air, and is consequently brought to the ear almost with the same velocity with which it was first breathed out. Whereas, in the open air, the first motion diffuseth itself every way into circles, such as are made by the throwing of a stone into a standing water, where the velocity grows less and less as the undulation proceeds further and further from the beginning of its motion.

The second is this, that if the trunk be short, and the end which is applied to the mouth be wider than that which is applied to the ear, thus also the sound will be stronger than if it were made in the open air. And the cause is the same, namely, that by how much the wider end of the trunk is less distant from the beginning of the sound, by so much the less is the diffusion.

The third, that it is easier for one, that is within a chamber, to hear what is spoken without, than for him, that stands without, to hear what is spoken within. For the windows and other inlets of the

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moved air are as the wide end of the trunk. And for this reason some creatures seem to hear the better, because nature has bestowed upon them wide and capacious ears.

The fourth is this, that though he, which standeth upon the sea-shore, cannot hear the collision of the two nearest waves, yet nevertheless he hears the roaring of the whole sea. And the cause seems to be this, that though the several collisions move the organ, yet they are not severally great enough to cause sense; whereas nothing hinders but that all of them together may make sound.

The difference
between sounds
acute and grave

3. That bodies when they are stricken do yield some a more grave, others a more acute sound, the cause may consist in the difference of the times in which the parts stricken and forced out of their places return to the same places again. For in some bodies, the restitution of the moved parts is quick, in others slow. And this also may be the cause, why the parts of the organ, which are moved by the medium, return to their rest again, sometimes sooner, sometimes later. Now, by how much the vibrations or the reciprocal motions of the parts are more frequent, by so much doth the whole sound made at the same time by one stroke consist of more, and consequently of smaller parts. For what is acute in sound, the same is subtle in matter; and both of them, namely acute sound and subtle matter, consist of very small parts, that of time, and this of the matter itself.

The third distinction of sounds cannot be conceived clearly enough by the names I have used of *clear* and *hoarse*, nor by any other that I know; and therefore it is needful to explain them by

examples. When I say hoarse, I understand whispering and hissing, and whatsoever is like to these, by what appellation soever it be expressed. And sounds of this kind seem to be made by the force of some strong wind, raking rather than striking such hard bodies as it falls upon. On the contrary, when I use the word clear, I do not understand such a sound as may be easily and distinctly heard; for so whispers would be clear; but such as is made by somewhat that is broken, and such as is clamour, tinkling, the sound of a trumpet, &c. and to express it significantly in one word, noise. And seeing no sound is made but by the concurrence of two bodies at the least, by which concurrence it is necessary that there be as well reaction as action, that is to say, one motion opposite to another; it follows that according as the proportion between those two opposite motions is diversified, so the sounds which are made will be different from one another. And whensoever the proportion between them is so great, as that the motion of one of the bodies be insensible if compared with the motion of the other, then the sound will not be of the same kind; as when the wind falls very obliquely upon a hard body, or when a hard body is carried swiftly through the air; for then there is made that sound which I call a hoarse sound, in Greek *συριγμος*. Therefore the breath blown with violence from the mouth makes a hissing, because in going out it rakes the superficies of the lips, whose reaction against the force of the breath is not sensible. And this is the cause why the winds have that hoarse sound. Also if two bodies, how hard soever, be rubbed together with no great pressure, they

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make a hoarse sound. And this hoarse sound, when it is made, as I have said, by the air raking the superficies of a hard body, seemeth to be nothing but the dividing of the air into innumerable and very small files. For the asperity of the superficies doth, by the eminences of its innumerable parts, divide or cut in pieces the air that slides upon it.

The difference between clear & hoarse sounds, whence.

4. *Noise*, or that which I call clear sound, is made two ways; one, by two hoarse sounds made by opposite motions; the other, by collision, or by the sudden pulling asunder of two bodies, whereby their small particles are put into commotion, or being already in commotion suddenly restore themselves again; which motion, making impression upon the medium, is propagated to the organ of hearing. And seeing there is in this collision or divulsion an endeavour in the particles of one body, opposite to the endeavour of the particles of the other body, there will also be made in the organ of hearing a like opposition of endeavours, that is to say, of motions; and consequently the sound arising from thence will be made by two opposite motions, that is to say, by two opposite hoarse sounds in one and the same part of the organ. For, as I have already said, a hoarse sound supposeth the sensible motion of but one of the bodies. And this opposition of motions in the organ is the cause why two bodies make a noise, when they are either suddenly stricken against one another, or suddenly broken asunder.

The sound of thunder and of a gun, whence it proceeds.

5. This being granted, and seeing withal that thunder is made by the vehement eruption of the air out of the cavities of congealed clouds, the

cause of the great noise or clap may be the sudden breaking asunder of the ice. For in this action it is necessary that there be not only a great concussion of the small particles of the broken parts, but also that this concussion, by being communicated to the air, be carried to the organ of hearing, and make impression upon it. And then, from the first reaction of the organ proceeds that first and greatest sound, which is made by the collision of the parts whilst they restore themselves. And seeing there is in all concussion a reciprocation of motion forwards and backwards in the parts stricken; for opposite motions cannot extinguish one another in an instant, as I have shown in the 11th article of chapter VIII; it follows necessarily that the sound will both continue, and grow weaker and weaker, till at last the action of the reciprocating air grow so weak, as to be imperceptible. Wherefore a possible cause is given both of the first fierce noise of the thunder, and also of the murmur that follows it.

The cause of the great sound from a discharged piece of ordnance is like that of a clap of thunder. For the gunpowder being fired doth, in its endeavour to go out, attempt every way the sides of the metal in such manner, as that it enlargeth the circumference all along, and withal shorteneth the axis; so that whilst the piece of ordnance is in discharging, it is made both wider and shorter than it was before; and therefore also presently after it is discharged its wideness will be diminished, and its length increased again by the restitution of all the particles of the matter, of which it consisteth, to their former position. And this is

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done with such motions of the parts, as are not only very vehement, but also opposite to one another ; which motions, being communicated to the air, make impression upon the organ, and by the reaction of the organ create a sound, which lasteth for some time ; as I have already shown in this article.

I note by the way, as not belonging to this place, that the possible cause why a gun recoils when it is shot off, may be this ; that being first swollen by the force of the fire, and afterwards restoring itself, from this restitution there proceeds an endeavour from all the sides towards the cavity ; and consequently this endeavour is in those parts which are next the breech ; which being not hollow, but solid, the effect of the restitution is by it hindered and diverted into the length ; and by this means both the breech and the whole gun is thrust backwards ; and the more forcibly by how much the force is greater, by which the part next the breech is restored to its former posture, that is to say, by how much the thinner is that part. The cause, therefore, why guns recoil, some more some less, is the difference of their thickness towards the breech ; and the greater that thickness is, the less they recoil ; and contrarily.

Whence it is that pipes, by blowing into them, have a clear sound.

