NICOLAUS COPERNICUS

OF TORUŃ

OŃ THE REVOLUTIONS OF THE HEAVENLY SPHERES

Translated by
CHARLES GLEN WALLIS

Diligent reader, in this work, which has just been created and published, you have the motions of the fixed stars and planets as these motions have been reconstituted on the basis of ancient as well as recent observations, and have moreover been embellished by new and marvellous hypotheses. You also have most convenient tables, from which you will be able to compute those motions with the utmost ease for any time whatever. Therefore buy, read, and enjoy [this work].

Let no one untrained in geometry enter here.

NUREMBERG
JOHANNES PETREIUS
1543

NICOLAUS COPERNICUS
ON THE REVOLUTIONS OF THE HEAVENLY SPHERES

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FOREWORD BY ANDREAS OSIANDER

To the Reader
Concerning the Hypotheses of this Work

There have already been widespread reports about the novel hypotheses of this work, which declares that the earth moves whereas the sun is at rest in the center of the universe. Hence certain scholars, I have no doubt, are deeply offended and believe that the liberal arts, which were established long ago on a sound basis, should not be thrown into confusion. But if these men are willing to examine the matter closely, they will find that the author of this work has done nothing blameworthy. For it is the duty of an astronomer to compose the history of the celestial motions through careful and expert study. Then he must conceive and devise the causes of these motions or hypotheses about them. Since he cannot in any way attain to the true causes, he will adopt whatever suppositions enable the motions to be computed correctly from the principles of geometry for the future as well as for the past. The present author has performed both these duties excellently. For these hypotheses need not be true nor even probable. On the contrary, if they provide a calculus consistent with the observations, that alone is enough. Perhaps there is someone who is so ignorant of geometry and optics that he regards the epicycle of Venus as probable, or thinks that it is the reason why Venus sometimes precedes and sometimes follows the sun by forty degrees and even more. Is there anyone who is not aware that from this assumption it necessarily follows that the diameter of the planet at perigee should appear more than four times, and the body of the planet more than sixteen times, as great as at apogee? Yet this variation is refuted by the experience of every age. In this science there are some other no less important absurdities, which need not be set forth at the moment. For this art, it is quite clear, is completely and absolutely ignorant of the causes of the apparent nonuniform motions. And if any causes are devised by the imagination, as indeed very many are, they are not put forward to convince anyone that they are true, but merely to provide a reliable basis for computation. However, since different hypotheses are sometimes offered for one and the same motion (for example, eccentricity and an epicycle for the sun's motion), the astronomer will take as his first choice that hypothesis which is the easiest to grasp. The philosopher will perhaps rather seek the semblance of the truth. But neither of them will understand or state anything certain, unless it has been divinely revealed to him.

Therefore alongside the ancient hypotheses, which are no more probable, let us permit these new hypotheses also to become known, especially since they are admirable as well as simple and bring with them a huge treasure of very skillful observations. So far as hypotheses are concerned, let no one expect anything certain from astronomy, which cannot furnish it, lest he accept as the truth ideas conceived for another purpose, and depart from this study a greater fool than when he entered it. Farewell.

LETTER OF NICHOLAS SCHÖNBERG

Nicholas Schönberg, Cardinal of Capua,
to Nicholas Copernicus, Greetings.

Some years ago word reached me concerning your proficiency, of which everybody constantly spoke. At that time I began to have a very high regard for you, and also to congratulate our contemporaries among whom you enjoyed such great prestige. For I had learned that you had not merely mastered the discoveries of the ancient astronomers uncommonly well but had also formulated a new cosmology. In it you maintain that the earth moves; that the sun occupies the lowest, and thus the central, place in the universe; that the eighth heaven remains perpetually motionless and fixed; and that, together with the elements included in its sphere, the moon, situated between the heavens of Mars and Venus, revolves around the sun in the period of a year. I have also learned that you have written an exposition of this whole system of astronomy, and have computed the planetary motions and set them down in tables, to the greatest admiration of all. Therefore with the utmost earnestness I entreat you, most learned sir, unless I inconvenience you, to communicate this discovery of yours to scholars, and at the earliest possible moment to send me your writings on the sphere of the universe together with the tables and whatever else you have that is relevant to this subject. Moreover, I have instructed Theodoric of Reden to have everything copied in your quarters at my expense and dispatched to me. If you gratify my desire in this matter, you will see that you are dealing with a man who is zealous for your reputation and eager to do justice to so fine a talent. Farewell.

Rome, 1 November 1536

DEDICATION
TO HIS HOLINESS, POPE PAUL III

PREFACE

I can readily imagine, Holy Father, that as soon as some people hear that in this volume, which I have written about the revolutions of the spheres of the universe, I ascribe certain motions to the terrestrial globe, they will shout that I must be immediately repudiated together with this belief. For I am not so enamored of my own opinions that I disregard what others may think of them. I am aware that a philosopher's ideas are not subject to the judgement of ordinary persons, because it is his endeavor to seek the truth in all things, to the extent permitted to human reason by God. Yet I hold that completely erroneous views should be shunned. Those who know that the consensus of many centuries has sanctioned the conception that the earth remains at rest in the middle of the heaven as its center would, I reflected, regard it as an insane pronouncement if I made the opposite assertion that the earth moves. Therefore I debated with myself for a long time whether to publish the volume which I wrote to prove the earth's motion or rather to follow the example of the Pythagoreans and certain others, who used to transmit philosophy's secrets only to kinsmen and friends, not in writing but by word of mouth, as is shown by Lysis' letter to Hipparchus. And they did so, it seems to me, not, as some suppose, because they were in some way jealous about their teachings, which would be spread around; on the contrary, they wanted the very beautiful thoughts attained by great men of deep devotion not to be ridiculed by those who are reluctant to exert themselves vigorously in any literary pursuit unless it is lucrative; or if they are stimulated to the non-acquisitive study of philosophy by the exhortation and example of others, yet because of their dullness of mind they play the same part among philosophers as drones among bees. When I weighed these considerations, the scorn which I had reason to fear on account of the novelty and unconventionality of my opinion almost induced me to abandon completely the work which I had undertaken.

But while I hesitated for a long time and even resisted, my friends drew me back. Foremost among them was the cardinal of Capua, Nicholas Schönberg, renowned in every field of learning. Next to him was a man who loves me dearly, Tiedemann Giese, bishop of Chełmno, a close student of sacred letters as well as of all good literature. For he repeatedly encouraged me and, sometimes adding reproaches, urgently requested me to publish this volume and finally permit it to appear after being buried among my papers and lying concealed not merely until the ninth year but by now the fourth period of nine years. The same conduct was recommended to me by not a few other very eminent scholars. They exhorted me no longer to refuse, on account of the fear which I felt, to make my work available for the general use of students of astronomy. The crazier my doctrine of the earth's motion now appeared to most people, the argument ran, so much the more admiration and thanks would it gain after they saw the publication of my writings dispel the fog of absurdity by most luminous proofs. Influenced therefore by these persuasive men and by this hope, in the end I allowed my friends to bring out an edition of the volume, as they had long besought me to do.

However, Your Holiness will perhaps not be greatly surprised that I have dared to publish my studies after devoting so much effort to working them out that I did not hesitate to put down my thoughts about the earth's motion in written form too. But you are rather waiting to hear from me how it occurred to me to venture to conceive any motion of the earth, against the traditional opinion of astronomers and almost against common sense. I have accordingly no desire to conceal from Your Holiness that I was impelled to consider a different system of deducing the motions of the universe's spheres for no other reason than the realization that astronomers do not agree among themselves in their investigations of this subject. For, in the first place, they are so uncertain about the motion of the sun and moon that they cannot establish and observe a constant length even for the tropical year. Secondly, in determining the motions not only of these bodies but also of the other five planets, they do not use the same principles, assumptions, and explanations of the apparent revolutions and motions. For while some employ only homocentrics, others utilize eccentrics and epicycles, and yet they do not quite reach their goal. For although those who put their faith in homocentrics showed that some nonuniform motions could be compounded in this way, nevertheless by this means they were unable to obtain any incontrovertible result in absolute agreement with the phenomena. On the other hand, those who devised the eccentrics seem thereby in large measure to have solved the problem of the apparent motions with appropriate calculations. But meanwhile they introduced a good many ideas which apparently contradict the first principles of uniform motion. Nor could they elicit or deduce from the eccentrics the principal consideration, that is, the structure of the universe and the true symmetry of its parts. On the contrary, their experience was just like some one taking from various places hands, feet, a head, and other pieces, very well depicted, it may be, but not for the representation of a single person; since these fragments would not belong to one another at all, a monster rather than a man would be put together from them. Hence in the process of demonstration or "method", as it is called, those who employed eccentrics are found either to have omitted something essential or to have admitted something extraneous and wholly irrelevant. This would not have happened to them, had they followed sound principles. For if the hypotheses assumed by them were not false, everything which follows from their hypotheses would be confirmed beyond any doubt. Even though what I am now saying may be obscure, it will nevertheless become clearer in the proper place.

For a long time, then, I reflected on this confusion in the astronomical traditions concerning the derivation of the motions of the universe's spheres. I began to be annoyed that the movements of the world machine, created for our sake by the best and most systematic Artisan of all, were not understood with greater certainty by the philosophers, who otherwise examined so precisely the most insignificant trifles of this world. For this reason I undertook the task of rereading the works of all the philosophers which I could obtain to learn whether anyone had ever proposed other motions of the universe's spheres than those expounded by the teachers of astronomy in the schools. And in fact first I found in Cicero that Hicetas supposed the earth to move. Later I also discovered in Plutarch that certain others were of this opinion. I have decided to set his words down here, so that they may be available to everybody:

Some think that the earth remains at rest. But Philolaus the Pythagorean believes that, like the sun and moon, it revolves around the fire in an oblique circle. Heraclides of Pontus and Ecphantus the Pythagorean make the earth move, not in a progressive motion, but like a wheel in a rotation from west to east about its own center.

Therefore, having obtained the opportunity from these sources, I too began to consider the mobility of the earth. And even though the idea seemed absurd, nevertheless I knew that others before me had been granted the freedom to imagine any circles whatever for the purpose of explaining the heavenly phenomena. Hence I thought that I too would be readily permitted to ascertain whether explanations sounder than those of my predecessors could be found for the revolution of the celestial spheres on the assumption of some motion of the earth.

Having thus assumed the motions which I ascribe to the earth later on in the volume, by long and intense study I finally found that if the motions of the other planets are correlated with the orbiting of the earth, and are computed for the revolution of each planet, not only do their phenomena follow therefrom but also the order and size of all the planets and spheres, and heaven itself is so linked together that in no portion of it can anything be shifted without disrupting the remaining parts and the universe as a whole. Accordingly in the arrangement of the volume too I have adopted the following order. In the first book I set forth the entire distribution of the spheres together with the motions which I attribute to the earth, so that this book contains, as it were, the general structure of the universe. Then in the remaining books I correlate the motions of the other planets and of all the spheres with the movement of the earth so that I may thereby determine to what extent the motions and appearances of the other planets and spheres can be saved if they are correlated with the earth's motions. I have no doubt that acute and learned astronomers will agree with me if, as this discipline especially requires, they are willing to examine and consider, not superficially but thoroughly, what I adduce in this volume in proof of these matters. However, in order that the educated and uneducated alike may see that I do not run away from the judgement of anybody at all, I have preferred dedicating my studies to Your Holiness rather than to anyone else. For even in this very remote corner of the earth where I live you are considered the highest authority by virtue of the loftiness of your office and your love for all literature and astronomy too. Hence by your prestige and judgement you can easily suppress calumnious attacks although, as the proverb has it, there is no remedy for a backbite.

Perhaps there will be babblers who claim to be judges of astronomy although completely ignorant of the subject and, badly distorting some passage of Scripture to their purpose, will dare to find fault with my undertaking and censure it. I disregard them even to the extent of despising their criticism as unfounded. For it is not unknown that Lactantius, otherwise an illustrious writer but hardly an astronomer, speaks quite childishly about the earth's shape, when he mocks those who declared that the earth has the form of a globe. Hence scholars need not be surprised if any such persons will likewise ridicule me. Astronomy is written for astronomers. To them my work too will seem, unless I am mistaken, to make some contribution also to the Church, at the head of which Your Holiness now stands. For not so long ago under Leo X the Lateran Council considered the problem of reforming the ecclesiastical calendar. The issue remained undecided then only because the lengths of the year and month and the motions of the sun and moon were regarded as not yet adequately measured. From that time on, at the suggestion of that most distinguished man, Paul, bishop of Fossombrone, who was then in charge of this matter, I have directed my attention to a more precise study of these topics. But what I have accomplished in this regard, I leave to the judgement of Your Holiness in particular and of all other learned astronomers. And lest I appear to Your Holiness to promise more about the usefulness of this volume than I can fulfill, I now turn to the work itself.

INTRODUCTION

"Free in mind must be he who desires to have understanding.

Alcinous

TO THE ILLUSTRIOUS JOHN SCHÖNER, as to his own revered father, G. Joachim Rheticus sends his greetings.

On May 14th I wrote you a letter from Posen in which I informed you that I had undertaken a journey to Prussia,[1] and I promised to declare, as soon as I could, whether the actuality answered to report and to my own expectation. However, I have been able to devote scarcely[2] ten weeks to mastering the astronomical work of the learned man to whom I have repaired; for I had a slight illness and, on the honorable invitation of the Most Reverend Tiedemann Giese, bishop of Kulm, I went with my teacher to Löbau and there rested from my studies for several weeks.[3] Nevertheless, to fulfill my promises at last and gratify your desires, I shall set forth, as briefly and clearly as I can, the opinions of my teacher on the topics which I have studied.

First of all I wish you to be convinced, most learned Schöner, that this man whose work I am now treating is in every field of knowledge and in mastery of astronomy not inferior to Regiomontanus. I rather compare him with Ptolemy, not because I consider Regiomontanus inferior to Ptolemy, but because my teacher shares with Ptolemy the good fortune of completing, with the aid of divine kindness, the reconstruction of astronomy which he began, while Regiomontanus—alas, cruel fate—departed this life before he had time to erect his columns.

My teacher has written a work of six books in which, in imitation of Ptolemy, he has embraced the whole of astronomy,

[1] The basic study for the biography of Rheticus will be found in *Vierteljahrsschrift für Geschichte und Landeskunde Vorarlbergs*, neue Folge, II(1918), 5-46. For subsequent work consult *Forschungen zur Geschichte Vorarlbergs und Liechtensteins*, I(1920), 128-30; *Schriften des Vereines für Geschichte des Bodensees*, LV(1927), 122-37; and Martin Bilgeri, *Das Vorarlberger Schrifttum* (Vienna, 1936), pp. 64-70.

[2] Reading *vix* (Th 447.8) instead of *viri* (PII, 295.7).

[3] In the light of this remark, we must regard as incorrect Prowe's statement (PI[2], 395) that the *Narratio prima* was written at Löbau. Prowe himself declares that Rheticus's trip to Löbau kept him from his studies (PI[2], 428).

stating and proving individual propositions mathematically and by the geometrical method.

The first book contains the general description of the universe and the foundations by which he undertakes to save the appearances and the observations of all ages. He adds as much of the doctrine of sines and plane and spherical triangles as he deemed necessary to the work.

The second book contains the doctrine of the first motion[4] and the statements about the fixed stars which he thought he should make in that place.

The third book treats of the motion of the sun. And because experience has taught him that the length of the year measured by the equinoxes depends, in part, on the motion of the fixed stars, he undertakes in the first portion of this book to examine by right reason and with truly divine ingenuity the motions of the fixed stars and the mutations of the solstitial and equinoctial points.

The fourth book treats of the motion of the moon and eclipses; the fifth, the motions of the remaining planets; the sixth, latitudes.

I have mastered the first three books, grasped the general idea of the fourth, and begun to conceive the hypotheses of the rest. So far as the first two books are concerned, I have thought it unnecessary to write anything to you, partly because I have a special plan,[5] partly because my teacher's doctrine of the first motion does not differ from the common and received opinion,[6] save that he has so constructed anew the tables of declinations, right ascensions, ascensional differences, and the other tables belonging to this branch of the science that they can be brought by the method of proportional parts into agree-

[4] The apparent daily rotation of the heavens; see p. 41, above.

[5] Rheticus doubtless refers to his plan for writing a "Second Account." For an explanation why this "Second Account" was never written see p. 10, above.

[6] But in the common and received opinion the first motion was real; in Copernicus's system, apparent. Rheticus ignores the distinction, for it involves the motion of the earth. Throughout the first third of this *Account* he withholds all reference to Copernicus's principal alteration of astronomical theory, the shift from a stationary to a moving earth, and from geocentrism to heliocentrism (cf. below, pp. 135-36, n. 115).

ment with the observations of all ages. Therefore I shall set forth clearly to you, God willing, the subjects treated in the third book together with the hypotheses of all the remaining motions, so far as at present with my meager mental attainments I have been able to understand them.

The Motions of the Fixed Stars

My teacher made observations with the utmost care at Bologna, where he was not so much the pupil as the assistant and witness of observations of the learned Dominicus Maria;[7] at Rome, where, about the year 1500, being twenty-seven years of age more or less, he lectured on mathematics before a large audience of students and a throng of great men and experts in this branch of knowledge; then here in Frauenburg,[8] when he had leisure for his studies. From his observations of the fixed stars he selected the one which he made of Spica Virginis in 1525. He determined its distance from the autumnal point[9] as about 17°21′, and its declination as not less than 8°40′ south of the equator. Then comparing all the observations of previous writers with his own, he found that a revolution of the anomaly or of the circle of inequality had been completed and that the second revolution extends from Timocharis to our own time. Thereby he geometrically determined the mean motion of the fixed stars and the equations of their unequal motion.

Timocharis's observation of Spica in the 36th year[10] of the first Callippic cycle, when compared with his observation in the 48th year of the same cycle, shows us that the stars moved 1° in 72 years in that era.[11] From Hipparchus to

[7] Concerning whom Lino Sighinolfi has assembled some material, chiefly biographical, in his article "Domenico Maria Novara e Nicolò Copernico" (*Studi e memorie per la storia dell' università di Bologna*, V [1920], 211-35).

[8] Cf. Th 193, note to line 9.

[9] The first point of Libra (cf. Th 161.24-25).

[10] 295/4 B.C. A Callippic cycle contained 76 years (HII, 25.16-17; Th 159.11). See F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie* (Leipzig, 1906-14), II, 409-19; and J. K. Fotheringham in *Monthly Notices of the Royal Astronomical Society*, LXXXIV (1924), 387-92.

[11] HII, 28.11-30.17.

Menelaus they regularly completed 1° in 100 years.[12] My teacher therefore concluded that Timocharis's observations fell in the last quadrant of the circle of inequality,[13] in which the motion appears mean-diminishing, and that between Hipparchus and Menelaus the motion of inequality was slowest. A comparison of Menelaus's observations with Ptolemy's shows that the stars then moved 1° in 86[14] years. Therefore Ptolemy's

[12] Ptolemy accepts this estimate as the approximate value for the entire period from Hipparchus to himself (HII, 23.11-16); and he regards the rate of precession as constant (HII, 34.11-17).

[13] Copernicus held that the rate of precession varied. To represent the variation he constructs a "circle of inequality," (Fig. 26) in which *a* is the point of slowest

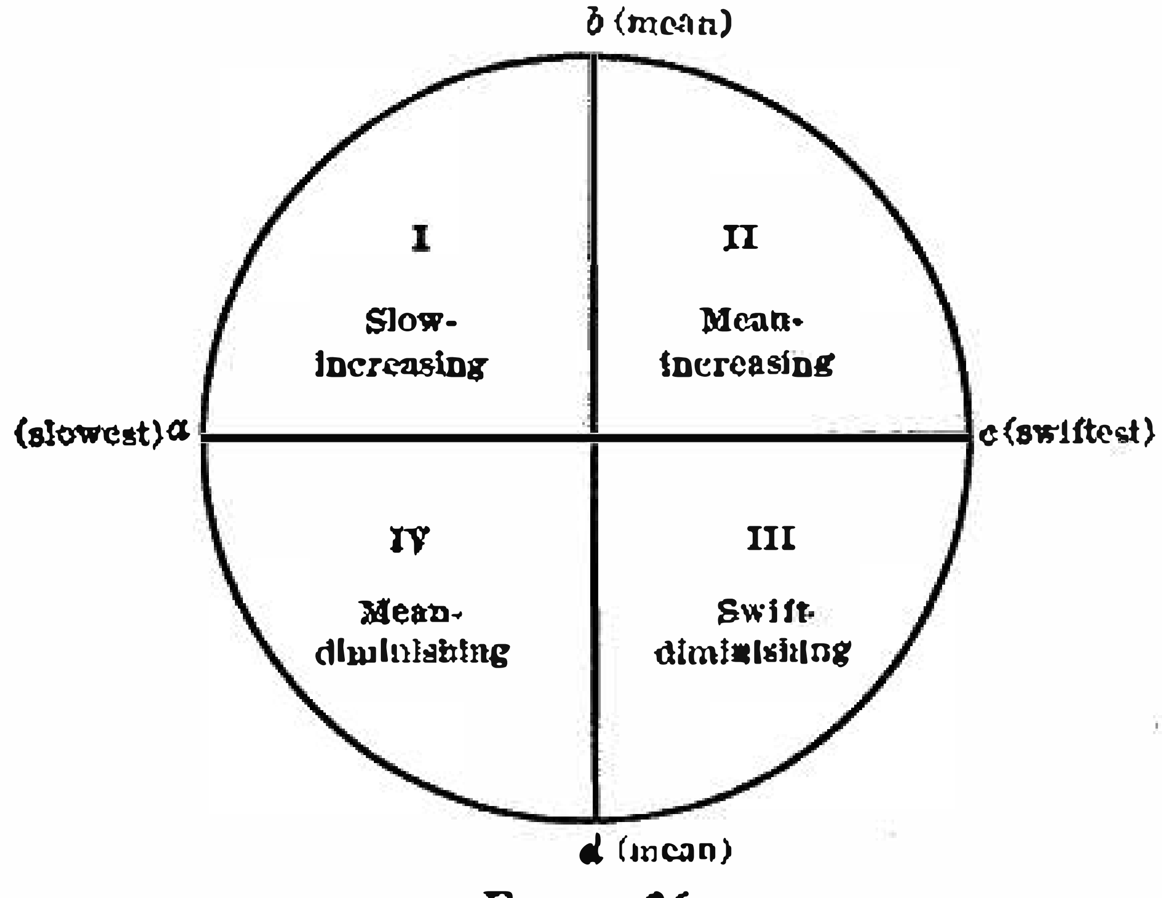

FIGURE 26

motion; *c*, the point of swiftest motion; *b* and *d*, the points of mean motion. The first quadrant *ab* is the quadrant of slow-increasing motion; the second quadrant *bc*, of mean-increasing motion; the third quadrant *cd*, swift-diminishing; the fourth quadrant *da*, mean-diminishing. See Th 169.25-170.4, and p. 100, above.

[14] This should be 96, as an examination of Menzzer's chart (p. 21 of his notes) shows. Menelaus determined the longitudinal distance of Spica from the summer solstice as 86° 15′, and of β Scorpii from the autumnal equinox as 35° 55′ (HII, 30.18-31.16, 33.3-24; Th 159.24-29). For Ptolemy, 40 years later, the corresponding values were 86° 40′ and 36° 20′ (HII, 103.16, 109.18; Th 159.29-160.4; *LXXXVI s.* in Th 160.1 is wrong, as can be seen in the *Letter against Werner* [cf. p. 104, above] and in the three editions of the *Syntaxis* available to Copernicus: 1515, p. 83r; 1528, p. 78r; 1538, p. 187; cf. Th 161.28-31). For both stars the motion is 25′ in 40 years, or 1° in 96 years. It is more likely that an X has fallen out of LXXXXVI than that Rheticus made an error in his computations.

observations were made when the motion of anomaly was in the first quadrant, and the stars then moved with a slow-increasing[15] or -augmenting motion. Further, from Ptolemy to Albategnius, 66 years correspond to 1°;[16] a comparison of our observations with those of Albategnius shows that the stars in their unequal motion again completed 1° in 70 years;[17] and a comparison of the observation which I mentioned above with the others which my teacher made in Italy shows that the fixed stars in their unequal motion are once more passing through 1° in 100 years. Therefore it is clearer than sunlight that between Ptolemy and Albategnius the motion of inequality passed the first boundary of mean motion and the entire quadrant of mean-increasing motion, and about the time of Albategnius was in the region of swiftest motion. Between Albategnius and ourselves the third quadrant of unequal motion was completed (during this time the stars moved with a swift-diminishing motion) and the other boundary of mean motion was passed. In our era the anomaly has again entered the fourth quadrant of mean-diminishing motion, and hence the unequal motion is once more approaching the point of slowest motion.

To reduce these calculations to a definite system in which they would agree with all the observations, my teacher computed that the unequal motion is completed in 1,717 Egyptian years,[18] the maximum correction is about 70′, the mean motion

[15] *addito* (Th 449.3) has dropped out of Prowe's text (PII, 298.14).

[16] Nallino, *Al-Battānī*, I, 124.32-33, 128.2-4. A translation of Albategnius's work into Latin was included in a book printed at Nuremberg in 1537 and was presumably available to Copernicus and Rheticus, as may be inferred from the latter's remark about Albategnius on p. 124, below. The volume opened with a treatise entitled in some copies *Rudimenta astronomica Alfragani*, and in others, *Brevis ac perutilis compilatio Alfragani*. I have been unable to consult this translation, but the relevant passage was excerpted by Menzzer (n. 81). Rheticus ignores the distinction made by Copernicus (Th 162.7-11) that the rate of precession was 1° in 66 years from Menelaus to Albategnius, and 1° in 65 years from Ptolemy to Albategnius. The distinction is based on the observations cited in Th 159.24–160.10.

[17] Copernicus states this rate as 1° in 71 years (Th 162.12-14); calculation from his data gives the fractional result 1° in 70¼ years.

[18] The changing rate of precession requires 1717 years to pass through the four quadrants of the circle of inequality.

of the stars in an Egyptian year is about 50″,[19] and the complete revolution of the mean motion will take 25,816 Egyptian years.[20]

General Consideration of the Tropical Year

This theory of the motions of the fixed stars is supported by the length of the year reckoned from the equinoctial points. It is quite clear why from Hipparchus[21] to Ptolemy there was a deficiency of 19/20 of a day;[22] from Ptolemy to Albategnius, of about 7 days;[23] and from Albategnius to the observations which my teacher made in 1515, of about 5 days.[24] These discrepancies are not at all caused by a defect in the instruments, as was heretofore believed, but occur according to a definite and completely self-consistent law. Hence equality of motion must be measured, not by the equinoxes, but by the fixed stars, as observations of the motions not merely of the sun and moon but of the other planets as well testify with a remarkable unanimity of all ages.

[19] The mean rate of precession is about 50″ a year, or 1° in about 72 years; and the greatest difference between the mean equinox and the true equinox is about 70′ (Th 179.4-7).

(*a*) The slowest rate of precession is 1° in 100 years, or 36″ a year. The difference between the slowest rate and the mean rate is 14″ a year, and in 300 years (the three centuries before Menelaus) the maximum difference of 70′ between mean and true equinox is attained.

(*b*) The swiftest rate is 1° in 66 years, or slightly more than 54½″ a year. The difference between the swiftest rate and the mean rate is about 4½″ a year, and in 743 years (between Ptolemy and Albategnius) about 4/5 of the maximum difference of 70′ is attained; the remaining 1/5 accumulates because the rate of precession during the 620 years between Albategnius and Copernicus's observations in Italy is slightly more rapid than the mean rate.

Copernicus's estimate of the mean precession, about 50″ a year, agrees quite closely with the determination accepted at present. His belief in the cyclic variation of the rate of precession is of course erroneous.

[20] The complete passage of the stars around the celestial sphere requires 25,816 years.

[21] Michael Mästlin, editor of the fourth (1596) and fifth (1621) editions of the *Narratio prima,* correctly substituted "Hipparchus" for the older reading "Timocharis." Unfortunately the incorrect reading was revived in Th 449.24-25.

[22] HI, 203.22–204.18. The tropical year (t) is less than 365¼ days; the "deficiency" from year x to year y is (y — x) (365¼ days — t).

[23] Nallino, *Al-Battānī,* I, 42.10-14.

[24] Th 193.20-21.

It is the accepted opinion that because from Timocharis to Ptolemy the stars moved very slowly the year was less than 365¼ days by only 1/300 of a day;[25] and from Ptolemy to Albategnius, because the stars moved rapidly, by 1/106 of a day.[26] If the observations of our age are compared with those of Albategnius, it is clear that the difference is 1/128 of a day.[27] Therefore a greater length of the tropical year apparently corresponds to a slow motion of the stars, a lesser length to a swift motion, and the lengthening of the year to a diminishing velocity; so that if the length of the tropical year in our era is accurately determined, it will again be almost the same as Ptolemy's value. Hence we must say that the equinoctial points, like the nodes of the moon,[28] move in precedence, and not that the stars move in consequence.[29]

We must accordingly imagine a mean equinox moving in precedence from the first star of Aries in the sphere of the fixed stars, and displacing them by its uniform motion. The true equinox deviates to either side of this mean equinox in an unequal and regular motion; but the radius of the distance between the true equinox and the mean equinox does not much exceed 70′. Thus a definite law governing the length of the tropical year has existed in all ages, and it can be ascertained today. It agrees very closely, moreover, and almost to the minute with the observations which all scholars have made of the fixed stars.

To offer you some taste of this matter, most learned Schöner, I have computed for you the true precession of the equinoxes at certain times of observation.

[25] HI, 205.9-14, 207.24–208.1.

[26] Albategnius's estimate of the length of the tropical year was $365^d 14^m 26^s$ (Nallino, *op. cit.*, I, 42.17). The difference between this value and 365¼ days ($= 365^d 15^m$) is 34^s or 34/3600 of a day. Albategnius expresses this difference as $\frac{3\frac{2}{5}}{360}$ (*op. cit.*, I, 127.19-20). It is much closer to 1/106 than to 1/105 of a day, and Copernicus writes the more accurate fraction (Th 193.2-3). Hence I have followed Mästlin in changing our text from 1/105 to 1/106.

[27] Th 193.20-21, 194.4-5.

[28] See p. 73, above.

[29] Copernicus interprets precession as a motion of the equator.

Egyptian Year		*True Precession*		*Period*
		°	′	
B.C.	293	2	24	Timocharis
	127	4	3	Hipparchus
C.E.	138	6	40	Ptolemy
	880	18	10	Albategnius
	1076	19	37[30]	Arzachel
	1525	27	21	present

Ptolemy's precession subtracted from the positions of the stars as given by Ptolemy leaves a remainder equal to the distance of the stars from the first star of Aries; then the addition of Albategnius's precession gives the true position of the observed star. A similar procedure is followed in all the other cases. The results thus obtained coincide to the utmost degree of exactness with the observations of all scholars, even where the minutes are noted, or are derived from recorded declinations or from the motion of the moon reduced to greater precision, as a comparison of our observations with those of the ancients shows us. For when the minutes are neglected, as you see, at least a part of a degree is cut off,[31] ½° or ⅓° or ¼°, etc. However, these results do not agree with the motions of the planetary apsides, and therefore an independent motion had to be assigned to them, as will be clear from solar theory.[32]

Realizing that equality of motion must be measured by the fixed stars, my teacher carefully investigated the sidereal year. He finds that it is 365 days, 15 minutes, and about 24 seconds[33] and that it has always been of this length from the time of

[30] Prowe states (PII, 300 n) that the first edition read incorrectly 12° 37′, and that the change to 19° 37′ was made by Mästlin. But the Basel edition of 1566 gave 19° 37′ (p. 198r); and that number is suspect, for it would make the rate of precession (*a*) between Albategnius and Arzachel too slow (1° 27′ in 196 years, or 1° in 135 years); and (*b*) between Arzachel and Copernicus too fast (7° 44′ in 449 years, or 1° in 58 years). To be consistent with the theory and the rest of the table, the true precession for Arzachel must be about 20° 57′.

[31] Reading *recidant* (Th 450.28) instead of *recitant* (PII, 301.9).

[32] See pp. 119-21, below.

[33] The minute of the text is a *minutum diei* = 1/60 of a day = 24^m; in like manner, the second = 1/60 of a *minutum diei* = 24^s. The length of the sidereal year, then, is given here as about $365^d6^h9^m36^s$. In *De rev.* it is given as $365^d6^h9^m40^s$; cf. p. 67, above.

the earliest observations. For the fact that the Babylonians, according to Albategnius, assign 3 seconds more,[34] and Thābit 1 second less,[35] can be safely ascribed to either the instruments and observations, which, as you know, cannot have been entirely accurate, or to the inequality in the motion of the sun, or even to the circumstance that the ancients, having no sure theory of eclipses, neglected to take account of the solar parallax in their observations. In any case, this discrepancy over the entire period from the Babylonians to ourselves cannot be compared with the discrepancy of 22 seconds between Ptolemy and Albategnius.[36] That there necessarily was a deficiency of $^{19}/_{20}$ of a day from Hipparchus to Ptolemy, and from Ptolemy to Albategnius of about 7 days, I have deduced, not without the greatest pleasure, most learned Schöner, from the foregoing theory of the motions of the stars and from my teacher's treatment of the motion of the sun, as you will see a little further on.[37]

The Change in the Obliquity of the Ecliptic

My teacher found that the cycle of maximum obliquity stands in the following relation: while the unequal motion of the fixed stars is once completed, half of the change in the obliquity occurs. He therefore concluded that the entire period of the change in the obliquity is 3,434 Egyptian years.[38]

[34] The Latin translation of Albategnius gave $365\tfrac{1}{4}^d + \tfrac{1}{131}^d = 365^d15^m27\tfrac{1}{2}^s$ (see above, p. 67, n. 25). However, the Arabic MS on which Nallino based his text reads (I, 40.28-29) $365\tfrac{1}{4}^d + \tfrac{1}{120}^d = 365^d15^m30^s$.

[35] Rheticus and Copernicus (Th 194.8-12) probably drew this information about Thābit from the *Epitome*, Bk. III, Prop. 2 (see above, p. 65, n. 19), which gave Thābit's value as $365^d6^h9^m12^s$ ($= 365^d15^m23^s$). Proof that Rheticus used the *Epitome* is afforded by two references to it (p. 124, below) and by a quotation from it (pp. 133-34, below). For Thābit see George Sarton, *Introduction to the History of Science* (Baltimore, 1927–), I, 599-600.

[36] Rheticus is referring to the difference in their determinations of the length of the tropical year: Ptolemy $365^d14^m48^s$ (HI, 208.11-12)

Albategnius $\underline{365^d14^m26^s}$ (See above, p. 115, n. 26)

22^s

[37] Pages 128-30, below.

[38] It was stated above (p. 113) that the period of the unequal motion of the fixed stars is 1,717 Egyptian years.

In the time of Timocharis, Aristarchus, and Ptolemy the change in the obliquity was very slow, so that they believed the maximum arc of declination to be invariable, always having the value of $^{11}/_{83}$ of a great circle.[39] After them, Albategnius announced the obliquity for his own era as about 23°35';[40] Arzachel, about 190 years after him, 23°34'; and Prophatius Judaeus, 230 years later, 23°32'. In our own era it appears not greater than 23°28½'.[41] Accordingly it is clear that in the 400 years before Ptolemy the change in the obliquity was very slow. But from Ptolemy to Albategnius, a period of about 750 years, the obliquity decreased by 17', and from Albategnius to ourselves, a period of 650 years, by only 7'. Hence it follows that the variation of the obliquity, like the deviation of the planets from the ecliptic,[42] is governed by a motion in libration or motion along a straight line. It is a property of such motion that in the middle the motion is quickest, and slowest at the ends. Then about the time of Albategnius the pole of the equator or of the ecliptic was approximately in the middle of this motion in libration, while at present it is near the second limit of slowest motion, where the poles approach each other most closely. But I stated above[43] that the motions of the fixed stars and the variation in the length of the tropical year are saved by the motion of the equator. Now the poles of the equator

[39] $^{11}/_{83} \times 360° = 47° 42' 40''$, which makes the obliquity 23° 51' 20" (HI, 68.4-6, 81.50).

[40] Nallino, *Al-Battānī*, I, 12.20-22; Menzzer, n. 87.

[41] Th 162.24-25, 171.31-172.4; cf. above, p. 64, n. 15. The foregoing statement about the history of the determinations of the obliquity is virtually identical with the scholion in Reinhold's 1542 edition of Peurbach's *Theoricae novae planetarum*, fol. e8r-v; cf. Boncompagni, *Bulletino di bibliografia e di storia delle scienze*, XX(1887), 594-95. Since the statement in our text is earlier than Reinhold's, but Reinhold's contains additional items, apparently they both drew from some common source. For Arzachel and Prophatius Judaeus see Sarton, *Introduction*, I, 758-59; II, 850-53. Copernicus believed that Prophatius obtained his value of 23° 32' by a direct determination; but it was rather a calculation from Arzachel's tables, according to Duhem (*Le Système du monde*, III, 311). J. Millàs i Vallicrosa has published Prophatius's translation of Arzachel's *Saphea* in *Don Profeit Tibbon, Tractat de l'assafea d'Azarquiel* (Barcelona, 1933).

[42] Cf. pp. 80-81, 84-85, above and pp. 180, 182-85, below.

[43] See above, p. 115, n. 29.

are the prolongations of the earth's axis, and it is from them that the altitude of the pole is measured. Let me in passing call your attention, most learned Schöner, to the sort of hypotheses or theories of motion that the observations require; but you will hear clearer evidence.

Furthermore, my teacher assumes that the minimum obliquity will be 23°28′, and the difference between the minimum and the maximum, 24′. On this basis he geometrically constructed a table of[44] proportional minutes, from which the maximum obliquity of the ecliptic may be derived for all ages. Thus the proportional minutes were, in the time of Ptolemy 58, Albategnius 18, Arzachel 15, and in our own time 1.[45] If, using these figures, we take proportional parts of the 24′ difference between minimum and maximum, we shall have a sure rule for the change in the obliquity.[46]

The Eccentricity of the Sun and the Motion of the Solar Apogee

Since every difficulty in the motion of the sun is connected with the variable and unstable length of the year, I must first speak of the change in the apogee and eccentricity, in order that all the causes of the inequality of the year may be enumerated. However, by the assumption of theories suitable to the purpose, my teacher shows that these causes are regular and certain.

When Ptolemy declared that the apogee of the sun was fixed,[47] he preferred accepting the common opinion to trusting his own observations, which differed perhaps but little from the common opinion. But it can be definitely established from

[44] In his 1621 edition (p. 99), Mästlin inserted "sixty."

[45] Thus in the case of Arzachel, $^{15}/_{60} \times 24' = 6' + 23° 28' = 23° 34'$. For Albategnius, the editions of our text put the number of proportional minutes at 24; I have emended this obviously incorrect number to 18.

[46] For modern astronomy the change in the obliquity is a progressive diminution. The evidence available to Copernicus warranted only the same conclusion (Th 76.27-28). But he believed that after the obliquity had decreased to 23° 28′, it would increase to 23° 52′, completing a cycle which would then be repeated.

[47] HI, 232.18-233.16; cf. n. 13, pp. 62-63, above.

his own account that about the time of Hipparchus, that is, 200 years before his own time, the eccentricity was 417[48] of the units of which 10,000 constitute the radius of the eccentric, and in his own time 414.[49] In the time of Arzachel (in whom Regiomontanus had great faith) the eccentricity was about 346, according to the maximum inequality.[50] But in our own time it is 323, since my teacher states that he finds the maximum inequality not greater than 1° 50½′.[51]

Furthermore, carefully investigating the motions of the apsides of the sun and of the other planets, he learned, as you see from what has been said above,[52] that the apsides have independent motions in the sphere of the fixed stars. We are no more justified in attributing the apparent motions of the fixed stars and apsides, and the change in the obliquity to a single motion and a single cause than is one of your experts, who speak of the motions of the planets as self-moving, in attempting to produce the motions and appearances of each of the planets by one and the same device; or than anyone undertaking to defend the view that the foot, hand, and tongue exercise all their functions by means of the same muscle and by the same motive force. Therefore my teacher assigned two motions to the solar apogee, one mean and the other unequal, with which it moves in the eighth sphere. Moreover, since the true equinox moves with a regular unequal motion in the reverse

[48] HI, 233.5-8; Hipparchus determined the eccentricity as approximately 1/24 of the radius of the eccentric: 1/24 × 10,000 = 416⅔.

[49] HI, 236.15-18. Ptolemy's value for the eccentricity is slightly smaller than Hipparchus's; but since he believed the eccentricity to be constant (see above, p. 61, n. 9), he ignored the small difference between the two values, which he denotes as approximate (ἔγγιστα) in any case. Copernicus held that the eccentricity varies, and hence utilized the difference. Ptolemy's value would be more accurately expressed as 415 than as 414 (Th 209. n. to line 12).