6. Also the cause why the sound of a pipe, which is made by blowing into it, is nevertheless clear, is the same with that of the sound which is made by collision. For if the breath, when it is blown into a pipe, do only rake its concave superficies, or fall upon it with a very sharp angle of incidence, the sound will not be clear, but hoarse. But if the

angle be great enough, the percussion, which is made against one of the hollow sides, will be reverberated to the opposite side; and so successive repercussions will be made from side to side, till at last the whole concave superficies of the pipe be put into motion; which motion will be reciprocated, as it is in collision; and this reciprocation being propagated to the organ, from the reaction of the organ will arise a clear sound, such as is made by collision, or by breaking asunder of hard bodies.

In the same manner it is with the sound of a man's voice. For when the breath passeth out without interruption, and doth but lightly touch the cavities through which it is sent, the sound it maketh is a hoarse sound. But if in going out it strike strongly upon the *larynx*, then a clear sound is made, as in a pipe. And the same breath, as it comes in divers manners to the palate, the tongue, the lips, the teeth, and other organs of speech, so the sounds into which it is articulated become different from one another.

7. I call that *primary sound*, which is generated by motion from the sounding body to the organ in a strait line without reflection; and I call that *reflected sound*, which is generated by one or more reflections, being the same with that we call *echo*, and is iterated as often as there are reflections made from the object to the ear. And these reflections are made by hills, walls, and other resisting bodies, so placed as that they make more or fewer reflections of the motion, according as they are themselves more or fewer in number; and they make them more or less frequently, according

Of reflected
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as they are more or less distant from one another. Now the cause of both these things is to be sought for in the situation of the reflecting bodies, as is usually done in sight. For the laws of reflection are the same in both, namely, that the angles of incidence and reflection be equal to one another. If, therefore, in a hollow elliptic body, whose inside is well polished, or in two right parabolical solids, which are joined together by one common base, there be placed a sounding body in one of the burning points, and the ear in the other, there will be heard a sound by many degrees greater than in the open air; and both this, and the burning of such combustible things, as being put in the same places are set on fire by the sun-beams, are effects of one and the same cause. But, as when the visible object is placed in one of the burning points, it is not distinctly seen in the other, because every part of the object being seen in every line, which is reflected from the concave superficies to the eye, makes a confusion in the sight; so neither is sound heard articulately and distinctly when it comes to the ear in all those reflected lines. And this may be the reason why in churches which have arched roofs, though they be neither elliptical nor parabolical, yet because their figure is not much different from these, the voice from the pulpit will not be heard so articulately as it would be, if there were no vaulting at all.

From whence
it is that sound
is uniform and
lasting.

8. Concerning the *uniformity* and *duration* of sounds, both which have one common cause, we may observe, that such bodies as being stricken yield an unequal or harsh sound, are very heterogeneous, that is to say, they consist of parts which

are very unlike both in figure and hardness, such as are wood, stones, and others not a few. When these are stricken, there follows a concussion of their internal particles, and a restitution of them again. But they are neither moved alike, nor have they the same action upon one another; some of them recoiling from the stroke, whilst others which have already finished their recoilings are now returning; by which means they hinder and stop one another. And from hence it is that their motions are not only unequal and harsh, but also that their reciprocations come to be quickly extinguished. Whensoever, therefore, this motion is propagated to the ear, the sound it makes is unequal and of small duration. On the contrary, if a body that is stricken be not only sufficiently hard, but have also the particles of which it consisteth like to one another both in hardness and figure, such as are the particles of glass and metals, which being first melted do afterwards settle and harden; the sound it yieldeth will, because the motions of its parts and their reciprocations are like and uniform, be uniform and pleasant, and be more or less lasting, according as the body stricken hath greater or less magnitude. The possible cause, therefore, of sounds uniform and harsh, and of their longer or shorter duration, may be one and the same likeness and unlikeness of the internal parts of the sounding body, in respect both of their figure and hardness.

Besides, if two plane bodies of the same matter and of equal thickness, do both yield an uniform sound, the sound of that body, which hath the greatest extent of length, will be the longest heard.

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For the motion, which in both of them hath its beginning from the point of percussion, is to be propagated in the greater body through a greater space, and consequently that propagation will require more time; and therefore also the parts which are moved, will require more time for their return. Wherefore all the reciprocations cannot be finished but in longer time; and being carried to the ear, will make the sound last the longer. And from hence it is manifest, that of hard bodies which yield an uniform sound, the sound lasteth longer which comes from those that are round and hollow, than from those that are plane, if they be like in all other respects. For in circular lines the action, which begins at any point, hath not from the figure any end of its propagation, because the line in which it is propagated returns again to its beginning; so that the figure hinders not but that the motion may have infinite progression. Whereas in a plane, every line hath its magnitude finite, beyond which the action cannot proceed. If, therefore, the matter be the same, the motion of the parts of that body whose figure is round and hollow, will last longer than of that which is plane.

Also, if a string which is stretched be fastened at both ends to a hollow body, and be stricken, the sound will last longer than if it were not so fastened; because the trembling or reciprocation which it receives from the stroke, is by reason of the connection communicated to the hollow body; and this trembling, if the hollow body be great, will last the longer by reason of that greatness. Where-

fore also, for the reason above mentioned, the sound will last the longer.

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9. In hearing it happens, otherwise than in seeing, that the action of the medium is made stronger by the wind when it blows the same way, and weaker when it blows the contrary way. The cause whereof cannot proceed from anything but the different generation of sound and light. For in the generation of light, none of the parts of the medium between the object and the eye are moved from their own places to other places sensibly distant; but the action is propagated in spaces imperceptible; so that no contrary wind can diminish, nor favourable wind encrease the light, unless it be so strong as to remove the object further off or bring it nearer to the eye. For the wind, that is to say the air moved, doth not by its interposition between the object and the eye work otherwise than it would do, if it were still and calm. For, where the pressure is perpetual, one part of the air is no sooner carried away, but another, by succeeding it, receives the same impression, which the part carried away had received before. But in the generation of sound, the first collision or breaking asunder beateth away and driveth out of its place the nearest part of the air, and that to a considerable distance, and with considerable velocity; and as the circles grow by their remoteness wider and wider, so the air being more and more dissipated, hath also its motion more and more weakened. Whensoever therefore the air is so stricken as to cause sound, if the wind fall upon it, it will move it all nearer to the ear, if it blow

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may be helped
and hindered
by the wind.

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that way, and further from it if it blow the contrary way; so that according as it blows from or towards the object, so the sound which is heard will seem to come from a nearer or remoter place; and the reaction, by reason of the unequal distances, be strengthened or debilitated.

From hence may be understood the reason why the voice of such as are said to speak in their bellies, though it be uttered near hand, is nevertheless heard, by those that suspect nothing, as if it were a great way off. For having no former thought of any determined place from which the voice should proceed, and judging according to the greatness, if it be weak they think it a great way off, if strong near. These ventriloqui, therefore, by forming their voice, not as others by the emission of their breath, but by drawing it inwards, do make the same appear small and weak; which weakness of the voice deceives those, that neither suspect the artifice nor observe the endeavour which they use in speaking; and so, instead of thinking it weak, they think it far off.

Not only air, but other bodies, how hard soever they be, convey sound.

10. As for the medium, which conveys sound, it is not air only. For water, or any other body how hard soever, may be that medium. For the motion may be propagated perpetually in any hard continuous body; but by reason of the difficulty, with which the parts of hard bodies are moved, the motion in going out of hard matter makes but a weak impression upon the air. Nevertheless, if one end of a very long and hard beam be stricken, and the ear be applied at the same time to the other end, so that, when the action goeth out of the beam, the

air, which it striketh, may immediately be received by the ear, and be carried to the tympanum, the sound will be considerably strong.

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In like manner, if in the night, when all other noise which may hinder sound ceaseth, a man lay his ear to the ground, he will hear the sound of the steps of passengers, though at a great distance; because the motion, which by their treading they communicate to the earth, is propagated to the ear by the uppermost parts of the earth which receiveth it from their feet.

11. I have shown above, that the difference between grave and acute sounds consisteth in this, that by how much the shorter the time is, in which the reciprocations of the parts of a body stricken are made, by so much the more acute will be the sound. Now by how much a body of the same bigness is either more heavy or less stretched, by so much the longer will the reciprocations last; and therefore heavier and less stretched bodies, if they be like in all other respects, will yield a graver sound than such as be lighter and more stretched.

The causes of grave and acute sounds, and of concert.

As for the concert of sounds, it is to be considered that the reciprocation or vibration of the air, by which sound is made, after it hath reached the drum of the ear, imprinteth a like vibration upon the air that is inclosed within it; by which means the sides of the drum within are stricken alternately. Now the concert of two sounds consists in this, that the tympanum receives its sounding stroke from both the sounding bodies in equal and equally frequent spaces of time; so that when two strings make their vibrations in the same

Again, the vibration of the string A B remaining still the same it was, let C D be stretched till its vibration be swifter than that of the string A B in the proportion of 3 to 2, and let E F be divided into three equal parts in K and L. In what time therefore the string C D makes one third part of its vibration, which third part is from G to H, the string A B will make one third part of its vibration, that is to say, two-thirds of E F, namely, E L. And in what time the string C D makes another third part of its vibration, namely H G, the string A B will make another third part of its vibration, namely from L to F, and back again from F to L. Lastly, whilst the string C D makes the last third part of its vibration, that is from G to H, the string A B will make the last third part of its vibration from L to E. But the points E and H are both in the sides of the organ. Wherefore, at every third time, the organ will be stricken by the vibration of both the strings together, and make that concord which is called a *fifth*.

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12. For the finding out the cause of *smells*, I shall make use of the evidence of these following phenomena. First, that smelling is hindered by cold, and helped by heat. Secondly, that when the wind bloweth from the object, the smell is the stronger ; and, contrarily, when it bloweth from the sentient towards the object, the weaker ; both which phenomena are, by experience, manifestly found to be true in dogs, which follow the track of beasts by the scent. Thirdly, that such bodies, as are less pervious to the fluid medium, yield less smell than such as are more pervious ; as may be seen in stones and metals, which, compared with

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plants and living creatures, and their parts, fruits and excretions, have very little or no smell at all. Fourthly, that such bodies, as are of their own nature odorous, become yet more odorous when they are bruised. Fifthly, that when the breath is stopped, at least in men, nothing can be smelt. Sixthly, that the sense of smelling is also taken away by the stopping of the nostrils, though the mouth be left open.

The first organ
and the gene-
ration of
smelling.

13. By the fifth and sixth phenomenon it is manifest, that the first and immediate organ of smelling is the innermost cuticle of the nostrils, and that part of it, which is below the passage common to the nostrils and the palate. For when we draw breath by the nostrils we draw it into the lungs. That breath, therefore, which conveys smells is in the way which passeth to the lungs, that is to say, in that part of the nostrils which is below the passage through which the breath goeth. For, nothing is smelt, neither beyond the passage of the breath within, nor at all without the nostrils.

And seeing that from different smells there must necessarily proceed some mutation in the organ, and all mutation is motion; it is therefore also necessary that, in smelling, the parts of the organ, that is to say of that internal cuticle and the nerves that are inserted into it, must be diversely moved by different smells. And seeing also, that it hath been demonstrated, that nothing can be moved but by a body that is already moved and contiguous; and that there is no other body contiguous to the internal membrane of the nostrils but breath, that is to say attracted air, and such little solid invisible bodies, if there be any such, as

are intermingled with the air; it follows necessarily, that the cause of smelling is either the motion of that pure air or ethereal substance, or the motion of those small bodies. But this motion is an effect proceeding from the object of smell, and, therefore, either the whole object itself or its several parts must necessarily be moved. Now, we know that odorous bodies make odour, though their whole bulk be not moved. Wherefore the cause of odour is the motion of the invisible parts of the odorous body. And these invisible parts do either go out of the object, or else, retaining their former situation with the rest of the parts, are moved together with them, that is to say, they have simple and invisible motion. They that say, there goes something out of the odorous body, call it an effluvium; which effluvium is either of the ethereal substance, or of the small bodies that are intermingled with it. But, that all variety of odours should proceed from the effluvia of those small bodies that are intermingled with the ethereal substance, is altogether incredible, for these considerations; first, that certain unguents, though very little in quantity, do nevertheless send forth very strong odours, not only to a great distance of place, but also for a great continuance of time, and are to be smelt in every point both of that place and time; so that the parts issued out are sufficient to fill ten thousand times more space, than the whole odorous body is able to fill; which is impossible. Secondly, that whether that issuing out be with strait or with crooked motion, if the same quantity should flow from any other odorous body with the same motion, it would follow that all odorous bodies would yield the same smell. Thirdly,

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that seeing those effluvia have great velocity of motion (as is manifest from this, that noisome odours proceeding from caverns are presently smelt at a great distance) it would follow, that, by reason there is nothing to hinder the passage of those effluvia to the organ, such motion alone were sufficient to cause smelling; which is not so; for we cannot smell at all, unless we draw in our breath through our nostrils. Smelling, therefore, is not caused by the effluvium of atoms; nor, for the same reason, is it caused by the effluvium of ethereal substance; for so also we should smell without the drawing in of our breath. Besides, the ethereal substance being the same in all odorous bodies, they would always affect the organ in the same manner; and, consequently, the odours of all things would be alike.

It remains, therefore, that the cause of smelling must consist in the simple motion of the parts of odorous bodies without any efflux or diminution of their whole substance. And by this motion there is propagated to the organ, by the intermediate air, the like motion, but not strong enough to excite sense of itself without the attraction of air by respiration. And this is a possible cause of smelling.

How smelling is
helped by heat
and by wind.

14. The cause why smelling is hindered by cold and helped by heat may be this; that heat, as hath been shown in chapter XXI, generateth simple motion; and therefore also, wheresoever it is already, there it will increase it; and the cause of smelling being increased, the smell itself will also be increased. As for the cause why the wind blowing from the object makes the smell the stronger, it is all one with that for which the at-

traction of air in respiration doth the same. For, he that draws in the air next to him, draws with it by succession that air in which is the object. Now, this motion of the air is wind, and, when another wind bloweth from the object, will be increased by it.