[50] For the method of computing the eccentricity from the maximum inequality, see above, p. 61, n. 11. The information that Arzachel had put the maximum inequality at 1° 59′ 10″ (cf. Th 212.15-16) was obtained by Copernicus and Rheticus from the *Epitome*, Book III, Prop. 13. By the Table of Chords (Th 44.18-19), this inequality would correspond to an eccentricity of 346 (cf. Th 210.1-6).

[51] Th 211.16-19; 212.16; 224.8, 37.

[52] Page 116.

order of the signs, the apogees of the sun and of the other planets, like the fixed stars,[53] are displaced eastward. Consequently, to harmonize the observations of all ages in a consistent law, my teacher was compelled to distinguish these three motions.

To understand this analysis, assume a maximum eccentricity of 417 units, and a minimum of 321. Let the difference, 96, be the diameter of a small circle, on whose circumference the center of the eccentric moves from east to west. The distance from the center of the universe, then, to the center of the small circle will be 369 units. You will recall that 10,000 of these units constitute the radius of the eccentric. This is the device which my teacher derived from the three above-mentioned eccentricities, in a manner closely resembling the surely divine discovery by which the uniform motions of the moon are determined from three lunar eclipses.[54]

My teacher further established that the velocity with which the center of the eccentric revolves is the same as that with which each value of the changing obliquity recurs. This discovery is indeed worthy of the highest admiration, since it is achieved with such great and remarkable agreement.

The eccentricity was greatest about 60 B.C., when the declination of the sun was also at its maximum. The eccentricity has decreased, moreover, in accordance with this single law, similar to no other. This and other[55] like sports of nature often bring me great solace in the fluctuating vicissitudes of my fortunes, and gently soothe my troubled mind.

The Kingdoms of the World Change with the Motion of the Eccentric

I shall add a prediction. We see that all kingdoms have had their beginnings when the center of the eccentric was at some special point on the small circle. Thus, when the eccentricity of the sun was at its maximum, the Roman government be-

[53] See p. 115, above.
[54] HI, 265.16-19, 268.3-12; Th 236.15-17, 28-32; 246.3-8.
[55] Reading *alii* (Th 453.7) instead of *alibi* (PII, 305.1).

came a monarchy; as the eccentricity decreased, Rome too declined, as though aging, and then fell. When the eccentricity reached the boundary and quadrant of mean value, the Mohammedan faith was established; another great empire came into being and increased very rapidly, like the change in the eccentricity. A hundred years hence, when the eccentricity will be at its minimum, this empire too will complete its period. In our time it is at its pinnacle from which equally swiftly, God willing, it will fall with a mighty crash. We look forward to the coming of our Lord Jesus Christ when the center of the eccentric reaches the other boundary of mean value, for it was in that position at the creation of the world. This calculation does not differ much from the saying of Elijah, who prophesied under divine inspiration that the world would endure only 6,000 years,[56] during which time nearly two revolutions are completed. Thus it appears that this small circle is in very truth the Wheel of Fortune, by whose turning the kingdoms of the world have their beginnings and vicissitudes. For in this manner are the most significant changes in the entire history of the world revealed, as though inscribed upon this circle. Moreover, I shall soon, God willing, hear from your own lips how it may be inferred from important conjunctions and other learned prognostications, of what nature these empires were destined to be, whether governed by just or oppressive laws.[57]

[56] From Rheticus's language it appears that he attributed the dire prophecy to the prophet Elijah. But the Old Testament contains no such prediction by Elijah; however, the late Prof. Ralph Marcus kindly called my attention to the following passage in the *Babylonian Talmud:* "The Tanna debe Eliyyahu teaches: The world is to exist six thousand years" (*Babylonian Talmud,* English translation, ed. Isidore Epstein [London, 1935–], *Sanhedrin,* Vol. II [= *Nezikin,* Vol. VI], p. 657).

[57] Rheticus again displays his devotion to astrological superstition in the Preface to Werner's *De triangulis sphaericis.* He there declares: "The changes in empires depend upon celestial phenomena. Lands formerly distinguished for their culture, fertile soil, and possessions now lie barren and desolate, inhabited by barbarians, oppressed by tyranny . . . The fiercest nations become civilized, unproductive land is brought under cultivation, from heaven are sent down new forms of earth, culture, and physical type of man. And we see that at intervals of about three hundred and fifty years there always occurs some significant change in the sub-

Now while the center of the eccentric descends toward the center of the universe, the center of the small circle, it is clear, moves in the order of the signs about 25″ each Egyptian year. And starting from the point of its greatest distance from the center of the universe, the center of the eccentric moves in precedence. Hence the inequality arising from the motion of the anomaly for any specified time is subtracted from the mean motion, until a semicircle is completed; but in the other semicircle it is added, in order to obtain the true[58] motion of the apogee. Now the greatest difference between the true and mean apogee is deduced, in the proper geometrical manner, from the above-mentioned data as 7°24′;[59] the other differences are determined, in the customary way, from the position of the center of the eccentric on the small circle. The unequal motion is known, since three positions are given. With regard to the mean motion there is some doubt, since we do not have for these three positions the true place of the solar apogee on the ecliptic. The doubt arises from the disagreement between

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lunar world, corresponding to some important alteration in the motion of the sphere of the fixed stars" (*Abh. zur Gesch. d. math. Wiss.*, XXIV, 1, fol. a2v). Later in the same Preface he asserts: "As far as the stars are concerned, I have no doubt that for the Turkish empire there is impending disaster, momentous, sudden, and unforeseen, since the influence of the fiery Triangle is approaching, and the strength of the watery Triangle is declining. Moreover, the anomaly of the sphere of the fixed stars is nearing its third boundary. Whenever it reaches any such boundary, there always occur the most significant changes in the world and in the empires, as history makes clear" (*ibid.*, fol. a5r).

In a letter to Tycho Brahe, Christopher Rothmann censures Rheticus and asks: "How can the variation in the eccentricity of the sun produce a change of empires?" (*Tychonis Brahe opera omnia*, ed. Dreyer, VI, 160.28-29). I know of no evidence indicating that Copernicus shared the astrological views of Rheticus. Dreyer would perhaps not have advanced this suggestion (*Planetary Systems*, p. 333), had he been familiar with the aforementioned Preface by Rheticus.

Schöner's *Opera mathematica* appeared at Nuremberg in 1551, and again in 1561. The first paper is an introduction to judicial astrology (*Isagoge astrologiae iudiciariae*), and the second is a fearfully thorough essay in genethlialogy (*De iudiciis nativitatum*).

[58] Reading *verus* (Th 453.35) instead of *versus* (PII, 306.3).

[59] In *De rev.* Copernicus puts the greatest difference at about 7½° (Th 223.5-8); while an earlier passage gives 7° 28′ (Th 221.3-5), Rheticus has chosen to follow Copernicus's tables, which give 7° 24′ (Th 224.8-10).

Albategnius and Arzachel, pointed out by Regiomontanus in the *Epitome*, Book III, Proposition 13.[60]

Albategnius is too free in his treatment of the inner secrets of astronomy, as can be seen in many passages. Did he commit this fault in his determination of the solar apogee? Let us grant that he had the correct time of the equinox. Nevertheless, it is impossible, as Ptolemy states,[61] by means of instruments to determine with precision the times of the solstices. For a single minute of declination, which of course easily escapes the eye, may deceive us in this matter by about 4°, to which four days correspond. How was Albategnius able to determine the position of the solar apogee? If he used the method of intermediate positions on the ecliptic, explained by Regiomontanus in the *Epitome*, Book III, Proposition 14, he failed to employ a more trustworthy procedure. He is therefore himself to blame for going astray, since he selected eclipses occurring not near the apogee, but near the middle longitudes of the eccentric of the sun, where the solar apogee, even if mistakenly located 6° from its true position, could produce no noticeable error in eclipses.

According to Regiomontanus,[62] Arzachel boasts that he made 402 observations, and determined from them the position of the apogee. We grant that by this diligence he found the true eccentricity. But since it is not clear that he took into account lunar eclipses occurring near the apsides of the sun, it is apparent that we must no more accept his[63] determination of the higher apse than that of Albategnius.

[60] "Albategnius determined the eccentricity as 2° 4′ 45″, and the arc BH as 7° 43′. Arzachel, however . . . found the same eccentricity as Albategnius, but his value for the arc BH was 12° 10′. This is certainly surprising, since Arzachel lived after Albategnius." The arc BH is the distance from the apogee to the summer solstice.

[61] HI, 196.21–197.11.

[62] *Epitome,* Book III, Prop. 13: "Arzachel, 193 years after Albategnius, made 402 observations [*considerationes*] of the four points midway between the equinoctial and solstitial points, and found BH to be 12° 10′." It should be noted, with reference to the equivalence of *consideratio* and *observatio* (see above, p. 99, n. 28), that in citing this passage Rheticus altered *considerationes* to *observationes*.

[63] Reading *ei* (Th 454.20) instead of *eis* (PII, 306.32).

Now you see what great effort my teacher had to put forth to determine the mean motion of the apogee. For nearly 40 years in Italy and here in Frauenburg, he observed eclipses and the motion of the sun. He selected the observation by which he established that in C.E. 1515 the solar apogee was at 6⅔° of Cancer.[64] Then examining all the eclipses in Ptolemy and comparing them with his own very careful observations, he concluded that the mean annual[65] motion of the apogee with reference to the fixed stars was about 25″,[66] and with reference to the mean equinox about 1′15″.[67] Through this result it is established, by applying the true precession to both the mean and the unequal motions, that the true[68] position of the apogee was in the time of Hipparchus 63° from the true equinox, Ptolemy 64½°, Albategnius 76½°, and Arzachel 82°, while in our time all the calculations agree with experience. These figures are surely more satisfactory than the Alfonsine, which put the solar apogee at 12° of Gemini in the time of Ptolemy, and at the beginning of Cancer in our time.[69] We are 2° closer than the Alfonsine Tables to the estimate of Arzachel.[70] Albategnius's computation of the position of the apogee exceeds the Alfonsine by 1°, while we, for a good reason, fall short of his figure by 6°.[71] For my teacher cannot depart from Ptolemy and from his own observations, not only because he made and

[64] Th 210.10–211.26.

[65] Reading *annuum* (Th 454.26) instead of *annum* (PII, 307.8).

[66] Th 221.32–222.3.

[67] The mean annual motion of the equinox (mean precession) is about 50″ (see pp. 113-14, above; cf. Th 172.14-17), and it is retrograde (see p. 115, above). The motion of the apogee is direct (see p. 123, above). Hence, to obtain the motion of the apogee relative to the equinox, the two mean annual rates must be added: 25″ + 50″ = 1′ 15″.

[68] Reading *verus* (Th 454.29) instead of *versus* (PII, 307.12).

[69] That is, at 72° for Ptolemy's time, and at 90° for Copernicus's time.

[70] As we saw above (notes 60 and 62 on p. 124), the *Epitome* stated that Arzachel found the apogee 12° 10′ from the summer solstice = 77° 50′ from the equinox; cf. Th 210.5–8.

[71] Albategnius located the apogee 7° 43′ from the summer solstice = 82° 17′ from the equinox; cf. above, p. 124, n. 60 and Nallino, *Al-Battānī*, I, 44.29-33. The version of the Alfonsine Tables to which Rheticus refers evidently contained the following values: for Ptolemy's time, 72°; Albategnius's, 81°; Arzachel's, 72° (or 84°?); Copernicus's, 90°.

noted his own observations with his own eyes, but also because he knows that Ptolemy, working with the utmost care and making use of eclipses, accurately investigated the motions of the sun and the moon and established them correctly, so far as he could. We are compelled, nevertheless, to differ from him by about 1°,[72] as the motion of the apogee has made clear to us. For Ptolemy regarded the apogee as fixed and therefore showed little care in his treatment of this topic.

You have the opinion of my teacher regarding the motion of the sun. He has accordingly drawn up tables in which he collects for any specified time the true position of the solar apogee, the true eccentricity, the true inequalities, the uniform motions of the sun with reference to the fixed stars and to the mean equinoxes, and hence the true position of the sun corresponding to the observations of all ages. Clearly the tables of Hipparchus, Ptolemy, Theon, Albategnius, and Arzachel, and the Alfonsine Tables, which are to some extent a composite of the others, are temporary only and can endure at most 200 years, until, that is, the discrepancy in the length of the year, eccentricity, inequality, etc., becomes evident, a thing which occurs in the motions and appearances of the other planets for a similar definite reason. Not undeservedly, therefore, could the astronomy of my teacher be called perpetual, as the observations of all ages testify, and the observations of posterity will doubtless confirm. But he calculates his motions and the positions of the apsides from the first star of Aries,[73] since equality of motion is measured by the fixed stars. Then by adding the true precession, he computes and determines the distance in each age of the true positions of the planets from the true equinox.

If such an account of the celestial phenomena had existed a

[72] Hipparchus found the apogee 24½° from the summer solstice (HI, 233.8-10) = 65½° from the equinox, and Ptolemy accepted his determination (HI, 237.6-11).

[73] γ Arietis (Th 130.6-7), not α Arietis (Th 130.22) as Berry thought (*A Short History of Astronomy*, p. 110 n); cf. above, p. 63, n. 13; Rudolf Wolf, *Geschichte der Astronomie* (Munich, 1877), p. 240; Dreyer, *Planetary Systems*, p. 330; Armitage, *Copernicus*, pp. 105-6.

little before our time, Pico would have had no opportunity, in his eighth and ninth books,[74] of impugning not merely astrology but also astronomy. For we see daily how markedly common calculation departs from the truth.

Special Consideration of the Length of the Tropical Year

In improving the calendar most scholars enumerate various lengths of the year as computed by writers. But they do this in a confused way and come to no conclusion—surely a remarkable procedure for such great mathematicians.

From what has been said above, however, most learned Schöner, you see the four causes of the unequal motion of the sun as measured by the equinoxes: the inequality of the precession of the equinoxes, the inequality of the motion of the sun in the ecliptic, the decrease of the eccentricity, and last, the motion of the apogee for a twofold reason. By virtue of the same causes, the tropical year cannot be equal.

We may readily pardon Ptolemy for measuring equality of motion by the equinoxes,[75] since he held that the fixed stars move in consequence,[76] the position of the apogee is fixed,[77] and the eccentricity of the sun does not decrease.[78] How others would excuse themselves, I do not know. Let us even grant them that the stars and the solar apogee have the same motion in consequence; that therefore time measured by the true equinox in reality does not change; and that the entire inequality (though to assert this in our time would be most absurd) is caused by the defect in the instruments, since the motion of the solar apogee produces only a slight change in the length of the year. Nevertheless, it will not therefore follow that the sun regularly returns to the true equinox always in equal times, as we say that the moon regularly increases its distance from the mean apogee of the epicycle, and returns to the same position in equal times—a statement quoted by the

[74] Pico della Mirandola, *Disputationes adversus astrologos*, Books VIII–IX (pp. 457-82 in the Venice, 1498, edition of Pico's *Opera omnia*).

[75] See above, p. 65, n. 18.

[76] HI, 193.14-16; cf. above, p. 63, n. 13.

[77] See p. 119, above.

[78] See above, p. 120, n. 49.

learned Marcus Beneventanus[79] from the Alfonsine Tables. For since we surely cannot deny that the eccentricity of the sun changes, how can they assert that the variation of the angle of anomaly from the mean motion does not alter the length of the tropical year? I heartily congratulate the state and all scholars, whom the work of my teacher will advantage, that we have a sure understanding of the inequality of the year.

But that you may the more readily grasp all these ideas, most learned Schöner, I set them forth numerically before your eyes, in order that I may at length fulfill the pledge I made above.[80]

Let the sun be at the mean vernal equinoctial point, which was 3°29′ west of the first star of Aries at the time of the observation of the autumnal equinox made by Hipparchus in 147 B.C.[81] Let the sun move from this point in the eighth sphere and return to it in a sidereal year (365 days, 15 minutes, and about 24 seconds).[82] However, because the mean equinox in a sidereal year moves about 50″ in the direction opposite to that of the sun, the result[83] is that the sun reaches the new position of the mean vernal equinoctial point before it reaches the starting point, where the sun and the mean equinox had occupied the same position on the ecliptic. Therefore the year as measured by the mean equinox is shorter than the sidereal year,[84] and is computed to be, on the basis of our hypotheses, 365 days, 14 minutes, and about 34 seconds.[85] Now if, for the year measured by the mean equinox, we inquire what the excess[86] in days and fractions of days amounted to in the

[79] For a brief account of the life and work of Marco da Benevento see L. Birkenmajer in *Bulletin international de l'académie des sciences de Cracovie, classe des sciences math. et naturelles* (1901), pp. 63-71; and A. Birkenmajer in *Philosophisches Jahrbuch*, XXXVIII(1925), 336-44.

[80] Page 117.

[81] HI, 195.17-20, 204.1-6.

[82] See above, p. 116, n. 33.

[83] Reading *fit* (Th 456.17) instead of *sit* (PII, 310.11).

[84] See p. 46, above.

[85] That is, $365^d 5^h 49^m 36^s$. Newcomb's determination (1900) is $365^d.24219879 = 365^d 5^h 48^m 46^s$ (*American Ephemeris for 1940*, p. xx).

[86] That is, the length of time by which the tropical year exceeds the Egyptian year of exactly 365 days.

285[87] years between Hipparchus and Ptolemy, we shall find that it was about $69^d\ 9^m$.[88] Then there would be a deficiency of $2^d\ 6^m$,[89] if we assumed that each year exceeded 365 days by ¼ of a day. Let us therefore consider the remaining causes, until we find a deficiency of only $^{19}/_{20}$ of a day.

At the time of Hipparchus's observation, the true equinox was about 21′ of the starry ecliptic west of the mean equinox, and the sun was then in the same position as the mean equinox. But in the time of Ptolemy the true equinox was about 47′ east of the mean equinox. Therefore when the sun in the time of Ptolemy arrived at the point 21′ west of the mean equinox, where the true equinoctial point had been in the time of Hipparchus, the equinox did not occur. Nor did it occur when the sun reached the mean equinox. But after it had moved 47′ beyond the mean equinox, it came to the center of the earth, as Pliny says,[90] that is, to the true equinoctial point. The sun, then, had to pass through 1°8′,[91] an arc which it completed in its true motion in $1^d\ 8^m$. Retaining this as a side, I ask how much the angle of anomaly decreased in this instance, and I find that about 1 minute of a day corresponds to it. Thus it is clear that to the excess computed for the year as measured by the mean equinox, there is an addition of $1^d\ 9^m$.[92] Ptolemy correctly stated,[93] then, that between his own observation and that of Hipparchus, from true equinox to true equinox, there were $285^y\ 70^d\ 18^m$. Therefore there was a deficiency of 57 minutes of a day, the result of subtracting $1^d\ 9^m$ from $2^d\ 6^m$, the deficiency which appeared above for the year as measured by the mean equinox.

Let us now consider the deficiency of 7 days between

[87] In the table on p. 116, above, the interval between Hipparchus and Ptolemy is 265 years, because Hipparchus is assigned to 127 B.C. The present passage uses an observation made by Hipparchus in 147 B.C.

[88] $285 \times 14^m 34^s = 69^d 11\tfrac{1}{2}^m$.

[89] $$\begin{array}{rr} 285 \times \tfrac{1}{4}^d = & 71^d 15^m \\ - & \underline{69\ \ 9} \\ & 2^d\ 6^m \end{array}$$

[90] *Natural History* ii.19(17).81.

[91] $21' + 47' = 1°\ 8'$.   [92] $69^d 9^m + 1^d 9^m = 70^d 18^m$.   [93] HI, 204.11-16.

Ptolemy and Albategnius. The situation is clear because the interval of time, 743 years, is greater, and hence all the causes will be more obvious. In the time of Ptolemy the mean equinox was about $7°28'$ west of the first star of Aries.[94] But since the mean equinox moved from that position, as has been explained, in the direction opposite to that of the sun, the result is that between Ptolemy and Albategnius there was an excess of about $180^d\ 14^m$ for the year as measured by the mean equinox.[95] Then there will be a deficiency of $5^d\ 31^m$, if we compare the year as measured by the mean equinox with the result obtained by adding a day every four years.[96] Whereas in the time of Ptolemy the true equinox was $47'$ east of the mean equinox, in the time of Albategnius it was $22'$ west of the mean equinox. Therefore the sun reached the true equinox before it reached the mean equinox or the former position of the true equinox,[97] in contrast with our previous example. Hence the time corresponding to $1°9'$[98] will be subtracted from the excess for the year as measured by the mean equinox, and added to the deficiency of $5^d\ 31^m$. We must deal in the same way with the variation in the angle of anomaly caused by the decrease in the eccentricity, to which 30 minutes of a day correspond. Then the change in the angle of anomaly, and the unequal motion of precession, combined with the other two causes of the unequal motion of the sun, produce a further deficiency of $1^d\ 30^m$ to be subtracted from the excess for the year as measured by the mean equinox. Hence the true excess from the time of Ptolemy to the time of Albategnius's observation becomes $178^d\ 44^m$.[99] But the addition of this further deficiency to $5^d\ 31^m$ shows that the total deficiency was $7^d\ 1^m$.[100] Q.E.D.

[94] For it had moved $3°\ 59'$ from its position at the time of Hipparchus: $285 \times 50'' = 3°\ 57\frac{1}{2}'$.

[95] $743 \times 14^m 34^s = 180^d 23^m$.

[96] $743 \times \frac{1}{4}^d = 185^d 45^m$

$$\begin{array}{r} 185^d 45^m \\ -\ 180\ \ 14 \\ \hline 5^d 31^m \end{array}$$

[97] Prowe states (PII, 312 n) that Mästlin substituted *aequinoctium* for the older reading *aequinoctialem*. But both of Mästlin's editions show *aequinoctialem*.

[98] $47' + 22' = 1°\ 9'$.

[99] $180^d 14^m - 1^d 30^m = 178^d 44^m$.

[100] $5^d 31^m + 1^d 30^m = 7^d 1^m$.

It was a difficult task to recover by this analysis the motions of the fixed stars and of the sun, and through the computation of these motions to attain a correct understanding of the length of the tropical year. A boundless kingdom in astronomy has God granted to my learned teacher. May he, as its ruler, deign to govern, guard, and increase it, to the restoration of astronomic truth. Amen.

I intended to report briefly to you, most learned Schöner, the entire treatment of the motions of the moon and of the remaining planets, as well as of the fixed stars and sun, in order that you might understand what benefits to students of mathematics and to all posterity will flow from the writings of my teacher, as from a most plentiful spring. But when I saw that my book was already growing excessively long, I decided to compose a special "Account"[101] of these topics. However, the material which I thought must precede and prepare the way, as it were, I shall set forth at this point. And I shall interweave with my teacher's hypotheses for the motions of the moon and of the remaining planets certain general considerations, in order that you may conceive greater hope for the entire work, and understand why he was compelled to assume other hypotheses or theories.

Having stated at the beginning of this *Account*[102] that my teacher in writing his book imitated Ptolemy, I see that there is practically nothing left for me to take up with you in reference to his method of improving the motions. For Ptolemy's tireless diligence in calculating, his almost superhuman accuracy in observing, his truly divine procedure in examining and investigating all the motions and appearances, and finally his completely consistent method of statement and proof cannot be sufficiently admired and praised by anyone to whom Urania is gracious.

In one respect, however, a burden greater than Ptolemy's confronts my teacher. For he must arrange in a certain and consistent scheme or harmony the series and order of all the

[101] Cf. above, p. 110, n. 5.
[102] Pages 109-10.

motions and appearances, marshalled on the broad battlefield of astronomy by the observations of 2,000 years, as by famous generals. Ptolemy, on the other hand, had the observations of the ancients, to which he could safely entrust himself, for scarcely a quarter of this period. Time, the true god and teacher of the laws of the celestial state, discloses the errors of astronomy to us. For an imperceptible or unnoticed error at the foundation of astronomical hypotheses, principles, and tables is revealed or greatly increased by the passage of time. Therefore my teacher must not so much restore astronomy as build it anew.

Ptolemy was able to harmonize satisfactorily most of the hypotheses of the ancients—Timocharis, Hipparchus, and others—with every inequality in the motions known to him from so small an elapsed period of observation. Therefore he quite rightly and wisely—a praiseworthy action—selected those hypotheses which seemed to be in better agreement with reason and our senses, and which his greatest predecessors had employed.[103] Nevertheless, the observations of all scholars and heaven itself and mathematical reasoning convince us that Ptolemy's[104] hypotheses and those commonly accepted do not suffice to establish the perpetual and consistent connection and harmony of celestial phenomena and to formulate that harmony in tables and rules. It was therefore necessary for my teacher to devise new hypotheses, by the assumption of which he might geometrically and arithmetically deduce with sound logic systems of motion like those which the ancients and Ptolemy, raised on high, once perceived "with the divine eye of the soul,"[105] and which careful observations reveal as existing in the heavens to those today who study the remains of the ancients. Surely students hereafter will see the value of Ptolemy and the other ancient writers, so that they will recall these men who have been until now excluded from the schools, and restore them, like returned exiles, to their ancient place of honor. The

[103] Reading *fuerant* (Th 458.27) instead of *fuerunt* (PII, 314.6).
[104] Reading *Ptolemaei* (Th 458.28) instead of *Ptolemaeo* (PII, 314.8).
[105] A Greek phrase quoted from the pseudo-Aristotelian *De mundo* 391a15.

poet says: "No one desires the unknown."[106] Hence it is not strange that heretofore Ptolemy together with all antiquity has lain ignored in obscurity, as doubtless you, most excellent Schöner, together with other good and learned men have often grieved.

General Considerations Regarding the Motions of the Moon, Together with the New Lunar Hypotheses

The theory of eclipses all by itself seems to maintain respect for astronomy among uneducated people; yet we see daily how much it differs nowadays from common calculation in the prediction of both the duration and extent of eclipses. In constructing astronomical tables we should not, as we see certain writers doing,[107] reject the precise observations of Ptolemy and other excellent authorities as false and untrustworthy, unless the passage of time discloses to us that some manifest error has crept in. For what is more human than sometimes to be mistaken and deceived even by the appearance of truth, especially in these difficult, abstruse, and by no means obvious matters?

In his exposition of the motion of the moon, my teacher assumes such theories and schemes of motion as make it clear that the eminent ancient philosophers were not at all blind in their observations. Just as we showed above that the increase and decrease of the tropical year are regular, so, from a careful investigation of the motions of the sun and moon, it is possible to deduce for each age the true distances of the sun, moon, and earth from one another, or the reason why the diameters of the sun, moon, and earth's shadow have been found different at different times, and thus, in addition, to attain a correct understanding of the solar and lunar parallax.

In the *Epitome*, Book V, Proposition 22, Regiomontanus says: "But it is noteworthy that the moon does not appear so great at quadrature, when it is in the perigee of the epicycle, whereas, if the entire disk were visible, it should appear four

[106] Ovid *Ars amatoria* iii.397.

[107] Doubtless Rheticus intended to include Werner in the group; cf. Copernicus's sharp protests in the *Letter against Werner*, pp. 99-100, 103-5, above.

times its apparent size at opposition, when it is in the apogee of the epicycle."[108] This difficulty was noticed by Timocharis and Menelaus, who always use the same lunar diameter[109] in their observations of the stars. But experience has shown my teacher that the parallax and size of the moon, at any distance from the sun, differ little or not at all from those which occur at conjunction and opposition, so that clearly the traditional eccentric cannot be assigned to the moon. He supposes therefore that the lunar sphere encloses the earth together with[110] its adjacent elements, and that the center of the deferent is the center of the earth, about which the deferent revolves uniformly, carrying the center of the lunar epicycle.

The second inequality, which appears in the distance of the moon from the sun, he saves as follows. He assumes that the moon moves on an epicycle of an epicycle of a concentric; that is, to the first epicycle, which in general is in evidence at conjunction and opposition, he joins a second small[111] epicycle which carries the moon; and he shows that the ratio of the diameter of the first epicycle to the diameter of the second is as 1,097:237. The scheme of motions is as follows. The inclined circle has the same motion as heretofore, save that its equal periods are measured by the fixed stars. The deferent, which is concentric, moves regularly and uniformly about its own center (which is also the center of the earth), at the same time rotating uniformly and regularly from the line of mean motion of the sun. The first epicycle also revolves uniformly about its own center; in its upper circumference it carries the center of the small second epicycle in precedence, in its lower circumference, in consequence.[112] My teacher computes this uniform and regular motion from the true apogee. This point

[108] The quotation is substantially correct. But the original has *aux* and *oppositum augis*, for which Rheticus substitutes *apogium* and *perigium* (cf. above, p. 34, n. 117).

[109] ½° (Th 235.8-11); the observations referred to are cited in HII, 25.15-34.8.

[110] Prowe's text (PII, 315.29) omits *cum* between *terram* and *adiacentibus* (Th 459.30).

[111] Reading *parvum* (Th 459.35) for *parum* (PII, 316.3).

[112] Cf. above, pp. 68-69, n. 28.

is marked on the upper circumference of the first epicycle by a line drawn from the center of the earth through the center of the first epicycle to its circumference. Starting from the small epicycle's true apogee, which is indicated on its circumference by a line drawn from the center of the first epicycle through the center of the second epicycle, the moon also moves regularly and uniformly on the circumference of the small second epicycle. The rule governing this motion is the following: the moon revolves twice on its epicycle[113] in one period of the deferent, so that at every conjunction and opposition the moon is found in the perigee of the small epicycle, but at the quadratures in its apogee. This is the device or hypothesis by which my teacher removes all the aforementioned incongruities, and which satisfies all the appearances, as he clearly shows, and as can be inferred also from his tables.

Furthermore, most learned Schöner, you see that here in the case of the moon we are liberated from an equant by the assumption of this theory, which, moreover, corresponds to experience and all the observations. My teacher dispenses with equants for the other planets as well, by assigning to each of the three superior planets only one epicycle and eccentric; each of these moves uniformly about its own center, while the planet revolves on the epicycle in equal periods with the eccentric. To Venus and Mercury, however, he assigns an eccentric on an eccentric. The planets are each year observed as direct, stationary, retrograde, near to and remote from the earth, etc.[114] These phenomena, besides being ascribed to the planets, can be explained, as my teacher shows, by a regular motion of the spherical earth; that is, by having the sun occupy the center of the universe, while the earth revolves instead of the sun on the eccentric,[115] which it has pleased him to name the great

[113] While discarding an unnecessary emendation of Mästlin's, Prowe's text (PII, 316.20) inserts *parvo*, for which there is no warrant in the Basel edition of 1566 (p. 201v).

[114] Reading *etc.* with the editions of 1566 (p. 202r), 1596 (p. 110), and 1621 (p. 107), instead of *et cum* (PII, 317.10; Th 460.24).

[115] This is the first indication in the *Narratio prima* that the astronomy of Copernicus involves heliocentrism and a moving earth. Rheticus evidently deemed

circle. Indeed, there is something divine in the circumstance that a sure understanding of celestial phenomena must depend on the regular and uniform motions of the terrestrial globe alone.

The Principal Reasons Why We Must Abandon the Hypotheses of the Ancient Astronomers

In the first place, the indisputable precession of the equinoxes, as you have heard, and the change in the obliquity of the ecliptic persuaded my teacher to assume that the motion of the earth could produce most of the appearances in the heavens, or at any rate save them satisfactorily.

Secondly, the diminution of the eccentricity of the sun is observed, for a similar reason and proportionally, in the eccentricities of the other planets.

Thirdly, the planets evidently have the centers of their deferents in the sun, as the center of the universe. That the ancients, not to mention the Pythagoreans for the moment, were aware of this fact is sufficiently clear for example from Pliny's statement, following undoubtedly the best authorities, that Venus and Mercury do not recede further from the sun than fixed, ordained limits because their paths encircle the sun;[116] hence these planets necessarily share the mean motion of the sun. As Pliny says,[117] the course of Mars is hard to trace. In addition to the other difficulties in the correction of its motion, Mars unquestionably shows a parallax sometimes greater

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it advisable, before introducing these ideas, to paint the portrait of Copernicus as a great astronomer, who made careful observations and painstaking calculations, who studied thoroughly the work of his predecessors and respected, in particular, the authority of Ptolemy. The cautiousness of Rheticus stands in striking contrast to the forthright procedure of Copernicus in the *Commentariolus* (cf. pp. 57-59, above).

[116] *Natural History* ii.17(14).72. It is more likely that Pliny's *conversas absidas* meant simply "different courses," i.e., orbits unlike those of the superior planets; cf. Rackham's translation in the Loeb Classical Library (London, 1938). Rheticus's understanding of the passage was governed by Th 27.18-25. For Kepler's comments on this obscure section in Pliny and on Copernicus's interpretation of it see his *Opera*, ed. Frisch, I, 271-72.

[117] *Natural History* ii.17(15).77.

than the sun's, and therefore it seems impossible that the earth should occupy the center of the universe. Although Saturn and Jupiter, as they appear to us at their morning and evening rising, readily yield the same conclusion, it is particularly and especially supported by the variability of Mars when it rises. For Mars, having a very dim light, does not deceive the eye as much as Venus or Jupiter, and the variation of its size is related to its distance from the earth. Whereas at its evening rising Mars seems to equal Jupiter in size, so that it is differentiated only by its fiery splendor, when it rises in the morning just before the sun and is then extinguished in the light of the sun, it can scarcely be distinguished from stars of the second magnitude. Consequently at its evening rising it approaches closest to the earth, while at its morning rising it is furthest away; surely this cannot in any way occur on the theory of an epicycle. Clearly then, in order to restore the motions of Mars and the other planets, a different place must be assigned to the earth.

Fourthly, my teacher saw that only on this theory could all the circles in the universe be satisfactorily made to revolve uniformly and regularly about their own centers, and not about other centers—an essential property of circular motion.

Fifthly, mathematicians as well as physicians must agree with the statements emphasized by Galen here and there: "Nature does nothing without purpose"[118] and "So wise is our Maker that each of his works has not one use, but two or three or often more."[119] Since we see that this one motion of the earth satisfies an almost infinite number of appearances, should we not attribute to God, the creator of nature, that skill which we observe in the common makers of clocks? For they carefully avoid inserting in the mechanism any superfluous wheel or any whose function could be served better by another with a slight change

[118] *De usu partium* x.14 (ed. Helmreich, Leipzig, 1907–9, II, 109.2). Rheticus quotes the Greek text, the first edition (Venice, 1525) of which was available to him, as was also the Basel edition of 1538. The words quoted appear in the 1525 edition, Vol. I, fol. H7v.47 (= fol. 63v of the separate pagination for the *De usu partium*).

[119] *Ibid.* x.15 (ed. Helmreich, II, 111.5–8; 1525 ed., Vol. I, fol. H8r.14–16). Rheticus accommodates the quotation to the structure of his own sentence.

of position. What could dissuade my teacher, as a mathematician, from adopting a serviceable theory of the motion of the terrestrial globe, when he saw that on the assumption of this[120] hypothesis there sufficed, for the construction of a sound science of celestial phenomena, a single eighth sphere, and that motionless, the sun at rest in the center of the universe, and for the motions of the other planets, epicycles on an eccentric or eccentrics on an eccentric or epicycles on an epicycle? Moreover, the motion of the earth in its circle produces the inequalities of all the planets except the moon; this one motion alone seems to be the cause of every apparent inequality at a distance from the sun, in the case of the three superior planets, and in the neighborhood of the sun, in the case of Venus and Mercury. Finally, this motion makes it possible to satisfy each of the planets by only one deviation in latitude of the deferent of the planet. Hence it is particularly the planetary motions that require such hypotheses.

Sixthly and lastly, my teacher was especially influenced by the realization that the chief cause of all the uncertainty in astronomy was that the masters of this science (no offense is intended to divine Ptolemy, the father of astronomy) fashioned their theories and devices for correcting the motion of the heavenly bodies with too little regard for the rule which reminds us that the order and motions of the heavenly spheres agree in an absolute system. We fully grant these distinguished men their due honor, as we should. Nevertheless, we should have wished them, in establishing the harmony of the motions, to imitate the musicians who, when one string has either tightened or loosened, with great care and skill regulate and adjust the tones of all the other strings, until all together produce the desired harmony, and no dissonance is heard in any. If Albategnius, to speak of him for the moment, had followed this precept in his work, we should doubtless have today a surer understanding of all the motions. For it is likely that the Alfonsine Tables drew heavily from him; and since this one rule was neglected, we should have had to face at some time, if we intend to speak the truth, the collapse of all astronomy.

[120] Reading *tali* (Th 461.31) instead of *talia* (PII, 319.13).

Under the commonly accepted principles of astronomy, it could be seen that all the celestial phenomena conform to the mean motion of the sun and that the entire harmony of the celestial motions is established and preserved under its control. Hence the sun was called by the ancients leader, governor of nature, and king. But whether it carries on this administration as God rules the entire universe, a rule excellently described by Aristotle in the *De mundo*,[121] or whether, traversing the entire heaven so often and resting nowhere, it acts as God's administrator in nature, seems not yet altogether explained and settled. Which of these assumptions is preferable, I leave to be determined by geometers and philosophers (who are mathematically equipped). For in the trial and decision of such controversies, a verdict must be reached in accordance with not plausible opinions but mathematical laws (the court in which this case is heard). The former manner of rule has been set aside, the latter adopted. My teacher is convinced, however, that the rejected method of the sun's rule in the realm of nature must be revived, but in such a way that the received and accepted method retains its place. For he is aware that in human affairs the emperor need not himself hurry from city to city in order to perform the duty imposed on him by God; and that the heart does not move to the head or feet or other parts of the body to sustain[122] a living creature, but fulfills its function through other organs designed by God for that purpose.

Now my teacher concluded that the mean motion of the sun must be the sort of motion that is not only established by the imagination, as in the case of the other planets, but is self-caused, since it appears to be truly "both choral dancer and choral leader." He then showed that his opinion was sound and not inconsistent with the truth, for he saw that by his hypotheses the efficient cause of the uniform motion of the sun could be geometrically deduced and proved. Hence the mean motion of the sun would necessarily be perceived in all the

[121] Chap. vi. This work is now athetized; see Wilhelm Capelle, *Neue Jahrbücher für das klassische Altertum*, XV(1905), 532.

[122] Reading *conservationem* (Th 462.37) instead of *conversationem* (PII, 321.4).

motions and appearances of the other planets in a definite manner, as appears in each of them. Thus the assumption of the motion of the earth on an eccentric provides a sure theory of celestial phenomena, in which no change should be made without at the same time re-establishing the entire system, as would be fitting, once more on proper ground. While we were unable from our common theories even to surmise this rule by the sun in the realm of nature, we ignored most of the ancient encomia of the sun as poetry. You see, then, what sort of hypotheses for saving the motions my teacher had to assume under these circumstances.

Transition to the Explanation of the New Hypotheses for the Whole of Astronomy

I interrupt your thoughts, distinguished sir, for I am aware that while you listen to the reasons, investigated by my teacher with remarkable learning and great devotion, for revising the astronomical hypotheses, you thoughtfully consider what foundation may finally prove to be suitable for the hypotheses of astronomy reborn. But the men who endeavor to pull all the stars together around in the ether in accordance with their own opinion, as though they had put chains upon them, merit pity rather than resentment, in your judgment as in that of other true mathematicians and all good men. You are not unacquainted with the importance to astronomers of hypotheses or theories, and with the difference between a mathematician and a physicist.[123] Hence you agree, I feel, that the results to which the observations and the evidence of heaven itself lead us again and again must be accepted, and that every difficulty must be faced and overcome with God as our guide and mathematics and tireless study as our companions.

Accordingly, anyone who declares that he must be mindful of the highest and principal end of astronomy will be grateful

[123] Presumably a reference to Aristotle's *Physics* 193b22-23, and to Simplicius's *Commentary on Aristotle's Physics*, second comment on Book ii.2 (*Commentaria in Aristotelem Graeca*, Vol. IX, ed. H. Diels, Berlin, 1882, pp. 290-93); the first edition of Simplicius's *Commentary* (Venice, 1526) was available to Rheticus and Schöner.

with us to my teacher and will consider as applicable to himself Aristotle's remark: "When anyone shall succeed in finding proofs of greater precision, gratitude will be due to him for the discovery."[124] The examples of Callippus and Aristotle[125] assure us that, in the effort to ascertain the causes of the phenomena, astronomy must be revised as unequal motions of the heavenly bodies are encountered. Hence I may hope that Averroes, who played the role of the severe Aristarchus[126] to Ptolemy, would not receive the hypotheses of my teacher harshly, if only he would examine natural philosophy patiently. In my opinion, Ptolemy was not so bound and sworn to his own hypotheses that, were he permitted to return to life, upon seeing the royal road blocked and made impassable by the ruins of so many centuries, he would not seek another road over land and sea to the construction of a sound science of celestial phenomena, since he could not rise through the air and open sky to the desired goal. For what else shall I say of the man who wrote the following words:

> Propositions assumed without proof, if once they are perceived to be in agreement with the phenomena, cannot be established without some method and reflection; and the procedure for apprehending them is hard to explain, since in general, of first principles, there naturally is either no cause or one difficult to set forth.[127]

How modestly and wisely Aristotle speaks on the subject of the celestial motions can be seen everywhere in his works. He says in another connection: "It is the mark of an educated man to look for precision in each class of things just so far as the

[124] *De caelo* ii.5 287b34–288a1 (J. L. Stocks's rendering in the Oxford translations of the works of Aristotle, 1930, Vol. II).