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15. That bodies which contain the least quantity of air, as stones and metals, yield less smell than plants and living creatures; the cause may be, that the motion, which causeth smelling, is a motion of the fluid parts only; which parts, if they have any motion from the hard parts in which they are contained, they communicate the same to the open air, by which it is propagated to the organ. Where, therefore, there are no fluid parts as in metals, or where the fluid parts receive no motion from the hard parts, as in stones, which are made hard by accretion, there can be no smell. And therefore also the water, whose parts have little or no motion, yieldeth no smell. But, if the same water, by seeds and the heat of the sun, be together with particles of earth raised into a plant, and be afterwards pressed out again, it will be odorous, as wine from the vine. And as water passing through plants is by the motion of the parts of those plants made an odorous liquor; so also of air, passing through the same plants whilst they are growing, are made odorous airs. And thus also it is with the juices and spirits, which are bred in living creatures.

Why such bodies are least smelt, which have the least intermixture of air in them.

16. That odorous bodies may be made more odorous by contrition proceeds from this, that being broken into many parts, which are all odorous, the air, which by respiration is drawn from the object towards the organ, doth in its passage

Why odorous things become more odorous, when bruised.

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touch upon all those parts, and receive their motion. Now, the air toucheth the superficies only ; and a body having less superficies whilst it is whole, than all its parts together have after it is reduced to powder, it follows that the same odorous body yieldeth less smell whilst it is whole, than it will do after it is broken into smaller parts. And thus much of smells.

The first organ of tasting ; and why some savours cause nauseousness.

17. The taste follows ; whose generation hath this difference from that of the sight, hearing, and smelling, that by these we have sense of remote objects ; whereas, we taste nothing but what is contiguous, and doth immediately touch either the tongue or palate, or both. From whence it is evident, that the cuticles of the tongue and palate, and the nerves inserted into them are the first organ of taste ; and (because from the concussion of the parts of these, there followeth necessarily a concussion of the *pia mater*) that the action communicated to these is propagated to the brain, and from thence to the farthest organ, namely, the heart, in whose reaction consisteth the nature of sense.

Now, that savours, as well as odours, do not only move the brain but the stomach also, as is manifest by the loathings that are caused by them both ; they, that consider the organ of both these senses, will not wonder at all ; seeing the tongue, the palate and the nostrils, have one and the same continued cuticle, derived from the *dura mater*.

And that effluvia have nothing to do in the sense of tasting, is manifest from this, that there is no taste where the organ and the object are not contiguous.

By what variety of motions the different kinds of tastes, which are innumerable, may be distinguished, I know not. I might with others derive them from the divers figures of those atoms, of which whatsoever may be tasted consisteth; or from the diverse motions which I might, by way of supposition, attribute to those atoms; conjecturing, not without some likelihood of truth, that such things as taste sweet have their particles moved with slow circular motion, and their figures spherical; which makes them smooth and pleasing to the organ; that bitter things have circular motion, but vehemement, and their figures full of angles, by which they trouble the organ; and that sour things have strait and reciprocal motion, and their figures long and small, so that they cut and wound the organ. And in like manner I might assign for the causes of other tastes such several motions and figures of atoms, as might in probability seem to be the true causes. But this would be to revolt from philosophy to divination.

18. By the *touch*, we feel what bodies are cold or hot, though they be distant from us. Others, as hard, soft, rough, and smooth, we cannot feel unless they be contiguous. The organ of touch is every one of those membranes, which being continued from the *pia mater* are so diffused throughout the whole body, as that no part of it can be pressed, but the *pia mater* is pressed together with it. Whatsoever therefore presseth it, is felt as hard or soft, that is to say, as more or less hard. And as for the sense of rough, it is nothing else but innumerable perceptions of hard and hard succeeding one another by short intervals both of

The first organ of feeling; and how we came to the knowledge of such objects as are common to the touch and to other senses.

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time and place. For we take notice of rough and smooth, as also of magnitude and figure, not only by the touch, but also by memory. For though some things are touched in one point, yet rough and smooth, like quantity and figure, are not perceived but by the flux of a point, that is to say, we have no sense of them without time; and we can have no sense of time without memory.

CHAPTER XXX.

OF GRAVITY.

1. A thick body doth not contain more matter, unless also more place, than a thin.—2. That the descent of heavy bodies proceeds not from their own appetite, but from some power of the earth.—3. The difference of gravities proceedeth from the difference of the impetus with which the elements, whereof heavy bodies are made, do fall upon the earth.—4. The cause of the descent of heavy bodies.—5. In what proportion the descent of heavy bodies is accelerated.—6. Why those that dive do not, when they are under water, feel the weight of the water above them.—7. The weight of a body that floateth, is equal to the weight of so much water as would fill the space, which the immersed part of the body takes up within the water.—8. If a body be lighter than water, then how big soever that body be, it may float upon any quantity of water, how little soever.—9. How water may be lifted up and forced out of a vessel by air.—10. Why a bladder is heavier when blown full of air, than when it is empty.—11. The cause of the ejection upwards of heavy bodies from a wind-gun.
12. The cause of the ascent of water in a weather-glass.
13. The cause of motion upwards in living creatures.—14. That there is in nature a kind of body heavier than air, which nevertheless is not by sense distinguishable from it.—15. Of the cause of magnetical virtue.

A thick body doth not contain more matter as that place required, so, as that by thick was

signified a more resisting body, and by thin, a body less resisting; following the custom of those that have before me discoursed of refraction. Now if we consider the true and vulgar signification of those words, we shall find them to be names collective, that is to say, names of multitude; as *thick* to be that, which takes up more parts of a space given, and *thin* that, which contains fewer parts of the same magnitude in the same space, or in a space equal to it. Thick therefore is the same with frequent, as a thick troop; and thin the same with unfrequent, as a thin rank, thin of houses; not that there is more matter in one place than in another equal place, but a greater quantity of some named body. For there is not less matter or body, indefinitely taken, in a desert, than there is in a city; but fewer houses, or fewer men. Nor is there in a thick rank a greater quantity of body, but a greater number of soldiers, than in a thin. Wherefore the multitude and paucity of the parts contained within the same space do constitute density and rarity, whether those parts be separated by vacuum or by air. But the consideration of this is not of any great moment in philosophy; and therefore I let it alone, and pass on to the search of the causes of *gravity*.

2. Now we call those bodies *heavy*, which, unless they be hindered by some force, are carried towards the centre of the earth, and that by their own accord, for aught we can by sense perceive to the contrary. Some philosophers therefore have been of opinion, that the descent of heavy bodies proceeded from some internal appetite, by which, when they were cast upwards, they descended

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more place,
than a thin.

That the descent of heavy bodies proceeds not from their own appetite, but from some power of the earth.

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again, as moved by themselves, to such place as was agreeable to their nature. Others thought they were attracted by the earth. To the former I cannot assent, because I think I have already clearly enough demonstrated that there can be no beginning of motion, but from an external and moved body; and consequently, that whatsoever hath motion or endeavour towards any place, will always move or endeavour towards that same place, unless it be hindered by the reaction of some external body. Heavy bodies, therefore, being once cast upwards, cannot be cast down again but by external motion. Besides, seeing inanimate bodies have no appetite at all, it is ridiculous to think that by their own innate appetite they should, to preserve themselves, not understanding what preserves them, forsake the place they are in, and transfer themselves to another place; whereas man, who hath both appetite and understanding, cannot, for the preservation of his own life, raise himself by leaping above three or four feet from the ground. Lastly, to attribute to created bodies the power to move themselves, what is it else than to say that there be creatures which have no dependance upon the Creator? To the latter, who attribute the descent of heavy bodies to the attraction of the earth, I assent. But by what motion this is done, hath not as yet been explained by any man. I shall therefore in this place say somewhat of the manner and of the way by which the earth by its action attracteth heavy bodies.

The difference
of gravities
proceedeth
from the differ-

3. That by the supposition of simple motion in the sun, homogeneous bodies are congregated and heterogeneous dissipated, has already been demon-

strated in the 5th article of chapter XXI. I have also supposed, that there are intermingled with the pure air certain little bodies, or, as others call them, atoms; which by reason of their extreme smallness are invisible, and differing from one another in consistence, figure, motion, and magnitude; from whence it comes to pass that some of them are congregated to the earth, others to other planets, and others are carried up and down in the spaces between. And seeing those, which are carried to the earth, differ from one another in figure, motion, and magnitude, they will fall upon the earth, some with greater, others with less impetus. And seeing also that we compute the several degrees of gravity no otherwise than by this their falling upon the earth with greater or less impetus; it follows, that we conclude those to be the more heavy that have the greater impetus, and those to be less heavy that have the less impetus. Our inquiry therefore must be, by what means it may come to pass, that of bodies, which descend from above to the earth, some are carried with greater, others with less impetus; that is to say, some are more heavy than others. We must also inquire, by what means such bodies, as settle upon the earth, may by the earth itself be forced to ascend.

4. Let the circle made upon the centre C (in fig. 2) be a great circle in the superficies of the earth, passing through the points A and B. Also let any heavy body, as the stone AD, be placed anywhere in the plane of the equator; and let it be conceived to be cast up from AD perpendicularly, or to be carried in any other line to E, and supposed to rest there. Therefore, how much

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ence of the impetus with which the elements, whereof heavy bodies are made, do fall upon the earth.

The cause of the descent of heavy bodies.

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the descent of
heavy bodies.

space soever the stone took up in A D, so much space it takes up now in E. And because all place is supposed to be full, the space A D will be filled by the air which flows into it first from the nearest places of the earth, and afterwards successively from more remote places. Upon the centre C let a circle be understood to be drawn through E; and let the plane space, which is between the superficies of the earth and that circle, be divided into plane orbs equal and concentric; of which let that be the first, which is contained by the two perimeters that pass through A and D. Whilst therefore the air, which is in the first orb, filleth the place A D, the orb itself is made so much less, and consequently its latitude is less than the strait line A D. Wherefore there will necessarily descend so much air from the orb next above. In like manner, for the same cause, there will also be a descent of air from the orb next above that; and so by succession from the orb in which the stone is at rest in E. Either therefore the stone itself, or so much air, will descend. And seeing air is by the diurnal revolution of the earth more easily thrust away than the stone, the air, which is in the orb that contains the stone, will be forced further upwards than the stone. But this, without the admission of vacuum, cannot be, unless so much air descend to E from the place next above; which being done, the stone will be thrust downwards. By this means therefore the stone now receives the beginning of its descent, that is to say, of its gravity. Furthermore, whatsoever is once moved, will be moved continually (as hath been shown in the 19th article of chapter VIII) in the same way, and

with the same celerity, except it be retarded or accelerated by some external movent. Now the air, which is the only body that is interposed between the earth A and the stone above it E, will have the same action in every point of the strait line EA, which it hath in E. But it depressed the stone in E; and therefore also it will depress it equally in every point of the strait line EA. Wherefore the stone will descend from E to A with accelerated motion. The possible cause therefore of the descent of heavy bodies under the equator, is the diurnal motion of the earth. And the same demonstration will serve, if the stone be placed in the plane of any other circle parallel to the equator. But because this motion hath, by reason of its greater slowness, less force to thrust off the air in the parallel circles than in the equator, and no force at all at the poles, it may well be thought (for it is a certain consequent) that heavy bodies descend with less and less velocity, as they are more and more remote from the equator; and that at the poles themselves, they will either not descend at all, or not descend by the axis; which whether it be true or false, experience must determine. But it is hard to make the experiment, both because the times of their descents cannot be easily measured with sufficient exactness, and also because the places near the poles are inaccessible. Nevertheless, this we know, that by how much the nearer we come to the poles, by so much the greater are the flakes of the snow that falls; and by how much the more swiftly such bodies descend as are fluid and dissippable, by so much the smaller are the particles into which they are dissipated.

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In what proportion the descent of heavy bodies is accelerated.

5. Supposing, therefore, this to be the cause of the descent of heavy bodies, it will follow that their motion will be accelerated in such manner, as that the spaces, which are transmitted by them in the several times, will have to one another the same proportion which the odd numbers have in succession from unity. For if the strait line EA be divided into any number of equal parts, the heavy body descending will, by reason of the perpetual action of the diurnal motion, receive from the air in every one of those times, in every several point of the strait line EA, a several new and equal impulsion; and therefore also in every one of those times, it will acquire a several and equal degree of celerity. And from hence it follows, by that which Galileus hath in his *Dialogues of Motion* demonstrated, that heavy bodies descend in the several times with such differences of transmitted spaces, as are equal to the differences of the square numbers that succeed one another from unity; which square numbers being 1, 4, 9, 16, &c. their differences are 3, 5, 7, &c.; that is to say, the odd numbers which succeed one another from unity. Against this cause of gravity which I have given, it will perhaps be objected, that if a heavy body be placed in the bottom of some hollow cylinder of iron or adamant, and the bottom be turned upwards, the body will descend, though the air above cannot depress it, much less accelerate its motion. But it is to be considered that there can be no cylinder or cavern, but such as is supported by the earth, and being so supported is, together with the earth, carried about by its diurnal motion. For by this means the bottom of

the cylinder will be as the superficies of the earth; and by thrusting off the next and lowest air, will make the uppermost air depress the heavy body, which is at the top of the cylinder, in such manner as is above explicated.

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6. The gravity of water being so great as by experience we find it is, the reason is demanded by many, why those that dive, how deep soever they go under water, do not at all feel the weight of the water which lies upon them. And the cause seems to be this, that all bodies by how much the heavier they are, by so much the greater is the endeavour by which they tend downwards. But the body of a man is heavier than so much water as is equal to it in magnitude, and therefore the endeavour downwards of a man's body is greater than that of water. And seeing all endeavour is motion, the body also of a man will be carried towards the bottom with greater velocity than so much water. Wherefore there is greater reaction from the bottom; and the endeavour upwards is equal to the endeavour downwards, whether the water be pressed by water, or by another body which is heavier than water. And therefore by these two opposite equal endeavours, the endeavour both ways in the water is taken away; and consequently, those that dive are not at all pressed by it.