[125] *Metaphysics* xii.8 1073b32–1074a5.

[126] For Averroes as an adversary of the Ptolemaic astronomy see pp. 194-95, below, and Duhem, *Le Système du monde*, II, 133-39. In a long article devoted to the relation between Copernicus and the astronomer Aristarchus of Samos, Brachvogel failed to recognize that the Aristarchus of our passage is unquestionably Aristarchus of Alexandria, the severe critic of Homer, not Aristarchus of Samos (ZE, XXV[1933-35], 703, n. 15).

[127] HII, 212.11-16. Rheticus presented to Copernicus (PI², 411) a copy of the first edition of the Greek text of the *Syntaxis* (Basel, 1538); the passage quoted begins at the foot of the page numbered (incorrectly) 219.

nature of the subject admits."[128] Now in physics as in astronomy, one proceeds as much as possible from effects and observations to principles. Hence I am convinced that Aristotle, who wrote careful discussions of the heavy and the light, circular motion, and the motion and rest of the earth,[129] if he could hear the reasons for the new hypotheses, would doubtless honestly acknowledge what he proved in these discussions, and what he assumed as unproved principle. I can therefore well believe that he would support my teacher, inasmuch as the well-known saying attributed to Plato[130] is certainly correct: "Aristotle is the philosopher of the truth." On the other hand, were he to burst forth in harsh language, it would be only to lament bitterly, I am persuaded, the condition of this most beautiful part of philosophy in the following terms: "It has been said very well by Plato[131] that 'geometry and the studies that accompany it dream about being, but the clear waking vision of it is impossible for them as long as they leave the assumptions which they employ undisturbed and cannot give any account of them' "; and he would add: "We must be deeply grateful to the immortal gods for the knowledge of such a theory of the phenomena."

But since these remarks are less appropriate here than in a certain other treatise,[132] I shall proceed to set forth the remaining hypotheses of my teacher in an open and orderly manner, in an endeavor to throw some light on my previous statements.

The Arrangement of the Universe

Aristotle says: "That which causes derivative truths to be true is most true."[133] Accordingly, my teacher decided that he

[128] *Nicomachean Ethics* 1094b23-25 (W. D. Ross's translation, Oxford, 1925).

[129] *De caelo* i.2-4; ii.3, 13-14; *Physics* viii.8-9.

[130] The authentic works of Plato contain no reference to the philosopher Aristotle.

[131] *Republic* vii.13 533B-C, slightly altered (P. Shorey's translation, Loeb Classical Library, London, 1930-35).

[132] Probably Rheticus has in mind his projected "Second Account."

[133] *Metaphysics* i minor.1 993b26-27 (W. D. Ross's translation, Oxford, 1928). Although Rheticus usually quotes from Greek authors in the original language, in the present instance he is using Bessarion's Latin translation of the *Metaphysics*.

must assume such hypotheses as would contain causes capable of confirming the truth of the observations of previous centuries, and such as would themselves cause, we may hope, all future astronomical predictions of the phenomena to be found true.

First, surmounting no mean difficulties, he established by hypothesis that the sphere of the stars, which we commonly call the eighth sphere, was created by God to be the region which would enclose within its confines the entire realm of nature, and hence that it was created fixed and immovable as the place of the universe. Now motion is perceived only by comparison with something fixed; thus sailors on the sea, to whom

> land is no longer
> Visible, only the sky on all sides and on all sides the water[134]

are not aware of any motion of their ship when the sea is undisturbed by winds, even though they are borne along at such high speed that they pass over several long miles in an hour. Hence this sphere was studded by God for our sake with a large number of twinkling stars, in order that by comparison with them, surely fixed in place, we might observe the positions and motions of the other enclosed spheres and planets.

Then, in harmony with these arrangements, God stationed in the center of the stage His governor of nature, king of the entire universe, conspicuous by its divine splendor, the sun

> To whose rhythm the gods move, and the world
> Receives its laws and keeps the pacts ordained.[135]

The other spheres are arranged in the following manner. The first place below the firmament or sphere of the stars falls to the sphere of Saturn, which encloses the spheres of first Jupiter, then Mars; the spheres of first Mercury, then Venus

[134] Vergil *Aeneid* iii.192-93.

[135] Giovanni Gioviano Pontano, *Urania* or *De stellis* i.240-41 (Florence, 1514, p. 7r); Pontano's poems were reprinted in *Ioannis Ioviani Pontani carmina* (ed. Benedetto Soldati, Florence, 1902), where the quoted lines appear on p. 10. Copernicus owned a copy of the selection of Pontano's prose works which was printed at Venice in 1501 (PI², 417).

surround the sun; and the centers of the spheres of the five planets are located in the neighborhood of the sun. Between the concave surface of Mars' sphere and the convex of Venus', where there is ample space, the globe of the earth together with its adjacent elements, surrounded by the moon's sphere, revolves in a great circle which encloses within itself, in addition to the sun, the spheres of Mercury and Venus, so that the earth moves among the planets as one of them.

As I carefully consider this arrangement of the entire universe according to the opinion of my teacher, I realize that Pliny set down an excellent and accurate statement when he wrote: "To inquire what is beyond the universe or heaven, by which all things are overarched, is no concern of man, nor can the human mind form any conjecture concerning this question." And he continues: "The universe is sacred, without bounds, all in all; indeed, it is the totality, finite yet similar to the infinite, etc."[136] For if we follow my teacher, there will be nothing beyond the concave surface of the starry sphere for us to investigate, except insofar as Holy Writ has vouchsafed us knowledge, in which case again the road will be closed to placing anything beyond this concave surface. We will therefore gratefully admire and regard as sacrosanct all the rest of nature, enclosed by God within the starry heaven. In many ways and with innumerable instruments and gifts He has endowed us, and enabled us[137] to study and know nature; we will advance to the point to which He desired us to advance, and we will not attempt to transgress the limits imposed by Him.

That the universe is boundless up to its concave surface, and truly similar to the infinite is known, moreover, from the fact that we see all the heavenly bodies twinkle, with the exception of the planets including Saturn, which, being the nearest of them to the firmament, revolves on the greatest circle. But this conclusion follows far more clearly by deduction from the hypotheses of my teacher. For the great circle which carries the

[136] Abridged from *Natural History* ii.1.1-2.

[137] Prowe's text (PII, 326.18) omits *nos* between *idoneos* and *effecit* (Th 466.20).

earth has a perceptible ratio to the spheres of the five planets, and hence every inequality in the appearances of these planets is demonstrably derived from their relations to the sun. Every horizon on the earth, being a great circle of the universe, divides the sphere of the stars into equal parts. Equal periods in the revolutions of the spheres are shown to be measured by the fixed stars. Consequently it is quite clear that the sphere of the stars is, to the highest degree, similar to the infinite, since by comparison with it the great circle vanishes, and all the phenomena are observed exactly as if the earth were at rest in the center of the universe.

Moreover, the remarkable symmetry and interconnection of the motions and spheres, as maintained by the assumption of the foregoing hypotheses, are not unworthy of God's workmanship and not unsuited to these divine bodies. These relations, I should say, can be conceived by the mind (on account of its affinity with the heavens) more quickly than they can be explained by any human utterance, just as in demonstrations they are usually impressed upon our minds, not so much by words as by the perfect and absolute ideas, if I may use the term, of these most delightful objects. Nevertheless it is possible, in a general survey of the hypotheses, to see how the inexpressible harmony and agreement of all things manifest themselves.

For in the common hypotheses there appeared no end to the invention of spheres; moreover, spheres of an immensity that could be grasped by neither sense nor reason were revolved with extremely slow and extremely rapid motions. Some writers stated that the daily motion of all the lower spheres is caused by the highest movable sphere;[138] but when a great storm of controversy raged over this question, they could not explain why a higher sphere should have power over a lower. Others, like Eudoxus[139] and those who followed him, assigned to each planet a special sphere, the motion of which caused the

[138] See Dreyer, *Planetary Systems*, p. 91.

[139] See Aristotle *Metaphysics* xii.8 1073b17-26; and Simplicius's *Commentary on Aristotle's De caelo* (*Commentaria in Aristotelem Graeca*, Vol. VII, ed. Hei-

planet to revolve about the earth once in a natural day. Moreover, ye immortal gods, what dispute, what strife there has been until now over the position of the spheres of Venus and Mercury, and their relation to the sun. But the case is still before the judge. Is there anyone who does not see that it is very difficult and even impossible ever to settle this question while the common hypotheses are accepted? For what would prevent anyone from locating even Saturn below the sun, provided that at the same time he preserved the mutual proportions of the spheres and epicycle, since in these same hypotheses there has not yet been established the common measure of the spheres of the planets, whereby each sphere may be geometrically confined to its place? I refrain from mentioning here the vast commotion which those who defame this most beautiful and most delightful part of philosophy have stirred up on account of the great size of the epicycle of Venus, and on account of the unequal motion, on the assumption of equants, of the celestial spheres about their own centers.

However, in the hypotheses of my teacher, which accept, as has been explained, the starry sphere as boundary, the sphere of each planet advances uniformly with the motion assigned to it by nature and completes its period without being forced into any inequality by the power of a higher sphere. In addition, the larger spheres revolve more slowly, and, as is proper, those that are nearer to the sun, which may be said to be the source of motion and light, revolve more swiftly. Hence Saturn, moving freely in the ecliptic, revolves in thirty years, Jupiter in twelve, and Mars in two. The center of the earth measures the length of the year by the fixed stars. Venus passes through the zodiac in nine months, and Mercury, revolving about the sun on the smallest sphere, traverses the universe in eighty days. Thus there are only six moving spheres which revolve about

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berg, Berlin, 1894, 493.11-15; 494.12-18, 23-26; 495.8-9, 17-22; 496.15-19). The first edition of Simplicius's *Commentary* (Venice, 1526) was available to Rheticus; it was a Greek version, done by Bessarion or one of his circle, of William of Moerbeke's Latin translation (*Sitzungsberichte der Akademie der Wissenschaften zu Berlin*, 1892, pp. 74-75).

the sun, the center of the universe. Their common measure is the great circle which carries the earth, just as the radius of the spherical earth is the common measure of the circles of the moon, the distance of the sun from the moon, etc.

Who could have chosen a more suitable and more appropriate number than six? By what number could anyone more easily have persuaded mankind that the whole universe was divided into spheres by God the Author and Creator of the world? For the number six is honored beyond all others in the sacred prophecies of God and by the Pythagoreans and the other philosophers. What is more agreeable to God's handiwork than that this first and most perfect work should be summed up in this first and most perfect number?[140] Moreover, the celestial harmony is achieved by the six aforementioned movable spheres. For they are all so arranged that no immense interval is left between one and another; and each, geometrically defined, so maintains its position that if you should try to move any one at all from its place, you would thereby disrupt the entire system.

But now that we have touched on these general considerations, let us proceed to an exposition of the circular motions which are appropriate to the several spheres and to the bodies that cleave to and rest upon them. First we shall speak of the hypotheses for the motions of the terrestrial globe, on which we have our being.

The Motions Appropriate[141] to the Great Circle and Its Related Bodies. The Three Motions of the Earth: Daily, Annual, and the Motion in Declination

Following Plato and the Pythagoreans, the greatest mathematicians of that divine age, my teacher thought that in order

[140] This passage, in which Rheticus reveals his acceptance of number mysticism, finds no parallel in the works of Copernicus; cf. above, p. 122, n. 57. For an excellent discussion of the metamathematical superstructure, erected in the early modern period on the basis of Pythagorean and Platonic philosophy, see Edward W. Strong, *Procedures and Metaphysics* (Berkeley, Calif., 1936), chap. viii.

[141] Reading *competant* (Th 468.9) instead of *computant* (PII, 329.23).

to determine the causes of the phenomena circular motions must be ascribed to the spherical earth.[142] He saw (as Aristotle also points out[143]) that when one motion is assigned to the earth, it may properly have other motions, by analogy with the planets. He therefore decided to begin with the assumption that the earth has three motions, by far the most important of all.

For in the first place, having assumed the general arrangement of the universe described above, he showed that, enclosed by its poles within the lunar sphere, the earth, like a ball on a lathe, rotates from west to east, as God's will ordains; and that by this motion, the terrestrial globe produces day and night and the changing appearances of the heavens, according as it is turned toward the sun. In the second place, the center of the earth, together with its adjacent elements and the lunar sphere, is carried uniformly in the plane of the ecliptic by the great circle, which I have already mentioned more than once,[144] in the order of the signs. In the third place, the equator and the axis of the earth have a variable inclination to the plane of the ecliptic and move in the direction opposite to that of the motion of the center, so that on account of this inclination of the earth's axis and the immensity of the starry sphere, no matter where the center of the earth may be, the equator and the poles of the earth are almost invariably directed to the same points in the heavens. This result will ensue if the ends of the earth's axis, that is, the poles of the earth, are understood to move daily in precedence a distance almost exactly equal to the motion of the center of the earth in consequence on the great circle, and to describe about the axis and poles of the great circle or ecliptic small circles equidistant from them.

But to these motions we should add, in the opinion of my teacher, two librations of the poles of the earth and the two motions, the one uniform and the other unequal, with which

[142] Whether Plato held that the earth is at rest or in motion is much disputed; see Thomas L. Heath, *Aristarchus of Samos* (Oxford, 1913), pp. 174-85.

[143] *De caelo* ii.14 296a24-296b3.

[144] Pages 135-36, 144, 145, 147.

the center of the great circle advances in the ecliptic;[145] let us also recall what was said above[146] concerning the motions of the moon about the center of the earth. We shall then have, most learned Schöner, a true system of hypotheses for deducing in its entirety what the moderns call the doctrine of the first motion, which at present is derived from all sorts of motions of the starry sphere; and for determining the causes of the motions and phenomena of the sun and moon, as they have been carefully observed by scholars for the past two thousand years. I may merely mention, since I shall have occasion to deal with the topic more fully below,[147] that the motion of the great circle unquestionably affects the appearances of the other five planets. With so few motions and, as it were, with a single circle is so vast a subject comprehended.

In the doctrine of the first motion nothing need be changed. For, utilizing the properties of things which are interrelated, we shall determine the maximum obliquity and in the same way investigate the declinations of the remaining parts of the ecliptic, right ascensions, the theory of shadows and gnomons in all regions of the earth, the lengths of days, oblique ascensions, the rising and setting of the stars, etc. However, our hypotheses differ from those of antiquity in that in ours, as opposed to the views[148] of the ancients, no circle except the ecliptic is properly described by the imagination on the starry sphere. The other circles, to wit, the equator, the two tropics, arctics and antarctics, horizons, meridians, and all the others connected with the doctrine of the first motion, e.g., vertical circles, parallels of altitude, colures etc. are properly traced upon the globe of the earth, and transferred by a certain relation to the heavens.

In addition to the apparent daily revolution about the earth, which the sun shares with all the stars and the other planets, there are those phenomena related to the sun which Ptolemy and the moderns have attributed to the sun's own motions and also those which are observed to occur in connection with the

[145] See pp. 121, 123, above. [146] Pages 134-35. [147] Pages 168-85.
[148] Reading *praescriptum* (Th 469.9) instead of *praeceptum* (PII, 331.12).

shift of the solstitial and equinoctial points, the distance of the stars from them, and the motion of the apogee among the fixed stars. All these phenomena present themselves to our eyes as if the sun and the sphere of the stars move. For the way in which, according to common belief, these bodies emerge in the east or rise, gradually climb above the horizon until they reach the meridian, from which they descend in like manner, and then traverse the lower hemisphere, daily completing their diurnal revolutions, is caused, clearly enough, by the first motion which my teacher, in company with Plato,[149] assigns to the earth.

The sun seems to us to move in the order of the signs, and we persuade ourselves that by this motion it describes the ecliptic and determines the length of the year. But these phenomena can be produced by the second motion which my teacher attributes to the earth. For as the earth moves on the great circle and comes to a position between the constellation Libra and the sun, those of us who suppose the earth to be at rest think that the sun is in the constellation Aries, because a line drawn from the center of the earth through the sun to the sphere of the stars strikes that constellation. Then, as the earth advances to Scorpio, the sun seems to be in Taurus, and so to traverse the zodiac.[150] I assert, however, that with the sun at rest this motion is properly the earth's. And the sidereal year is the time in which the center of the earth or, in appearance, of the sun completes a single revolution from a star to the same star.

The third motion of the earth produces the regular, cyclic changes of season on the whole earth; for it causes the sun and the other planets to appear to move on a circle oblique to the equator, and the sun to appear to the several regions of the earth exactly as it would if the earth were by hypothesis at the center of the universe and the planets moved on an oblique circle. For on account of the above-mentioned motion of its

[149] Cf. above, p. 148, n. 142.

[150] Prowe's text (PII, 332.9) inserts *totum*, for which there is no warrant in the Basel edition of 1566 and Mästlin's editions of 1596 and 1621.

poles, the plane of the equator, in comparison with the sun, turns away from the plane of the ecliptic and returns toward it, or as the Greeks say λοξεύεται καὶ ἐγκλίνει.[151] Hence the same inclination of the equator to the ecliptic recurs at almost the same points on the ecliptic, and the poles of the daily rotation are always in very nearly the same spot on the starry sphere.

Now when the equator attains its greatest inclination to the plane of the ecliptic, that is, to the sun, the line drawn from the center of the sun to the center of the earth cuts a cone in the globe of the earth as it performs its daily rotation, thereby describing the tropics. Furthermore, when the plane of the equator returns to the plane of the ecliptic, that is, to the sun, all over the earth the equinox occurs, since the line of which I just spoke divides the globe of the earth along the equator into two hemispheres. But the other parallels of latitude are marked on the earth according as the motions of the equator away from and toward the sun (or to use Ptolemy's terms λόξωσις καὶ ἔγκλισις) are combined. The arctics and antarctics are described by their points of contact with the horizons.[152] The poles of the ecliptic, in the opinion of my teacher, describe the polar circles equidistant from the poles of the equator. The great circle of the earth's globe which passes through the poles of the equator and the aforesaid equidistant poles of the ecliptic is the solstitial colure; and another great circle, intersecting the first in the poles of the equator at spherical right angles, is the equinoctial colure. And it is to be understood that in this manner the circles of any point at all and any other circles whatsoever are readily traced upon the earth and thence transferred to the overarching heavens.

Moreover, in obedience to the command given by the observations, the globe of the earth has risen to the circumference of the eccentric, while the sun has descended to the center of the universe. Now, in the common hypotheses the center of the eccentric was situated in our age between the center of the entire universe (which in these hypotheses was also the center

[151] For example, *Syntaxis* xiii.1 (HII, 528.11-16) and xiii *passim*.
[152] Cf. Th 74.23-28.

of the earth) and the constellation Gemini. Conversely, in my teacher's hypotheses the center of the great circle, which I referred to in the beginning of this *Account*[153] as the center of the eccentric, is found between the sun, which is the center of the universe according to my teacher, and the constellation Sagittarius; and the diameter of the great circle that passes through the center of the earth represents the line of mean motion of the sun. Since the line drawn from the center of the earth through the center of the sun to the ecliptic determines the true place of the sun, it is not difficult to see how in the system of Ptolemy and the moderns the sun is conceived to move unequally in the ecliptic and how the angle of inequality from the mean motion is investigated geometrically. When the earth is in the higher apse of the great circle, the sun is thought to be at the apogee on the eccentric, and, conversely, when the earth is in the lower apse, the sun seems to be in perigee.

But the manner in which the fixed stars appear to alter their distance from the equinoctial and solstitial points, and the greatest obliquity of the sun to vary, etc. (my treatment of these topics at the beginning of the *Account* is drawn from Book III of my teacher's work) has been shown by him to depend on the motion in declination, which I have set forth in a general way, and on two mutually interacting librations. From the poles which were referred to just above as the equidistant poles of the ecliptic, in both hemispheres let 23°40′[154] of a great circle be measured off, and let two points be marked there in order to designate the poles of the mean equator. Let the two colures be drawn in the proper manner to indicate the mean solstices and equinoxes. For purposes of study, let these points be imagined and indicated on a small sphere which encloses the globe of the earth and which, by its uniform motion, produces the third motion assigned to the earth.

Now, with the center of the earth between the sun and the constellation Virgo, let the mean equator be inclined or oblique to the sun, and let the line of the true place of the sun pass

[153] Page 121.
[154] The mean value between the maximum of 23° 52′ and minimum of 23° 28′.

through the common intersection of the plane of the ecliptic, the mean equator, and the mean equinoctial colure. Let the mean vernal equinox and true vernal equinox occur simultaneously where required by the scheme of motions, as will be crystal clear from what follows. The center of the earth advances from its position $59'8''11'''$[155] each day with uniform motion as reckoned by the fixed stars. In addition to this motion of the center of the earth, let the mean vernal point move an equal distance in precedence; and since it moves at a slightly faster rate, let it describe an angle greater by about $8'''$. This is the reason why I said just above that the motion in declination is almost exactly equal to the uniform motion of the center of the earth as reckoned by the fixed stars. But there is a continual increase in the angle made by the vernal point of the mean equator as compared with the center of the earth (in accordance with the rule given above). Hence, before the center of the earth finally returns to the point on the ecliptic whence it set out, the line of the true place of the sun reaches the mean equinox, and the stars seem to us to move with a mean or uniform motion in consequence, to the amount of the precession. This precession, as I stated in the beginning,[156] is about $50''$[157] in an Egyptian year, and in 25,816 Egyptian years it performs a complete revolution. Thus it is clear what the mean equinox is, what the mean precession is, and how these phenomena can be presented to the eyes as though by a mechanical device.

Librations

Let there be[158] a straight line *a b* of finite length, for example $24'$, divided at *d* into two equal parts. Then with the point of the compass placed at *d*, describe a circle *c e* with the radius *d c* directed to *a* and $6'$ long (that is, a quarter of the entire length). Construct a second circle of the same size in this

[155] Although PII, 334.14 and Th 471.12 give $59'\ 8''\ 2'''$, the correct reading is unquestionably $59'\ 8''\ 11'''$ (cf. Th 196.9-11, 198.5). The error undoubtedly arose because 11 was interpreted as a Roman numeral, whereas it was Arabic (cf. 1566 ed., p. 205v; 1596 ed., p. 124; and 1621 ed., p. 123).

[156] Pages 113-14. [157] $8''' \times 365 = 48\frac{2}{3}''$. [158] Reading *Sit* (Th 471.27).

figure;[159] and let the two small circles (to use this term for the moment) be so placed that each is attached to the circumference of the other and can move freely about its own center. Call that circle the first which carries the other on its circumference, and let it be fastened to the center of the line *a b* at the point *d*. Denote the center of the second small circle by *f*, and any point chosen at random on its circumference by *h*. Place the point *h* of the second small circle upon *a*, the end of the given line; and *f* upon *c*. Let *h* describe in one direction, about *f* as center, an angle twice as great as that described in equal time by *f* about *d* in the opposite direction. Clearly, then, in one revolution of the first small circle the point *h* twice describes and traverses the line *a b*, and the second small circle revolves twice.[160]

While thus describing a straight line through the combination of two circular motions, the point *h* moves most slowly near the ends *a* and *b*, and more rapidly near the center *d*. It has therefore pleased my teacher to name this motion of the point *h* along the line *a b* a "libration," because it resembles the motion of objects hanging in the air.[161] It is also called

[159] Prowe altered "ab" (Th 471.30) to *a b* (PII, 335.6), thereby making a difficult passage hopeless.

[160] For a detailed explanation of this device see above, p. 88, n. 100.

[161] In the corresponding passage of *De rev.* Copernicus wrote *motus . . . pendentibus similes librationibus* and *pendentium instar* (Th 163.13,19). Menzzer (p. 136) rendered the former expression by: "Pendelschwingungen ähnliche Bewegungen" (motions like the swinging of a pendulum); and the latter by: "den Pendeln ähnlich" (like a pendulum). Accepting Menzzer's interpretation, Dreyer stated that the librations were so named because they are "like the motion of a pendulum" (*Planetary Systems*, p. 330).

Are we justified in attributing the pendulum to Copernicus? I think not. In the sentence under discussion Rheticus's language is *ad similitudinem pendentium in aere*, "it resembles the motion of objects hanging in the air." This formulation is modeled after a phrase used by Copernicus in a wholly different context, *in aere pendentibus* (Th 22.14). Here Menzzer (p. 21) translated by: "in der Luft Schwebende" (objects suspended in the air). We may safely conclude that Copernicus is not referring to the pendulum, but in general to the kind of motion which is quickest in the middle and slowest at the ends (cf. p. 118, above).

E. Wiedemann corrected a false attribution of the pendulum to the Arabs (*Verhandlungen der deutschen physikalischen Gesellschaft*, XXI[1919], 663-64 and *Zeitschrift für Physik*, X[1922], 267-68); his strictures were overlooked by Edmund Hoppe, *Geschichte der Physik* (Braunschweig, 1926), p. 25.

motion along the diameter; for if you imagine a circle with diameter *a b* and center *d*, the position on the diameter *a b*, to which the point *h* is brought by the aforesaid combined motion of the small circles, is determined from the doctrine of chords; and by this method the table of prosthaphaereses[162] is constructed.

My teacher calls the motion of the first small circle about *d* the anomaly, since the prosthaphaeresis is derived from this motion.[163] Thus let *f*, the center of the second small circle, describe an angle by starting from the point *c*[164] and moving to the left on the circumference of the first small circle; let the angle *c d f* be 30°. The line *d f g*, drawn from the center *d*, will cut off, on the circumference of the circle *a b*, an arc *a g* of the same number of degrees as the arc *c f* of the first small circle. Since the point *h* of the second small circle moves from *g* to the right at twice the speed of *f*, a straight line drawn from the point *g* to the point *h* clearly subtends half of double the arc *a g*, and *h d* half of double the arc which remains when the arc *a g* is subtracted from a quadrant.[165] Therefore *a h*, that is, the distance of *h* from *a* along the diameter *a b*, is 1,340[166] units, of which the radius constitutes 10,000. But if *a b* is di-

[162] The varying differences between an apparent and mean motion. When the mean motion is smaller than the apparent, the difference is added (prosthesis) to the mean motion, in order to get the apparent motion; conversely, when the mean motion is greater than the apparent, the difference is subtracted (aphaeresis). The Latin equivalent for this Greek term is *aequatio* (Th 180.14-19).

[163] Reading *motu* (Th 472.17) instead of *motus* (PII, 336.18).

[164] Copernicus's discussion of this topic (*De rev.* iii,4) is accompanied by a diagram, which Rheticus follows, save that he interchanges *c* with *d*, and *g* with *h*. Now the first three editions of the *Narratio prima* contained no figures, and Mästlin supplied them from *De rev.* To eliminate disagreement between Copernicus's diagram and Rheticus's lettering, Mästlin adopted the simple expedient of transposing the letters in the diagram. But Prowe, following Th, resolved to adhere faithfully to Copernicus's figure, and therefore to alter the text of the *Narratio prima* wherever necessary. In the present instance *d* was left unchanged, although it should have been replaced by *c* (Th 472.18; PII, 336.19).

[165] Cf. Th 167.4-7. If we employ the notation used by Manitius in his *Ptolemäus Handbuch* (I, 47 n), we should write:

$$\mathrm{gh} = \tfrac{1}{2}\, s2b\ \mathrm{ag}$$
$$\mathrm{hd} = \tfrac{1}{2}\, s2b\ (90^\circ - \mathrm{ag}).$$

[166] Because *h d*, subtending an arc equal to 60° on the circle *a b*, is 8,660 (Th 49.37): 10,000 − 8,660 = 1,340.

vided into 60 units, *a h* will be 4,[167] and *h b* 56. Then by taking the proportional part of 24′, we shall know the position of the point *h* on the given finite straight[168] line in this case.

Now that we understand this argument in a rough way, it will be easy to see how the greatest obliquity of the equator to the plane of the ecliptic varies and how the true precession of the equinoxes becomes unequal. Since short arcs do not differ sensibly from straight lines, let us begin by imagining that the point *d* is placed upon the north pole of the mean equator and that the line *a b* is an arc of the mean solstitial colure. Lying between the north pole of the mean equator and the nearby pole,[169] which is one of the poles that move at a uniform distance from the poles of the ecliptic, *b*[170] marks the least distance of the pole of the daily rotation, or pole of the earth, from the aforesaid pole of the ecliptic.[171] And *a*, lying between the north pole of the mean equator and the plane of the ecliptic, marks the greatest distance of the pole of the earth from the pole of the ecliptic. Then with the two small circles properly fitted into place by means of the line *a b*, it may be understood what part of the 24′ of the line *a b* is described at the present time by the north pole of the earth in the point *h* by reason of the combined motion of the two small circles. Observing the law of opposition, the south pole moves by a similar device, as the shifting universe alters the greatest obliquity.

Assume that the first small circle completes its revolution in 3,434[172] Egyptian years and that the terminus from which

[167] Since *a b* = 60, the radius = 30. And 1,340 : 10,000 = 4.02 : 30.

[168] Reading *rectae* (Th 472.27) instead of *recta* (PII, 336.30); and *subsistat.*

[169] The true pole of the equator.

[170] Reading *quare et* (Th 472.36) instead of *Quare* b *est* (PII, 337.7).

[171] In a note Prowe cites this passage as it appeared in the first edition and declares it to be corrupt. It is, however, entirely sound. But a textual difficulty was introduced by the 1566 edition, which gives *et* instead of *ab* after *terrae* (p. 206r); and the difficulty was aggravated by Th, which keeps *et* and inserts *a* after *dictum* (472.36-37).

[172] Reading *IIIMCCCCXXXIIII* (Th 473.4) instead of *XXXIIIIMCCCCXXXIIII* (PII, 337.17). From two other passages in the *Narratio prima* (PII, 302.5-6, 339.5-7), Prowe should have seen that the number he gives here is incorrect (cf. p. 117, above and p. 158, below).

the motion of anomaly begins is the point *a* on the circumference of the circle whose diameter is described by the first libration. If the poles of the earth had no libration other than this one, and did not deviate from the mean solstitial colure, it will be clear at once to anyone that only the angle of inclination of the plane of the true equator to the plane of the ecliptic would vary on account of this motion of the poles of the earth, decreasing when they move from *a* through *d* to *b* and increasing while they complete the opposite movement from *b* through *d* to *a*; and that hence no inequality would appear in the precession of the equinoxes.

However, it is certainly clear from the observations that the true equinoctial points move 70′ to either side of the mean equinoctial points in the greatest prosthaphaeresis and that the change in the obliquity takes twice as long as this motion. My teacher was therefore persuaded to introduce,[173] in addition to the first, a second lesser libration, whereby the poles of the earth deviate from the mean solstitial colure toward the sides of the universe in such a way that the arc or straight line *a d b* of the second libration forms four right angles with the mean solstitial colure. In the north let *a* lie to the right side of the universe, *b* to the left; and in the south *a* to the left, *b* to the right. Through the points *h* of the first libration let *d* of the second libration describe lines of 24′ to either side of *a d b*. Finally, let the poles of the earth be in reality fixed to the points *h* of the second libration, and let them be deflected by the second libration only 28′ to either side of the said colure, with *a* and *b* taken as the outermost points. For when the poles are at these points, the true solstitial colure makes with the mean solstitial colure an angle not perceptibly greater than 70′.

Now the prosthaphaereses of precession must be taken in relation to the mean vernal point. Hence my teacher's analysis of the second libration deals with the relation of the true vernal point to the mean, especially since this method of examining the prosthaphaereses is rather easy. Then the line *a b* will be 140′ long; and it will be so placed that it corresponds to the

[173] Reading *ad* before *constituendam* (Th 473.15; PII, 338.6).

north line of the second libration, with *d* at the mean vernal point, the true vernal point at *h*, and the radius of either small circle 35′. Moreover, the terminus from which the motion begins is the mean vernal point, from which the true vernal point moves to the right toward *a*. But the anomaly is measured from the northernmost point of the circle whose diameter is described by the true vernal point; and the northernmost point is marked on the circumference of the circle by the mean equinoctial colure. And since in one cycle of the obliquity the inequality of the precession is twice completed, the anomaly of the second libration has a period of 1,717 Egyptian years.[174] Therefore the anomaly of the obliquity, as taken from the tables and doubled, equals the anomaly of the precession. The name "simple anomaly" is given to the former, "double anomaly" to the latter.

But if the second libration alone were to be assumed, the angle of inclination of the planes of the true equator and ecliptic clearly would not vary; and this would be a serious fault, for every inequality of the phenomena would be observed only in connection with the inequality of the precession of the equinox.[175] However, since both librations occur together and since, as has been said, their motions interact, the poles of the earth describe about the poles of the mean equator the figure of twisted rings.[176]

When the poles of the earth cross the mean solstitial colure, the true[177] colure lies in the same plane with the mean, and the true vernal point coincides with the mean; however, unless the poles of both equators coincide, the planes of the equators and of the mean and true solstitial and equinoctial colures do not[178] completely coincide. Now, when the north pole lies between *d* of the second libration and *a*, the outermost point to the right, the south pole occupying the opposite point, the true equinox

[174] Cf. above, p. 156, n. 172.

[175] Omitting *veri* (Th 474.5-6; PII, 339.14).

[176] That is, the figure 8 (cf. Th 163.30–165.15).

[177] Reading *verus* (Th 474.9) instead of *versus* (PII, 339.19).

[178] Reading with Mästlin (1596 ed., p. 128; 1621 ed., p. 126) *non* before *omnino* (PII, 339.23).

follows the mean, and the sun comes to the mean equator before it comes to the true. But when the poles of the earth cross over to the opposite sides of the universe, so that the north pole lies to the left of the mean solstitial colure and the south to the right, the true equinox precedes the mean, and the sun meets the true equator before it meets the mean. Besides, when the poles of the earth move from *a* toward *b*, the tropical year decreases, because the true equinox advances, as it were, to meet the sun; but when the poles move from *b* toward *a*, since the equinox, as it were, flees from the sun, the tropical year increases. And when the poles of the earth are near *d*, for a brief span of years the increase or decrease in the year is distinctly perceptible. Moreover, since the apparent motion of the fixed stars is bound up with the length of the tropical year, in the same way the motion in precedence of the solstitial and equinoctial points among the fixed stars is observed as swifter and slower.

So far as the solar apogee is concerned,[179] and the distance of the vernal equinox from it, the conclusions which in the beginning[180] I drew from the observations in accordance with my teacher's opinion are clarified by the preceding discussion. The motion of the apogee in the ecliptic depends on the motion of the center of the small circle and on the uniform motion of the center of the great circle in the circumference of the small circle. The diameter of the great circle or ecliptic that passes through the centers of the sun and small circle is the mean apse-line of the sun; but the diameter through the centers of the sun and great circle is the true apse-line. The center of the great circle is found between the sun and the point on the ecliptic where the sun is thought to be in perigee.[181] Similarly, the center of the small circle is situated between the point of mean perigee and the sun.

In the time of Ptolemy the true apse-line was at one end, the point of apparent apogee, 57°50′ from the first star of

[179] Reading with Mästlin *ad* instead of the first *ab* (1621 ed., p. 127; PII, 340.13).

[180] Pages 119-26.

[181] The point of apparent perigee.

Aries; and at the other end, the perigee, 237° 50′.[182] But for the mean apse-line this distance was 60° 16′, and in the opposite point, 240° 16′. For, starting from that point on the small circle which is at the greatest distance from the center of the sun, the center of the great circle had moved about 21⅓° in precedence; and the simple anomaly, that is, the anomaly of the obliquity, had at that time an equal value.[183] But since the center of the small circle moves uniformly about the center of the sun, and the center of the great circle moves uniformly on the circumference of the small circle, the higher apse of the sun, at the time of the observation made by my teacher, was found to be 69° 25′ from the first star of Aries.[184] Because at that time the simple anomaly was almost exactly 165°, the prosthaphaeresis was determined as almost exactly 2° 10′,[185] and the center of the small circle fixed the point of mean perigee between the sun and 251° 35′.[186] Furthermore, the eccentricity of the great circle, or eccentric of the sun if this term is preferred, which Ptolemy computed as 1/24 of the radius of the great circle, is in our time about 1/31,[187] as the observations show, and as is readily deduced if the hypotheses of my teacher are adopted and mathematics applied.

The manner in which the eccentricities of the five planets vary on account of the motion of the center of the great circle on the small circle, as I pointed out in the reasons for revising the hypotheses,[188] can be understood with no great effort. In the investigation of the five planets two considerations are of special importance: first, in what manner and to what extent the center of the earth approaches to or withdraws from the centers of the deferents of the planets; second, what relation this ap-

[182] For the true apogee was 64° 30′ from the true equinox (see p. 125, above);
subtract 6° 40′, the true precession (see p. 116, above)
57° 50′ + 180° = 237° 50′ for the perigee.

[183] Hence the prosthaphaeresis was about 2° 26′ (Th 224.14-15): 57° 50′ + 2° 26′ = 60° 16′ + 180° = 240° 16′.

[184] Th 221.23-28.

[185] Th 221.28-30, 224.32.

[186] 69° 25′ + 2° 10′ + 180° = 251° 35′.

[187] Cf. above, p. 61, n. 9.

[188] See p. 136.

proach or withdrawal bears to the radius of the deferent of each planet. The causes will not be far to seek.

In the case of Saturn the entire diameter of the small circle has no perceptible ratio whatever to the radius of the deferent, since Saturn is the first planet beneath the starry sphere. Hence observations can reveal no variation in the eccentricity of Saturn. As for Jupiter, its apogee is about a quadrant from the apogee of the sun. Hence the motion of the center of the great circle produces no observable change in the eccentricity at the present time, even though the ratio of the diameter of the small circle to the radius of the deferent is perceptible and measurable. And this is the reason why in the case of Mercury also no change is observed in the eccentricity, since its apogee is at a similar distance from the apogee of the sun.

Because the apogee of Mars is about 50° to the left of the sun's apogee, and the apogee of Venus 42° to the right, the centers of their deferents are suitably placed to reveal the change in the eccentricity;[189] and the diameter of the small circle has a perceptible ratio to the deferent of each. By a trigonometrical analysis of the observations of these two planets, my teacher found that the eccentricity of Mars has decreased by 1/42, of Venus by 1/5,[190] on account of the approach of the center of the great circle to the sun.

Lest any of the motions attributed to the earth should seem to be supported by insufficient evidence, our wise Maker expressly provided that they should all be observed equally perceptibly in the apparent motions of all the planets; with so few motions was it feasible to satisfy most of the necessary phenomena of nature. Therefore the motion of the center of the great circle affects not only the sun and the planets revolving about it but also the phenomena of the moon. For Ptolemy

[189] Cf. Th 327.16-18.

[190] Prowe's text (PII, 342.9) omits *partem* before *propter* (Th 475.35). According to Copernicus's findings, the eccentricity of Mars had decreased from 1500 to 1460 (cf. above, p. 77, n. 58), a decrease of 1/37 or 1/38 rather than 1/42, as Rheticus has it. As for the eccentricity of Venus, Copernicus explicitly reports a diminution of somewhat more than 1/6, not 1/5 (from 416 to 350; cf. Th 369.8-11).

computed the greatest distance of the sun from the earth to be 1,210 units, of which the radius of the earth is one, and the axis of the earth's shadow 268;[191] and my teacher shows that in our time the greatest distance of the sun from the earth is 1,179 units, and the axis of the cone of shadow 265.[192] But I have decided to reserve the other related topics[193] for a "Second Account" to follow this one, wherein I shall examine the motions and phenomena of the sun and moon by the light of the change in the hypotheses.

THE SECOND PART OF THE HYPOTHESES

The Motions of the Five Planets

When I reflect on this truly admirable structure of new hypotheses wrought by my teacher, I frequently recall, most learned Schöner, that Platonic dialogue which indicates the qualities required in an astronomer and then adds "No nature except an extraordinary one could ever easily formulate a theory."[194]

When I was with you last year and watched your work and that of other learned men in the improvement of the motions of Regiomontanus and his teacher Peurbach, I first began to understand what sort of task and how great a difficulty it was to recall this queen of mathematics, astronomy, to her palace, as she deserved, and to restore the boundaries of her kingdom. But from the time that I became, by God's will, a spectator and witness of the labors which my teacher performs with energetic mind and has in large measure already accomplished, I realized that I had not dreamed of even the shadow of so great a burden of work. And it is so great a labor that it is not any hero who can endure it and finally complete it. For this reason, I suppose,

[191] HI, 425.17-21.

[192] Th 282.25-26.

[193] Omitting *his* (PII, 342.24; Th 476.6).

[194] *Epinomis* 990B; θεωρῆσαι here means "observe" rather than "theorize," as Rheticus interpreted it. The authenticity of the *Epinomis* is disputed; for the view that it is genuine see J. Harward, *The Epinomis of Plato* (Oxford, 1928), pp. 26-58 and, for the opposing view, J. Geffcken, *Griechische Literaturgeschichte* (Heidelberg, 1926–34), II, 174-76.

the ancients related that Hercules, sprung of Jupiter most high, no longer trusting his own shoulders, replaced the heavens upon Atlas, who, being long accustomed to the burden, resumed it with stout heart and undiminished vigor, as he had borne it in former days.

Moreover, divine Plato, master of wisdom as Pliny styles him,[195] affirms not indistinctly in the *Epinomis* that astronomy was discovered under the guidance of God.[196] Others perhaps interpret this opinion of Plato's otherwise. But when I see that my teacher always has before his eyes the observations of all ages together with his own, assembled in order as in catalogues; then when some conclusion must be drawn or contribution made to the science and its principles, he proceeds from the earliest observations to his own, seeking the mutual relationship which harmonizes them all; the results thus obtained by correct inference under the guidance of Urania he then compares with the hypotheses of Ptolemy and the ancients; and having made a most careful examination of these hypotheses, he finds that astronomical proof requires their rejection; he assumes new hypotheses, not indeed without divine inspiration and the favor of the gods; by applying mathematics, he geometrically establishes the conclusions which can be drawn from them by correct inference; he then harmonizes the ancient observations and his own with the hypotheses which he has adopted; and after performing all these operations he finally writes down the laws of astronomy—when, I say, I behold this procedure, I think that Plato must be understood as follows.

The mathematician who studies the motions of the stars is surely like a blind man who, with only a staff to guide him, must make a great, endless, hazardous journey that winds through innumerable desolate places. What will be the result? Proceeding anxiously for a while and groping his way with his staff, he will at some time, leaning upon it, cry out in

[195] *Natural History* vii.30(31).110.

[196] This proposition is not expressly formulated anywhere in the *Epinomis*, but is derived from the argument in 989D-990A. For the question of the authenticity of the *Epinomis* see n. 194, above.

despair to heaven, earth, and all the gods to aid him in his misery. God will permit him to try his strength for a period of years, that he may in the end learn that he cannot be rescued from threatening danger by his staff. Then God compassionately stretches forth His hand to the despairing man, and with His hand conducts him to the desired goal.

The staff of the astronomer is mathematics or geometry, by which he ventures at first to test the road and press on. For in the examination from afar of those divine objects so remote from us, of what avail is the strength of the human mind? Of what avail[197] dim-sighted eyes? Accordingly, if God in His kindness had not endowed the astronomer with heroic ambitions and led him by the hand, as it were, along a road otherwise inaccessible to the human intellect, the astronomer would not be, I think, in any respect better circumstanced and more fortunate than the blind man, save that trusting in his reason and offering divine honors to his staff, he will one day rejoice in the recall of Urania from the underworld. When, however, he considers the matter aright, he will perceive that he is not more blessed than Orpheus, who was aware that Eurydice was following him as he danced his way up from Orcus; but when he reached the jaws of Avernus, she whom he dearly longed to possess disappeared from view and descended once more to the infernal regions. Let us then examine, as we set out to do, my teacher's hypotheses for the remaining planets, to see whether with unremitting devotion and under the guidance of God, he has led Urania back to the upper world and restored her to her place of honor.

With regard to the apparent motions of the sun and moon, it is perhaps possible to deny what is said about the motion of the earth, although I do not see how the explanation of precession is to be transferred to the sphere of the stars. But if anyone desires to look either to the principal end of astronomy and the order and harmony of the system of the spheres or to

[197] Reading *quid* (Th 477.15) instead of *quam* (PII, 344.24). In a note, Prowe attributes the change from *quam* to *quid* to Mästlin; but the 1566 edition has *quid* (p. 207v).

ease and elegance and a complete explanation of the causes of the phenomena, by the assumption of no other hypotheses will he demonstrate the apparent motions of the remaining planets more neatly and correctly. For all these phenomena appear to be linked most nobly together, as by a golden chain; and each of the planets, by its position and order and every inequality of its motion, bears witness that the earth moves and that we who dwell upon the globe of the earth, instead of accepting its changes of position, believe that the planets wander in all sorts of motions of their own. And if it is possible anywhere else to see how God has left the universe for our discussion, it surely is eminently clear in this matter. No one can be affected, I think, by the argument that God permits Ptolemy and other famous heroes to dissent on this point. For it is not the sort of opinion which Socrates in the *Gorgias*[108] declares to be evil for men; and it does not cause any harm to either the science itself or the divining art derived therefrom.

The ancients attributed to the epicycles of the three superior planets the entire inequality of motion which they discovered that these planets had with respect to the sun. Then they saw that the remaining apparent inequality in these planets did not occur simply on the theory of an eccentric. The results obtained by calculating the motions of these planets in imitation of the hypotheses for Venus agreed with experience and the observations. Hence they decided to assume for the second apparent inequality a device like that which their analyses established for Venus. As in the case of Venus, the center of the epicycle of each planet was to move at a uniform distance from the center of the eccentric, but at a uniform rate with respect to the center of the equant; and this point was to be the center of uniform motion also for the planet, as it moved on the epicycle with its own motion, starting from the mean apogee. So long as the ancients strove to retain the earth in the center of the universe,

[108] 458A; Rheticus is quoting not the original Greek but a Latin translation. Copernicus used the translation of Marsilio Ficino (*Stromata Copernicana*, pp. 306–7). But *perniciosas* in our text shows that Rheticus used Simon Grynaeus's revision of Ficino's translation, for Grynaeus replaced Ficino's *malum* by *perniciosum* (Basel, 1532, p. 342).

they were compelled by the observations to affirm that, just as Venus revolved with its own special motion on the epicycle, but by reason of the eccentric advanced with the mean motion of the sun, so conversely the superior planets in the epicycle were related to the sun, but moved with special motions on the eccentric. But in[199] the theory of Mercury, the ancients thought that they had to accept, in addition to the devices which they deemed adequate to save the appearances of Venus, a different position for the equant, and revolution on a small circle for the center from which the epicycle was equidistant. All these arrangements were shrewdly devised, like most of the work of antiquity, and would agree satisfactorily with the motions and appearances if we granted that the celestial circles admit an inequality about their centers—a relation which nature abhors—and if we regarded the especially notable first inequality of apparent motion as essential to the five planets, although it is clearly accidental.

Moreover, in the latitudes of the planets, the ancients seem to neglect the axiom that all the motions of the heavenly bodies either are circular or are composed of circular motions; unless perhaps it is proposed to explain the reflexions and declinations of Venus and Mercury, the inclinations[200] of the epicycles in the three superior planets, and the deviations in the inferior planets by motions in libration, as was done just above for the earth's motion in declination. We may admit this for the reflexions and declinations of Venus and Mercury, inasmuch as the angles of inclination of the planes of their eccentrics and epicycles remain everywhere unchanged. But common calculation shows that the inclinations of the epicycles in the three superior planets, and the deviations of Venus and Mercury do not occur through librations. Let me speak only of the deviations. The proportional minutes, by which we compute the deviations in relation to the distance of the center of the epicycle from the nodes and apsides, have been investigated and determined by the same method by which the declinations

[199] Omitting *in* (PII, 346.19; Th 478.21).

[200] Reading *declinationes* (Th 478.33) instead of *declinationis* (PII, 347.3).

of the parts of the ecliptic are examined in the doctrine of the first motion. Therefore, when the center of the epicycle of Venus is 60° from any of the apsides of the eccentric, we infer a deviation of 5′, and for Mercury 22½′. But if the deferent were assumed to oscillate by means of librations, true science would require for this position of the epicycle of Venus a deviation not greater than 2½′ and for Mercury 11¼′. For in this position of the center of the epicycle the angle of inclination of the plane of the eccentric to the plane of the ecliptic would be found not greater than 5′ for Venus and 22½′ for Mercury, on account of the properties of motion in libration. Perhaps for this reason John Regiomontanus thought it advisable to caution his readers that calculation of latitudes is concerned only with the approximate truth.[201]

Finally, as Aristotle points out at length in another connection,[202] men by nature desire to know. Hence it is quite vexing that the causes of phenomena are nowhere else so hidden and wrapped, as it were, in Cimmerian darkness, a feeling which Ptolemy shares with us. Concerning the hypotheses of the ancients for the five planets I shall say no more for the present than is required perhaps by an explanation of the new hypotheses (if I may so term them) and a comparison of them with the ancient hypotheses. I sincerely cherish Ptolemy and his followers equally with my teacher, since I have ever in mind and memory that sacred precept of Aristotle: "We must esteem both parties but follow the more accurate."[203] And yet somehow I feel more inclined to the hypotheses of my teacher.

[201] As Mästlin (1596 ed., p. 136) indicates, the reference is to the *Epitome*, Book XIII, Prop. 21: "But to find the inclinations of this kind for every position of the epicycle on the eccentric is no mean task. Hence attention was necessarily directed toward another means whereby the latitudes for the remaining positions of the epicycle would be readily determined approximately."

[202] *Metaphysics* i.1 980a21. Rheticus is quoting, not the original Greek, but some Latin translation which is neither the *antiqua translatio* nor Bessarion's; cf. above, p. 142, n. 133.

[203] *Metaphysics* xii.8 1073b16-17 (W. D. Ross's translation). The precept perhaps came to the attention of Rheticus because it was quoted in Simplicius's *Commentary on Aristotle's De caelo* (ed. Heiberg, 506.2-3); cf. above, p. 145, n. 139.

This is so perhaps partly because I am persuaded that now at last I have a more accurate understanding of that delightful maxim which on account of its weightiness and truth is attributed to Plato: "God ever geometrizes";[204] but partly because in my teacher's revival of astronomy I see, as the saying is, with both eyes and as though a fog had lifted and the sky were now clear, the force of that wise statement of Socrates in the *Phaedrus:* "If I think any other man is able to see things that can naturally be collected into one and divided into many, him I follow after and 'walk in his footsteps as if he were a god.' "[205]

The Hypotheses for the Motions in Longitude of the Five Planets

What has been said thus far regarding the motion of the earth has been demonstrated by my teacher. Consequently (as I pointed out[206] in the reasons for revising the hypotheses) the entire inequality in the apparent motion of the planets which seems to occur in their positions with respect to the sun[207] is caused by the annual motion of the earth on the great circle. It likewise follows that the planets in reality have a single inequality, which is observed in relation to the parts of the zodiac, and is one of the two recognized heretofore. Hence only those hypotheses are acceptable which can explain both inequalities of motion. Just as my teacher chose to employ an epicycle on an epicycle for the moon,[208] so, for the purpose of demonstrating conveniently the order of the planets and the measurement of their motion, he has selected, for the three superior planets, epicycles on an eccentric, but for Venus and Mercury eccentrics on an eccentric.

[204] This celebrated maxim is not found in the Platonic corpus. See Plutarch's *Moralia: Quaestiones convivales* Book viii, Question 2 (ed. Bernardakis, IV [Leipzig, 1892], 307.11–308.2). The first edition of this work (Venice, 1509) was available to Rheticus; the passage cited appears on p. 882.

[205] *Phaedrus* 266B (H. N. Fowler's translation, Loeb Classical Library, 1913).

[206] Page 138.

[207] A Greek phrase borrowed from Ptolemy; cf., e.g., HII, 250.14-15.

[208] Cf. above, pp. 68-69, 134.

Now since we look up at the motions of the three superior planets as from the center of the earth, but regard the revolutions of the inferior planets as below us, the centers of the deferents of the planets may properly be brought into relation with the center of the great circle; and from this point we may then quite correctly transfer all the motions and phenomena to the center of the earth. Therefore there must be understood for the five planets an eccentric, the center of which lies outside the center of the great circle.

But to gain a better understanding of the method of establishing the new hypotheses, in short to place everything in an increasingly clearer light, let us suppose first that the planes of the eccentrics of the five planets are in the plane of the ecliptic, and that the centers of the deferents and equants are related to the center of the great circle, as with the ancients they were related to the center of the earth. Then let us divide into four equal parts the distances between the center of the great circle and the points or centers of the equants. Next let us place the center of the eccentric of each of the three superior planets at the third dividing point, as you move upward from the center of the great circle toward the apogee. With the remaining fourth part as radius, let us describe an epicycle with its center on the circumference of the eccentric, and the scheme of real motion in longitude will become apparent for each of these planets.[209]

Then, in the opinion of my teacher, as the epicycle revolves, the planet moves in its upper circumference in consequence, in its lower in precedence, so that when the center of the epicycle is in the apogee of the eccentric, the planet is found in the perigee of the epicycle; and conversely, when the center of the epicycle is in the perigee of the eccentric, the planet is in the apogee of the epicycle. By this similarity of motions, the planet completes its periods on the epicycle in equal time with the center of the epicycle on the eccentric. If the equants are removed, the inequality in the motion of the superior planets with respect to the center of the great circle is clearly regular

[209] Cf. above, p. 74, n. 50.

and composed of uniform motions. For the epicycle assumed in this theory succeeds to the function of the equant; and the eccentric describes equal angles about its own center in equal times, while the planet, moving on the epicycle to which it is attached, likewise describes equal angles about the center of the epicycle in equal times.

But the motion of Venus will be established as follows. Rejecting the deferent, which is replaced by the great circle, describe a small circle about the third dividing point, with the remaining quarter of the line as radius. Then let the center of the epicycle of Venus, which will here be called eccentric on the eccentric, second eccentric, and movable eccentric, move on the circumference of the said small circle[210] according to this law, that whenever the center of the earth crosses the apse-line, the center of the eccentric is in the point of the small circle that is nearest to the center of the great circle; and whenever the earth is midway on its circle between the two apsides, the center of the eccentric of Venus is in the point of the small circle that is most remote from the center of the great circle. The center of the eccentric moves in the same direction as the earth, that is, in the order of the signs; but, as follows from the foregoing, it revolves twice in each period of the earth.

While the scheme of motions for Mercury agrees in general with the theory of Venus, on account of the remaining inequality, there is an additional epicycle,[211] whose diameter Mercury describes by a libration. To put the scheme in terms of the earth's motion, the length of the radius of the movable deferent is 3,573,[212] the eccentricity of the first deferent 736, the length of the radius of the small circle, which carries the movable center of the deferent, 211, and the diameter of the said epicycle 380 units, of which 10,000 constitute the line from the center of the great circle to the center of the earth. But in the motion of Mercury the following law is observed:

[210] Cf. above, p. 81, n. 69.

[211] Cf. above, p. 88, n. 96.

[212] Rheticus has chosen to give the minimum value (cf. Th 382.27–383.2, and above, p. 86, n. 90).

the center of the movable eccentric, in contrast with the case of Venus, is most remote from the center of the great circle whenever the earth is in the line of the planet's apsides; and nearest, whenever the earth is at a quadrant's distance from the apsides of the planet. Mercury will have, as is apparent, a fixed epicycle. The diameter of this epicycle is directed to the center of the movable deferent and is described by a motion in libration of the planet moving along it in a straight line according to the following law. Whenever the center of the movable eccentric is most remote from the center of the great circle, the planet is in the perigee of the epicycle, which is the lower limit of the diameter described by the planet. Conversely, Mercury is at the other limit, which may be called the apogee, whenever the center of the movable eccentric is nearest to the center of the great circle. But the motions of the apsides of the planets, like certain other topics, are reserved for the "Second Account."

The foregoing is very nearly the whole system of hypotheses for saving the entire real inequality of the motion in longitude of the planets. Therefore, if our eye were at the center of the great circle, lines of sight drawn from it through the planets to the sphere of the stars would, as the lines of the true motions, be rotated in the ecliptic by the planets exactly as the schemes of the aforementioned circles and motions require, so that they would reveal the real inequalities of these motions in the zodiac. But we, as dwellers upon the earth, observe the apparent motions in the heavens from the earth. Hence we refer all the motions and phenomena to the center of the earth as the foundation and inmost part of our abode, by drawing lines from it through the planets, as though our eye had moved from the center of the great circle to the center of the earth. Clearly it is from this latter point that the inequalities of all the phenomena, as they are seen by us, must be calculated. But if it is our purpose to deduce the true and real inequalities in the motion of the planets, we must use the lines drawn from the center of the great circle, as has been explained. To smooth our way through the topics in planetary phenomena which remain to be discussed and to make the whole treatise easier and more agreeable, let us imagine not only the lines of true apparent motion drawn from the center of the earth through the planets to the ecliptic but also those drawn from the center of the great circle and therefore properly called the lines of the inequality of motion.

When, as the earth advances with the motion of the great circle, it reaches a position where it is on a straight line between the sun and one of the three superior planets, the planet will be seen at its evening rising; and because the earth, when so situated, is at its nearest to the planet, the ancients said that the planet was at its nearest to the earth and in the perigee of its epicycle. But when the sun approaches the line of the true and apparent place of the planet—this occurs when the earth reaches the point opposite the above-mentioned position—the planet begins to disappear by setting in the evening and to attain its greatest distance from the earth, until the line of the true place of the planet passes also through the center of the sun. Then the sun lies between the planet and the earth, and the planet is occulted. After occultation, since the motion of the earth continues uninterrupted and since the line of the true place of the sun withdraws from the line of the true place of the planet, the planet reappears at its morning rising, when it has attained the proper distance from the sun required by the arc of vision.

Moreover, in the hypotheses of the three superior planets, the great circle takes the place of the epicycle attributed to each of the planets by the ancients. Hence the true apogee and perigee of the planet with respect to the great circle will be found on the diameter of the great circle prolonged to meet the planet. But the mean apogee and perigee will be found on the diameter of the great circle that moves[213] parallel to the line drawn from the center of the eccentric to the center of the epicycle. Since in the semicircle closer to the planet the earth approaches the planet, and in the other, opposite semicircle recedes from it, in the former semicircle the ends of the

[213] Reading *movetur* (Th 482.11) instead of *moventur* (PII, 352.11).

diameters of the great circle are the perigees, but in the latter the apogees. For the former semicircle takes the place of the lower part of the epicycle, but the latter, the upper.

Imagine that a conjunction of sun and planet is not far off. Let the center of the earth be in the true place of the apogee of the planet with respect to the great circle; and let the line of the real inequality coincide with the line of the apparent place of the planet. However, as the earth in its motion moves away from this position, the line[214] of the real inequality and the line of the true place of the planet begin to intersect in the planet. The former advances with the regular unequal motion of the planet in the order of the signs; and the latter, as it separates from the former, makes the planet seem to us to move more rapidly in the ecliptic than it really does with its own motion.

But when the earth reaches the part of the great circle that is nearer[215] to the planet, the direction of its motion at once becomes westward, so that the apparent motion of the planet forthwith seems slower to us. Moreover, because the earth mounts toward the planet, the line of the true motion of the sun moves away from the planet, and the planet is thought to approach us, as though it were descending from its upper circumference. However, the motion of the planet seems to be direct, until the center of the earth reaches the point on the great circle with respect to the planet[216] where the angle through which the line of the true place of the planet moves daily in precedence equals the diurnal angle of the real inequality in consequence. For there, since the two motions neutralize each other, the planet appears to remain at its first stationary point for a number of days, depending on the ratio of the great circle to the eccentric of the planet under consideration, the position of the planet on its circle, and the real rate of its motion. Then as the earth moves from this position nearer to the planet, we believe that the planet retrogrades

[214] Reading *linea* (Th 482.18) instead of *lineae* (PII, 352.20).
[215] Reading *propiorem* (Th 482.22) instead of *propriorem* (PII, 352.25).
[216] Reading *planetam* (Th 482.27) instead of *planetae* (PII, 352.31).

and moves in precedence, since the regression of the line of the true place of the planet perceptibly exceeds the real motion of the planet. This apparent retrogradation continues until the earth reaches the true perigee of the planet with respect to the great circle, where the planet, at the mid-point of regression, is in opposition to the sun and nearest to the earth. When Mars is found in this position, it has, in addition to the common retrogradation or parallax caused by the great circle, another parallax caused by the sensible ratio of the radius of the earth to the distance of Mars, as careful observation will testify.

Finally, as the earth moves in consequence from this central conjunction with the planet, so to say, the westward regression diminishes exactly as it had previously increased, until when the motions are again equal, the planet reaches its second stationary point. Then as the real motion of the planet exceeds the motion of the line of the true place of the planet, and as the earth advances, the situation is reached where the planet at length appears at the mid-point of its direct motion; and the earth again comes to the true apogee of the planet, whence we started its motion, and produces for us in order all the abovementioned phenomena of each of the planets.

The foregoing is the first use made of the great circle in the study of the planetary motions; by it we are freed from the three large epicycles in Saturn, Jupiter, and Mars. What the ancients called the argument of the planet, my teacher calls the planet's motion in commutation,[217] for by means of it we explain the phenomena arising from the motion of the earth on the great circle. These phenomena are clearly caused by the great circle, as the parallaxes of the moon are caused by the ratio of the radius of the earth to the lunar circles. The motion of the center of the epicycle of each planet, when subtracted from the uniform motion of the earth, which is also the mean motion of the sun, leaves as a remainder the uniform motion of commutation; and it is computed from the mean apogee, from which the earth also moves uniformly. Hence the true and apparent motion of each planet in the ecliptic is

[217] Th 308.2-8; cf. p. 48, above.

readily obtained from my teacher's tables of the prosthaphaereses of the planets.

Moreover, we shall find the second of the uses[218] of the great circle, no less important than the first, in the theory of Venus and Mercury. For since we observe these two planets from the earth as from a lookout, even if they should remain fixed like the sun, nevertheless, because we are carried about them by the motion of the great circle, we would think that they, like the sun, traverse the zodiac in motions of their own. Now the observations testify that Venus and Mercury move on their circles in independent motions of their own. Hence, in addition to the mean motion of the sun, by which they are carried in the order of the signs, other accidental phenomena caused by the great circle are observed in them. For in the first place we will consider their circles as epicycles which, as though on their own deferents, traverse the zodiac at an equal rate with the sun. Thus when the earth is in the perigee of the first deferents, their entire circles will be thought to be in the apogee of the eccentric, and conversely in the perigee with the earth in apogee. Moreover, just as in the superior planets the apogees and perigees with respect to the planets are designated on the great circle, so conversely they are marked on the circles of Venus and Mercury with respect to the center of the earth, wherever it may be; and, by reason of the annual motion of the earth, are drawn through all the points on the deferents. The ends of the diameter of the movable deferent that moves[219] parallel to the line of the mean motion of the sun, that is, the line from the center of the great circle to the center of the earth, are the mean apsides. The apsides in the part of the movable deferent that is more remote from the earth are called, not without reason, the higher apsides; those in the nearer part, the lower.

Venus revolves in nine months, as was stated above,[220] and

[218] Reading *utilitatum* (Th 483.20) instead of *utilitatem* (PII, 354.24).

[219] Reading *movetur* (1566 ed., p. 210v; 1596 ed., p. 143; 1621 ed., p. 137) instead of *moventur* (PII, 355.19).

[220] Page 146.

Mercury in approximately three. Hence, if[221] the annual motion of the earth should cease, each planet would appear to us on the earth to be in each period twice in conjunction with the sun, twice stationary, and twice at the outermost points in the curvature[222] of the deferents, and once morning, evening, retrograde, direct, in apogee, and in perigee. Moreover, if our eye were at the center of the great circle, only the independent unequal motions of Venus and Mercury, as of the other planets, would[223] appear; and as the planets traversed the entire zodiac by their own motions, they would come to be in opposition to the sun and would be seen in the other configurations with respect to it.[224]

But since we do not observe the motions of the planets from the center of the great circle, nor does the annual motion of the earth cease, it will be quite clear why these phenomena appear in such great variety to us who inhabit the earth. In accordance with the size of their circles, Venus and Mercury outrun the earth by their swifter motion, while the earth follows them in its annual motion. Therefore Venus overtakes the earth in about sixteen months,[225] and Mercury in four; with these intervals as their period, the planets show us again and again all the phenomena which God desired to be seen from the earth.

The lines of the real inequalities of motion move[226] regularly, revolving about the center of the great circle in the period allotted to them by God; but the lines of the true places, which are drawn from the center of the earth through Venus and Mercury, move in an altogether different manner, not only because they are drawn from a point outside the orbits, but also because the point is movable. We think that Venus and

[221] Reading *Si* (Th 483.39) instead of *Sic* (PII, 355.22).

[222] Reading *curvaturis* (Th 484.3; PII, 355.26).

[223] Reading *offerrent* (Th 484.5-6) instead of *offerent* (PII, 356.3).

[224] Reading *eum* (Th 484.7) instead of *cum* (PII, 356.4).

[225] This estimate of the synodic period of Venus is too low. Mästlin's correction, nineteen months (1596 ed., p. 143; 1621 ed., p. 137), should be unhesitatingly adopted, since it agrees with Th 310.1-7 and the tables (Th 318-19).

[226] Reading *incedunt* (Th 484.15) instead of *incedant* (PII, 356.14).

Mercury move on their circles with the motion with which the ancients said that they moved on the epicycle. But since this motion is merely the difference by which the swifter planet exceeds the mean motion of the earth or sun, my teacher calls this excess the motion in commutation, for exactly the same reasons as in the three superior planets. Consequently all the phenomena of Venus and Mercury which would appear if the earth were fixed recur more slowly on account of the earth's motion; and they occur at all the parts of the deferents and points on the ecliptic where their motions of every sort would be observed. For even without the earth fixed in Cancer, Ptolemy would have found that Mercury has its least elongations from the sun in Libra, and Venus in Taurus.[227] No matter where the earth may be on the great circle, Venus and Mercury seem to us to have their greatest elongation from the sun when they are observed at the sides of the deferent. If both tangents are drawn from the center of the earth to the deferents of Venus and Mercury, the planets will move in the order of the signs in the upper circumference, upper, that is, with reference to the earth; but in the opposite direction in the lower circumference, which is nearer to the earth. For here they appear to the senses to be stationary and retrograde, since the line of the true place of the planet makes about the center of the earth a diurnal angle in precedence equal to the angle of the mean motion, which is also the motion of the earth, in consequence, or a greater angle, etc. It is clear from these considerations why Venus and Mercury are seen to revolve about the sun.

It is also clearer than sunlight that the circle which carries the earth is rightly called the great circle. If generals have received the surname "Great" on account of successful exploits in war or conquests of peoples, surely this circle deserved to have that august name applied to it. For almost alone it makes us share in the laws of the celestial state, corrects all the errors of the motions, and restores to its rank this most beautiful

[227] For Ptolemy found the apogee of Mercury in Libra, and of Venus in Taurus (HII, 271.2-4, 300.15-16; cf. above, p. 85, n. 87 and p. 81, n. 71).

part of philosophy. Moreover, it is called the great circle because it has, in comparison with the circles of both the superior and inferior planets, a sensible magnitude which is the explanation of the principal phenomena.

The Apparent Deviation of the Planets from the Ecliptic

In the latitudes of the planets the first point to observe is that the name "great" is correctly assigned to the circle that carries the center of the earth. This circle deserves even higher commendation for the reason that the views of the ancients regarding the latitudes are quite involved and obscure, as is well known. The motions in longitude of the planets offer excellent evidence that the center of the earth describes what we call the great circle; but in the latitudes of the planets, the uses of this circle, as if placed in some well-lighted spot, are more obvious, since the great circle is the principal cause of every inequality of the appearances in latitude, even though it nowhere departs from the plane of the ecliptic. You see, most learned Schöner, that this circle should be honored and embraced with the greatest affection; for when all the causes have been set forth, it puts the whole subject of motion in latitude so briefly and so clearly before our eyes.

First, let the deferents of the three superior planets be inclined to the ecliptic as in Ptolemy's system; let their apogees be found to the north, their perigees to the south; and let the planets revolve on their deferents like the moon on its oblique circle, out of the plane of which it does not move. The lines of the real inequality, the dragons[228] of the planets, as they are commonly called, indicate the inclinations of the deferents

[228] This term was employed to designate the deviation of the moon and the planets from the ecliptic. The point on the ecliptic where the moon or planet passes from south latitude to north (ascending node) was called the "dragon's head," *caput Draconis* (Th 261.29); the point where it passes from north latitude to south (descending node) was called the "dragon's tail," *cauda Draconis* (Th 261.30). The usage survives in (*a*) the modern name, draconitic month, for the average time between two successive passages of the moon through the same node, and (*b*) the symbols still used to denote the nodes (for these symbols in a MS of the fourteenth century see Paul Tannery, *Mémoires scientifiques*, Toulouse and Paris, 1912- , IV, 356-57, plate II).

to the plane of the ecliptic and its intersections with the motions of the planets. Intersecting these lines in the centers of the planets are the lines of the true places. The latter, according to the position of the earth's center[229] on the great circle in relation to the planet, and the position of the planet on its oblique circle, mark the true places of the planets as nearer[230] to and remoter from the line through the middle of the signs,[231] in accordance with the size of the angles which the lines of the true places make with the plane of the ecliptic, as mathematical theory requires. Therefore, no matter what part of its deferent and epicycle the planet is in on the oblique circle, when the center of the earth is in the half of the great circle that is more remote from the planet—the half which the ancients called the upper part of the epicycle—the apparent latitudes clearly must be smaller than the angle of inclination of the deferent to the plane of the ecliptic; for in this position of the center of the earth in relation to the planet, the angle of apparent latitude is smaller than the angle of inclination, being an interior angle in comparison with the exterior and opposite. Furthermore, when the center of the earth reaches the half of the great circle that is nearer to the planet, conversely the apparent latitude is seen to be greater than the angle of inclination, obviously for the same reasons; for what was previously the exterior and opposite angle is now the interior angle.

This is the reason why the ancients thought that when the center of the epicycle was outside the nodes, the upper part of the epicycle was always between the planes of the deferent and ecliptic; that the other half of the epicycle was tilted in the same direction as the half of the deferent occupied by the center of the epicycle; that the diameter which passed through the middle longitudes of the epicycle moved parallel to the plane of the ecliptic; and that when the epicycle was in

[229] Reading with Mästlin (1596 ed., p. 145) *centri* instead of *centro* (PII, 358.12); cf. *in tali centri terrae situ* (PII, 358.22).

[230] Reading *propiora* (Th 485.27) instead of *propriora* (PII, 358.14).

[231] A term for the ecliptic; *per signorum medium*, "through the middle of the signs," is a literal translation of the standard Greek term διὰ μέσων τῶν ζῳδίων (cf. HI, 68.17-18).

the nodes, the planet had no latitude wherever it might be on the epicycle. In our hypotheses, the planet has no latitude when it is in one of the nodes, no matter where the earth may be found on the great circle. If the angle between the planes of the epicycle and deferent had been found, in the hypotheses of the ancients, invariably equal to the angle of inclination of the planes of the deferent and ecliptic; that is, if the plane of the epicycle had been found always parallel to the ecliptic, the aforementioned theory of latitudes would be sufficient. But an inequality is implied in the observations geometrically examined, as can be seen in the last book of Ptolemy's *Great Syntaxis*.[232] Therefore, using a motion in libration, my teacher makes the angle of inclination of the deferent to the ecliptic increase and diminish in a definite relation to the mean motion of the planet on its oblique circle, and of the earth on the great circle. This result will be obtained if in each period of the motion in commutation the diameter along which the libration takes place is twice described by the outermost limits of the oblique circle, and if the following condition is observed: that when the planet is at its evening rising the angle of inclination is greatest, and hence the angle of apparent latitude is even greater; but with the planet at its morning rising, minimal, and hence the apparent latitude, as is consistent, even smaller.[233]

But the appearances of Venus and Mercury in latitude, with the single exception of the deviation, are more easily understood than the theories of the superior planets. Let us examine the latitudes of Venus first. Within the great circle the sphere of Venus is the first to occur. According to my teacher, the plane in which Venus moves is inclined to the plane of the ecliptic or great circle along the diameter through the true apsides of the first deferent, so that the eastern half rises northward from the plane surface of the ecliptic by the angle of inclination which would be contained, in Ptolemy's hypotheses, between the planes of the epicycle and deferent; and the western half dips southward. By "eastern half" is to be understood the half that

[232] HII, 537.7–542.15.
[233] Cf. pp. 79-80, above.

extends in consequence from the place of the higher apse, etc. By this simple hypothesis alone we can easily derive all the rules for the declinations and reflexions, together with their causes, from the relation of the position of the earth to the plane of the planet.

For when by the annual motion of the earth we reach the place opposite the higher apse of the first deferent, where we think that the circle of Venus is like an epicycle in the apogee of its deferent, the plane in which Venus moves seems to us to have a reflexion from the plane of the ecliptic, because in this position we see the plane of Venus crosswise. And because we look at this plane from below, the part that rises northward will be to the left, and the other part, that dips southward, to the right, for us whose eyes are directed southward. But as the earth moves upward toward the higher apse of the planet, the circle of Venus is thought to descend from the apogee of its eccentric, and we begin to look down as from above upon the inclined plane of the deferent of Venus. Therefore the reflexion gradually changes into a declination, so that when the earth is at a quadrant's distance from its former position, no matter where the planet may be seen in the part of its path that tilts upward, it has only a declination from the ecliptic. In this position, since we on the earth are opposite the half of the deferent that extends in consequence from the higher apse and rises northward from the plane of the ecliptic, the ancients said that the epicycle of Venus was in the descending node and that the apogee of the epicycle reached its greatest northern declination, and the perigee its greatest southern.

Then, as the earth in its annual motion carries us upward toward the place of the higher apse of Venus, its circle, like an epicycle, seems to approach the lower apse of its deferent; the plane of the epicycle, which is for us the plane in which Venus moves, and which previously had a declination to the plane of the ecliptic, again appears to have a reflexion to us; and the northern half of the deferent, rising from the plane of the ecliptic, lies to the right because we see Venus from above. But when the center of the earth reaches the place of the higher apse of Venus, no declination and only a reflexion is seen; and the circle of Venus is believed to be in the lower apse of its deferent, as the ancients would have said. This is the order of the phenomena while the center of the earth completes half a revolution, as it mounts in the order of the signs from the place of the lower apse of Venus to the place of the higher apse of Venus.

When the earth descends in the same way, the reflexion, to our eyes, gradually changes into a declination; and because the half of the plane of the deferent that extends in precedence from the higher apse becomes, through this motion of the earth, opposite to us, the apogee of the deferent of Venus begins to have a southward declination from the plane of the ecliptic, until when the earth is 90° from the place of the apse both halves are seen in declination to the plane of the ecliptic and the circle of Venus like an epicycle is thought to be in the ascending node at the higher apse. As the earth moves on from this position, the declination again changes into a reflexion; and when the earth reaches the place of the lower apse of Venus, it begins to produce once more the same phenomena of latitude in Venus. From these considerations it is clear that when the earth is on the apse-line of Venus, the plane of the deferent of the planet appears to have a reflexion; when the earth is at a quadrant's distance from the apsides, a declination; and when the earth is at the intervening points, mixed latitudes are seen.[234]

Mingled with these latitudes, which the ancients assigned to the epicycle of Venus, there is still another, called "deviation" by the ancients, by Ptolemy "tilting of the eccentric circles,"[235] which they demonstrated by the center of the deferent of the epicycle of Venus, now eliminated. Hence my teacher has decided that another theory must be constructed in better agreement with the observations. To make this theory of my teacher for saving the deviation easier to understand, like the other ideas heretofore set forth, let us define the plane

[234] Cf. pp. 83-84, above.

[235] For example, HII, 535.6-7.

discussed above as the mean plane, and therefore fixed; from it the true plane deviates in a definite way, now to one side, now to the other. We comprehend all motions with less effort and expenditure of time by directing our attention to their poles. We should therefore begin with the statement that one of the poles of the mean plane lies north of the plane of the ecliptic by the amount of the angle of inclination; the other pole on the opposite side lies an equal distance to the south; and what we shall prove with regard to the north pole, or the phenomena related to it, must be understood in like manner with regard to the south pole, the law of opposition being, of course, observed.

Accordingly, let us assume that about the north pole of the mean plane there is a movable circle, whose radius equals the greatest inclinations of the mean and true planes. Let the north pole of the true plane describe the diameter of the said circle by a motion in libration. Furthermore, let the movable circle follow the motion of the planet, so that as Venus proceeds with its own motion it observes the following rule: it leaves behind one of the two intersections that follow it, and exactly in a year overtakes the intersection left behind. Draw a great circle through the poles of both planes, mark off 90° on each side of[236] its intersection with the true plane, and the nodes or intersections, as I have called them, are determined when the poles of the true and mean planes do not coincide. While a periodic return of Venus to either one of the nodes is being completed, let the pole of the true plane twice describe the diameter of the said movable circle by a motion in libration. Let these phenomena so occur that the planet appears to have entered into a covenant with the center of the earth whereby, whenever the earth is at the apsides of the deferent, no matter where Venus is on its true deferent, it has its greatest northward deviation from the mean plane, that is, it is at its greatest distance from its mean course; moreover, when the earth is at a quadrant's distance from the apsides of the deferent, the planet, together with its entire true plane, lies in the plane of the mean

[236] Reading *ab* (Th 488.11) instead of *ad* (PII, 362.18).

deferent; and when the earth passes through the intervening points, the path of the planet likewise has intermediate deviations. That this covenant of earth and planet might be everlasting, God ordained that the first small circle of libration, to use this term, should revolve once in the time in which one return of Venus to either of the movable nodes occurs.

Let us make these relations clearer by an example. If at any beginning of the motion of deviation the north pole of the true plane is at its greatest southward distance from the pole of the adjacent mean plane, and if Venus is at the limit of its deviation, which lies to the north, the center of the earth being in one of the apsides of Venus, in the fourth part of a year the earth in its annual motion will come to the mid-point between the apsides, and in the same time the planet will reach its movable intersection or node. Because the motion in libration is commensurable with the periodic return of the planet to its nodes or intersections, the first small circle of libration will likewise complete a quadrant; and the second small circle, which moves at twice the rate of the first, will join the pole of the true plane to the pole of the mean plane, and therefore the two planes will coincide. But as the planet moves away from the node, the earth proceeds toward the other apse of the first eccentric, and the pole of the true plane moves northward in libration from the pole of the mean plane. Thus it happens that even though Venus is in south latitude, as in our example, the latitude, if south, nevertheless diminishes, if north, increases. When the earth reaches the other apse, the pole of the true plane attains the northern limit of its motion in libration; and the planet, midway between the two intersections in its annual return to the nodes, again has its greatest northward deviation. It is therefore clear that the motion of the circle which has been assumed has this advantage, that the revolution of Venus with respect to the nodes occurs in a year; and that when the earth is in the apse-line, no matter where the planet is in its true plane, it always has its greatest deviation from the mean plane; and that when the earth is midway between the apsides, the planet is in the nodes. Moreover, by reason of the motion in libration, it happens that when Venus is in one of the nodes, the two planes coincide; and that part of the true plane in which Venus is moving always deviates northward from the mean plane, so that this latitude, as is proper, always remains a north latitude.

The mean plane of Venus, as we have called it, is intersected by the ecliptic in the apse-line of the first eccentric; and the half of this plane that lies in consequence from the higher apse rises northward, and the other half, by the law of opposition, dips southward. In Mercury there is a mean plane of a similar nature. It is inclined to each side of the plane of the ecliptic along the apse-line, as is proper, so that conversely the half of the mean plane that lies in precedence from the higher apse extends northward. Therefore, in the annual revolution of the center of the earth the declinations and reflexions in Mercury will be found interchanged, as compared with those of Venus. To make this contrast more striking, God arranged the deviation of the true plane of Mercury from the mean plane so that the half in which Mercury is moving always deviates southward from the mean plane; and when the earth is at the apsides, the planet lies with its true plane in the mean plane. Consequently Mercury has only the above-mentioned differences in latitude from Venus, except that this deviation is greater in Mercury than in Venus,[237] as the former has also the greater angle of inclination.[238] The other changes of latitude in Mercury will quite easily be found exactly as in Venus.

> A part of the task remains, and part is done;
> Here let the anchor drop and moor our boat,

to conclude this *First Account* with the words of the poet.[239]

[237] The traditional estimates were, respectively, 45′ and 10′; although Copernicus departed from them somewhat, he did not alter their relative value (cf. above, p. 90, n. 102).

[238] Cf. pp. 83, 89, above.

[239] Ovid *Ars amatoria* i.771-72. Kepler also used this couplet to close Part III of his work on Mars (*Opera*, ed. Frisch, III, 325).

Just as soon as I have read the entire work of my teacher with sufficient application, I shall begin to fulfill the second part of my promise. I hope that both will be more acceptable to you, because you will see clearly that when the observations of scholars have been set forth, the hypotheses of my teacher agree so well with the phenomena that they can be mutually interchanged, like a good definition and the thing defined.

Most illustrious and most learned Schöner, whom I shall always revere like a father, it now remains for you to receive this work of mine, such as it is, kindly and favorably. For although I am not unaware what burden my[240] shoulders can carry and what burden they refuse to carry, nevertheless your unparalleled and, so to say, paternal affection for me has impelled me to enter this heaven not at all fearfully and to report everything to you to the best of my ability. May Almighty and Most Merciful God, I pray, deem my venture worthy of turning out well, and may He enable me to conduct the work I have undertaken along the right road to the proposed goal. If I have said anything with youthful enthusiasm (we young men are always endowed, as he says, with high, rather than useful, spirit) or inadvertently let fall any remark which may seem directed against venerable and sacred antiquity more boldly than perhaps the importance and dignity of the subject demanded, you surely, I have no doubt, will put a kind construction upon the matter and will bear in mind my feeling toward you rather than my fault.