Why those that
dive, do not,
when they are
under water,
feel the weight
of the water
above them.

Coroll. From hence also it is manifest, that water in water hath no weight at all, because all the parts of water, both the parts above, and the parts that are directly under, tend towards the bottom with equal endeavour and in the same strait lines.

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The weight of a body that floateth, is equal to the weight of so much water as would fill the space which the immersed part of the body takes up within the water.

7. If a body float upon the water, the weight of that body is equal to the weight of so much water as would fill the place which the immersed part of the body takes up within the water.

Let EF (in fig. 3) be a body floating in the water $ABCD$; and let the part E be above, and the other part F under the water. I say, the weight of the whole body EF is equal to the weight of so much water as the space F will receive. For seeing the weight of the body EF forceth the water out of the space F , and placeth it upon the superficies AB , where it presseth downwards; it follows, that from the resistance of the bottom there will also be an endeavour upwards. And seeing again, that by this endeavour of the water upwards, the body EF is lifted up, it follows, that if the endeavour of the body downwards be not equal to the endeavour of the water upwards, either the whole body EF will, by reason of that inequality of their endeavours or moments, be raised out of the water, or else it will descend to the bottom. But it is supposed to stand so, as neither to ascend nor descend. Wherefore there is an equilibrium between the two endeavours; that is to say, the weight of the body EF is equal to the weight of so much water as the space F will receive; which was to be proved.

If a body be lighter than water, then how big soever that body be, it may float upon any quantity of water, how little soever.

8. From hence it follows, that any body, of how great magnitude soever, provided it consist of matter less heavy than water, may nevertheless float upon any quantity of water, how little soever.

Let $ABCD$ (in fig. 4) be a vessel; and in it let $EFGH$ be a body consisting of matter which is less heavy than water; and let the space $AGCF$

be filled with water. I say, the body EFGH will not sink to the bottom DC. For seeing the matter of the body EFGH is less heavy than water, if the whole space without ABCD were full of water, yet some part of the body EFGH, as EFIK, would be above the water; and the weight of so much water as would fill the space IGHK would be equal to the weight of the whole body EFGH; and consequently GH would not touch the bottom DC. As for the sides of the vessel, it is no matter whether they be hard or fluid; for they serve only to terminate the water; which may be done as well by water as by any other matter how hard soever; and the water without the vessel is terminated somewhere, so as that it can spread no farther. The part therefore EFIK will be extant above the water AGCF which is contained in the vessel. Wherefore the body EFGH will also float upon the water AGCF, how little soever that water be; which was to be demonstrated.

9. In the 4th article of chapter XXVI, there is brought for the proving of vacuum the experiment of water enclosed in a vessel; which water, the orifice above being opened, is ejected upwards by the impulsion of the air. It is therefore demanded, seeing water is heavier than air, how that can be done. Let the second figure of the same, chapter XXVI be considered, where the water is with great force injected by a syringe into the space FGB. In that injection, the air (but pure air) goeth with the same force out of the vessel through the injected water. But as for those small bodies, which formerly I supposed to be intermingled with air and to be moved with simple motion, they

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How water may be lifted up and forced out of a vessel by air.

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cannot, together with the pure air, penetrate the water; but remaining behind are necessarily thrust together into a narrower place, namely into the space which is above the water FG. The motions therefore of those small bodies will be less and less free, by how much the quantity of the injected water is greater and greater; so that by their motions falling upon one another, the same small bodies will mutually compress each other, and have a perpetual endeavour of regaining their liberty, and of depressing the water that hinders them. Wherefore, as soon as the orifice above is opened, the water which is next it will have an endeavour to ascend, and will therefore necessarily go out. But it cannot go out, unless at the same time there enter in as much air; and therefore both the water will go out, and the air enter in, till those small bodies which were left within the vessel have recovered their former liberty of motion; that is to say, till the vessel be again filled with air, and no water be left of sufficient height to stop the passage at B. Wherefore I have shown a possible cause of this phenomenon, namely, the same with that of thunder. For as in the generation of thunder, the small bodies enclosed within the clouds, by being too closely pent together, do by their motion break the clouds, and restore themselves to their natural liberty; so here also the small bodies enclosed within the space which is above the strait line FG, do by their own motion expel the water as soon as the passage is opened above. And if the passage be kept stopped, and these small bodies be more vehemently compressed

by the perpetual forcing in of more water, they will at last break the vessel itself with great noise.

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10. If air be blown into a hollow cylinder, or into a bladder, it will increase the weight of either of them a little, as many have found by experience, who with great accurateness have tried the same. And it is no wonder, seeing, as I have supposed, there are intermingled with the common air a great number of small hard bodies, which are heavier than the pure air. For, the ethereal substance, being on all sides equally agitated by the motion of the sun, hath an equal endeavour towards all the parts of the universe; and, therefore, it hath no gravity at all.

Why a bladder is heavier when blown full of air, than when it is empty.

11. We find also by experience, that, by the force of air enclosed in a hollow cannon, a bullet of lead may with considerable violence be shot out of a gun of late invention, called the wind-gun. In the end of this cannon there are two holes, with their valves on the inside, to shut them close; one of them serving for the admission of air, and the other for the letting of it out. Also, to that end which serves for the receiving in of air, there is joined another cannon of the same metal and bigness, in which there is fitted a rammer which is perforated, and hath also a valve opening towards the former cannon. By the help of this valve the rammer is easily drawn back, and letteth in air from without; and being often drawn back and returned again with violent strokes, it forceth some part of that air into the former cannon, so long, till at last the resistance of the enclosed air is greater than the force of the stroke. And by this

The cause of the ejection upwards of heavy bodies from a wind-gun.

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means men think there is now a greater quantity of air in the cannon than there was formerly, though it were full before. Also, the air thus forced in, how much soever it be, is hindered from getting out again by the aforesaid valves, which the very endeavour of the air to get out doth necessarily shut. Lastly, that valve being opened which was made for the letting out of the air, it presently breaketh out with violence, and driveth the bullet before it with great force and velocity.

As for the cause of this, I could easily attribute it, as most men do, to condensation, and think that the air, which had at the first but its ordinary degree of rarity, was afterwards, by the forcing in of more air, condensed, and last of all, rarified again by being let out and restored to its natural liberty. But I cannot imagine how the same place can be always full, and, nevertheless, contain some times a greater, sometimes a less quantity of matter; that is to say, that it can be fuller than full. Nor can I conceive how fulness can of itself be an efficient cause of motion. For both these are impossible. Wherefore we must seek out some other possible cause of this phenomenon. Whilst, therefore, the valve which serves for the letting in of air, is opened by the first stroke of the rammer, the air within doth with equal force resist the entering of the air from without; so that the endeavours between the internal and external air are opposite, that is, there are two opposite motions whilst the one goeth in and the other cometh out; but no augmentation at all of air within the cannon. For there is driven out by the stroke as much pure air, which passeth between the rammer and the sides

of the cannon, as there is forced in of air impure by the same stroke. And thus, by many forcible strokes, the quantity of small hard bodies will be increased within the cannon, and their motions also will grow stronger and stronger, as long as the matter of the cannon is able to endure their force; by which, if it be not broken, it will at least be urged every way by their endeavour to free themselves; and as soon as the valve, which serves to let them out, is opened, they will fly out with violent motion, and carry with them the bullet which is in their way. Wherefore, I have given a possible cause of this phenomenon.