Furthermore, concerning my learned teacher I should like you to hold the opinion and be fully convinced that for him there is nothing better or more important than walking in the footsteps of Ptolemy and following, as Ptolemy did, the ancients and those who were much earlier than himself. However, when he became aware that the phenomena, which control the astronomer, and mathematics compelled him to make certain assumptions even against his wishes, it was enough, he thought,

[240] Reading *mei* (Th 489.36) instead of *me* (PII, 364.30).

if he aimed his arrows by the same method to the same target as Ptolemy, even though he employed a bow and arrows of far different type of material from Ptolemy's. At this point we should recall the saying "Free in mind must he be who desires to have understanding."[241] But my teacher especially abhors what is alien to the mind of any honest man, particularly to a philosophic nature; for he is far from thinking that he should rashly depart, in a lust for novelty, from the sound opinions of the ancient philosophers, except for good reasons and when the facts themselves coerce him. Such is his time of life, such his seriousness of character and distinction in learning, such, in short, his loftiness of spirit and greatness of mind that no such thought can take hold of him. It is rather the mark of youth or of "those who pride themselves on some trifling speculation," to use Aristotle's words,[242] or of those passionate intellects that are stirred and swayed by any breeze and their own moods, so that, as though their pilot had been washed overboard, they snatch at anything that comes to hand and struggle on bravely. But may truth prevail, may excellence prevail, may the arts ever be honored, may every good worker bring to light useful things in his own art, and may he search in such a manner that he appears to have sought the truth. Never will my teacher avoid the judgment of honest and learned men, to which he plans of his own accord to submit.

[241] This sentence serves as motto for the *Narratio prima* (see p. 108, above) and also for Kepler's *Dissertatio cum nuncio sidereo* (Prague, 1610; see *Kepleri opera omnia*, ed. Frisch, II, 485). It is quoted substantially correctly from the *Didaskalikos* (C. F. Hermann's Teubner edition of Plato, VI [Leipzig, 1892], 152). This elementary textbook of Platonic philosophy was available to Rheticus in the Aldine editions of Iamblichus (1516) and Apuleius (1521); in the latter work the words quoted appear on fol. 1 2r. The *Didaskalikos* was formerly attributed to Alcinous, but now it is held that its author was Albinus, who flourished in Smyrna during the middle of the second century C.E., and was a teacher of Galen (R. E. Witt, *Albinus and the History of Middle Platonism*, Cambridge, 1937, pp. 104-9).

[242] *De mundo* 391a23-24; Rheticus has adapted the original to the structure of his sentence and has shifted the meaning of θεωρία from "spectacle" to "speculation" (cf. above, p. 162, n. 194). The *De mundo* is pseudo-Aristotelian (cf. above, p. 139, n. 121).

In the ode[244] which is said to be preserved in golden letters in the temple of Minerva and which celebrates the Olympic victory of the boxer Diagoras of Rhodes, Pindar says that Diagoras's native land is the daughter of Venus and the dearly beloved spouse of the sun; that Jupiter, moreover, rained much gold there, inasmuch as the Rhodians worshipped his daughter Minerva; and that in consequence, through Minerva, the Rhodians gained a reputation for wisdom and education, to which they were deeply devoted.

I am not aware that anyone could apply this resounding praise of the Rhodians to any region of our time more suitably than to Prussia, concerning which I propose to say a few words that perhaps you desired to hear. Doubtless the same divinities would be found to be presiding over this region, should some skillful astrologer make careful inquiry about the stars that rule over this most beautiful, most fertile, and most fortunate area. As Pindar says:[245]

But the tale is told in ancient story that, when Zeus and the immortals were dividing the earth among them, the isle of Rhodes was not yet to be seen in the open main, but was hidden in the briny depths of the sea; and that, as the Sun-god was absent, no one put forth a lot on his behalf, and so they left him without any allotment of land, though the god himself was pure from blame. But when that god made mention of it, Zeus was about to order a new casting of the lot, but the Sun-god would not suffer it. For, as he said, he could see a plot of land rising from the bottom of the foaming main, a plot that was destined to prove rich in substance for men, and kindly for pasture.

[243] This section was omitted from the Basel edition of 1566, the Warsaw edition of 1854, and Th. It was included in the following editions: Danzig, 1540; Basel, 1541; Tübingen, 1596; Frankfurt, 1621; and PII, 367-77. It was also printed, in incomplete form, in *Acta Borussica*, II (Königsberg and Leipzig, 1731), 413-25; and completely in Hipler, *Spicilegium Copernicanum*, pp. 215-22. It was translated into German by Franz Beckmann (ZE, III[1866], 5-17) and Prowe (PI², 448-63).

[244] Pindar, seventh Olympian ode.

[245] *Ibid.*, 54-63; the translation is by J. E. Sandys in the Loeb Classical Library (London, 1915).

Doubtless the sea once covered Prussia, too. What more definite and more important[246] evidence could anyone produce than that today amber is found inland, at a very great distance from the coast? Therefore, on the principle that it rose from the sea, by an act of the gods Prussia passed into the hands of Apollo, who cherishes it now, as once he cherished Rhodes, his spouse. Cannot the sun reach Prussia as well as Rhodes with vertical rays? I grant that it cannot. But it makes up for this in many other ways; and what it accomplishes in Rhodes by its vertical rays, it performs in Prussia by lingering above the horizon. Moreover, amber is a special gift of God, with which He desired to adorn this region above all others, as I think nobody will deny. Indeed, anyone who considers the nobility of amber and its use in medicine will regard it, not without reason, as sacred to Apollo and as a magnificent gift, an abundance of which he presents, like a most valuable jewel, to his spouse Prussia.

But besides the medical and prophetic arts, which Apollo invented and first practiced, he is filled with a passion for hunting. For this reason he seems to have chosen this land before all others. And since he long foresaw that the savage Turks would despoil Rhodes, he transferred his abode to these parts and migrated hither with his sister Diana, as seems not improbable. For no matter where you turn your eyes, if you look at the woods, you will say that they are game preserves ("paradise" in Greek) and beehives stocked by Apollo; if you look at the orchards and fields, rabbit warrens and birdhouses, lakes, ponds, and springs, you will say that they are the holy places of Diana and the fisheries of the gods. And Apollo appears to have chosen Prussia before other regions, I say, as his paradise. Besides stag, doe, bear, boar, and the kind of wild beast that is commonly known elsewhere, he brought in also urus, elk, bison, etc., species scarcely to be found in other places, to say nothing of the numerous and quite rare types of bird and fish.

The progeny which Apollo received from his spouse Prussia

[246] Reading *propiusque* (1596 ed., p. 153) instead of *propriusque* (PII, 369.13).

is as follows: Königsberg, seat of the illustrious prince, Albrecht, duke of Prussia, margrave of Brandenburg, etc., patron of all the learned and renowned men of our time; Thorn, once quite famous for its market, but now for its foster-son, my teacher; Danzig, metropolis of Prussia, eminent for the wisdom and dignity of its Council, for the wealth and splendor of its renascent literature; Frauenburg, residence of a large body of learned and pious men, famous for its eloquent and wise Bishop, the Most Reverend John Dantiscus; Marienburg, treasury of His Serene Majesty, the king of Poland; Elbing, ancient settlement in Prussia, where, too, the sacred pursuit of literature is undertaken; Kulm, famous for its literature, where the Law of Kulm had its origin.

You might say that the buildings and the fortifications are palaces and shrines of Apollo; that the gardens, the fields, and the entire region are the delight of Venus, so that it could be called, not undeservedly, Rhodes. What is more, Prussia is the daughter of Venus, as is clear if you examine either the fertility of the soil or the beauty and charm of the whole land.

As Venus is said to have risen from the sea, so Prussia is the daughter of Venus and of the sea. And therefore it is fertile enough to feed Holland and Zealand with its crops and to serve as granary for the neighboring kingdoms and also for England and Portugal. Besides this excellent produce it exports quantities of fish of every sort and other valuable resources, with which it abounds. But Venus is interested in the things that promote culture, dignity, and the good and humane life. These could not grow and develop in this region, for the character of the country forbade it. So she saw to it that with the aid of the sea they could be successfully imported into Prussia from abroad.

But since these facts are so well known to you, most learned Schöner, that there is no need for me to speak of them at greater length; and since they are treated in other books, wholly devoted to this subject, I refrain from further praise. I add only this item, that by the grace of the presiding divinity the Prussians are a numerous people and also possess an unusual talent for culture. Moreover, they worship Minerva with every type of art and for this reason receive the kindness of Jupiter. For, not to speak of the lesser arts attributed to Minerva, like architecture and its allied disciplines, the revival of literature in the world is everywhere welcomed with keenest interest, as befits heroes, by the illustrious duke most of all, and also by all the dignitaries and nobles of Prussia, in whose hands lies the direction of affairs, and by the rulers of states. They strive to encourage and support it, both independently and jointly. Therefore Jupiter forms a yellow cloud and rains much gold. This means, as I interpret it, that because Jupiter is said to preside over kingdoms and states, when the mighty undertake to support studies, learning, and the muses, then God gathers the minds of his subject and neighboring kings, princes, and peoples into a golden cloud; from it he distils peace and all the blessings of peace, like drops of gold; minds in love with tranquillity and public order; cities governed by just laws; wise men; upright and devout education of children; pious and pure spread of religion, etc.

The story is frequently told of the shipwreck of Aristippus, which they say occurred off the island of Rhodes. Upon being washed ashore, he noticed certain geometrical diagrams on the beach; exclaiming that he saw the traces of men, he bade his companions be of good cheer. And his belief did not play him false. For through his great learning he easily obtained for himself and his comrades from educated and humane men the things necessary for sustaining life.

So may the gods love me, most learned Schöner, it has not yet happened to me that I should enter the home of any distinguished man in this region—for the Prussians are a most hospitable people—without immediately seeing geometrical diagrams at the very threshold or finding geometry present in their minds. Hence nearly all of them, being men of good will, bestow upon the students of these arts every possible benefit and service, since true knowledge and learning are never separated from goodness and kindness.

In particular, I am wont to marvel at the kindness of two distinguished men toward me, since I readily recognize how slight is my scholarly equipment, measuring myself by my own abilities. One of them is the illustrious prelate whom I mentioned at the outset,[247] the Most Reverend Tiedemann Giese, bishop of Kulm. His Reverence mastered with complete devotion the set of virtues and doctrine, required of a bishop by Paul. He realized that it would be of no small importance to the glory of Christ if there existed a proper calendar of events in the Church and a correct theory and explanation of the motions. He did not cease urging my teacher, whose accomplishments and insight he had known for many years, to take up this problem, until he persuaded him to do so.

Since my teacher was social by nature and saw that the scientific world also stood in need of an improvement of the motions, he readily yielded to the entreaties of his friend, the reverend prelate. He promised that he would draw up astronomical tables with new rules and that if his work had any value he would not keep it from the world, as was done by John Angelus,[248] among others. But he had long been aware that in their own right the observations in a certain way required hypotheses which would overturn the ideas concerning the order of the motions and spheres that had hitherto been discussed and promulgated and that were commonly accepted and believed to be true; moreover, the required hypotheses would contradict our senses.

He therefore decided that he should imitate the Alfonsine Tables rather than Ptolemy and compose tables with accurate rules but no proofs. In that way he would provoke no dispute among philosophers; common mathematicians would have a correct calculus of the motions; but true scholars, upon whom Jupiter had looked with unusually favorable eyes, would easily arrive, from the numbers set forth, at the principles and sources from which everything was deduced. Just as heretofore learned men had to work out the true hypothesis of the motion of the

[247] Page 109.

[248] For Angelus see Gesner, *Bibliotheca universalis*, pp. 382v–383r; and *Allgemeine deutsche Biographie*, I, 457.

starry sphere from the Alfonsine doctrine, so the entire system would be crystal clear to learned men. The ordinary astronomer, nevertheless, would not be deprived of the use of the tables, which he seeks and desires, apart from all theory. And the Pythagorean principle would be observed that philosophy must be pursued in such a way that its inner secrets are reserved for learned men, trained in mathematics, etc.[249]

Then His Reverence pointed out that such a work would be an incomplete gift to the world, unless my teacher set forth the reasons for his tables and also included, in imitation of Ptolemy, the system or theory and the foundations and proofs upon which he relied to investigate the mean motions and prosthaphaereses and to establish epochs as initial points in the computation of time. The bishop further argued that such a procedure had produced great inconvenience and many errors in the Alfonsine Tables, since we were compelled to assume and to approve their ideas on the principle that, as the Pythagoreans used to say, "The Master said so"—a principle which has absolutely no place in mathematics.

Moreover, contended the bishop, since the required principles and hypotheses are diametrically opposed to the hypotheses of the ancients, among scholars there would be scarcely anyone who would hereafter examine the principles of the tables and publish them after the tables had gained recognition as being in agreement with the truth. There was no place in science, he asserted, for the practice frequently adopted in kingdoms, conferences, and public affairs, where for a time plans are kept secret until the subjects see the fruitful results and remove from doubt the hope that they will come to approve the plans.

So far as the philosophers are concerned, he continued, those of keener insight and greater information would carefully study Aristotle's extensive discussion and would note that after convincing himself that he had established the immobility of the earth by many proofs Aristotle finally takes refuge in the following argument:

[249] At this point the text as printed in *Acta Borussica* breaks off.

We have evidence for our view in what the mathematicians say about astronomy. For the phenomena observed as changes take place in the figures by which the arrangement of the stars is marked out occur as they would on the assumption that the earth is situated at the center.[250]

Accordingly the philosophers would then decide:

If this concluding statement by Aristotle cannot be linked with his previous discussion, we shall be compelled, unless we are to waste the time and effort which we have invested, rather to assume the true basis of astronomy. Moreover, we must work out appropriate solutions for the remaining problems under discussion. By returning to the principles with greater care and equal assiduity, we must determine whether it has been proved that the center of the earth is also the center of the universe. If the earth were raised to the lunar sphere, would loose fragments of earth seek, not the center of the earth's globe, but the center of the universe, inasmuch as they all fall at right angles to the surface of the earth's globe? Again, since we see that the magnet by its natural motion turns north, would the motion of the daily rotation or the circular motions attributed to the earth necessarily be violent motions? Further, can the three motions, away from the center, toward the center, and about the center, be in fact separated? We must analyze other views which Aristotle used as fundamental propositions with which to refute the opinions of the *Timaeus* and the Pythagoreans.

They will ponder the foregoing questions and others of the same kind if they desire to look to the principal end of astronomy and to the power and the efficacy of God and nature.

But if it is to be the intention and decision of scholars everywhere to hold fast to their own principles passionately and insistently, His Reverence warned, my teacher should not anticipate a fate more fortunate than that of Ptolemy, the king of this science. Averroes, who was in other respects a philosopher of the first rank, concluded that epicycles and eccentrics could not possibly exist in the realm of nature and that Ptolemy did not know why the ancients had posited motions of rotation. His final judgment is: "The Ptolemaic astronomy is nothing, so far as existence is concerned; but it is convenient

[250] *De caelo* ii.14 297a2-6; the translation is from Thomas L. Heath, *Greek Astronomy* (London, 1932), p. 91.

for computing the nonexistent."[251] As for the untutored, whom the Greeks call "those who do not know theory, music, philosophy, and geometry,"[252] their shouting should be ignored, since men of good will do not undertake any labors for their sake.

By these and many other contentions, as I learned from friends familiar with the entire affair, the learned prelate won from my teacher a promise to permit scholars and posterity to pass judgment on his work. For this reason men of good will and students of mathematics will be deeply grateful with me to His Reverence, the bishop of Kulm, for presenting this achievement to the world.

In addition, the benevolent prelate deeply loves these studies and cultivates them earnestly. He owns a bronze armillary sphere for observing equinoxes, like the two somewhat larger ones which Ptolemy says were at Alexandria[253] and which learned men from everywhere in Greece came to see. He has also arranged that a gnomon truly worthy of a prince should be brought to him from England. I have examined this instrument with the greatest pleasure, for it was made by an excellent workman who knew his mathematics.

The second of my patrons is the esteemed and energetic John of Werden, burgrave of Neuenburg, etc., mayor of the famous city of Danzig. When he heard about my studies from certain friends, he did not disdain to greet me, undistinguished though I am, and to invite me to meet him before I left Prussia. When I so informed my teacher, he rejoiced for my sake and drew such a picture of the man that I realized I was being invited by Homer's Achilles, as it were. For besides his distinction in the arts of war and peace, with the favor of the muses he also cultivates music. By its sweet harmony he refreshes and inspires his spirit to undergo and to endure the burdens

[251] Averroes, *Commentary on Aristotle's Metaphysics*, Book xii, *summa* ii, *caput* iv, No. 45. A Latin version of Averroes's *Commentary* was printed (Padua, 1473) with the *Metaphysics* in Latin translation (GW, 2,419; see also 2,337-40).

[252] Aulus Gellius *Noctes Atticae* i.9.8. It is at this point that the text as printed in *Acta Borussica* is resumed.

[253] HI, 195.5-7; 197.17-20.

of office. He is worthy of having been made by Almighty and Most Merciful God a "shepherd of the people."[254] Happy the state over which God has appointed such rulers!

In the *Phaedo*[255] Socrates rejects the opinion of those who called the soul a "harmony." And he did so rightly if by harmony they understood nothing but a mixture of the elements in the body. But if they defined the soul as a harmony because in addition to the gods only the human mind understands harmony—just as it alone knows number, wherefore certain thinkers did not fear to call it a number—and also because they knew that souls suffering from the deadliest diseases are sometimes healed by musical harmonies, then their opinion will not seem unfortunate, inasmuch as it is principally the soul of a heroic man that is called a harmony. Hence we might correctly call those states happy whose rulers have harmonious souls, that is, philosophical natures. Surely the Scythian had no such soul who preferred hearing a horse's neighing to a talented musician whom others listened to in amazement. Would that all kings, princes, prelates, and other dignitaries of the realms had souls chosen from the vessel of harmonious souls. Then these excellent studies and those which are chiefly to be pursued for their own sake would doubtless achieve a worthy station.

The foregoing, most distinguished sir, are the things which I thought I should for the present write to you regarding the hypotheses of my teacher, Prussia, and my patrons. Farewell, most learned sir, and do not disdain to guide my studies with your advice. For you know that we young men greatly need the counsel of older and wiser men. And you have not forgotten that charming sentiment of the Greeks, "The opinions of older men are better."[256]

From my library at Frauenburg
September 23, 1539

[254] A familiar epithet of kings and chiefs in Homer, e.g., *Iliad* ii.243.

[255] 86B-C, 92A-95A.

[256] This line, from a lost play of Euripedes, is preserved in Stobaeus, *Florilegium* cxv.2.

A BRIEF COMMENTARY ON THE HYPOTHESES FOR THE MOTIONS OF THE HEAVENLY BODIES

OUR ANCESTORS assumed, I observe, a large number of celestial spheres for this reason especially, to explain the apparent motion of the planets by the principle of regularity. For they thought it altogether absurd that a heavenly body that is a perfect sphere, should not always move uniformly.[1] They saw that by connecting and combining regular motions in various ways they could make any body appear to move to any position.

Callippus and Eudoxus, who endeavored to solve the problem by the use of concentric spheres, were unable to account for all the planetary movements; they had to explain not merely the apparent revolutions of the planets but also the fact that these bodies appear to us sometimes to mount higher in the heavens, sometimes to descend; and this fact is incompatible with the principle of concentricity. Therefore it seemed better to employ eccentrics and epicycles, a system which most scholars finally accepted.

Yet the planetary theories of Ptolemy and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind.

Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uni-

[1] Perhaps this sentence should be translated: For they thought it altogether absurd that a heavenly body should not always move with uniform velocity in a perfect circle.

formly about its proper center, as the rule of absolute motion requires. After I had addressed myself to this very difficult and almost insoluble problem, the suggestion at length came to me how it could be solved with fewer and much simpler constructions than were formerly used, if some assumptions (which are called axioms) were granted me. They follow in this order.

Assumptions[2]

1. There is no one center of all the celestial circles or spheres.

2. The center of the earth is not the center of the universe, but only of gravity[3] and of the lunar sphere.

3. All the spheres revolve about the sun as their mid-point, and therefore the sun is the center of the universe.

4. The ratio of the earth's distance from the sun to the height of the firmament is so much smaller than the ratio of the earth's radius to its distance from the sun that the distance from the earth to the sun is imperceptible in comparison with the height of the firmament.

5. Whatever motion appears in the firmament arises not from any motion of the firmament, but from the earth's motion. The earth together with its circumjacent elements[4] performs a complete rotation on its fixed poles in a daily motion, while the firmament and highest heaven abide unchanged.

6. What appear to us as motions of the sun arise not from

[2] In his description of the *Commentariolus* Dreyer incorrectly states the number of assumptions as six (*Planetary Systems*, p. 317). The source of his mistake is probably the oversight in PI², 291, which Prowe himself calls attention to and corrects (PII, 187 n).

[3] "Now the element of earth is the heaviest; and all heavy objects are borne to the earth, tending toward its inmost center. In accordance with their nature, heavy objects are borne from all directions at right angles to the surface of the earth; and since the earth is spherical, they would come together at its center, were they not checked at its surface. For a straight line which is at right angles to the tangential plane at the point of tangency leads to the center" (Th 19.28-20.3).

[4] These are (*a*) the atmosphere and (*b*) the waters that lie upon the surface of the earth. See p. 63, below and in *De rev.*: ". . . not only does the earth so move together with the watery element that is joined with it, but also no small part of the air and whatever else is related in the same way to the earth" (Th 22.15-17).

its motion but from the motion of the earth and our sphere, with which we revolve about the sun like any other planet. The earth has, then, more than one motion.

7. The apparent retrograde and direct motion of the planets arises not from their motion but from the earth's. The motion of the earth alone, therefore, suffices to explain so many apparent inequalities in the heavens.

Having set forth these assumptions, I shall endeavor briefly to show how uniformity of the motions can be saved in a systematic way. However, I have thought it well, for the sake of brevity, to omit from this sketch mathematical demonstrations, reserving these for my larger work.[5] But in the explanation of the circles I shall set down here the lengths of the radii; and from these the reader who is not unacquainted with mathematics will readily perceive how closely this arrangement of circles agrees with the numerical data and observations.

Accordingly, let no one suppose that I have gratuitously asserted, with the Pythagoreans, the motion of the earth; strong proof will be found in my exposition of the circles. For the principal arguments by which the natural philosophers attempt to establish the immobility of the earth rest for the most part on the appearances; it is particularly such arguments that collapse here, since I treat the earth's immobility as due to an appearance.

The Order of the Spheres

The celestial spheres are arranged in the following order. The highest is the immovable sphere of the fixed stars, which contains and gives position to all things. Beneath it is Saturn, which Jupiter follows, then Mars.[6] Below Mars is the sphere on which we revolve; then Venus; last is Mercury. The lunar sphere revolves about the center of the earth and moves with

[5] From this reservation we may infer that when Copernicus wrote the *Commentariolus* he had already planned *De rev.* or was at work upon it.

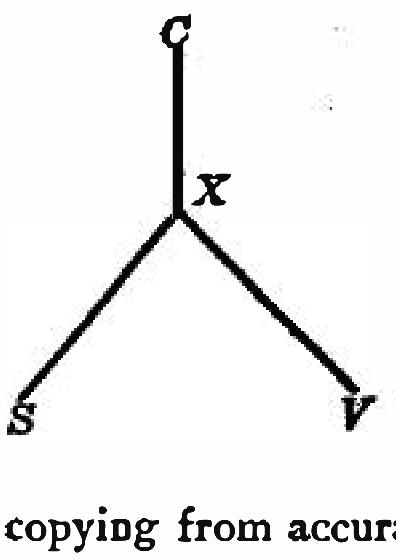
[6] S, V: *sub eo Saturnus; hunc sequitur Martius.* In S, after *Saturnus,* the words *quem sequitur Iovius* have been inserted above the line by a second hand. These readings provide a clue to the relationship between S and V; see the following note.

the earth like an epicycle. In the same order also, one planet surpasses another in speed of revolution, according as they trace greater or smaller circles. Thus Saturn completes its revolution in thirty years, Jupiter in twelve, Mars in two and one-half,[7] and the earth in one year; Venus in nine months, Mercury in three.

[7] S: *Sic quidem Saturnus anno 30, Jupiter 12, Mars, tellus annua revolutione restituuntur;* V: *Sic quidem Saturnus anno trigesimo, Iuppiter duodecimo, Mars, tellus annua revolutione restituitur.* The number for Mars has dropped out of both S and V, but it may be restored from a later section in the *Commentariolus*, where the sidereal period of Mars is given as twenty-nine months (p. 74, below). In *De rev.* Copernicus reduced the period to two years, bringing it closer to the true value of 687 days, or one year and ten and one-half months (Th 29.6: *Deinde Mars, qui biennio circuit;* the explanatory figure likewise has *Martis bima revolutio;* confirmation from Rheticus on p. 146, below).

In his edition of V, Curtze filled the lacuna by inserting, without any supporting argument, [*tertio*] after *Mars* (MCV, I, 7.27). This erroneous reading was accepted by Prowe, who unwisely dropped the square brackets (PII, 188.13). Adolf Müller evidently accepted Prowe's text unquestioningly (ZE, XII, 360); hence his translation of the *Commentariolus* assigned the grossly inaccurate value of three years to Mars' sidereal period (ZE, XII, 364). With no more warrant L. Birkenmajer brought Copernicus into close agreement with modern astronomy; for his translation runs: "Mars revolves in not quite two years" (*Mikołaj Kopernik Wybór pism*, p. 9). It will be observed that none of these scholars noted the later passage in the *Commentariolus* from which the lacuna may be filled without hesitation.

The omission of the number of years or months in Mars' sidereal period furnishes a clue to the relation between S and V. Since they share this omission, they are both derived from a copy of the *Commentariolus* in which the error had already appeared. Now it is most unlikely that the *Commentariolus* came from the hands of Copernicus with so glaring a defect in it. Let us assume C as the text issued by Copernicus. Between C, as the original text, and S and V, as later copies, there intervenes X, one or more copies in which the omission occurred. The stemma here proposed may be represented by an inverted Y. The foregoing analysis is supported by the readings cited in the preceding note, where both S and V omit Jupiter from the list of the celestial spheres. Certainly no two independent scribes, copying from accurate texts, would both of them have omitted Jupiter and dropped the number for Mars. A study of the other variants makes it equally unlikely that S was copied from V, or V from S.



In the preface to his edition of S, Lindhagen reports the opinion of paleographers that S was written in Switzerland or northern Italy during the late sixteenth or early seventeenth century. Curtze thought that V was written in the late sixteenth century (MCV, I, 2).

The earth has three motions. First, it revolves annually in a great circle[8] about the sun in the order of the signs, always describing equal arcs in equal times; the distance from the center of the circle to the center of the sun is 1/25 of the radius of the circle.[9] The radius is assumed to have a length imperceptible in comparison with the height of the firmament;[10] consequently the sun appears to revolve with this motion, as if the earth lay in the center of the universe. However, this appearance is caused by the motion not of the sun but of the earth, so that, for example, when the earth is in the sign of Capricornus, the sun is seen diametrically opposite in Cancer, and so on. On account of the previously mentioned distance of the sun from the center of the circle, this apparent motion of the sun is not uniform, the maximum inequality being $2\frac{1}{6}°$.[11]

[8] This great circle is the *orbis magnus* discussed above (p. 16).

[9] Here Copernicus accepts Ptolemy's view that the eccentricity was fixed (HI, 233.11-16). However, Ptolemy had put the eccentricity at 1/24 (HI, 236.19-21). Hence we may say that in the *Commentariolus* Copernicus retains a fixed eccentricity, but offers an improved determination of it. On the other hand, in *De rev.* he finds that the eccentricity is 1/31 (Th 211.23-25; cf. p. 160, below). Consequently he there abandons the idea of a fixed eccentricity (Th 209.27-210.1), and holds that it varies between a maximum of 1/24 and a minimum of 1/31 (Th 219.31-220.6, 209.11-13, 211.18).

[10] See Assumption 4, above.

[11] Let the apparent motion of the sun (or real motion of the earth) take place on the great circle (*orbis magnus*) AEP (Fig. 20). Let the motion be uniform

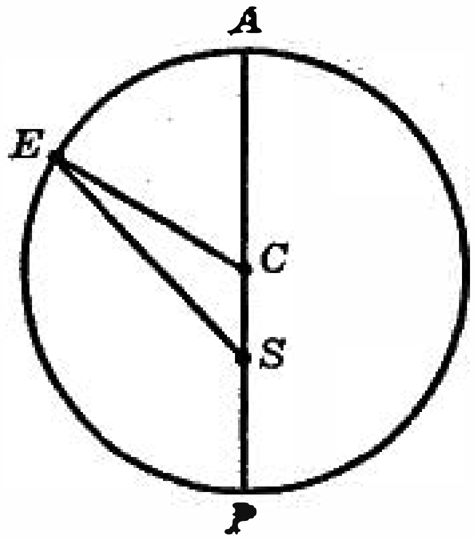


FIGURE 20

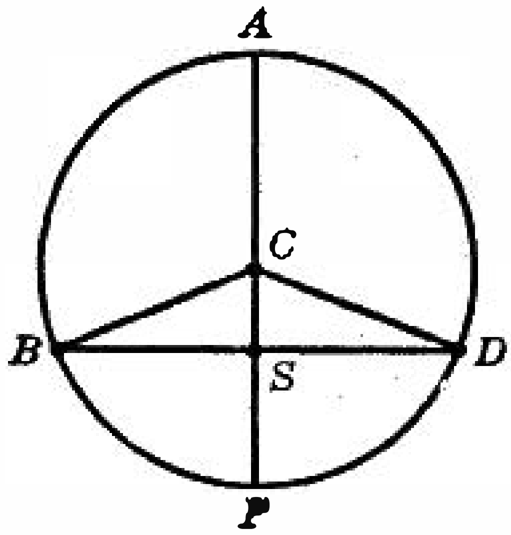


FIGURE 21

with respect to the center at C. Let the sun be at S. Let the apogee be at A, and the perigee at P. Assume that the earth starts from A and has reached any point E on the circumference. Then the line of sight ES will give the observed place of the sun, and $\angle$ ASE will measure the observed motion. But $\angle$ ACE will measure

The line drawn from the sun through the center of the circle is invariably directed toward a point of the firmament about 10° west of the more brilliant of the two bright stars in the head of Gemini;[12] therefore when the earth is opposite this point, and the center of the circle lies between them, the sun is seen at its greatest distance from the earth.[13] In this circle, then,

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the uniform or mean motion. Now the inequality to which Copernicus refers is the difference between the uniform and the observed motions; and it is measured by ∠ CES. It is evident that when the earth (or the observed place of the sun) is at A or P, the inequality is zero.

If we draw BD ⊥ ACSP at S (Fig. 21), the inequality attains its maximum at B and D (Th 207.15-208.7; cf. HI, 220.12-16, 221.9-223.3). It is obvious that the smaller the eccentricity CS is, the smaller the maximum inequality will be. Now Ptolemy had put the maximum inequality at 2° 23′ (HI, 238.22-239.1), corresponding to an eccentricity of 1/24 (CS:AC=1:24). Since in the *Commentariolus* Copernicus reduces the eccentricity to 1/25, he diminishes the maximum inequality to 2° 10′. And in *De rev.*, where he further reduces the eccentricity to 1/31, the maximum inequality is 1° 51′ (Th 212.14-16).

When he states in the *Commentariolus* that the maximum inequality, corresponding to an eccentricity of 1/25, is 2⅙° (*duobus gradibus et sextante unius*), he is evidently writing a convenient fraction. For 1/25 × 100,000 = 4,000; and by his Table of Chords, 4,000 subtends 2° 17½′ (Th 44.20-21). For the equivalence of Copernicus's Table of Chords with a modern table of sines see Armitage, *Copernicus*, pp. 171-73.

[12] With what fixed star are we to identify "the more brilliant of the two bright stars in the head of Gemini" (*stella lucida quae est in capite Gemelli splendidior*)? Both Gemini 1 (Castor, α Geminorum) and Gemini 2 (Pollux, β Geminorum) were described as being in the head of Gemini. They were usually differentiated as western and eastern (HII, 20.17-18, 21; 92.3-4; Th 132.29-32); thus in a later section of the *Commentariolus*, where Copernicus refers to "the star which is described as being in the head of the eastern of the two Gemini" (*stellam quae in capite Geminorum orientalis dicitur*), he is speaking of Pollux (p. 78). But in the present passage he does not employ the customary designation, and he relies on *splendidior* to indicate whether he is referring to Castor or to Pollux. Now, in the catalogues these stars were both listed as of the the second magnitude; and it therefore seems impossible to decide the question by appealing to a difference in brilliancy. However, Pollux is distinguished by its color; and it is perhaps possible that Copernicus is using *splendidior* as a color term. On this uncertain basis let us tentatively identify the star of our text with Pollux.

[13] If the preceding note is correct, then Copernicus is here locating the solar apogee about 10° west of Pollux. Now Ptolemy had put the longitude of the solar apogee at 65° 30′ (HI, 237.9-11) and the longitude of Pollux at 86° 40′ (HII, 93.4); hence the apogee was 21° 10′ west of Pollux. He held that the

the earth revolves together with whatever else is included within the lunar sphere.

The second motion, which is peculiar to the earth, is the daily rotation on the poles in the order of the signs, that is, from west to east. On account of this rotation the entire universe appears to revolve with enormous speed. Thus does the earth rotate together with its circumjacent waters and encircling atmosphere.[14]

The third is the motion in declination. For the axis of the daily rotation is not parallel to the axis of the great circle, but is inclined to it at an angle that intercepts a portion of a circum-

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apogee was fixed in relation to the vernal equinox (HI, 232.18-233.16), that the equinoctial and solstitial points were constant (HI, 192.12-22), and that the fixed stars moved eastward 1° in 100 years (HII, 15.15-17). Had Ptolemy been right, the apogee should have been found, at the time of Copernicus, about 35° west of Pollux.

Copernicus reverses Ptolemy's explanation of precession; for he regards the fixed stars as constant (Assumption 5) and attributes the precessional motion to the equinoctial and solstitial points (p. 67, below). Hence, when in the present passage he asserts that the solar apogee is fixed, he means that it is fixed in relation to the fixed stars; but its distance from the vernal equinox increases, because the equinoctial points move steadily westward.

Furthermore, so long as the vernal equinox was regarded as constant, it had served as the point from which celestial longitude was measured (HII, 36.16-17). Hence if Copernicus is to utilize without error the work of the ancient astronomers, he must first reconstruct the entire history of precession. In the next section of the *Commentariolus* he lays down two of the main propositions. But it is evident that he has not yet completely formulated the theory which is outlined briefly in the *Letter against Werner* (pp. 99-101, below) and expounded fully in *De rev.* (Th 157-173; cf. pp. 111-17, below). Moreover, since celestial longitude can no longer be reckoned from the vernal equinox, some fixed star must be selected in its stead. But Copernicus has apparently not yet made a choice; throughout the *Commentariolus*, when he states a celestial position, he gives it in terms of neighboring stars, never in terms of longitude reckoned from a fixed origin. Because he does not give us sufficient data to make the correction for precession, we cannot say with precision what he then believed the longitude of the solar apogee to be.

In *De rev.* he chooses a fixed star from which to measure longitude (Th 114.22-33, 130.6-7); he determines the longitude of the solar apogee as 96° 40′ (Th 211.20-21, 25-26); and he no longer regards the doctrine of a fixed apogee as tenable: "There now emerges the more difficult problem of the motion of the solar apse . . . which Ptolemy thought was fixed . . ." (Th 216.3-4).

[14] Cf. above, p. 58, n. 4.

ference in our time about 23½°.[15] Therefore, while the centre of the earth always remains in the plane of the ecliptic, that is, in the circumference of the great circle, the poles of the earth rotate, both of them describing small circles about centers equidistant from the axis of the great circle.[16] The period of this motion is not quite a year and is nearly equal to the annual revolution on the great circle. But the axis of the great circle is invariably directed toward the points of the firmament which are called the poles of the ecliptic. In like manner the motion in declination, combined with the annual motion in their joint effect upon the poles of the daily rotation, would keep these poles constantly fixed at the same points of the heavens, if the periods of both motions were exactly equal.[17] Now with the long passage of time it has become clear that this inclination of the earth to the firmament changes. Hence it is the common opinion that the firmament has several motions in conformity with a law not yet sufficiently understood. But the motion of

[15] In *De rev.* Copernicus states that he and certain of his contemporaries have found this angle (which is equal to the obliquity of the ecliptic) to be not greater than 23° 29′ (Th 76.29-77.1); and again that "in our times it is found to be not greater than 23° 28½′" (Th 162.24-25). Newcomb's determination of the obliquity for 1900 was 23° 27′ 8″.26; on the basis of an annual diminution of 0″.4684, the value for 1540 would be 23° 29′ 57″ (*American Ephemeris and Nautical Almanac for 1940*, Washington, D. C., 1938, p. xx).

[16] Müller's version is faulty. He translated: "beschreiben die beiden Pole der Erdachse bei stets gleichbleibendem Abstand kleine Kreise um die Pole der Ekliptik" (the two poles of the earth's axis, always maintaining an equal distance, describe small circles about the poles of the ecliptic; ZE, XII, 367). But Copernicus says plainly enough that it is the centers of the small circles that are equidistant from the poles of the ecliptic: *circulos utrobique parvos describentes in centris ab axe orbis magni aequidistantibus;* hence the poles of the earth are not, as Müller thought, equidistant from the poles of the ecliptic. This blunder led Müller into another error, as we shall see below (p. 73, n. 45).

[17] This obviously requires the direction of the motion in declination to be opposite to the direction of the annual motion. The explicit statement appears in *De rev.* (where the annual revolution of the earth about the sun is termed "the annual motion of the center" or more briefly "the motion of the center"): "Then there follows the third motion of the earth, the motion in declination, which is also an annual revolution but which takes place in precedence, that is, in the direction opposite to that of the motion of the center. Since the two motions are nearly equal in period and opposite in direction. . ." (Th 31.22-25; cf. p. 148, below).

the earth can explain all these changes in a less surprising way. I am not concerned to state what the path of the poles is. I am aware that, in lesser matters, a magnetized iron needle always points in the same direction. It has nevertheless seemed a better view to ascribe the changes to a sphere, whose motion governs the movements of the poles. This sphere must doubtless be sublunar.

Equal Motion Should Be Measured Not by the Equinoxes but by the Fixed Stars

Since the equinoxes and the other cardinal points of the universe shift considerably, whoever attempts to derive from them the equal length of the annual revolution necessarily falls into error.[18] Different determinations of this length were made in different ages on the basis of many observations. Hipparchus computed it as 365¼ days, and Albategnius the Chaldean as $365^d\ 5^h\ 46^m$,[19] that is, $13\frac{3}{5}^m$ or $13\frac{1}{3}^m$ less than Ptolemy.[20] Hispalensis increased Albategnius's estimate by the

[18] This assertion is directed against the Ptolemaic doctrine that the length of the year must be measured by the solstices and equinoxes (HI, 192.12-22; cf. Th 309.4-9).

[19] In *De rev.* Copernicus cites Albategnius's estimate more fully as $365^d5^h46^m24^s$ (Th 193.7-8). It is to this value that he adds $13\frac{3}{5}^m$ ($= 13^m36^s$), in order to obtain the sum $365\frac{1}{4}^d$ ($= 365^d6^h$). For Albategnius's determination see C. A. Nallino, *Al-Battānī sive Albatenii opus astronomicum* (*Pubblicazioni del Reale osservatorio di Brera in Milano*, No. 40, 1899-1907), Pt. I, 42.17. It seems clear that Copernicus did not draw from a single source the historical statements made in this section. But it is altogether likely that they were in large part based upon the *Epitome in Almagestum Ptolemaei* (Venice, 1496), begun by George Peurbach and completed by Regiomontanus (for Rheticus's use of this work, see below, p. 117, n. 35). For the *Epitome* (Bk. III, Prop. 2) gave Albategnius's determination as $365^d5^h46^m24^s$ or $13\frac{3}{5}^m$ less than $365\frac{1}{4}^d$.

[20] When Copernicus wrote the *Commentariolus*, he was misinformed about the value accepted by Hipparchus and Ptolemy, for he put it at 365¼ days. But in *De rev.* he correctly states that they found the year less than 365¼ days by 1/300th of a day, or $365^d5^h55^m12^s$ (Th 191.31-192.3, 192.21-23, 237.13-15; HI, 207.24-208.14). The *Epitome* (*loc. cit.*) cited Hipparchus's determination as $365\frac{1}{4}^d$, but quoted Ptolemy's value correctly. It should be noted that a work contemporary with the *Commentariolus* states: "Hipparchus thought that the year consisted of 365¼ days. Although he says that it was a fraction less than the complete quarter, he ignored the fraction, since he judged it to be imperceptible" (Augustinus Ricius, *De motu octavae sphaerae*, Trino, 1513, fol. e6r; Paris, 1521, p. 40 r).

20th part of an hour, since he determined the tropical year as $365^d\ 5^h\ 49^m$.[21]

[21] Prowe (PII, 191 n) and Müller (ZE, XII, 368, n. 41; the reference to *De rev.* should be III, xiii, not III, liii) followed Curtze (MCV, I, 10 n) in supposing that Hispalensis, i.e., from Hispalis = Seville, here means Isidore of Seville. In Copernicus's view precession attained its greatest rapidity in the time of Albategnius; thereafter diminution set in: "From these computations it is clear that in the 400 years before Ptolemy the precession of the equinoxes was less rapid than in the period from Ptolemy to Albategnius, and that in this same period it was more rapid than in the interval from Albategnius to our times" (Th 162.14-17; cf. p. 113, below). Therefore the shortest length of the tropical year fell in the time of Albategnius; and the increase noted by Hispalensis must be associated with a later astronomer. This chronological consideration rules out Isidore immediately. Moreover, an examination of the astronomical portions of his extant works (J. P. Migne, *Patrologia Latina*, Vols. LXXXI–LXXXIV) shows that he gives 365 days as the length of both the tropical and sidereal years.

Who, then, is Hispalensis? Jābir ibn Aflaḥ? In 1534 Peter Apian's *Instrumentum primi mobilis* was published together with Gebri filii Affla Hispalensis . . . *Libri IX de astronomia*. A copy was given by Rheticus to Copernicus (MCV, I, 36), and hence it did not get into his hands before 1539 (PII, 377.11-12). But all our evidence points to 1533 as the very latest year in which the *Commentariolus* could have been written. Moreover, Jābir (*op. cit.*, pp. 38-39) simply repeats the Hipparchus-Ptolemy estimate of the length of the tropical year. Clearly he is not the Hispalensis to whom Copernicus refers.

In his *Stromata Copernicana* (Cracow, 1924), p. 353, Birkenmajer correctly identified Copernicus' "Hispalensis" with Alfonso de Cordoba Hispalensis. The latter, who usually called himself *Alfonsus artium et medicinae doctor*, corrected Abraham Zacuto's *Almanach perpetuum exactissime nuper emendatum omnium celi motuum cum additionibus in eo factis tenens complementum* (Venice, 1502). On fol. a2r a letter is addressed to him as *Alfonso hispalensi de corduba artium et medicinae doctori*. His correction of Zacuto's *Almanach perpetuum* was published by Peter Liechtenstein at Venice on July 15, 1502, while Copernicus was a student at the nearby University of Padua. Alfonso Hispalensis' statement concerning the length of the year occurs on fol. a1v, where he corrects a computation of Zacuto and says: . . . *dividas per numerum dierum anni .365. et quartam minus undecim minutis hore* . . . (divide by the number of days in a year, 365¼ minus eleven minutes = $365^d5^h49^m$). This direct statement was overlooked by Birkenmajer, who thought he found nearly the same length of the tropical year by implication in the tables (which, however, were due to Zacuto and not to Alfonso Hispalensis). Birkenmajer also misread the second word in the volume's title, where "perpetuu3" = "perpetuum," not "perpetuum et" (Adriano Cappelli, *Lexicon abbreviaturarum*, 5th ed., Milan, 1954, p. XXXII). The *Almanach perpetuum* belonging to the library of the Ermland cathedral chapter (ZE, V, 375) may or may not have been a copy of the Venice, 1502 edition. The copy of that edition in the library of Upsala University (Pehr Fabian Aurivillius, *Catalogus librorum impressorum bibliothecae r. academiae Upsaliensis*, Upsala, 1814, p. 1002) lacks the page on which the entry *Liber capit. Varm.* would have appeared, had the volume

Lest these differences should seem to have arisen from errors of observation, let me say that if anyone will study the details carefully, he will find that the discrepancy has always corresponded to the motion of the equinoxes. For when the cardinal points moved 1° in 100 years, as they were found to be moving in the age of Ptolemy,[22] the length of the year was then what Ptolemy stated it to be. When however in the following centuries they moved with greater rapidity, being opposed to lesser motions, the year became shorter; and this decrease corresponded to the increase in precession. For the annual motion was completed in a shorter time on account of the more rapid recurrence of the equinoxes. Therefore the derivation of the equal length of the year from the fixed stars is more accurate. I used Spica Virginis[23] and found that the year has always been 365 days, 6 hours, and about 10 minutes,[24] which is also the estimate of the ancient Egyptians.[25] The same method must be employed also with the other motions of the

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once belonged to the library of the Ermland chapter (Birkenmajer, *Stromata*, p. 300).

Since "Hispalensis" in the *Commentariolus* means the *Almanach perpetuum* of 1502, it follows that Copernicus wrote the *Commentariolus* after July 15 of that year. If the entry ... *sexternus Theorice asserentis Terram moveri, Solem vero quiescere* ... (a manuscript of six leaves expounding the theory of an author who asserts that the earth moves while the sun stands still) in the catalogue of his books drawn up on May 1, 1514, by Matthew of Miechow (1457–1523), professor at the university of Cracow, refers to the *Commentariolus*, then its date of composition is narrowed down to the dozen years between July 15, 1502 and May 1, 1514.

[22] HII, 15.6–16.2. [23] Virgo 14 (HII, 102.16; Th 136.10), α Virginis.

[24] Copernicus's estimate of the length of the sidereal year is stated more exactly in *De rev.* as $365^d6^h9^m40^s$ (Th 195.29–196.2); Curtze misquotes the estimate as $365^d6^h8^m40^s$ (MCV, I, 10 n), and Prowe repeats the misstatement (PII, 191 n). Newcomb's determination (1900) is $365^d.25636042 = 365^d6^h9^m9^s.54$ (*American Ephemeris for 1940*, p. xx).

[25] Copernicus apparently derived this information from the *Epitome*. It stated (*loc. cit.*) that the value found by the ancient Egyptians was $365\frac{1}{4}^d + \frac{1}{130}^d$ ($= 365^d6^h11^m$). The Latin translation of Albategnius, which was printed at Nuremberg in 1537, likewise ascribed to certain ancient Egyptian and Babylonian astronomers a year consisting of $365\frac{1}{4}^d + \frac{1}{131}^d = 365^d6^h11^m$ (Nallino, *Al-Battānī*, I, 40.28–29, 204–9; cf. below, p. 117, n. 34). So far as I am aware, no determination of the length of the year more precise than $365\frac{1}{4}^d$ has been discovered among the papyri or other documents surviving from ancient Egypt.

planets, as is shown by their apsides, by the fixed laws of their motion in the firmament, and by heaven itself with true testimony.

The Moon

The moon seems to me to have four motions in addition to the annual revolution which has been mentioned. For it revolves once a month on its deferent circle about the center of the earth in the order of the signs.[26] The deferent carries the epicycle which is commonly called the epicycle of the first inequality or argument, but which I call the first or greater epicycle.[27] In the upper portion of its circumference this greater epicycle revolves in the direction opposite to that of the deferent,[28] and its period is a little more than a month. Attached

[26] The loss of a leaf from V creates a lacuna which begins at this point and ends near the close of the present section. For the intervening text we must rely on S alone.

[27] The meaning of *anni* is not clear to me, and I have omitted it from the translation. Müller rendered the passage as follows: "wir nennen ihn einfach den ersten, den Haupt- oder Jahres-Epicykel" (but which I call the first, the chief, or annual epicycle; ZE, XII, 370). There are three objections to Müller's version of *anni*. It is syntactically unsound; in Copernicus's system the first lunar epicycle has no connection with the year; Copernicus regularly employs in his lunar theory the terms "first epicycle" and "greater epicycle," but never "annual epicycle" or "epicycle of the year" (cf. Th 235.14-15, 257.7-8, 262.26, 277.22, 288.23).

[28] When the motion of a circle, in the upper portion of its circumference, is in precedence, i.e., from east to west, in the lower portion it is in consequence, from west to east; and vice versa. "Now let *abc* (Fig. 22) be the epicycle . . .

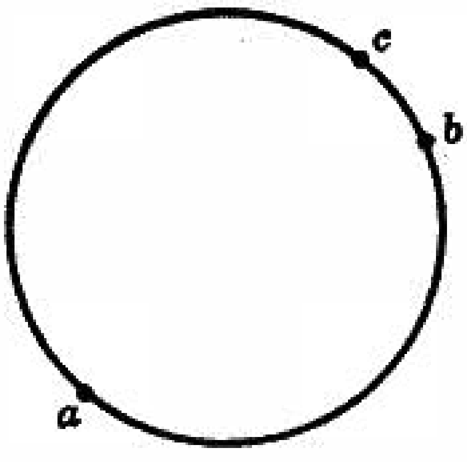

FIGURE 22

and let the motion of the epicycle be understood to be from *c* to *b* and from *b* to *a*, that is, in precedence in the upper portion and in consequence in the lower portion" (Th 251.26–252.1; cf. also Th 323.26-28, 325.21-23; PII, 349.14-16). When the direction of a motion is stated without reference to the portion of the circumference, it is the upper circumference that is understood.

to it is a second epicycle. The moon, finally, moving with this second epicycle, completes two revolutions a month in the direction opposite to that of the greater epicycle, so that whenever the center of the greater epicycle crosses the line drawn from the center of the great circle through the center of the earth (I call this line the diameter of the great circle), the moon is nearest to the center of the greater epicycle. This occurs at new and full moon; but contrariwise at the quadratures, midway between new and full moon,[29] the moon is most remote from the center of the greater epicycle. The length of the radius[30] of the greater epicycle is to the radius of the def-

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Müller was evidently unfamiliar with this usage, for he detached *in superiore quidem portione* from *contra motum orbis reflexus.* He translated: "dabei führt er auf seiner Aussenseite einen ferneren Epicykel mit sich" (as the first epicycle revolves, it carries with it on its surface another epicycle; ZE, XII, 370). But Rheticus explicitly states: "As the first epicycle revolves uniformly about its own center, in its upper circumference it carries the center of the small second epicycle in precedence, in its lower circumference, in consequence" (p. 134, below).

29 Here, too, Müller blundered. For he translated *in quadraturis mediantibus iisdem* by: "zur Zeit der mittleren Quadraturen" (at the time of the mean quadratures; ZE, XII, 371). This version ignores *iisdem* and mistakes *mediare* (to halve) for *medius* (the technical astronomical term for "mean"). But Copernicus has not yet begun to discuss the lunar inequalities; all that he is stating here is the elementary fact (see p. 47, above) that the quadratures are midway between new and full moon (*iisdem*).

30 Although *diametri*, the reading of S, cannot be checked on account of the lacuna in V, it is certainly wrong and must be changed to *semidiametri.* Computational support for this emendation is adduced in n. 32. Additional support comes from a calculation jotted down by Copernicus in his copy of the Tables of Regiomontanus (see Curtze in *Zeitschrift für Mathematik und Physik,* XIX (1874), 454-56). The note reads: *Semidiametrus orbis lunae ad epicyclium a* $\frac{10}{1\frac{1}{18}}$; *epicyclus a ad b* $\frac{19}{4}$ (PII, 211); "Radius of deferent of moon to first epicycle 10:1 1/18; first epicycle to second epicycle 19:4." Throughout this series of calculations Copernicus is comparing radius with radius, never diameter with radius.

While the note was properly used by Curtze to emend another false reading (*parte* for *quarta*) in this same sentence of S, he overlooked *diametri.* Curiously enough, in citing Curtze's work Prowe speaks of the note as containing "values calculated by Copernicus for the radii of the planetary epicycles" (PII, 193 n); yet he too failed to notice the discrepancy. Had Müller compared his computations (ZE, XII, 372, n. 51) for the *Commentariolus* with the lunar numerical ratios in *De rev.,* he would surely have caught the copyist's error. It should be observed that Rheticus compares diameter with diameter when he gives the ratio of the lunar epicycles (p. 134, below).

erent as 1 1/18:10;[31] and to the radius of the smaller epicycle as 4 3/4:1.[32]

By reason of these arrangements the moon appears, at times rapidly, at times slowly, to descend and ascend; and to this first inequality the motion of the smaller epicycle adds two irregularities.[33] For it withdraws the moon from uniform motion on the circumference of the greater epicycle, the maximum inequality being 12 1/4° of a circumference of corresponding size or diameter;[34] and it brings the center of the greater epicycle

[31] I have adopted this form for the sake of clarity and compactness. What Copernicus actually wrote may be literally translated as follows: "The length of the radius of the greater epicycle contains a tenth part of the radius of the deferent plus one-eighteenth of such tenth part." This ratio may be numerically represented by the expression $1/10 + 1/18 \cdot 1/10 : 1$ or $1\ 1/18 : 10$.

[32] Literally: "(The length of the radius of the greater epicycle) contains the radius of the smaller epicycle five times minus one-fourth of the smaller radius." While Copernicus incorporated in *De rev.* the lunar theory sketched in this section, he altered the numerical components slightly (Th 258.10-11). The ratio of first epicycle to deferent is given here as 1 1/18 : 10, which may be written 1055:10,000; in *De rev.* it has been changed to 1097:10,000, which may be written 1 1/10 : 10. The ratio of first epicycle to second epicycle appears there as 1097:237, which may be written 4.63:1; it is given above as 4.75:1.

[33] Although the meaning of the passage is clear, the text is faulty and simply does not parse. We might have expected *et primae quidem diversitati dupliciter variationem motus epicycli minoris ingerit* (cf. Th 257.20-21). The distance from the moon to the center of the earth varies, because the moon's orbit around the earth is really an ellipse; and the rate of the moon's apparent motion varies for the same reason. Copernicus uses the term "first inequality" to denote the variation in the moon's distance from the center of the earth and employs the first epicycle to account for it. Both the term and the geometrical device were traditional (cf. HI, 300.16-301.1).

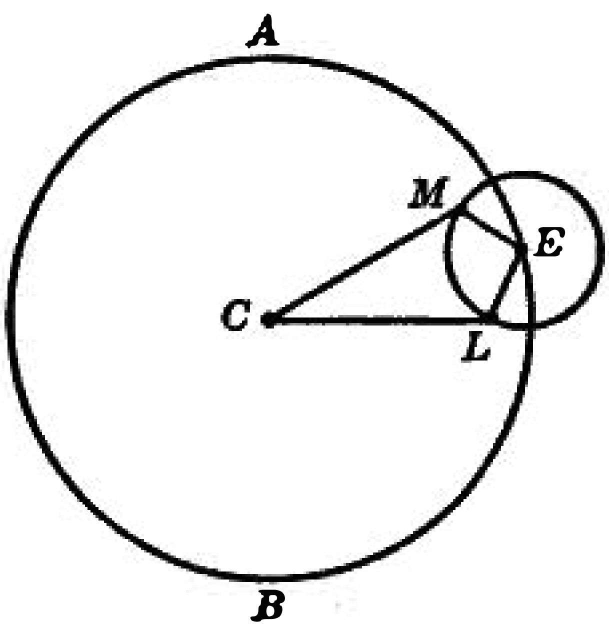

FIGURE 23

[34] The inequality is measured by an arc of the greater epicycle, or of a circle of equal dimensions. Let AB be the greater epicycle with center at C (Fig. 23). Choose any point E on the circumference, and with E as center describe the second epicycle. Draw CM and CL tangent to the second epicycle. When the moon is at M or L, the inequality attains its maximum. Now in the *Commentariolus* CE:EM = 4.75:1 = 100,000: 21,053. Then by the Table of Chords ∠ECM, which measures the maximum inequality, = 12° 9′ (Th 45.19-20). Hence the reading of S, *17 gradus et quadrantem*, is certainly wrong, and must be corrected to *12 gradus et quadrantem*. As in the case of the solar inequality (see above, p. 62, n. 11), Copernicus is writing a convenient fraction.

at times nearer the moon, at times further from it, within the limits of the radius of the smaller epicycle.[35] Therefore, since the moon describes unequal circles about the center of the greater epicycle, the first inequality varies considerably. In conjunctions and oppositions to the sun its greatest value does not exceed 4°56′, but in the quadratures it increases to 6°36′.[36] Those who employ an eccentric circle to account for this variation[37] improperly treat the motion on the eccentric as unequal,[38]

While the reading of S cannot be checked on account of the lacuna in V, the proposed emendation is confirmed by a comparison with *De rev.* We saw above (n. 32) that in the later work Copernicus diminished the ratio CE:EM, making it 1097:237 = 4.63:1 = 100,000:21,604. It is obvious that since CE has been shortened in relation to EM, ∠ECM must increase; by the Table of Chords, it is 12° 28′ (Th 45.21-22, 258.32–259.4, 264.31). Hence any such value as 17¼° for the maximum inequality in the *Commentariolus* must be rejected as a copyist's error.

[35] Reading *cum* for *eum* (PII, 193.11).

[36] The difference between the maximum in the quadratures and the maximum in conjunctions and oppositions is 6° 36′ — 4° 56′ = 1° 40′. According to Ptolemy, the difference was 2° 40′ (HI, 362.1-6). In *De rev.* it is 2° 44′ (Th 262.23-32, 265.10-11). Hence I suggest that the figure in our text should be changed from 6° 36′ to 7° 36′. Again the reading of S cannot be checked on account of the lacuna in V.

From the following table it can be seen how closely Copernicus adhered to the Ptolemaic determination of the lunar inequalities. The second column contains the maximum inequality in conjunctions and oppositions; the third column shows the greatest additional inequality in the quadratures; and the fourth column sums the second and third.

Ptolemy	5° 0′	2° 40′	7° 40′
Commentariolus	4° 56′	2° 40′	7° 36′
De revolutionibus	4° 56′	2° 44′	7° 40′

Although Ptolemy's table for the first lunar inequality gives 5° 1′ as the maximum (HI, 337.21, 390.24), he generally uses the round number 5° in his calculations (HI, 338.22–339.3, 362.1-6, 363.10-12, 364.20-22; cf. Th 257.26-29).

[37] Ptolemy is credited with having discovered the second inequality (HI, 294.9-14, 354.18–355.20); to account for it, he represented the center of the lunar epicycle as revolving on a circle eccentric to the earth (HI, 355.20-22; cf. Th 232.1-3).

[38] This charge that the representation employed by Ptolemy and his successors violates the axiom of uniform motion is amplified in *De rev.*: "For when they assert that the motion of the center of the epicycle is uniform with respect to the center of the earth, they must also admit that the motion is not uniform on the circle which it describes, namely, the eccentric" (Th 233.11-13). Müller was apparently puzzled by the words *praeter ineptam in ipso circulo motus inaequalitatem* and omitted them from his translation (ZE, XII, 373).

and, in addition, fall into two manifest errors. For the consequence by mathematical analysis is that when the moon is in quadrature, and at the same time in the lowest part of the epicycle, it should appear nearly four times greater (if the entire disk were luminous) than when new and full, unless its magnitude increases and diminishes in no reasonable way.[39] So too, because the size of the earth is sensible in comparison with its distance from the moon, the lunar parallax[40] should increase very greatly at the quadratures. But if anyone investigates these matters carefully, he will find that in both respects the quadratures differ very little from new and full moon, and accordingly will readily admit that my explanation is the truer.

With these three motions in longitude, then, the moon passes through the points of its motion in latitude.[41] The axes of the epicycles are parallel to the axis of the deferent, and therefore the moon does not move out of the plane of the deferent. But the axis of the deferent is inclined to the axis of the great circle

[39] Müller translated the last clause: "es sei denn, man behauptete thörichterweise ein wirkliches Wachsen und Abnehmen der Mondkugel" (unless they absurdly maintained that there is a real increase and decrease in the size of the moon; ZE, XII, 373). This version misses the point. The apparent size of the moon (as measured by its apparent diameter) varies, because the distance of the moon from the earth is not constant (see n. 33). The first of the "two manifest errors" produced by the eccentric is, not that it causes the apparent size of the moon to vary, but that it grossly exaggerates the variation (cf. Th 234.31–235.8).

[40] Müller failed to recognize *diversitas aspectus* as the technical term for parallax (see p. 51, above). Hence he was unable to distinguish the second of the "two manifest errors," and his translation (ZE, XII, 373) speaks only of the apparent variation in the size of the moon, "der scheinbare Unterschied in der Grösse." Consequently, in the next sentence, where Copernicus refers to both (*utrumque*) disagreements with the observational data which are produced by the eccentric (1. exaggeration of the variation in the apparent size of the moon; 2. exaggeration of the variation in the lunar parallax), Müller does not know how to render *utrumque*, and falls back on "Grössenunterschied" (variation in size). The explicit statement of Rheticus puts the matter beyond all question: "But experience has shown my teacher that the parallax and size of the moon, at any distance from the sun, differ little or not at all from those which occur at conjunction and opposition, so that clearly the traditional eccentric cannot be assigned to the moon" (p. 134, below).

[41] Here the lacuna in V ends.

or ecliptic;[42] hence the moon moves out of the plane of the ecliptic. Its inclination is determined by the size of an angle which subtends 5° of the circumference of a circle.[43] The poles of the deferent revolve at an equal distance from the axis of the ecliptic,[44] in nearly the same manner as was explained regarding declination.[45] But in the present case they move in the reverse order of the signs and much more slowly, the period of the revolution being nineteen years.[46] It is the com-

[42] Müller rendered *axi magni orbis sive eclipticae* by: "die Achse des grössten Kreises der Ekliptik" (the axis of the great circle of the ecliptic; ZE, XII, 373). This faulty translation shows that Müller did not quite grasp the meaning of *orbis magnus*, which he interpreted (ZE, XII, 365, n. 25) as meaning "great circle" in the geometrical sense, i.e., a circle drawn on the surface of a sphere with its center in the center of the sphere. However, Copernicus's term for "great circle" in the geometrical sense (see above, p. 12, n. 26) is not *orbis magnus* but *circulus maximus* (Th 57-66, *passim*). In the present passage *orbis magnus* bears its usual sense of the real annual revolution of the earth about the sun (see p. 16, above). The *orbis magnus* and the ecliptic lie in the same plane and have a common axis: "But the axis of the great circle is invariably directed toward the points of the firmament which are called the poles of the ecliptic" (p. 64, above).

[43] This estimate of 5° for the maximum latitude of the moon was derived from Ptolemy (HI, 388.11–389.7, 391.52; cf. Th 272.13-15, 274.8-9) and, subject to the correction mentioned in the following note, is retained in modern astronomy.

[44] Therefore the inclination of the moon's orbit to the ecliptic would be constant. That this inclination in fact varies was discovered by Tycho Brahe; see *Tychonis Brahe opera omnia*, ed. Dreyer, II, 121-30, 413.13-21; IV, 42.27-43.22; VI, 170.1–171.8; VII, 151.28–154.35; XI, 162-63; XII, 399-400; Dreyer's remarks on p. liv of the Introduction to Vol. I; and his *Tycho Brahe* (Edinburgh, 1890), pp. 342-44.

[45] See above, p. 64, n. 16. Müller missed the force of *propemodum sicut*, which he translated (ZE, XII, 373) by "ähnlich" (like); whereas "almost like," or something of the sort, is required. In the case of the moon, the poles of the deferent revolve at an equal distance from the axis of the ecliptic, *in aequidistantia axis eclipticae*; but in the case of the motion in declination, the poles of the earth revolve on circles having centers equidistant from the axis of the ecliptic.

[46] This estimate of nineteen years for the period during which the lunar nodes perform their regression was also derived from Ptolemy. He measured the rate of regression by subtracting the moon's mean motion in longitude from the mean motion in latitude (HI, 301.18-23, 356.4-9), the difference being about 3′ a day (HI, 356.25-357.6, 358.6-11). By reference to his tables for the moon's motion (HI, 282-293) we can determine the period required for the completion of the circuit as 18 years, 7 months, and 16 days. The discovery that the regression of the nodes is not uniform was made by Tycho Brahe (see the references cited in n. 44, above).

mon opinion that the motion takes place in a higher sphere, to which the poles are attached as they revolve in the manner described. Such a fabric of motions, then, does the moon seem to have.

The Three Superior Planets
Saturn—Jupiter—Mars

Saturn, Jupiter, and Mars have a similar system of motions, since their deferents completely enclose the great circle and revolve in the order of the signs about its center as their common center. Saturn's deferent revolves in 30 years, Jupiter's in 12 years, and that of Mars in 29 months;[47] it is as though the size of the circles delayed the revolutions. For if the radius of the great circle is divided into 25 units, the radius of Mars' deferent will be 38[48] units, Jupiter's 130 5/12, and Saturn's 230 1/6.[49] By "radius of the deferent" I mean the distance from the center of the deferent to the center of the first epicycle. Each deferent has two epicycles,[50] one of which carries the

[47] See above, p. 60, n. 7.

[48] Although both S and V read 30, I propose to substitute 38, for the reasons stated in n. 50, below.

[49] S: *230 et sextantem unius*; V: *236 et sextantem unius*. Prowe accepted V, but S is to be preferred, for the reasons given in n. 50, below.

[50] In the *Commentariolus* Copernicus employs for the planets what we have called the concentrobiepicyclic arrangement (see pp. 7, 37, above), consisting of two epicycles upon a deferent which is concentric with the great circle. In *De rev.* this device is replaced, for the three superior planets, by an eccentrepicyclic arrangement, i.e., by a single epicycle upon an eccentric deferent (Th 325.16-21); after indicating the geometric equivalence of the two devices (Th 325.11-16, 327.6-13), Copernicus points to the variation in the eccentricity of the great circle as the reason for his choice of the eccentrepicyclic arrangement (Th 327.13-16). When he wrote the *Commentariolus*, he regarded this eccentricity as constant (see above, p. 61, n. 9).

Now if the two arrangements are to produce identical results, then, as Copernicus points out, the radius (R) of the concentric deferent (*Commentariolus*) must be equal to the radius (R) of the eccentric deferent (*De rev.*). Let r denote the radius of the great circle. By a comparison of the ratio R:r, as given here, with the values in *De rev.*, we may discover whether in shifting from the concentrobiepicyclic to the eccentrepicyclic arrangement Copernicus altered the relative sizes of the deferent and great circle. In *De rev.*, for Saturn r = 1090 (Th 341.29), for Jupiter r = 1916 (Th 353.15-16), and for Mars r = 6580 (Th 364.8-9), when in each case R = 10,000.

other, in much the same way as was explained in the case of the moon,[51] but with a different arrangement. For the first epicycle revolves in the direction opposite to that of the deferent, the periods of both being equal. The second epicycle, carrying the planet, revolves in the direction opposite to that of the first with twice the velocity. The result is that whenever the second epicycle is at its greatest or least distance from the center of the deferent, the planet is nearest to the center of the first epicycle; and when the second epicycle is at the midpoints, a quadrant's distance from the two points just men-

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	R:r	
	De revolutionibus	*Commentariolus*
Saturn	10,000:1090 = 229⅓:25	230⅙:25
Jupiter	10,000:1916 = 130½:25	130$\frac{5}{12}$:25
Mars	10,000:6580 = 38:25	38:25

The table enables us to deal with a variant reading in this passage. For R in the case of Saturn, S has 230⅙, while V gives 236⅙ (Curtze's collation [MCV, IV, 7] inaccurately assigns to Jupiter the reading of S for Saturn). Prowe accepted the reading of V, but S is clearly preferable, as the following analysis will show.

I have already referred (see p. 69, n. 30) to the series of notes made by Copernicus in his copy of the Tables of Regiomontanus. Curtze correctly pointed out that the ratios contained in these notes are identical with those adopted in the *Commentariolus* (MCV, IV, 7 n); and he used the statement about the moon to emend a false reading in our text. However, he failed to make any further use of these entries. Now for the radius (not diameter, as MCV, I, 12 n. and PII, 195 n. have it; cf. PII, 211) of Saturn's deferent, they give 230⅚ (not 230⅝, as PII, 195 n). Hence we are justified in preferring the reading of S to that of V. This judgment is confirmed by the fact that Tycho Brahe's reference to the *Commentariolus* agrees with S (see his *Opera omnia*, ed. Dreyer, II, 428.40–429.2).

Moreover, these notes of Copernicus show that S and V agree on a false reading for R in the case of Mars. The statement in the Tables of Regiomontanus gives the radius of Mars' deferent as approximately 38 (*Martis semidiametrus orbis 38 fere*). Now a value of 30, which is the reading of both our MSS, would make the ratio R:r for Mars 30:25 = 10,000:8333, at wide variance from the corresponding ratio in *De rev.* But reference to the table will show that the agreement between *De rev.* and the *Commentariolus* for both Saturn and Jupiter is quite close. Hence I have adopted 38, the number written by Copernicus in his Tables of Regiomontanus, in place of 30. Writing o for 8 is not an uncommon error of copyists (cf. below, p. 82, n. 74).

[51] See the opening paragraph of the section on "The Moon."

tioned,[52] the planet is most remote from the center of the first epicycle. Through the combination of these motions of the deferent and epicycles, and by reason of the equality of their revolutions, the aforesaid withdrawals and approaches occupy absolutely fixed places in the firmament, and everywhere exhibit unchanging patterns of motion. Consequently the apsides are invariable;[53] for Saturn, near the star which is said to be on the elbow of Sagittarius;[54] for Jupiter, 8° east of the star which is called the end of the tail of Leo;[55] and for Mars, 6½° west of the heart of Leo.[56]

[52] Müller rendered *in quadrantibus autem mediantibus* by: "zur Zeit der mittleren Quadraturen" (at the time of the mean quadratures; *ZE*, XII, 374). With regard to *mediantibus*, this version repeats the blunder pointed out above in n. 29 on p. 69; and, in addition, it mistakes *quadrans* (quadrant, the fourth part of a circumference) for *quadratura* (quadrature; cf. above, p. 47, n. 163).

[53] This was Ptolemy's view. He held that the planetary apsides were fixed in relation to the sphere of the fixed stars, since, as measured by the equinoxes and solstices, both the apsides and the fixed stars moved in the same direction at the same slow rate (HII, 251.24–252.7, 252.11-18, 257.3-12, 269.3-11; cf. Th 308.20-24).

[54] The star is here described as *quae super cubitum esse dicitur Sagittatoris*. It is unquestionably to be identified with Sagittarius 19 in Ptolemy's catalogue (HII, 114.10), for that star was described in the first printed translation of the *Syntaxis* into Latin (Venice, 1515, p. 84r) as *quae est super cubitum dextrum*. In *De rev.* Copernicus uses instead the name *In dextro cubito* (Th 139.14).

[55] This star is Leo 27 in Ptolemy's catalogue and in *De rev.* (HII, 100.7; Th 135.12). Its Bayer name is β Leonis.

[56] This star is Leo 8 (HII, 98.6; Th 134.23-24). It was called Basiliscus or Regulus, and its Bayer name is α Leonis.

Ptolemy had put the apogee of Saturn at 23° of Scorpio; of Jupiter, at 11° of Virgo; and of Mars, at 25° 30′ of Cancer (HII, 412.12-17, 380.22–381.4, 345.12-20). In his catalogue of the fixed stars these places are, respectively, 31° 50′ west of Sagittarius 19, 16° 30′ east of Leo 27, and 7° west of Leo 8. From them Copernicus's determinations differ, respectively, by 31° 50′ eastward, 8° 30′ westward, and ½° eastward. Hence we may say that although in the *Commentariolus* Copernicus accepted Ptolemy's doctrine of the fixity of the planetary apsides, he intended to put forward improved determinations of them.

In *De rev.* the places are again altered. But now they are all east of Ptolemy's determinations; for Saturn's apogee is 17° 49′ west of Sagittarius 19; Jupiter's, 21° 10′ east of Leo 27; and Mars', 3° 50′ east of Leo 8 (Th 338.15-18, 350.15-16, 360.35). Hence Copernicus abandons the idea of the fixed apogee and enunciates the discovery that the longitude of the planetary apogees increases: "Moreover, the position of the higher apse of [Saturn's] eccentric has in the meantime advanced 13° 58′ in the sphere of the fixed stars. Ptolemy believed that this posi-

The radius of the great circle was divided above into 25 units. Measured by these units, the sizes of the epicycles are as follows. In Saturn the radius of the first epicycle consists of 19 units, 41 minutes; the radius of the second epicycle, 6 units, 34 minutes. In Jupiter the first epicycle has a radius of 10 units, 6 minutes; the second, 3 units, 22 minutes. In Mars the first epicycle, 5 units, 34 minutes; the second, 1 unit, 51 minutes.[57] Thus the radius of the first epicycle in each case is three times as great as that of the second.[58]

The inequality which the motion of the epicycles imposes upon the motion of the deferent is called the first inequality; it follows, as I have said, unchanging paths everywhere in the firmament. There is a second inequality, on account of which the planet seems from time to time to retrograde, and often to become stationary. This happens by reason of the motion, not of the planet, but of the earth changing its position in the great circle. For since the earth moves more rapidly than the planet, the line of sight directed toward the firmament regresses, and the earth more than neutralizes the motion of the planet. This regression is most notable when the earth is nearest to the planet, that is, when it comes between the sun and the planet at the evening rising of the planet. On the other hand, when

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tion, like the others, was fixed; but it is now clear that it moves about 1° in 100 years" (Th 339.7-11; cf. Th 351.2-5, 359.33–360.7; and Dreyer, *Planetary Systems*, p. 338).

[57] I resume the comparison instituted above in n. 50 on p. 74. As Copernicus points out (Th 327.7-8), the radius (E) of the first epicycle (*Commentariolus*) must be equal to the eccentricity (E) of the eccentric (*De rev.*). Now in *De rev.* for Saturn E = 854 (Th 330.18), for Jupiter E = 687 (Th 343.23-28), and for Mars E = 1460 (Th 358.28); we already have the values of r.

	r:E *De revolutionibus*	*Commentariolus*
Saturn	1090: 854 = 25:19^{p}35^m	25:19^{p}41^m
Jupiter	1916: 687 = 25: 8^{p}58^m	25:10^p 6^m
Mars	6580:1460 = 25: 5^{p}33^m	25: 5^{p}34^m

[58] Hence the radius of the second epicycle in the *Commentariolus* is equal to the radius of the single epicycle in *De rev.*, since both = ⅓ E (Th 325.19-20). An exception will be noted in the case of Mars, where Copernicus reduces the eccentricity from 1,500 (Th 354.29–355.2) to 1,460, but leaves the radius of the epicycle at 500 (Th 358.24-31, 360.7-11, 362.26-28).

the planet is setting in the evening or rising in the morning, the earth makes the observed motion greater than the actual. But when the line of sight is moving in the direction opposite to that of the planets and at an equal rate, the planets appear to be stationary, since the opposed motions neutralize each other; this commonly occurs when the angle at the earth between the sun and the planet is 120°.[59] In all these cases, the lower the deferent on which the planet moves, the greater is the inequality. Hence it is smaller in Saturn than in Jupiter, and again greatest in Mars, in accordance with the ratio of the radius of the great circle to the radii of the deferents. The inequality attains its maximum for each planet when the line of sight to the planet is tangent to the circumference of the great circle. In this manner do these three planets move.

In latitude they have a twofold deviation. While the circumferences of the epicycles remain in a single plane with their deferent, they are inclined to the ecliptic. This inclination is governed by the inclination of their axes, which do not revolve, as in the case of the moon,[60] but are directed always toward the same region of the heavens. Therefore the intersections of the deferent and ecliptic (these points of intersection are called the nodes) occupy eternal places in the firmament.[61] Thus the node where the planet begins its ascent toward the north is, for Saturn, 8½° east of the star which is described as being in the head of the eastern of the two Gemini;[62] for Jupiter, 4° west of

[59] Cf. Pliny *Natural History* ii.15(12).59: "In the trine aspect, that is, at 120° from the sun, the three superior planets have their morning stations, which are called the first stations . . . and again at 120°, approaching from the other direction, they have their evening stations, which are called the second stations"; cf. also ii.16(13).69-71. It has been shown that Copernicus read carefully a copy of the Rome, 1473 edition of Pliny's *Natural History* (L. A. Birkenmajer, *Stromata Copernicana*, Cracow, 1924, pp. 327-34); and also a copy of the Venice, 1487 edition (MCV, I, 40-41).

[60] See the closing paragraph of the section on "The Moon."

[61] Copernicus derived from Ptolemy the view that the nodes, like the apsides, are fixed (HII, 530.8-11; cf. Karl Manitius, *Des Claudius Ptolemäus Handbuch der Astronomie*, Leipzig, 1912-13, II, 426). But in *De rev.*, having discovered the motion of the apsides, Copernicus holds that this motion is shared by the nodes (Th 413.7-15, 415.20-25).

[62] Gemini 2 (HII, 92.4; Th 132.31-32), Pollux, β Geminorum; cf. above, p. 62, n. 12.

the same star; and for Mars, 6½° west of Vergiliae.[63] When the planet is at this point and its diametric opposite, it has no latitude. But the greatest latitude, which occurs at a quadrant's distance from the nodes,[64] is subject to a large inequality. For the inclined axes and circles seem to rest upon the nodes, as though swinging from them. The inclination becomes greatest when the earth is nearest to the planet, that is, at the evening rising of the planet; at that time the inclination of the axis is, for Saturn 2⅔°, Jupiter 1⅔°, and Mars 1⅚°.[65] On the other hand, near the time of the evening setting and morning rising, when the earth is at its greatest distance from the planet, the inclination is smaller,[66] for Saturn and Jupiter by 5/12°, and for

[63] Taurus 30 (HII, 90.2; Th 132.5-6). Authorities differ about the identification of Taurus 30; see Christian H. F. Peters and Edward B. Knobel, *Ptolemy's Catalogue of Stars* (Carnegie Institution of Washington, Publication No. 86, 1915), p. 115.

[64] Ptolemy had put the points of greatest northern latitude for Saturn and Jupiter at 0° of Libra, and for Mars at 30° of Cancer (HII, 526.6-11; cf. Th 413.7-11). If we compare these places with his determinations of the apogees (see above, p. 76, n. 56), we find that for Saturn the point of greatest northern latitude is 53° west of the apogee; for Jupiter, 19° east; and for Mars, 4° 30′ east. Ptolemy states these differences of position in round numbers as 50° west, 20° east, and 0° (HII, 587.5-9; cf. Manitius, *Ptolemäus Handbuch*, II, 425, n. 21).

In the present passage Copernicus gives the places of the ascending nodes. By adding 90° to these places, we obtain the points of greatest northern latitude. They turn out to be, for Saturn, 79° 40′ west of the apogee; for Jupiter, 20° 10′ east; and for Mars, 0° 20′ west. In the *Commentariolus*, then, Copernicus not only adheres to the Ptolemaic ideas of the fixed apogee and the fixed node, but he also retains Ptolemy's distance between apogee and node for Jupiter and Mars, although increasing the distance by 30° for Saturn.

In *De rev.*, although the apogee moves, the distance between apogee and node remains constant, since the node shares the motion of the apogee. Copernicus finds the points of greatest northern latitude, for Saturn at 7° of Scorpio; for Jupiter at 27° of Libra; and for Mars at 27° of Leo (Th 413.11-13). If we compare these places with his determinations of the apogees (see above, p. 76, n. 56), we find that for Saturn the point of greatest northern latitude is 23° 21′ west of the apogee; for Jupiter, 48° east; and for Mars, 27° 20′ east.

[65] S: *dextante;* V: *sextante.* Prowe, followed by Müller (ZE, XII, 377), adopted the reading of V; but *sextante* is clearly impossible, for the following sentence of the text states that the inclination diminishes in the case of Mars by 1⅔°.

[66] The inclination is greatest when the planet is in opposition, smallest when the planet is in conjunction; and the greatest difference between maximum and

Mars by 1⅜°. Thus this inequality is most notable in the greatest latitudes, and it becomes smaller as the planet approaches the node, so that it increases and decreases equally with the latitude.

The motion of the earth in the great circle also causes the observed latitudes to change, its nearness or distance increasing or diminishing the angle of the observed latitude, as mathematical analysis demands. This motion in libration occurs along a straight line, but a motion of this sort can be derived from two circles. These are concentric, and one of them, as it revolves, carries with it the inclined poles of the other. The lower circle revolves in the direction opposite to that of the upper, and with twice the velocity. As it revolves, it carries with it the poles of the circle which serves as deferent to the epicycles. The poles of the deferent are inclined to the poles of the circle

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minimum occurs at the points of greatest latitude (Th 415.9-14). The following table compares the maximum and minimum angles of inclination as given here with those in *De rev.* (Th 421.22-25; 421.31–422.1; 422.7-8, 10-11).

	Angles of Inclination	*Commentariolus*	*De revolutionibus*
Saturn	Greatest	2° 40′	2° 44′
	Least	2° 15′	2° 16′
Jupiter	Greatest	1° 40′	1° 42′
	Least	1° 15′	1° 18′
Mars	Greatest	1° 50′	1° 51′
	Least	0° 10′	0° 9′

From the table we see that the main inclinations and their limits of variation are as follows:

	Commentariolus	*De revolutionibus*
Saturn	2° 27½′ ± 12½′	2° 30′ ± 14′
Jupiter	1° 27½′ ± 12½′	1° 30′ ± 12′
Mars	1° ± 50′	1° ± 51′

In Ptolemy's treatment of the latitudes, for the three superior planets the angle at which the eccentric deferent was inclined to the ecliptic was constant (HII, 529.3-9). His values were: for Saturn 2° 30′, for Jupiter 1° 30′, and for Mars 1° (HII, 540.13-14, 542.5-9). But the epicycle was inclined to the eccentric at a varying angle (HII, 529.12–530.8). It will be observed that in Copernicus's theory the epicycles and deferent are coplanar; hence the angle at which the deferent is inclined to the ecliptic cannot be fixed, but must vary (Th 413.1-3, 29-31).

halfway[67] above at an angle equal to the inclination of these poles to the poles of the highest circle.[68] So much for Saturn, Jupiter, and Mars and the spheres which enclose the earth.