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12. Water, contrary to the custom of heavy bodies, ascendeth in the weather-glass; but it doth it when the air is cold: for when it is warm it descendeth again. And this organ is called a thermometer or thermoscope, because the degrees of heat and cold are measured and marked by it. It is made in this manner. Let A B C D (in fig. 5) be a vessel full of water, and E F G a hollow cylinder of glass, closed at E and open at G. Let it be heated, and set upright within the water to F; and let the open end reach to G. This being done, as the air by little and little grows colder, the water will ascend slowly within the cylinder from F towards E; till at last the external and internal air coming to be both of the same temper, it will neither ascend higher nor descend lower, till the temper of the air be changed. Suppose it, therefore, to be settled anywhere, as at H. If now the heat of the air be augmented, the water will descend below H; and if the heat be diminished, it will ascend above

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it. Which, though it be certainly known to be true by experience, the cause, nevertheless, hath not as yet been discovered.

In the sixth and seventh articles of chapter XXVIII, where I consider the cause of cold, I have shown, that fluid bodies are made colder by the pressure of the air, that is to say, by a constant wind that presseth them. For the same cause it is, that the superficies of the water is pressed at F; and having no place, to which it may retire from this pressure, besides the cavity of the cylinder between H and E, it is therefore necessarily forced thither by the cold, and consequently it ascendeth more or less, according as the cold is more or less increased. And again, as the heat is more intense or the cold more remiss, the same water will be depressed more or less by its own gravity, that is to say, by the cause of gravity above explained.

Cause of motion upwards in living creatures

13. Also living creatures, though they be heavy, can by leaping, swimming and flying, raise themselves to a certain degree of height. But they cannot do this except they be supported by some resisting body, as the earth, the water and the air. For these motions have their beginning from the contraction, by the help of the muscles, of the body animate. For to this contraction there succeedeth a distension of their whole bodies; by which distension, the earth, the water, or the air, which supporteth them, is pressed; and from hence, by the reaction of those pressed bodies, living creatures acquire an endeavour upwards, but such as by reason of the gravity of their bodies is presently

lost again. By this endeavour, therefore, it is, that living creatures raise themselves up a little way by leaping, but to no great purpose: but by swimming and flying they raise themselves to a greater height; because, before the effect of their endeavour is quite extinguished by the gravity of their bodies, they can renew the same endeavour again.

That by the power of the soul, without any antecedent contraction of the muscles or the help of something to support him, any man can be able to raise his body upwards, is a childish conceit. For if it were true, a man might raise himself to what height he pleased.

14. The diaphanous medium, which surrounds the eye on all sides, is invisible; nor is air to be seen in air, nor water in water, nor anything but that which is more opacous. But in the confines of two diaphanous bodies, one of them may be distinguished from the other. It is not therefore a thing so very ridiculous for ordinary people to think all that space empty, in which we say is air; it being the work of reason to make us conceive that the air is anything. For by which of our senses is it, that we take notice of the air, seeing we neither see, nor hear, nor taste, nor smell, nor feel it to be anything? When we feel heat, we do not impute it to the air, but to the fire: nor do we say the air is cold, but we ourselves are cold; and when we feel the wind, we rather think something is coming, than that any thing is already come. Also, we do not at all feel the weight of water in water, much less of air in air. That we come to know that to be a body, which we call air, it is by

That there is in nature a kind of body heavier than air, which nevertheless is not by sense distinguishable from it.

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nature, &c.

reasoning ; but it is from one reason only, namely, because it is impossible for remote bodies to work upon our organs of sense but by the help of bodies intermediate, without which we could have no sense of them, till they come to be contiguous. Wherefore, from the senses alone, without reasoning from effects, we cannot have sufficient evidence of the nature of bodies.

For there is underground, in some mines of coals, a certain matter of a middle nature between water and air, which nevertheless cannot by sense be distinguished from air ; for it is as diaphanous as the purest air ; and, as far as sense can judge, equally penetrable. But if we look upon the effect, it is like that of water. For when that matter breaks out of the earth into one of those pits, it fills the same either totally or to some degree ; and if a man or fire be then let down in it, it extinguishes them in almost as little time as water would do. But for the better understanding of this phenomenon, I shall describe the 6th figure. In which let A B represent the pit of the mine ; and let part thereof, namely C B, be supposed to be filled with that matter. If now a lighted candle be let down into it below C, it will as suddenly be extinguished as if it were thrust into water. Also, if a grate filled with coals thoroughly kindled and burning never so brightly, be let down, as soon as ever it is below C, the fire will begin to grow pale, and shortly after, losing its light, be extinguished, no otherwise than if it were quenched in water. But if the grate be drawn up again presently, whilst the coals are still very hot, the fire will, by little and little, be kindled

again, and shine as before. There is, indeed, between this matter and water this considerable difference, that it neither wetteth, nor sticketh to such things as are put down into it, as water doth; which, by the moisture it leaveth, hindereth the kindling again of the matter once extinguished. In like manner, if a man be let down below C, he will presently fall into a great difficulty of breathing, and immediately after into a swoon, and die unless he be suddenly drawn up again. They, therefore, that go down into these pits, have this custom, that as soon as ever they feel themselves sick, they shake the rope by which they were let down, to signify they are not well, and to the end that they may speedily be pulled up again. For if a man be drawn out too late, void of sense and motion, they dig up a turf, and put his face and mouth into the fresh earth; by which means, unless he be quite dead, he comes to himself again, by little and little, and recovers life by breathing out, as it were, of that suffocating matter, which he had sucked in whilst he was in the pit; almost in the same manner as they that are drowned come to themselves again by vomiting up the water. But this doth not happen in all mines, but in some only; and in those not always, but often. In such pits as are subject to it, they use this remedy. They dig another pit, as DE, close by it, of equal depth, and joining them both together with one common channel, EB, they make a fire in the bottom E, which carries out at D the air contained in the pit DE; and this draws with it the air contained in the channel EB; which, in like manner, is fol-

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lowed by the noxious matter contained in C B; and, by this means, the pit is for that time made healthful. Out of this history, which I write only to such as have had experience of the truth of it, without any design to support my philosophy with stories of doubtful credit, may be collected the following possible cause of this phenomenon; namely, that there is a certain matter fluid and most transparent, and not much lighter than water, which, breaking out of the earth, fills the pit to C; and that in this matter, as in water, both fire and living creatures are extinguished.

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of magnetical
virtue.