Venus

There remain for consideration the motions which are included within the great circle, that is, the motions of Venus and Mercury. Venus has a system of circles like the system of the superior planets,[69] but the arrangement of the motions is different. The deferent revolves in nine months, as was said above,[70] and the greater epicycle also revolves in nine months. By their composite motion the smaller epicycle is everywhere brought back to the same path in the firmament, and the higher apse is at the point where I said the sun reverses its course.[71] The period of the smaller epicycle is not equal to that of the deferent and greater epicycle,[72] but has a constant relation to

[67] S: *mediate;* V: *mediale.* Before S was known, Curtze emended V to *immediate,* which Prowe prints. But S is undoubtedly correct.

[68] Since motion in a straight line would violate the principle of circularity, Copernicus is at pains to prove that a rectilinear motion may be produced by a combination of two circular ones. A less concise account of this geometric device, employed in connection with the theory of precession, as well as an explanation of the term "libration," will be found in the *Narratio prima* (pp. 153-54, below; cf. Th 165.18–169.22).

[69] In *De rev.* Copernicus replaces the concentrobiepicyclic arrangement for Venus by an eccentreccentric arrangement, i.e., by two eccentrics (Th 368.23-29). The larger, outer eccentric which carries the planet has for its center a point which revolves on the smaller eccentric (Th 368.30–369.6).

[70] Page 60.

[71] In placing the apogee of Venus at the solar apogee Copernicus retains the Ptolemaic idea of the fixed apse, but he offers an improved determination. For Ptolemy had put the apogee of Venus at 25° of Taurus (HII, 300.15-16; cf. Th 365.20-25; 366.3-7, 17-20), and the solar apogee at 5° 30′ of Gemini (see above, p. 62, n. 13). Hence for him the apogee of Venus was 10° 30′ west of the solar apogee. Now we have already seen that in the *Commentariolus* Copernicus advances the solar apogee 11° 10′, as measured by the fixed stars, over Ptolemy's determination. Hence he advances the apogee of Venus 21° 40′, again as measured by the fixed stars, over Ptolemy's determination.

[72] This is the difference between the arrangement of the motions, on the one hand, for the three superior planets, and on the other hand, for Venus. In the former case the period of the smaller epicycle is one-half the period of the deferent and greater epicycle (see the opening paragraph of the section on "The

the motion of the great circle. For one revolution of the latter the smaller epicycle completes two. The result is that whenever the earth is in the diameter drawn through the apse, the planet is nearest to the center of the greater epicycle; and it is most remote, when the earth, being in the diameter perpendicular to the diameter through the apse, is at a quadrant's distance from the positions just mentioned. The smaller epicycle of the moon moves in very much the same way with relation to the sun.[73] The ratio of the radius of the great circle to the radius of the deferent of Venus is 25:18;[74] the greater epicycle has a value of ¾ of a unit, and the smaller ¼.[75]

Three Superior Planets"). Müller completely missed the distinction. His translation runs: "Die Umlaufszeit dieses kleineren Epicykels ist verschieden von der der oben genannten Kreise; so entsteht längst der Ekliptik eine ungleichförmige Bewegung. Vollführen jene einen Umlauf, so führt der kleinere einen doppelten aus" (The period of this smaller epicycle is different from that of the above-mentioned circles [i.e., deferent and greater epicycle]; thus there appears along the ecliptic an unequal motion. While those circles [i.e., deferent and greater epicycle] complete one revolution, the smaller epicycle completes two; ZE, XII, 378). The source of Müller's difficulty seems to have been the unusual expression *Minor autem epicyclus impares cum illis revolutiones habens, motui orbis magni imparitatem reservavit*. This may be literally translated as follows: "The smaller epicycle, having revolutions unequal with those of the deferent and greater epicycle, has reserved the inequality for the motion of the great circle." The next sentence in the text makes Copernicus's meaning clear beyond dispute. The revolution of the smaller epicycle takes half the time required by the motion on the great circle.

[73] See the opening paragraph of the section on "The Moon."

[74] S has the false reading 10, instead of 18 (Lindhagen reproduces this page of the MS). I call attention to the copyist's error of writing 0 for 8, in connection with the emendation proposed in the last paragraph of n. 50 (p. 75, above).

[75] To discover whether in shifting from the concentrobiepicyclic arrangement in the *Commentariolus* to the eccentreccentric arrangement in *De rev.* Copernicus altered the relative sizes of the circles, we may make the following comparisons. The radius (R) of the concentric deferent (*Commentariolus*) corresponds to the radius (R) of the outer eccentric (*De rev.*). Similarly, the radius (E) of the first epicycle (*Commentariolus*) corresponds to the eccentricity (E) of the outer eccentric (*De rev.*); and since the eccentricity varies, we take its mean value. Let r denote the radius of the great circle. Now in *De rev.* R = 7193, r = 10,000, and E = 312 (Th 367.13-14, 368.12-22, 371.11). Then in *De rev.* r:R = 10,000:7193 = 25:17.98, while in the *Commentariolus* r:R = 25:18; in *De rev.* r:E = 10,000:312 = 25:0.78, while in the *Commentariolus* r:E =

Venus seems at times to retrograde, particularly when it is nearest to the earth, like the superior planets, but for the opposite reason. For the regression of the superior planets happens because the motion of the earth is more rapid than theirs, but with Venus, because it is slower; and because the superior planets enclose the great circle, whereas Venus is enclosed within it. Hence Venus is never in opposition to the sun, since the earth cannot come between them, but it moves within fixed distances on either side of the sun. These distances are determined by tangents to the circumference drawn from the center of the earth, and never exceed 48° in our observations.[76] Here ends the treatment of Venus' motion in longitude.

Its latitude also changes for a twofold reason. For the axis of the deferent is inclined at an angle of 2½°,[77] the node whence the planet turns north being in the apse. However, the deviation which arises from this inclination, although in itself it is one and the same, appears twofold to us.[78] For when the earth is on the line drawn through the nodes of Venus, the deviations on the one side are seen above, and on the opposite

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25 : .75. The radius of the second epicycle = ⅓ E, a ratio which is applied in the *Commentariolus* to all the planets. In *De rev.* the radius of the smaller eccentric, being one-third of the mean eccentricity of the outer eccentric (Th 368.18-22), also = ⅓ E. Hence the second epicycle (*Commentariolus*) corresponds to the smaller eccentric (*De rev.*).

Despite *dodrantem* in the text, Müller's translation makes E = ⅔ (ZE, XII, 378). He was evidently confused by a misprint in Prowe's footnote (PII, 198). Yet in that same footnote, five lines below the misprint, the correct value of ¾ appears (cf. PII, 211 and MCV, I, 14-15 n).

[76] This value of 48° for the greatest elongation of Venus was derived from Ptolemy (HII, 522.14), and is accepted by modern astronomy.

[77] Müller wrote 2° (ZE, XII, 379). He was evidently unfamiliar with *s.* as the abbreviation of *semissis*, "one-half" (cf. Th 71.23, 167.4, 425.25). For in his note on the matter he misinterpreted *s.* as the abbreviation of *scrupula*, "minutes" (this word was not assigned to the masculine gender, as Müller thought). Had he consulted Curtze's collation of S and V, his difficulty would have been obviated. For Curtze, confronted by a variant reading (MCV, IV, 8), showed that 2½° is supported by *De rev.* (Th 424.23-24). Moreover, in Ptolemy's treatment of the latitude of Venus, there are two inclinations of the epicycle, and each is given as 2½° (HII, 535.15-18, 536.8-11).

[78] S, V: *duplex non ostenditur*. Müller correctly emended to *duplex nobis ostenditur* (ZE, XII, 379, n. 72).

side below; these are called the reflexions.[79] When the earth is at a quadrant's distance[80] from the nodes, the same natural inclinations of the deferent appear, but they are called the declinations. In all the other positions of the earth, both latitudes mingle and are combined, each in turn exceeding the other; by their likeness and difference they are mutually increased and eliminated.

The inclination of the axis is affected by a motion in libration that swings, not on the nodes as in the case of the superior planets,[81] but on certain other movable points. These points perform annual revolutions with reference to the planet. Whenever the earth is opposite the apse of Venus, at that time the amount of the libration attains its maximum for this planet, no matter where the planet may then be on the deferent. As a consequence, if the planet is then in the apse or diametrically opposite to it, it will not completely lack latitude, even though it is then in the nodes. From this point the amount of the libration decreases, until the earth has moved through a quadrant of a circle from the aforesaid position, and, by reason of the likeness of their motions, the point of maximum deviation[82] has moved an equal distance from the planet. Here no trace of the deviation is found.[83] Thereafter the descent of the deviation continues.[84] The initial point drops from north to south,

[79] An alternative name was obliquation: "They call this deviation of the planet the obliquation, but some call it the reflexion" (Th 418.22-23). In *De rev.* Copernicus generally uses obliquation, but in the *Narratio prima* Rheticus favors reflexion.

[80] Müller's translation: "in den Quadraturen" (in the quadratures; ZE, XII, 379) again confuses *quadrantibus* with *quadraturis* (cf. above, p. 76, n. 52). An inferior planet cannot come to quadrature (see above, p. 50); Copernicus has just stated that the greatest elongation of Venus is 48°.

[81] See the penultimate paragraph of the section on "The Three Superior Planets."

[82] Müller correctly emended *maxime* (S, V) to *maximae* (ZE, XII, 380, n. 75).

[83] Since the deviation vanishes when the earth is 90° from the apse-line of the planet, the deviation has no effect upon the declinations, but only upon the reflexions. Copernicus employs the deviation "because the angle of inclination . . . is found to be greater in the obliquation [reflexion] than in the declination" (Th 418.27-29).

[84] S, V: *continuato*. Prowe's *continuatio* is a misprint (PII, 200.3).

constantly increasing its distance from the planet in accordance with the distance of the earth from the apse. Thereby the planet is brought to the part of the circumference which previously was south. Now, however, by the law of opposition, it becomes north and remains so until the limit of the libration is again reached upon the completion of the circle. Here the deviation becomes equal to the initial deviation and once more attains its maximum. Thus the second semicircle is traversed in the same way as the first. Consequently this latitude, which is usually called the deviation, never becomes a south latitude. In the present instance, also, it seems reasonable that these phenomena should be produced by two concentric circles with oblique axes, as I explained in the case of the superior planets.[85]

Mercury

Of all the orbits in the heavens the most remarkable is that of Mercury, which traverses almost untraceable paths, so that it cannot be easily studied. A further difficulty is the fact that the planet, following a course generally invisible in the rays of the sun, can be observed for a very few days only. Yet Mercury too will be understood, if the problem is attacked with more than ordinary ability.

Mercury, like Venus, has two epicycles which revolve on the deferent.[86] The periods of the greater epicycle and deferent are equal, as in the case of Venus. The apse is located 14½° east of Spica Virginis.[87] The smaller epicycle revolves with twice the velocity of the earth. But by contrast with Venus, whenever the earth is above the apse or diametrically opposite

[85] For a fuller account of Copernicus's theory for the latitudes of Venus see pp. 180-85, below.

[86] In *De rev.* Copernicus replaces the concentrobiepicyclic arrangement for Mercury by an eccentreccentric arrangement (Th 377.2-3).

[87] Since Ptolemy had put the apogee of Mercury at 10° of Libra (HII, 264.12-14, 271.2-4; cf. Th 380.6-7), and Spica at 26° 40′ of Virgo (HII, 103.16), the apse was 13° 20′ east of Spica Virginis. Hence in the *Commentariolus* Copernicus retains the idea of the fixed apse and modifies its position slightly. But in *De rev.* he puts the apse 41° 30′ east of Spica (Th 136.10, 389.5-6, 393.5-8), and extends to Mercury the principle that the longitude of the planetary apogees increases (Th 393.16-19, 27-29; cf. n. 56 on pp. 76-77, above).

to it, the planet is most remote from the center of the greater epicycle; and it is nearest, whenever the earth is at a quadrant's distance[88] from the points just mentioned. I have said[89] that the deferent of Mercury revolves in three months, that is, in 88 days. Of the 25 units into which I have divided the radius of the great circle, the radius of the deferent of Mercury contains 9⅖. The first epicycle contains 1 unit, 41 minutes; the second epicycle is ⅓ as great, that is, about 34 minutes.[90]

But in the present case this combination of circles is not sufficient, though it is for the other planets. For when the earth passes through the above-mentioned positions with respect to the apse the planet appears to move in a much smaller path[91] than is required by the system of circles described above; and in a much greater path,[91] when the earth is at a quadrant's distance[92] from the positions just mentioned. Since no other inequality in longitude is observed to result from this, it may be reasonably explained by a certain approach of the planet to and withdrawal from the center of the deferent[93] along a

[88] Again Müller erroneously translates by "in the quadratures" (ZE, XII, 381). Mercury, like Venus, cannot come to quadrature (cf. above, p. 84, n. 80).

[89] Page 60, above.

[90] The analysis made above (p. 82, n. 75) for Venus is equally applicable here. In *De rev.* R (mean value) = 3,763, r = 10,000, and E (mean value) = 736 (Th 382.9-10, 382.27-383.2). Then in *De rev.* r:R = 10,000:3763 = 25:9.41, while in the *Commentariolus* r:R = 25:9.40; in *De rev.* r:E = 10,000:736 = 25:1.84, while in the *Commentariolus* r:E = 25:1.68. The radius of the second epicycle (*Commentariolus*) = ⅓ E. But the radius of the smaller eccentric (*De rev.*) = ⅓ E, only where E denotes the eccentricity of the outer eccentric (Th 377.11-15), as set down in conformity with the general planetary theory used in *De rev.* As in the case of Mars (see above, p. 77, n. 58), Copernicus modifies the ratio; the radius of the smaller eccentric = 212 (Th 382.8-9), or 2/7 E, where E denotes the mean eccentricity of the outer eccentric.

[91] Müller translates *longe minori apparet ambitu sidus moveri* by: "so scheint der Planet sich viel langsamer zu bewegen" (the planet appears to move much more slowly); and *longe etiam maiore* by: "viel schneller" (much more swiftly; ZE, XII, 381). However, Copernicus is concerned here with the variations, not in Mercury's velocity, but in its distance from the center of the great circle.

[92] Failing to recognize that *quadratura* is used here and again near the close of this paragraph in the sense of "quadrant" (see above, p. 47, n. 163), Müller inaccurately translates by "in the quadratures" (ZE, XII, 381, 382).

[93] S: *a centro orbis*; V: *centri orbis*. Prowe accepted V, although S is certainly correct.

straight line. This motion must be produced by two small circles stationed about the center of the greater epicycle, their axes being parallel to the axis of the deferent. The center of the greater epicycle, or of the whole epicyclic structure, lies on the circumference of the small circle that is situated between this center and the outer small circle. The distance from this center to the center of the inner circle is exactly[94] equal to the distance from the latter center to the center of the outer circle.[95] This distance has been found to be 14½ minutes[96] of one unit of the

[94] S, V: *asse*. For this sound reading Curtze incorrectly substituted *axe* (MCV, I, 17.5), which Prowe accepted (PII, 201.10). By ignoring the rules of syntax Müller contrived to incorporate *axe* in his translation.

[95] Let the dotted circumference (Fig. 24) represent the inner small circle with

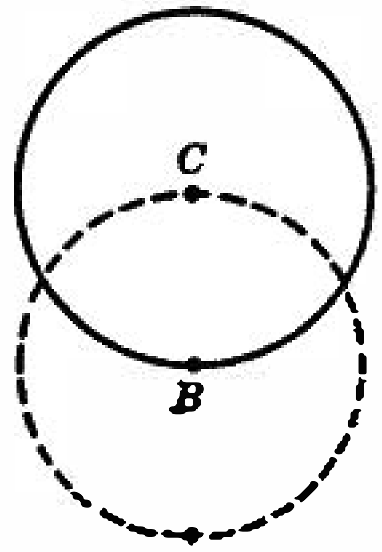

FIGURE 24

its center at B; and the unbroken circumference, the outer small circle with its center at C. The center of the greater epicycle is at A; and AB = BC.

[96] S: *minut. 14 et medio;* V: *minutibus 24 et medio*. Prowe accepted V, although S is certainly correct, as the closing words of this paragraph show. Again Copernicus's notations in his copy of the Tables of Regiomontanus aid us. For the entry concerning Mercury gives values for the deferent and epicycles that agree with those in our text. Then it adds that the inequality of the diameter is 29 minutes (*diversitas diametri 0.29*). Now Curtze, Prowe, and Müller quoted the entry in their notes (MCV, I, 16 n; PII, 201 n; ZE, XII, 381, n. 78). All three called attention to the agreement between the entry and our text with reference to the deferent and epicycles. But they failed to see that the "approach and withdrawal" of our text is identical with the "inequality of the diameter" in the entry; and that the value of 29 minutes given in both places establishes the correctness of S as against V.

This value varies but slightly from Ptolemy's. In his system, the inequality is produced by a small circle upon which the center of the eccentric revolves (HII, 252.26–253.6, 256.15-22; cf. Th 376.17-24). If we compare the radius of the small circle with the sum of the radii of the eccentric and epicycle (HII, 279.15-

25 by which I have measured the relative sizes of all the circles. The motion of the outer small circle performs two revolutions in a tropical year,[97] while the inner one completes four in the same time with twice the velocity in the opposite direction. By this composite motion the centers of the greater epicycle are carried along a straight line, just as I explained with regard to the librations in latitude.[98] Therefore, in the aforementioned positions of the earth with respect to the apse, the center of the greater epicycle is nearest to the center of the deferent; and it is most remote, when the earth is at a quadrant's distance[92] from these positions. When the earth is at the midpoints, that is, 45° from the points just mentioned, the center of the greater epicycle joins the center of the outer[99] small circle, and both centers coincide.[100] The amount of this with-

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18), we get the ratio 1:27½, while in the *Commentariolus* the corresponding ratio is 1:24 ($29^m : 9^p 24^m + 1^p 41^m + 34^m$).

In *De rev.* Copernicus represents the inequality by adding an epicycle to the outer eccentric (Th 377.4-8, 18-23); so that, if we include this refinement, his arrangement for Mercury in *De rev.* is bieccentrepicyclic rather than eccentreccentric (Th 377.23-26). But he does not alter the amount of the inequality. For he puts the diameter of the epicycle at 380, where r = 10,000 (Th 382.23-27, 384.9-14). Then the amount of the inequality is 190 (r = 10,000), or 28½ minutes, where r = 25.

[97] Müller failed to recognize *annus vertens* as the term for "tropical year" (see p. 46, above).

[98] See the closing paragraph of the section on "The Three Superior Planets."

[99] Müller omitted *exterioris* from his translation (ZE, XII, 382).

[100] Figure 25 may serve to clarify this motion in libration. In the initial position, the earth is at E1 on the produced apse-line, the center of the greater epicycle is at A, the center of the inner small circle is at B1, and the center of the outer small circle is at C. While the earth moves 45° from E1 to E2, the outer circle rotates through a quadrant, thereby moving the center of the inner circle from B1 to B2. But during this interval, the inner circle rotates through a semicircle, thereby bringing the center of the epicycle to C. As the earth moves 45° from E2 to E3, the center of the inner circle reaches B3, and the center of the epicycle comes to D. As the earth moves from E3 to E4, the center of the inner circle goes to B4, and the center of the epicycle to C. When the earth arrives at E5, the center of the inner circle returns to B1, and the center of the epicycle to A. While the earth completes the remaining semicircle E5-E6-E7-E8-E1, the small circles repeat their previous motion. Therefore, whenever the earth is on the produced apse-line (E1 or E5), the center of the greater epicycle is nearest (A) to the center of the deferent. When the earth is at a quadrant's distance from the apse-

drawal and approach is 29 minutes[96] of one of the above-mentioned units. This, then, is the motion of Mercury in longitude.

Its motion in latitude is exactly like that of Venus, but always in the opposite hemisphere. For where Venus is in north latitude, Mercury is in south. Its deferent is inclined to the ecliptic at an angle of 7°.[101] The deviation, which is always south, never

line (E_3 or E_7), the center of the epicycle is most remote (D) from the center of the deferent. When the earth is at E_2, E_4, E_6, or E_8, the center of the epicycle coincides with C, the center of the outer small circle.

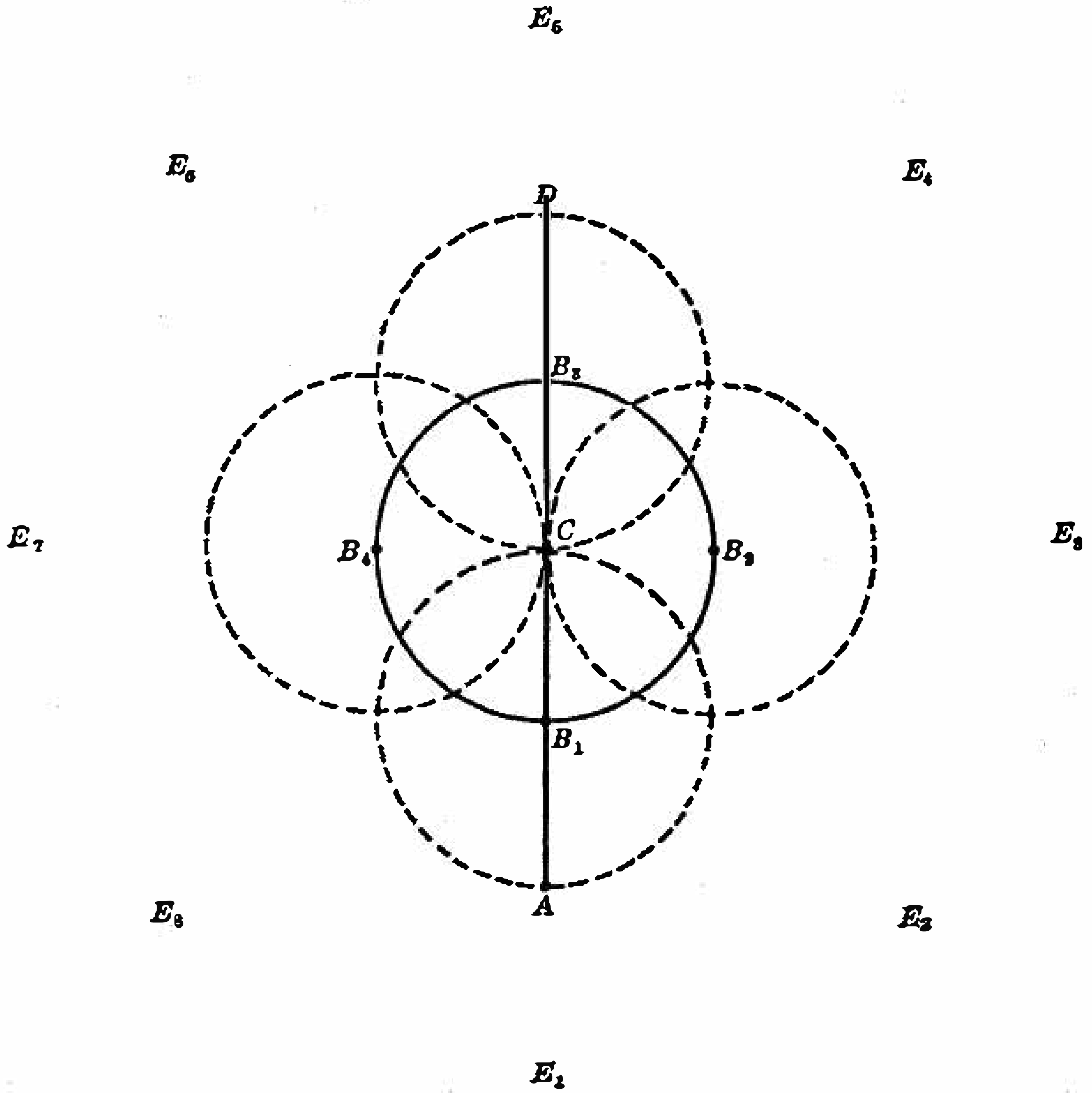


Figure 25

[101] In *De rev.* the angle is given as 6° 15′ for the declinations, and 7° for the reflexions (Th 424.23-27, 431.4-9). These were Ptolemy's values (HII, 536.20-22, 575.9-11).

exceeds ¾°.[102] For the rest, what was said about the latitudes of Venus may be understood here also, to avoid repetition.

Then Mercury runs on seven circles in all; Venus on five; the earth on three, and round it the moon on four; finally Mars, Jupiter, and Saturn on five each. Altogether, therefore, thirty-four circles suffice to explain the entire structure of the universe and the entire ballet of the planets.

[102] This was the traditional estimate (Th 433.20-25); but in *De rev.* Copernicus puts it at 51′ ± 18′ (Th 435.4-8; 440-41). Müller rendered this sentence by: "doch übersteigt die Ablenkung nach Süden nie den zwölften Teil eines Grades" (the southward deviation never exceeds 1/12°; ZE, XII, 382). This version omits *semper*, and puts *dodrantem* = 1/12. For 1/12 Copernicus wrote the usual word *uncia* (Th 159.28).

Book One

INTRODUCTION

Among the many various literary and artistic pursuits which invigorate men's minds, the strongest affection and utmost zeal should, I think, promote the studies concerned with the most beautiful objects, most deserving to be known. This is the nature of the discipline which deals with the universe's divine revolutions, the asters' motions, sizes, distances, risings and settings, as well as the causes of the other phenomena in the sky, and which, in short, explains its whole appearance. What indeed is more beautiful than heaven, which of course contains all things of beauty? This is proclaimed by its very names [in Latin], *caelum* and *mundus*, the latter denoting purity and ornament, the former a carving. On account of heaven's transcendent perfection most philosophers have called it a visible god. If then the value of the arts is judged by the subject matter which they treat, that art will be by far the foremost which is labeled astronomy by some, astrology by others, but by many of the ancients, the consummation of mathematics. Unquestionably the summit of the liberal arts and most worthy of a free man, it is supported by almost all the branches of mathematics. Arithmetic, geometry, optics, surveying, mechanics and whatever others there are all contribute to it.

Although all the good arts serve to draw man's mind away from vices and lead it toward better things, this function can be more fully performed by this art, which also provides extraordinary intellectual pleasure. For when a man is occupied with things which he sees established in the finest order and directed by divine management, will not the unremitting contemplation of them and a certain familiarity with them stimulate him to the best and to admiration for the Maker of everything, in whom are all happiness and every good? For would not the godly Psalmist [92:4] in vain declare that he was made glad through the work of the Lord and rejoiced in the works of His hands, were we not drawn to the contemplation of the highest good by this means, as though by a chariot?

The great benefit and adornment which this art confers on the commonwealth (not to mention the countless advantages to individuals) are most excellently observed by Plato. In the *Laws*, Book VII, he thinks that it should be cultivated chiefly because by dividing time into groups of days as months and years, it would keep the state alert and attentive to the festivals and sacrifices. Whoever denies its necessity for the teacher of any branch of higher learning is thinking foolishly, according to Plato. In his opinion it is highly unlikely that anyone lacking the requisite knowledge of the sun, moon, and other heavenly bodies can become and be called godlike.

However, this divine rather than human science, which investigates the loftiest subjects, is not free from perplexities. The main reason is that its principles and assumptions, called "hypotheses" by the Greeks, have been a source of disagreement, as we see, among most of those who undertook to deal with this subject, and so they did not rely on the same ideas. An additional reason is that the motion of the planets and the revolution of the stars could not be measured with numerical precision and completely understood except with the passage of time and the aid of many earlier observations, through which this knowledge was transmitted to posterity from hand to hand, so to say. To be sure, Claudius Ptolemy of Alexandria, who far excels the rest by his wonderful skill and industry, brought this entire art almost to perfection with the help of observations extending over a period of more than four hundred years, so that there no longer seemed to be any gap which he had not closed. Nevertheless very many things, as we perceive, do not agree with the conclusions which ought to follow from his system, and besides certain other motions have been discovered which were not yet known to him. Hence Plutarch too, in discussing the sun's tropical year, says that so far the motion of the heavenly bodies has eluded the skill of the astronomers. For, to use the year itself as an example, it is well known, I think, how different the opinions concerning it have always been, so that many have abandoned all hope that an exact determination of it could be found. The situation is the same with regard to other heavenly bodies.

Nevertheless, to avoid giving the impression that this difficulty is an excuse for indolence, by the grace of God, without whom we can accomplish nothing, I shall attempt a broader inquiry into these matters. For, the number of aids we have to assist our enterprise grows with the interval of time extending from the originators of this art to us. Their discoveries may be compared with what I have newly found. I acknowledge, moreover, that I shall treat many topics differently from my predecessors, and yet I shall do so thanks to them, for it was they who first opened the road to the investigation of these very questions.

THE UNIVERSE IS SPHERICAL *Chapter 1*

First of all, we must note that the universe is spherical. The reason is either that, of all forms, the sphere is the most perfect, needing no joint and being a complete whole, which can be neither increased nor diminished; or that it is the most capacious of figures, best suited to enclose and retain all things; or even that all the separate parts of the universe, I mean the sun, moon, planets and stars, are seen to be of this shape; or that wholes strive to be circumscribed by this boundary, as is apparent in drops of water and other fluid bodies when they seek to be self-contained. Hence no one will question the attribution of this form to the divine bodies.

THE EARTH TOO IS SPHERICAL *Chapter 2*

The earth also is spherical, since it presses upon its center from every direction. Yet it is not immediately recognized as a perfect sphere on account of the great height of the mountains and depth of the valleys. They scarcely alter the general sphericity of the earth, however, as is clear from the following considerations. For a traveler going from any place toward the north, that pole of the daily rotation gradually climbs higher, while the opposite pole drops down an equal amount. More stars in the north are seen not to set, while in the south certain stars are no longer seen to rise. Thus Italy does not see Canopus, which is visible in Egypt; and Italy does see the River's last star, which is unfamiliar to our area in the colder region. Such stars, conversely, move higher in the heavens for a traveller heading southward, while those which are high in our sky sink down. Meanwhile, moreover, the elevations of the poles have the same ratio everywhere to the portions of the earth that have been traversed. This happens on no other figure than the sphere. Hence the earth too is evidently enclosed between poles and is therefore spherical. Furthermore, evening eclipses of the sun and moon are not seen by easterners, nor morning eclipses by westerners, while those occurring in between are seen later by easterners but earlier by westerners.

The waters press down into the same figure also, as sailors are aware, since land which is not seen from a ship is visible from the top of its mast. On the other hand, if a light is attached to the top of the mast, as the ship draws away from land, those who remain ashore see the light drop down gradually until it finally disappears, as though setting. Water, furthermore, being fluid by nature, manifestly always seeks the same lower levels as earth and pusehs up from the shore no higher than its rise permits. Hence whatever land emerges out of the ocean is admittedly that much higher.

HOW EARTH FORMS A SINGLE SPHERE WITH WATER — *Chapter 3*

Pouring forth its seas everywhere, then, the ocean envelops the earth and fills its deeper chasms. Both tend toward the same center because of their heaviness. Accordingly there had to be less water than land, to avoid having the water engulf the entire earth and to have the water recede from some portions of the land and from the many islands lying here and there, for the preservation of living creatures. For what are the inhabited countries and the mainland itself but an island larger than the others?

We should not heed certain peripatetics who declared that the entire body of water is ten times greater than all the land. For, according to the conjecture which they accepted, in the transmutation of the elements as one unit of earth dissolves, it becomes ten units of water. They also assert that the earth bulges out to some extent as it does because it is not of equal weight everywhere on account of its cavities, its center of gravity being different from its center of magnitude. But they err through ignorance of the art of geometry. For they do not realize that the water cannot be even seven times greater and still leave any part of the land dry, unless earth as a whole vacated the center of gravity and yielded that position to water, as if the latter were heavier than itself. For, spheres are to each other as the cubes of their diameters. Therefore, if earth were the eighth part to seven parts of water, earth's diameter could not be greater than the distance from [their joint] center to the circumference of the waters. So far are they from being as much as ten times greater [than the land].

Moreover, there is no difference between the earth's centers of gravity and magnitude. This can be established by the fact that from the ocean inward the curvature of the land does not mount steadily in a continuous rise. If it did, it would keep the sea water out completely and in no way permit the inland seas and such vast gulfs to intrude. Furthermore, the depth of the abyss would never stop increasing from the shore of the ocean outward, so that no island or reef or any form of land would be encountered by sailors on the longer voyages. But it is well known that almost in the middle of the inhabited lands barely fifteen furlongs remain between the eastern Mediterranean and the Red Sea. On the other hand, in his *Geography* Ptolemy extended the habitable area halfway around the world. Beyond that meridian, where he left unknown land, the moderns have added Cathay and territory as vast as sixty degrees of longitude, so that now the earth is inhabited over a greater stretch of longitude than is left for the ocean. To these regions, moreover, should be added the islands discovered in our time under the rulers of Spain and Portugal, and especially America, named after the ship's captain who found it. On account of its still undisclosed size it is thought to be a second group of inhabited countries. There are also many other islands, heretofore unknown. So little reason have we to marvel at the existence of antipodes or antichthones. Indeed, geometrical reasoning about the location of America compels us to believe that it is diametrically opposite the Ganges district of India.

From all these facts, finally, I think it is clear that land and water together press upon a single center of gravity; that the earth has no other center of magnitude; that, since earth is heavier, its gaps are filled with water; and that consequently there is little water in comparison with land, even though more water perhaps appears on the surface.

The earth together with its surrounding waters must in fact have such a shape as its shadow reveals, for it eclipses the moon with the arc of a perfect circle. Therefore the earth is not flat, as Empedocles and Anaximenes thought; nor drum-shaped, as Leucippus; nor bowl-shaped, as Heraclitus; nor hollow in another way, as Democritus; nor again cylindrical, as Anaximander; nor does its lower side extend infinitely downward, the thickness diminishing toward the bottom, as Xenophanes taught; but it is perfectly round, as the philosophers hold.

THE MOTION OF THE HEAVENLY BODIES IS UNIFORM, ETERNAL, AND CIRCULAR OR COMPOUNDED OF CIRCULAR MOTIONS — *Chapter 4*

I shall now recall to mind that the motion of the heavenly bodies is circular, since the motion appropriate to a sphere is rotation in a circle. By this very act the sphere expresses its form as the simplest body, wherein neither beginning nor end can be found, nor can the one be distinguished from the other, while the sphere itself traverses the same points to return upon itself.

In connection with the numerous [celestial] spheres, however, there are many motions. The most conspicuous of all is the daily rotation, which the Greeks call ***nuchthemeron,*** that is, the interval of a day and a night. The entire universe, with the exception of the earth, is conceived as whirling from east to west in this rotation. It is recognized as the common measure of all motions, since we even compute time itself chiefly by the number of days.

Secondly, we see other revolutions as advancing in the opposite direction, that is, from west to east; I refer to those of the sun, moon, and five planets. The sun thus regulates the year for us, and the moon the month, which are also very familiar periods of time. In like manner each of the other five planets completes its own orbit.

Yet [these motions] differ in many ways [from the daily rotation or first motion]. In the first place, they do not swing around the same poles as the first motion, but run obliquely through the zodiac. Secondly, these bodies are not seen moving uniformly in their orbits, since the sun and moon are observed to be sometimes slow, at other times faster in their course. Moreover, we see the other five planets also retrograde at times, and stationary at either end [of the regression]. And whereas the sun always advances along its own direct path, they wander in various ways, straying sometimes to the south and sometimes to the north; that is why they are called "planets" [wanderers]. Furthermore, they are at times nearer to the earth, when they are said to be in perigee; at other times they are farther away, when they are said to be in apogee.

We must acknowledge, nevertheless, that their motions are circular or compounded of several circles, because these nonuniformities recur regularly according to a constant law. This could not happen unless the motions were circular, since only the circle can bring back the past. Thus, for example, by a composite motion of circles the sun restores to us the inequality of days and nights as well as the four seasons of the year. Several motions are discerned herein, because a simple heavenly body cannot be moved by a single sphere nonuniformly. For this nonuniformity would have to be caused either by an inconstancy, whether imposed from without or generated from within, in the moving force or by an alteration in the revolving body. From either alternative, however, the intellect shrinks. It is improper to conceive any such defect in objects constituted in the best order.

It stands to reason, therefore, that their uniform motions appear nonuniform to us. The cause may be either that their circles have poles different [from the earth's] or that the earth is not at the center of the circles on which they revolve. To us who watch the course of these planets from the earth, it happens that our eye does not keep the same distance from every part of their orbits, but on account of their varying distances these bodies seem larger when nearer than when farther away (as has been proved in optics). Likewise, in equal arcs of their orbits their motions will appear unequal in equal times on account of the observer's varying distance. Hence I deem it above all necessary that we should carefully scrutinize the relation of the earth to the heavens lest, in our desire to examine the loftiest objects, we remain ignorant of things nearest to us, and by the same error attribute to the celestial bodies what belongs to the earth.

DOES CIRCULAR MOTION SUIT THE EARTH? WHAT IS ITS POSITION? *Chapter 5*

Now that the earth too has been shown to have the form of a sphere, we must in my opinion see whether also in this case the form entails the motion, and what place in the universe is occupied by the earth. Without the answers to these questions it is impossible to find the correct explanation of what is seen in the heavens. To be sure, there is general agreement among the authorities that the earth is at rest in the middle of the universe. They hold the contrary view to be inconceivable or downright silly. Nevertheless, if we examine the matter more carefully, we shall see that this problem has not yet been solved, and is therefore by no means to be disregarded.

Every observed change of place is caused by a motion of either the observed object or the observer or, of course, by an unequal displacement of each. For when things move with equal speed in the same direction, the motion is not perceived, as between the observed object and the observer, I mean. It is the earth, however, from which the celestial ballet is beheld in its repeated performances before our eyes. Therefore, if any motion is ascribed to the earth, in all things outside it the same motion will appear, but in the opposite direction, as though they were moving past it. Such in particular is the daily rotation, since it seems to involve the entire universe except the earth and what is around it. However, if you grant that the heavens have no part in this motion but that the earth rotates from west to east, upon earnest consideration you will find that this is the actual situation concerning the apparent rising and setting of the sun, moon, stars and planets. Moreover since the heavens, which enclose and provide the setting for everything, constitute the space common to all things, it is not at first blush clear why motion should not be attributed rather to the enclosed than to the enclosing, to the thing located in space rather than to the framework of space. This opinion was indeed maintained by Heraclides and Ecphantus, the Pythagoreans, and by Hicetas of Syracuse, according to Cicero. They rotated the earth in the middle of the universe, for they ascribed the setting of the stars to the earth's interposition, and their rising to its withdrawal.

If we assume its daily rotation, another and no less important question follows concerning the earth's position. To be sure, heretofore there has been virtually unanimous acceptance of the belief that the middle of the universe is the earth. Anyone who denies that the earth occupies the middle or center of the universe may nevertheless assert that its distance [therefrom] is insignificant in comparison with [the distance of] the sphere of the fixed stars, but perceptible and noteworthy in relation to the spheres of the sun and the other planets. He may deem this to be the reason why their motions appear nonuniform, as conforming to a center other than the center of the earth. Perhaps he can [thereby] produce a not inept explanation of the apparent nonuniform motion. For the fact that the same planets are observed nearer to the earth and farther away necessarily proves that the center of the earth is not the center of their circles. It is less clear whether the approach and withdrawal are executed by the earth or the planets.

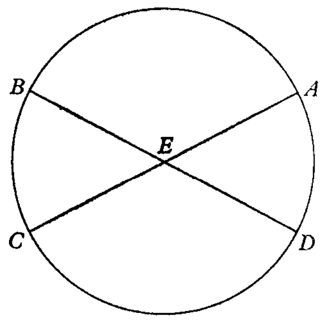
It will occasion no surprise if, in addition to the daily rotation, some other motion is assigned to the earth. That the earth rotates, that it also travels with several motions, and that it is one of the heavenly bodies are said to have been the opinions of Philolaus the Pythagorean. He was no ordinary astronomer, inasmuch as Plato did not delay going to Italy for the sake of visiting him, as Plato's biographers report.

But many have thought it possible to prove by geometrical reasoning that the earth is in the middle of the universe; that being like a point in relation to the immense heavens, it serves as their center; and that it is motionless because, when the universe moves, the center remains unmoved, and the things nearest to the center are carried most slowly.

THE IMMENSITY OF THE HEAVENS COMPARED TO THE SIZE OF THE EARTH *Chapter 6*

The massive bulk of the earth does indeed shrink to insignificance in comparison with the size of the heavens. This can be ascertained from the fact that the boundary circles (for that is the translation of the Greek term *horizons*) bisect the entire sphere of the heavens. This could not happen if the earth's size or distance from the universe's center were noteworthy in comparison with the heavens. For, a circle that bisects a sphere passes through its center, and is the greatest circle that can be described on it.