15. About the nature of heavy bodies, the greatest difficulty ariseth from the contemplation of those things which make other heavy bodies ascend to them; such as jet, amber, and the loadstone. But that which troubles men most is the loadstone, which is also called *Lapis Herculeus*; a stone, though otherwise despicable, yet of so great power that it taketh up iron from the earth, and holds it suspended in the air, as Hercules did Antæus. Nevertheless, we wonder at it somewhat the less, because we see jet draw up straws, which are heavy bodies, though not so heavy as iron. But as for jet, it must first be excited by rubbing, that is to say, by motion to and fro; whereas the loadstone hath sufficient excitation from its own nature, that is to say, from some internal principle of motion peculiar to itself. Now, whatsoever is moved, is moved by some contiguous and moved body, as hath been formerly demonstrated. And from hence it follows evidently, that the first endeavour, which iron hath towards the loadstone, is caused by the

motion of that air which is contiguous to the iron : PART IV.
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 also, that this motion is generated by the motion of Of the cause
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 the next air, and so on successively, till by this
 succession we find that the motion of all the inter-
 mediate air taketh its beginning from some motion
 which is in the loadstone itself; which motion,
 because the loadstone seems to be at rest, is in-
 visible. It is therefore certain, that the attractive
 power of the loadstone is nothing else but some
 motion of the smallest particles thereof. Sup-
 posing, therefore, that those small bodies, of which
 the loadstone is in the bowels of the earth com-
 posed, have by nature such motion or endeavour
 as was above attributed to jet, namely, a reciprocal
 motion in a line too short to be seen, both those
 stones will have one and the same cause of attrac-
 tion. Now in what manner and in what order of
 working this cause produceth the effect of attrac-
 tion, is the thing to be enquired. And first we
 know, that when the string of a lute or viol is
 stricken, the vibration, that is, the reciprocal mo-
 tion of that string in the same strait line, causeth
 like vibration in another string which hath like
 tension. We know also, that the dregs or small
 sands, which sink to the bottom of a vessel, will
 be raised up from the bottom by any strong and
 reciprocal agitation of the water, stirred with the
 hand or with a staff. Why, therefore, should not
 reciprocal motion of the parts of the loadstone con-
 tribute as much towards the moving of iron? For,
 if in the loadstone there be supposed such reciprocal
 motion, or motion of the parts forwards and back-
 wards, it will follow that the like motion will be

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propagated by the air to the iron, and consequently that there will be in all the parts of the iron the same reciprocations or motions forwards and backwards. And from hence also it will follow, that the intermediate air between the stone and the iron will, by little and little, be thrust away; and the air being thrust away, the bodies of the loadstone and the iron will necessarily come together. The possible cause therefore why the loadstone and jet draw to them, the one iron, the other straws, may be this, that those attracting bodies have reciprocal motion either in a strait line, or in an elliptical line, when there is nothing in the nature of the attracted bodies which is repugnant to such a motion.

But why the loadstone, if with the help of cork it float at liberty upon the top of the water, should from any position whatsoever so place itself in the plane of the meridian, as that the same points, which at one time of its being at rest respect the poles of the earth, should at all other times respect the same poles, the cause may be this; that the reciprocal motion, which I supposed to be in the parts of the stone, is made in a line parallel to the axis of the earth, and has been in those parts ever since the stone was generated. Seeing therefore, the stone, whilst it remains in the mine, and is carried about together with the earth by its diurnal motion, doth by length of time get a habit of being moved in a line which is perpendicular to the line of its reciprocal motion, it will afterwards, though its axis be removed from the parallel situation it had with the axis of the earth, retain its endeavour

of returning to that situation again; and all endeavour being the beginning of motion, and nothing intervening that may hinder the same, the loadstone will therefore return to its former situation. For, any piece of iron that has for a long time rested in the plane of the meridian, whensoever it is forced from that situation and afterwards left to its own liberty again, will of itself return to lie in the meridian again; which return is caused by the endeavour it acquired from the diurnal motion of the earth in the parallel circles which are perpendicular to the meridians.

If iron be rubbed by the loadstone drawn from one pole to the other, two things will happen; one, that the iron will acquire the same direction with the loadstone, that is to say, that it will lie in the meridian, and have its axis and poles in the same position with those of the stone; the other, that the like poles of the stone and of the iron will avoid one another, and the unlike poles approach one another. And the cause of the former may be this, that iron being touched by motion which is not reciprocal, but drawn the same way from pole to pole, there will be imprinted in the iron also an endeavour from the same pole to the same pole. For seeing the loadstone differs from iron no otherwise than as ore from metal, there will be no repugnance at all in the iron to receive the same motion which is in the stone. From whence it follows, that seeing they are both affected alike by the diurnal motion of the earth, they will both equally return to their situation in the meridian, whensoever they are put from the same. Also, of

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of magnetical
virtue.

the latter this may be the cause, that as the loadstone in touching the iron doth by its action imprint in the iron an endeavour towards one of the poles, suppose towards the North Pole; so reciprocally, the iron by its action upon the loadstone doth imprint in it an endeavour towards the other pole, namely towards the South Pole. It happens therefore in these reciprocations or motions forwards and backwards of the particles of the stone and of the iron betwixt the north and the south, that whilst in one of them the motion is from north to south, and the return from south to north, in the other the motion will be from south to north, and the return from north to south; which motions being opposite to one another, and communicated to the air, the north pole of the iron, whilst the attraction is working, will be depressed towards the south pole of the loadstone; or contrarily, the north pole of the loadstone will be depressed towards the south pole of the iron; and the axis both of the loadstone and of the iron will be situate in the same strait line. The truth whereof is taught us by experience.

As for the propagation of this magnetical virtue, not only through the air, but through any other bodies how hard soever, it is not to be wondered at, seeing no motion can be so weak, but that it may be propagated infinitely through a space filled with body of any hardness whatsoever. For in a full medium, there can be no motion which doth not make the next part yield, and that the next, and so successively without end; so that there is no effect whatsoever, but to the production thereof

something is necessarily contributed by the several motions of all the several things that are in the world.

PART IV.
30.

And thus much concerning the nature of body in general ; with which I conclude this my first section of the Elements of Philosophy. In the first, second, and third parts, where the principles of ratiocination consist in our own understanding, that is to say, in the legitimate use of such words as we ourselves constitute, all the theorems, if I be not deceived, are rightly demonstrated. The fourth part depends upon hypotheses ; which unless we know them to be true, it is impossible for us to demonstrate that those causes, which I have there explicated, are the true causes of the things whose productions I have derived from them.

Conclusion.

Nevertheless, seeing I have assumed no hypothesis, which is not both possible and easy to be comprehended ; and seeing also that I have reasoned aright from those assumptions, I have withal sufficiently demonstrated that they may be the true causes ; which is the end of physical contemplation. If any other man from other hypotheses shall demonstrate the same or greater things, there will be greater praise and thanks due to him than I demand for myself, provided his hypotheses be such as are conceivable. For as for those that say anything may be moved or produced by *itself*, by *species*, by *its own power*, by *substantial forms*, by *incorporeal substances*, by *instinct*, by *antiperistasis*, by *antipathy*, *sympathy*, *occult quality*, and other empty words of schoolmen, their saying so is to no purpose.

PART IV.
30.

Conclusion.

And now I proceed to the phenomena of man's body; where I shall speak of the *optics*, and of the *dispositions*, *affections*, and *manners* of men, if it shall please God to give me life, and show their causes.

END OF VOL. I.

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