Thus, let circle *ABCD* be a horizon, and let the earth, from which we do our observing, be *E*, the center of the horizon, which separates what is seen from what is not seen. Now, through a dioptra or horoscopic instrument or water level placed at *E*, let the first point of the Crab be sighted rising at point *C*, and at that instant the first point of the Goat is perceived to be setting at *A*. Then *A*, *E*, and *C* are on a straight line through the dioptra. This line is evidently a diameter of the ecliptic, since six visible signs form a semicircle, and *E*, the [line's] center, is identical with the horizon's center. Again, let the signs shift their position until the first point of the Goat rises at *B*. At that time the Crab will also be observed setting at *D*. *BED* will be a straight line and a diameter of the ecliptic. But, as we have already seen, *AEC* also is a diameter of the same circle. Its center, obviously, is the intersection [of the diameters]. A horizon, then, in this way always bisects the ecliptic, which is a great circle of the sphere. But on a sphere, if a circle bisects any great circle, the bisecting circle is itself a great circle. Consequently a horizon is one of the great circles, and its center is clearly identical with the center of the ecliptic.



Yet a line drawn from the earth's surface [to a point in the firmament] must be distinct from the line drawn from the earth's center [to the same point]. Nevertheless, because these lines are immense in relation to the earth, they become like parallel lines [III, 15]. Because their terminus is enormously remote they appear to be a single line. For in comparison with their length the space enclosed by them becomes imperceptible, as is demonstrated in optics. This reasoning certainly makes it quite clear that the heavens are immense by comparison with the earth and present the aspect of an infinite magnitude, while on the testimony of the senses the earth is related to the heavens as a point to a body, and a finite to an infinite magnitude.

But no other conclusion seems to have been established. For it does not follow that the earth must be at rest in the middle of the universe. Indeed, a rotation in twenty-four hours of the enormously vast universe should astonish us even more than a rotation of its least part, which is the earth. For, the argument that the center is motionless, and what is nearest the center moves the least, does not prove that the earth is at rest in the middle of the universe.

To take a similar case, suppose you say that the heavens rotate but the poles are stationary, and what is closest to the poles moves the least. The Little Bear, for example, being very close to the pole, is observed to move much more slowly than the Eagle or the Little Dog because it describes a smaller circle. Yet all these constellations belong to a single sphere. A sphere's movement, vanishing at its axis, does not permit an equal motion of all its parts. Nevertheless these are brought round in equal times, though not over equal spaces, by the rotation of the whole sphere. The upshot of the argument, then, is the claim that the earth as a part of the celestial sphere shares in the same nature and movement so that, being close to the center, it has a slight motion. Therefore, being a body and not the center, it too will describe arcs like those of a celestial circle, though smaller, in the same time. The falsity of this contention is clearer than daylight. For it would always have to be noon in one place, and always midnight in another, so that the daily risings and settings could not take place, since the motion of the whole and the part would be one and inseparable.

But things separated by the diversity of their situations are subject to a very different relation: those enclosed in a smaller orbit revolve faster than those traversing a bigger circle. Thus Saturn, the highest of the planets, revolves in thirty years; the moon, undoubtedly the nearest to the earth, completes its course in a month; and to close the series, it will be thought, the earth rotates in the period of a day and a night. Accordingly the same question about the daily rotation emerges again. On the other hand, likewise still undetermined is the earth's position, which has been made even less certain by what was said above. For that proof establishes no conclusion other than the heavens' unlimited size in relation to the earth. Yet how far this immensity extends is not at all clear. At the opposite extreme are the very tiny indivisible bodies called "atoms". Being imperceptible, they do not immediately constitute a visible body when they are taken two or a few at a time. But they can be multiplied to such an extent that in the end there are enough of them to combine in a perceptible magnitude. The same may be said also about the position of the earth. Although it is not in the center of the universe, nevertheless its distance therefrom is still insignificant, especially in relation to the sphere of the fixed stars.

WHY THE ANCIENTS THOUGHT THAT THE EARTH REMAINED AT REST IN THE MIDDLE OF THE UNIVERSE AS ITS CENTER *Chapter 7*

Accordingly, the ancient philosophers sought to establish that the earth remains at rest in the middle of the universe by certain other arguments. As their main reason, however, they adduce heaviness and lightness. Earth is in fact the heaviest element, and everything that has weight is borne toward it in an effort to reach its inmost center. The earth being spherical, by their own nature heavy objects are carried to it from all directions at right angles to its surface. Hence, if they were not checked at its surface, they would collide at its center, since a straight line perpendicular to a horizontal plane at its point of tangency with a sphere leads to the [sphere's] center. But things brought to the middle, it seems to follow, come to rest at the middle. All the more, then, will the entire earth be at rest in the middle, and as the recipient of every falling body it will remain motionless thanks to its weight.

In like manner, the ancient philosophers analyze motion and its nature in a further attempt to confirm their conclusion. Thus, according to Aristotle, the motion of a single simple body is simple; of the simple motions, one is straight and the other is circular; of the straight motions, one is upward and the other is downward. Hence every simple motion is either toward the middle, that is, downward; or away from the middle, that is, upward; or around the middle, that is, circular. To be carried downward, that is, to seek the middle, is a property only of earth and water, which are considered heavy; on the other hand, air and fire, which are endowed with lightness, move upward and away from the middle. To these four elements it seems reasonable to assign rectilinear motion, but to the heavenly bodies, circular motion around the middle. This is what Aristotle says [*Heavens*, I, 2; II, 14].

Therefore, remarks Ptolemy of Alexandria [*Syntaxis*, I, 7], if the earth were to move, merely in a daily rotation, the opposite of what was said above would have to occur, since a motion would have to be exceedingly violent and its speed unsurpassable to carry the entire circumference of the earth around in twenty-four hours. But things which undergo an abrupt rotation seem utterly unsuited to gather [bodies to themselves], and seem more likely, if they have been produced by combination, to fly apart unless they are held together by some bond. The earth would long ago have burst asunder, he says, and dropped out of the skies (a quite preposterous notion); and, what is more, living creatures and any other loose weights would by no means remain unshaken. Nor would objects falling in a straight line descend perpendicularly to their appointed place, which would meantime have been withdrawn by so rapid a movement. Moreover, clouds and anything else floating in the air would be seen drifting always westward.

THE INADEQUACY OF THE PREVIOUS ARGUMENTS AND A REFUTATION OF THEM — *Chapter 8*

For these and similar reasons forsooth the ancients insist that the earth remains at rest in the middle of the universe, and that this is its status beyond any doubt. Yet if anyone believes that the earth rotates, surely he will hold that its motion is natural, not violent. But what is in accordance with nature produces effects contrary to those resulting from violence, since things to which force or violence is applied must disintegrate and cannot long endure. On the other hand, that which is brought into existence by nature is well-ordered and preserved in its best state. Ptolemy has no cause, then, to fear that the earth and everything earthly will be disrupted by a rotation created through nature's handiwork, which is quite different from what art or human intelligence can accomplish.

But why does he not feel this apprehension even more for the universe, whose motion must be the swifter, the bigger the heavens are than the earth? Or have the heavens become immense because the indescribable violence of their motion drives them away from the center? Would they also fall apart if they came to a halt? Were this reasoning sound, surely the size of the heavens would likewise grow to infinity. For the higher they are driven by the power of their motion, the faster that motion will be, since the circumference of which it must make the circuit in the period of twenty-four hours is constantly expanding; and, in turn, as the velocity of the motion mounts, the vastness of the heavens is enlarged. In this way the speed will increase the size, and the size the speed, to infinity. Yet according to the familiar axiom of physics that the infinite cannot be traversed or moved in any way, the heavens will therefore necessarily remain stationary.

But beyond the heavens there is said to be no body, no space, no void, absolutely nothing, so that there is nowhere the heavens can go. In that case it is really astonishing if something can be held in check by nothing. If the heavens are infinite, however, and finite at their inner concavity only, there will perhaps be more reason to believe that beyond the heavens there is nothing. For, every single thing, no matter what size it attains, will be inside them, but the heavens will abide motionless. For, the chief contention by which it is sought to prove that the universe is finite is its motion. Let us therefore leave the question whether the universe is finite or infinite to be discussed by the natural philosophers.

We regard it as a certainty that the earth, enclosed between poles, is bounded by a spherical surface. Why then do we still hesitate to grant it the motion appropriate by nature to its form rather than attribute a movement to the entire universe, whose limit is unknown and unknowable? Why should we not admit, with regard to the daily rotation, that the appearance is in the heavens and the reality in the earth? This situation closely resembles what Vergil's Aeneas says:

Forth from the harbor we sail, and the land and the cities slip backward [*Aeneid*, III, 72].

For when a ship is floating calmly along, the sailors see its motion mirrored in everything outside, while on the other hand they suppose that they are stationary, together with everything on board. In the same way, the motion of the earth can unquestionably produce the impression that the entire universe is rotating.

Then what about the clouds and the other things that hang in the air in any manner whatsoever, or the bodies that fall down, and conversely those that rise aloft? We would only say that not merely the earth and the watery element joined with it have this motion, but also no small part of the air and whatever is linked in the same way to the earth. The reason may be either that the nearby air, mingling with earthy or watery matter, conforms to the same nature as the earth, or that the air's motion, acquired from the earth by proximity, shares without resistance in its unceasing rotation. No less astonishingly, on the other hand, is the celestial movement declared to be accompanied by the uppermost belt of air. This is indicated by those bodies that appear suddenly, I mean, those that the Greeks called "comets" and "bearded stars". Like the other heavenly bodies, they rise and set. They are thought to be generated in that region. That part of the air, we can maintain, is unaffected by the earth's motion on account of its great distance from the earth. The air closest to the earth will accordingly seem to be still. And so will the things suspended in it, unless they are tossed to and fro, as indeed they are, by the wind or some other disturbance. For what else is the wind in the air but the wave in the sea?

We must in fact avow that the motion of falling and rising bodies in the framework of the universe is twofold, being in every case a compound of straight and circular. For, things that sink of their own weight, being predominantly earthy, undoubtedly retain the same nature as the whole of which they are parts. Nor is the explanation different in the case of those things, which, being fiery, are driven forcibly upward. For also fire here on the earth feeds mainly on earthy matter, and flame is defined as nothing but blazing smoke. Now it is a property of fire to expand what it enters. It does this with such great force that it cannot be prevented in any way by any device from bursting through restraints and completing its work. But the motion of expansion is directed from the center to the circumference. Therefore, if any part of the earth is set afire, it is carried from the middle upwards. Hence the statement that the motion of a simple body is simple holds true in particular for circular motion, as long as the simple body abides in its natural place and with its whole. For when it is in place, it has none but circular motion, which remains wholly within itself like a body at rest. Rectilinear motion, however, affects things which leave their natural place or are thrust out of it or quit it in any manner whatsoever. Yet nothing is so incompatible with the orderly arrangement of the universe and the design of the totality as something out of place. Therefore rectilinear motion occurs only to things that are not in proper condition and are not in complete accord with their nature, when they are separated from their whole and forsake its unity.

Furthermore, bodies that are carried upward and downward, even when deprived of circular motion, do not execute a simple, constant, and uniform motion. For they cannot be governed by their lightness or by the impetus of their weight. Whatever falls moves slowly at first, but increases its speed as it drops. On the other hand, we see this earthly fire (for we behold no other), after it has been lifted up high, slacken all at once, thereby revealing the reason to be the violence applied to the earthy matter. Circular motion, however, always rolls along uniformly, since it has an unfailing cause. But rectilinear motion has a cause that quickly stops functioning. For when rectilinear motion brings bodies to their own place, they cease to be heavy or light, and their motion ends. Hence, since circular motion belongs to wholes, but parts have rectilinear motion in addition, we can say that "circular" subsists with "rectilinear" as "being alive" with "being sick". Surely Aristotle's division of simple motion into three types, away from the middle, toward the middle, and around the middle, will be construed merely as a logical exercise. In like manner we distinguish line, point, and surface, even though one cannot exist without another, and none of them without body.

As a quality, moreover, immobility is deemed nobler and more divine than change and instability, which are therefore better suited to the earth than to the universe. Besides, it would seem quite absurd to attribute motion to the framework of space or that which encloses the whole of space, and not, more appropriately, to that which is enclosed and occupies some space, namely, the earth. Last of all, the planets obviously approach closer to the earth and recede farther from it. Then the motion of a single body around the middle, which is thought to be the center of the earth, will be both away from the middle and also toward it. Motion around the middle, consequently, must be interpreted in a more general way, the sufficient condition being that each such motion encircle its own center. You see, then, that all these arguments make it more likely that the earth moves than that it is at rest. This is especially true of the daily rotation, as particularly appropriate to the earth. This is enough, in my opinion, about the first part of the question.

CAN SEVERAL MOTIONS BE ATTRIBUTED TO THE EARTH? THE CENTER OF THE UNIVERSE *Chapter 9*

Accordingly, since nothing prevents the earth from moving, I suggest that we should now consider also whether several motions suit it, so that it can be regarded as one of the planets. For, it is not the center of all the revolutions. This is indicated by the planets' apparent nonuniform motion and their varying distances from the earth. These phenomena cannot be explained by circles concentric with the earth. Therefore, since there are many centers, it will not be by accident that the further question arises whether the center of the universe is identical with the center of terrestrial gravity or with some other point. For my part I believe that gravity is nothing but a certain natural desire, which the divine providence of the Creator of all things has implanted in parts, to gather as a unity and a whole by combining in the form of a globe. This impulse is present, we may suppose, also in the sun, the moon, and the other brilliant planets, so that through its operation they remain in that spherical shape which they display. Nevertheless, they swing round their circuits in divers ways. If, then, the earth too moves in other ways, for example, about a center, its additional motions must likewise be reflected in many bodies outside it. Among these motions we find the yearly revolution. For if this is transformed from a solar to a terrestrial movement, with the sun acknowledged to be at rest, the risings and settings which bring the zodiacal signs and fixed stars into view morning and evening will appear in the same way. The stations of the planets, moreover, as well as their retrogradations and [resumptions of] forward motion will be recognized as being, not movements of the planets, but a motion of the earth, which the planets borrow for their own appearances. Lastly, it will be realized that the sun occupies the middle of the universe. All these facts are disclosed to us by the principle governing the order in which the planets follow one another, and by the harmony of the entire universe, if only we look at the matter, as the saying goes, with both eyes.

THE ORDER OF THE HEAVENLY SPHERES *Chapter 10*

Of all things visible, the highest is the heaven of the fixed stars. This, I see, is doubted by nobody. But the ancient philosophers wanted to arrange the planets in accordance with the duration of the revolutions. Their principle assumes that of objects moving equally fast, those farther away seem to travel more slowly, as is proved in Euclid's *Optics*. The moon revolves in the shortest period of time because, in their opinion, it runs on the smallest circle as the nearest to the earth. The highest planet, on the other hand, is Saturn, which completes the biggest circuit in the longest time. Below it is Jupiter, followed by Mars.

With regard to Venus and Mercury, however, differences of opinion are found. For, these planets do not pass through every elongation from the sun, as the other planets do. Hence Venus and Mercury are located above the sun by some authorities, like Plato's *Timaeus* [38 D], but below the sun by others, like Ptolemy [*Syntaxis*, IX, 1] and many of the moderns. Al-Bitruji places Venus above the sun, and Mercury below it.

According to Plato's followers, all the planets, being dark bodies otherwise, shine because they receive sunlight. If they were below the sun, therefore, they would undergo no great elongation from it, and hence they would be seen halved or at any rate less than fully round. For, the light which they receive would be reflected mostly upward, that is, toward the sun, as we see in the new or dying moon. In addition, they argue, the sun must sometimes be eclipsed by the interposition of these planets, and its light cut off in proportion to their size. Since this is never observed, these planets do not pass beneath the sun at all, according to those who follow Plato.

On the other hand, those who locate Venus and Mercury below the sun base their reasoning on the wide space which they notice between the sun and the moon. For the moon's greatest distance from the earth is $64\,^1/_6$ earth-radii. This is contained, according to them, about 18 times in the sun's least distance from the earth, which is 1160 earth-radii. Therefore between the sun and the moon there are 1096 earth-radii [$\cong 1160-64^1/_6$]. Consequently, to avoid having so vast a space remain empty, they announce that the same numbers almost exactly fill up the apsidal distances, by which they compute the thickness of those spheres. Thus the moon's apogee is followed by Mercury's perigee. Mercury's apogee is succeeded by the perigee of Venus, whose apogee, finally, almost reaches the sun's perigee. For between the apsides of Mercury they calculate about $177\,^1/_2$ earth-radii. Then the remaining space is very nearly filled by Venus' interval of 910 earth-radii.

Therefore they do not admit that these heavenly bodies have any opacity like the moon's. On the contrary, these shine either with their own light or with the sunlight absorbed throughout their bodies. Moreover, they do not eclipse the sun, because it rarely happens that they interfere with our view of the sun, since they generally deviate in latitude. Besides, they are tiny bodies in comparison with the sun. Venus, although bigger than Mercury, can occult barely a hundredth of the sun. So says Al-Battani of Raqqa, who thinks that the sun's diameter is ten times larger [than Venus'], and therefore so minute a speck is not easily descried in the most brilliant light. Yet in his *Paraphrase* of Ptolemy, Ibn Rushd reports having seen something blackish when he found a conjunction of the sun and Mercury indicated in the tables. And thus these two planets are judged to be moving below the sun's sphere.

But this reasoning also is weak and unreliable. This is obvious from the fact that there are 38 earth-radii to the moon's perigee, according to Ptolemy [*Syntaxis*, V, 13], but more than 49 according to a more accurate determination, as will be made clear below. Yet so great a space contains, as we know, nothing but air and, if you please, also what is called "the element of fire". Moreover, the diameter of Venus' epicycle which carries it 45° more or less to either side of the sun, must be six times longer than the line drawn from the earth's center to Venus' perigee, as will be demonstrated in the proper place [V, 21]. In this entire space which would be taken up by that huge epicycle of Venus and which, moreover, is so much bigger than what would accommodate the earth, air, aether, moon, and Mercury, what will they say is contained if Venus revolved around a motionless earth?

Ptolemy [*Syntaxis*, IX, 1] argues also that the sun must move in the middle between the planets which show every elongation from it and those which do not. This argument carries no conviction because its error is revealed by the fact that the moon too shows every elongation from the sun.

Now there are those who locate Venus and then Mercury below the sun, or separate these planets [from the sun] in some other sequence. What reason will they adduce to explain why Venus and Mercury do not likewise traverse separate orbits divergent from the sun, like the other planets, without violating the arrangement [of the planets] in accordance with their [relative] swiftness and slowness?

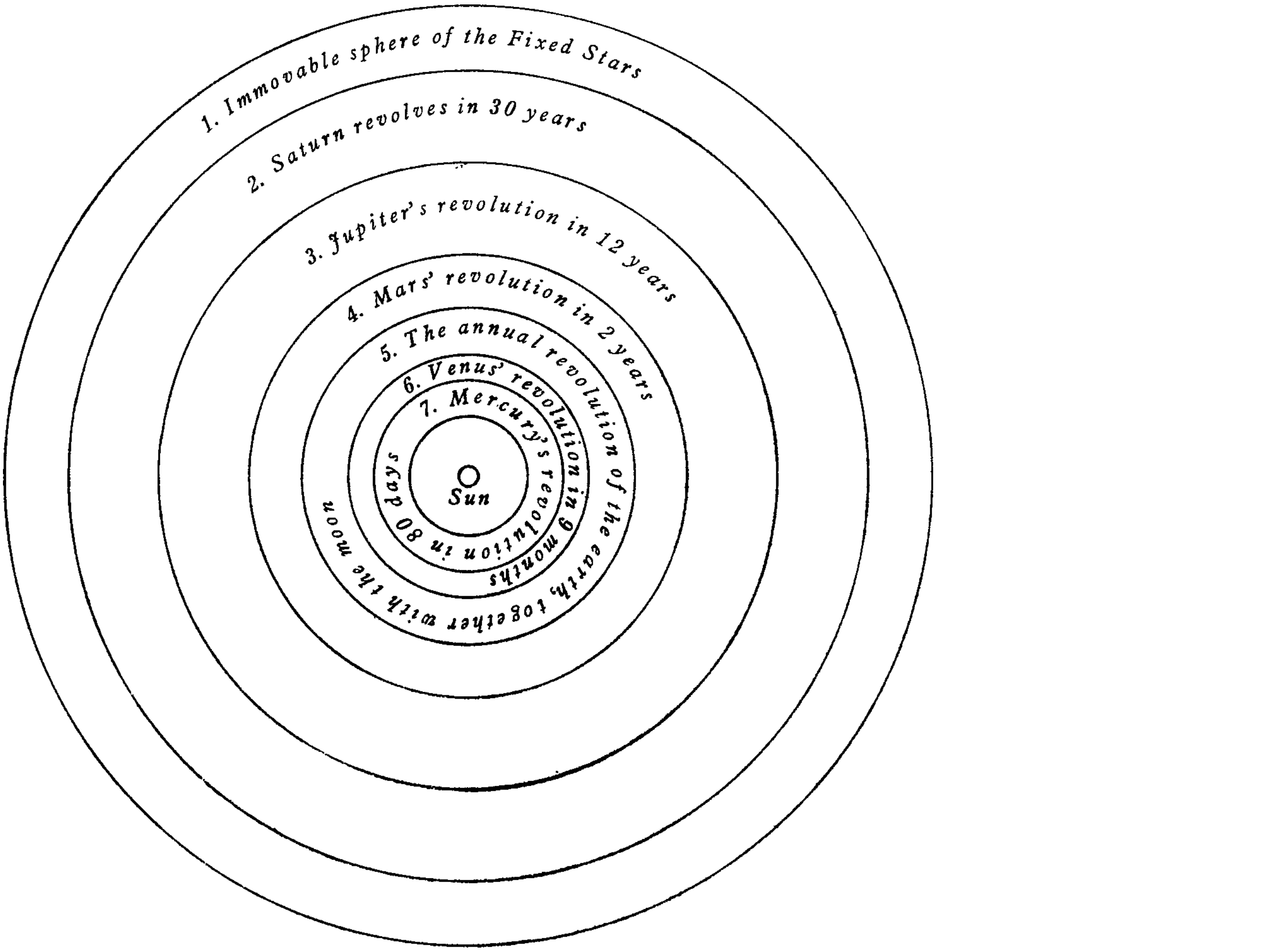
Then one of two alternatives will have to be true. Either the earth is not the center to which the order of the planets and spheres is referred, or there really is no principle of arrangement nor any apparent reason why the highest place belongs to Saturn rather than to Jupiter or any other planet.

In my judgement, therefore, we should not in the least disregard what was familiar to Martianus Capella, the author of an encyclopedia, and to certain other Latin writers. For according to them, Venus and Mercury revolve around the sun as their center. This is the reason, in their opinion, why these planets diverge no farther from the sun than is permitted by the curvature of their revolutions. For they do not encircle the earth, like the other planets, but "have opposite circles". Then what else do these authors mean but that the center of their spheres is near the sun? Thus Mercury's sphere will surely be enclosed within Venus', which by common consent is more than twice as big, and inside that wide region it will occupy a space adequate for itself. If anyone seizes this opportunity to link Saturn, Jupiter, and Mars also to that center, provided he understands their spheres to be so large that together with Venus and Mercury the earth too is enclosed inside and encircled, he will not be mistaken, as is shown by the regular pattern of their motions.

For [these outer planets] are always closest to the earth, as is well known, about the time of their evening rising, that is, when they are in opposition to the sun, with the earth between them and the sun. On the other hand, they are at their farthest from the earth at the time of their evening setting, when they become invisible in the vicinity of the sun, namely, when we have the sun between them and the earth. These facts are enough to show that their center belongs more to the sun, and is identical with the center around which Venus and Mercury likewise execute their revolutions.

But since all these planets are related to a single center, the space remaining between Venus' convex sphere and Mars' concave sphere must be set apart as also a sphere or spherical shell, both of whose surfaces are concentric with those spheres. This [intercalated sphere] receives the earth together with its attendant, the moon, and whatever is contained within the moon's sphere. Mainly for the reason that in this space we find quite an appropriate and adequate place for the moon, we can by no means detach it from the earth, since it is incontrovertibly nearest to the earth.

Hence I feel no shame in asserting that this whole region engirdled by the moon, and the center of the earth, traverse this grand circle amid the rest of the planets in an annual revolution around the sun. Near the sun is the center of the universe. Moreover, since the sun remains stationary, whatever appears as a motion of the sun is really due rather to the motion of the earth. In comparison with any other spheres of the planets, the distance from the earth to the sun has a magnitude which is quite appreciable in proportion to those dimensions. But the size of the universe is so great that the distance earth-sun is imperceptible in relation to the sphere of the fixed stars. This should be admitted, I believe, in preference to perplexing the mind with an almost infinite multitude of spheres, as must be done by those who kept the earth in the middle of the universe. On the contrary, we should rather heed the wisdom of nature. Just as it especially avoids producing anything superfluous or useless, so it frequently prefers to endow a single thing with many effects.



All these statements are difficult and almost inconceivable, being of course opposed to the beliefs of many people. Yet, as we proceed, with God's help I shall make them clearer than sunlight, at any rate to those who are not unacquainted with the science of astronomy. Consequently, with the first principle remaining intact, for nobody will propound a more suitable principle than that the size of the spheres is measured by the length of the time, the order of the spheres is the following, beginning with the highest.

The first and the highest of all is the sphere of the fixed stars, which contains itself and everything, and is therefore immovable. It is unquestionably the place of the universe, to which the motion and position of all the other heavenly bodies are compared. Some people think that it also shifts in some way. A different explanation of why this appears to be so will be adduced in my discussion of the earth's motion [I, 11].

[The sphere of the fixed stars] is followed by the first of the planets, Saturn, which completes its circuit in 30 years. After Saturn, Jupiter accomplishes its revolution in 12 years. Then Mars revolves in 2 years. The annual revolution takes the series' fourth place, which contains the earth, as I said [earlier in I, 10], together with the lunar sphere as an epicycle. In the fifth place Venus returns in 9 months. Lastly, the sixth place is held by Mercury, which revolves in a period of 80 days.

At rest, however, in the middle of everything is the sun. For in this most beautiful temple, who would place this lamp in another or better position than that from which it can light up the whole thing at the same time? For, the sun is not inappropriately called by some people the lantern of the universe, its mind by others, and its ruler by still others. [Hermes] the Thrice Greatest labels it a visible god, and Sophocles' Electra, the all-seeing. Thus indeed, as though seated on a royal throne, the sun governs the family of planets revolving around it. Moreover, the earth is not deprived of the moon's attendance. On the contrary, as Aristotle says in a work on animals, the moon has the closest kinship with the earth. Meanwhile the earth has intercourse with the sun, and is impregnated for its yearly parturition.

In this arrangement, therefore, we discover a marvelous symmetry of the universe, and an established harmonious linkage between the motion of the spheres and their size, such as can be found in no other way. For this permits a not inattentive student to perceive why the forward and backward arcs appear greater in Jupiter than in Saturn and smaller than in Mars, and on the other hand greater in Venus than in Mercury. This reversal in direction appears more frequently in Saturn than in Jupiter, and also more rarely in Mars and Venus than in Mercury. Moreover, when Saturn, Jupiter, and Mars rise at sunset, they are nearer to the earth than when they set in the evening or appear at a later hour. But Mars in particular, when it shines all night, seems to equal Jupiter in size, being distinguished only by its reddish color. Yet in the other configurations it is found barely among the stars of the second magnitude, being recognized by those who track it with assiduous observations. All these phenomena proceed from the same cause, which is in the earth's motion.

Yet none of these phenomena appears in the fixed stars. This proves their immense height, which makes even the sphere of the annual motion, or its reflection, vanish from before our eyes. For, every visible object has some measure of distance beyond which it is no longer seen, as is demonstrated in optics. From Saturn, the highest of the planets, to the sphere of the fixed stars there is an additional gap of the largest size. This is shown by the twinkling lights of the stars. By this token in particular they are distinguished from the planets, for there had to be a very great difference between what moves and what does not move. So vast, without any question, is the divine handiwork of the most excellent Almighty.

PROOF OF THE EARTH'S TRIPLE MOTION *Chapter 11*

In so many and such important ways, then, do the planets bear witness to the earth's mobility. I shall now give a summary of this motion, insofar as the phenomena are explained by it as a principle. As a whole, it must be admitted to be a threefold motion.

The first motion, named *nuchthemeron* by the Greeks, as I said [I, 4], is the rotation which is the characteristic of a day plus a night. This turns around the earth's axis from west to east, just as the universe is deemed to be carried in the opposite direction. It describes the equator, which some people call the "circle of equal days", in imitation of the designation used by the Greeks, whose term for it is *isemerinos*.

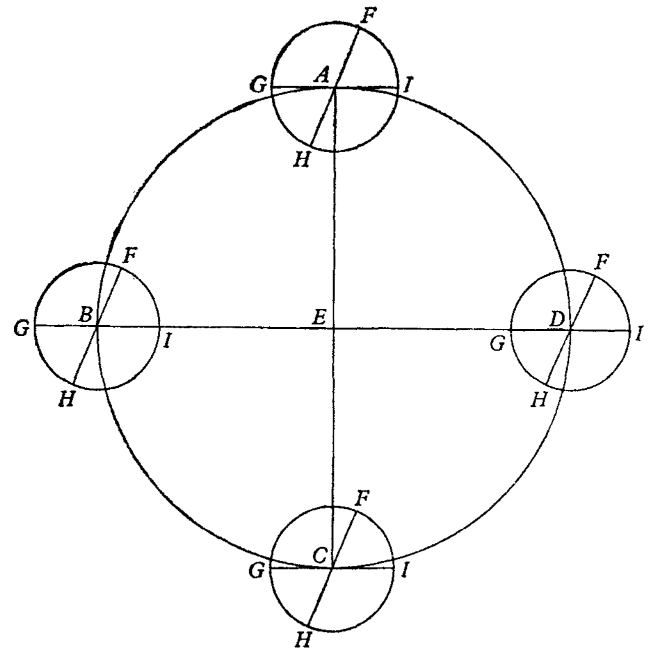
The second is the yearly motion of the center, which traces the ecliptic around the sun. Its direction is likewise from west to east, that is, in the order of the zodiacal signs. It travels between Venus and Mars, as I mentioned [I, 10], together with its associates. Because of it, the sun seems to move through the zodiac in a similar motion. Thus, for example, when the earth's center is passing through the Goat, the sun appears to be traversing the Crab; with the earth in the Water Bearer, the sun seems to be in the Lion, and so on, as I remarked.

To this circle, which goes through the middle of the signs, and to its plane, the equator and the earth's axis must be understood to have a variable inclination. For if they stayed at a constant angle, and were affected exclusively by the motion of the center, no inequality of days and nights would be observed. On the contrary, it would always be either the longest or shortest day or the day of equal daylight and darkness, or summer or winter, or whatever the character of the season, it would remain identical and unchanged.

The third motion in inclination is consequently required. This also is a yearly revolution, but it occurs in the reverse order of the signs, that is, in the direction opposite to that of the motion of the center. These two motions are opposite in direction and nearly equal in period. The result is that the earth's axis and equator, the largest of the parallels of latitude on it, face almost the same portion of the heavens, just as if they remained motionless. Meanwhile the sun seems to move through the obliquity of the ecliptic with the motion of the earth's center, as though this were the center of the universe. Only remember that, in relation to the sphere of the fixed stars, the distance between the sun and the earth vanishes from our sight forthwith.

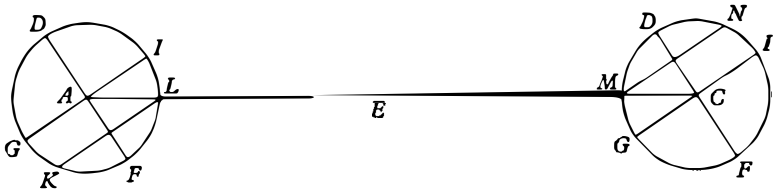
Since these are matters which crave to be set before our eyes rather than spoken of, let us describe a circle *ABCD*, which the annual revolution of the earth's center has traced in the plane of the ecliptic. Near its center let the sun be *E*. I shall divide this circle into four parts by drawing the diameters *AEC* and *BED*. Let *A* represent the first point of the Crab, *B* of the Balance, *C* of the Goat, and *D* of the Ram. Now let us assume that the earth's center is originally at *A*. About *A* I shall draw the terrestrial equator *FGHI*. This is not in the same plane [as the ecliptic], except that the diameter *GAI* is the intersection of the circles, I mean, of the equator and the ecliptic. Draw also the diameter *FAH* perpendicular to *GAI*, *F* being the limit of the [equator's] greatest inclination to the south, and *H* to the north. Under the conditions thus set forth, the earth's inhabitants will see the sun near the center *E* undergo the winter solstice in the Goat. This occurs because the greatest northward inclination, *H*, is turned toward the sun. For, the inclination of the equator to the line *AE*, through the agency of the daily rotation, traces the winter solstice parallel to the equator at an interval subtended by *EAH*, the angle of the obliquity.

Now let the earth's center start out in the order of the signs, and let *F*, the limit of maximum inclination, travel along an equal arc in the reverse order of the signs, until at *B* both have traversed a quadrant of their circles. In the interim the angle *EAI* always remains equal to *AEB*, on account of the equality of their revolutions; and the diameters always stay parallel to each other, *FAH* to *FBH*, and *GAI* to *GBI*, and the equator to the equator. In the immensity of the heavens,



for the reason already frequently mentioned, the same phenomena appear. Therefore from *B*, the first point of the Balance, *E* will seem to be in the Ram. The intersection of the circles will coincide with the single line *GBIE*, from which [the plane of the axis] will not be permitted by the daily rotation to deviate. On the contrary, the [axis'] inclination will lie entirely in the lateral plane. Accordingly the sun will be seen in the spring equinox. Let the earth's center proceed under the assumed conditions, and when it has completed a semicircle at *C*, the sun will appear to enter the Crab. But *F*, the southernmost inclination of the equator, will be turned toward the sun. This will be made to appear in the north, undergoing the summer solstice as measured by the angle of the obliquity, *ECF*. Again, when *F* turns away in the third quadrant of the circle, the intersection *GI* will once more fall on the line *ED*. From here the sun will be seen in the Balance undergoing the autumn equinox. Then as *H* by the same process gradually faces the sun, it will bring about a repetition of the initial situation, with which I began my survey.

Alternatively, let *AEC* be in the same way a diameter of the plane under discussion [the ecliptic] as well as the intersection of that plane with a circle perpendicular thereto. On *AEC*, around *A* and *C*, that is, in the Crab and the Goat, draw a circle of the earth in each case through the poles. Let this [meridian] be *DGFI*, the earth's axis *DF*, the north pole *D*, the south pole *F*, and *GI* the diameter of the equator. Now when *F* is turned toward the sun, which is near *E*, the equator's northward inclination being measured by the angle *IAE*, then the axial rotation will describe, parallel to the equator and to the south of it, at a distance *LI* and with diameter *KL*, the tropic of Capricorn as seen in the sun. Or, to speak more accurately, the axial rotation, as viewed from *AE*, generates a conic surface, having its vertex in the center of the earth, and its base in a circle parallel to the equator. Also at the opposite point, *C*, everything works out in like manner, but is reversed. It is clear therefore how the two motions, I mean, the motion of the center and



the motion in inclination, by their combined effect make the earth's axis remain in the same direction and in very much the same position, and make all these phenomena appear as though they were motions of the sun.

I said, however, that the annual revolutions of the center and of inclination are nearly equal. For if they were exactly equal, the equinoctial and solstitial points as well as the entire obliquity of the ecliptic would have to show no shift at all with reference to the sphere of the fixed stars. But since there is a slight variation, it was discovered only as it grew larger with the passage of time. From Ptolemy to us the precession of the equinoxes amounts to almost 21°. For this reason some people believed that the sphere of the fixed stars also moves, and accordingly they adopted a surmounting ninth sphere. This having proved inadequate, more recent writers now add on a tenth sphere. Yet they do not in the least attain their goal, which I hope to reach by the earth's motion. This I shall use as a principle and hypothesis in the demonstration of the other [motions].

[Here Copernicus originally planned to include a little more than two handwritten pages which he later deleted from his autograph. This deleted material, which was not printed in the first four editions of the *Revolutions* (1543, 1566, 1617, 1854), but was incorporated in those published after the recovery of Copernicus' autograph (1873, 1949, 1972), reads as follows].

The motion of the sun and moon can be demonstrated, I admit, also with an earth that is stationary. This is, however, less suitable for the remaining planets. Philolaus believed in the earth's motion for these and similar reasons. This is plausible because Aristarchus of Samos too held the same view according to some people, who were not motivated by the argumentation put forward by Aristotle and rejected by him [*Heavens*, II, 13-14]. But only a keen mind and persevering study could understand these subjects. They were therefore unfamiliar to most philosophers at that time, and Plato does not conceal the fact that there were then only a few who mastered the theory of the heavenly motions. Even if these were known to Philolaus or any Pythagorean, they nevertheless were probably not transmitted to posterity. For it was the Pythagoreans' practice not to commit the secrets of philosophy to writing nor divulge them to everybody, but to entrust them only to faithful friends and kinsmen, and pass them on from hand to hand. As evidence of this custom there is extant a letter from Lysis to Hipparchus. Because of its remarkable opinions and in order to make clear what value was attached to philosophy among themselves, I have decided to insert it here and to end this first Book with it. This, then, is a copy of the letter, which I translate from Greek as follows.

From Lysis to Hipparchus, greetings.

I would never have believed that after Pythagoras' death his followers' brotherhood would be dissolved. But now that we have unexpectedly been scattered hither and yon, as if our ship had been wrecked, it is still an act of piety to recall his godlike teachings and refrain from communicating the treasures of philosophy to those who have not even dreamed about the purification of the soul. For it is indecent to divulge to everybody what we achieved with such great effort, just as the Eleusinian goddesses' secrets may not be revealed to the uninitiated. The perpetrators of either of these misdeeds would be condemned as equally wicked and impious. On the other hand, it is worth considering how much time we spent wiping out the stains which clung to our hearts until we became receptive to his teachings after the course of five years. Dyers, having cleaned their fabrics, then apply their tincture with a mordant in order to fix the color indissolubly and prevent it from fading away easily thereafter. That godlike man prepared the lovers of philosophy in the same way, to avoid being disappointed in the hope he had conceived for the talents of any one of them. He did not sell his precepts for a price, and the snares with which young minds are entangled by many of the sophists were not set out by him because they are devoid of value. On the contrary, divine and human doctrines were promulgated by him.

Certain imitators of his teaching, however, perform at great length and out loud. Their instruction of the young follows a confused and improper procedure, thereby making their auditors impertinent and brash. For they mix disorderly and tainted morals with philosophy's lofty precepts. The result is like pouring pure fresh water into a deep well full of muck, since the muck is stirred up and the water is wasted. This is what happens to those who teach and are taught in this manner. For thick, dark woods obstruct the minds and hearts of those who were not correctly initiated, and completely damage the gentleness of their spirit and their reasonableness. These woods are infested with all sorts of vices, which by flourishing impede thought and prevent it from developing in any way.

As breeders of the interlopers I shall name principally self-indulgence and greed, both of which are extremely fertile. For, self-indulgence gives rise to incest, drunkenness, rape, unnatural pleasures, and certain violent impulses which lead as far as death and destruction. In fact, passion has inflamed some of these persons to so high a pitch that they spared neither their mothers nor their daughters. It has even carried them into conflict with their laws, country, government, and rulers. It has laid snares such that it brought them bound hand and foot to the final punishment. Greed, on the other hand, generates mayhem, murder, temple-robbery, poisoning, and other offspring of that sort. The lairs in those woods, where these urges lurk, must therefore be extirpated by fire and sword with all our might. When we have found the natural reason freed from these lusts, we shall then implant in it a most excellent and fruitful crop.

You too, Hipparchus, learned these rules with no small zeal. But, my good man, little did you heed them after you had tasted Sicilian luxury, for the sake of which you should have abandoned nothing. Many people even say that you are teaching philosophy publicly. This practice was forbidden by Pythagoras, who willed his notes to his daughter Damo with an order not to turn them over to anybody outside the family. Although she could have sold them for a lot of money, she refused to do so, considering poverty and her father's commands more precious than gold. They also say that when Damo died, she left the same obligation to her own daughter Bitale. Yet we of the male sex disobey our teacher and violate our oath. If, then, you mend your ways, I cherish you. But if you do not, as far as I am concerned, you are dead.

[The foregoing letter, the true nature of which was not suspected by Copernicus, ended Book I as originally planned. According to that plan, Book II began immediately after the letter with some introductory material, which was subsequently deleted. This deleted material, which was not printed in the first four editions of the *Revolutions*, but was included in those published after the recovery of Copernicus' autograph, reads as follows].

For what I have undertaken to do, those propositions of natural philosophy which seemed indispensable as principles and hypotheses, namely, that the universe is spherical, immense, and similar to the infinite, and that the sphere of the fixed stars as the container of everything is stationary, whereas all the other heavenly bodies have a circular motion, have been briefly reviewed. I have also assumed that the earth moves in certain revolutions, on which, as the cornerstone, I strive to erect the entire science of the stars.

[The rest of the material deleted here in the autograph was printed in the first four editions of the *Revolutions* as the following beginning of I, 12].

The proofs which I shall use in almost the entire work involve straight lines and arcs in plane and spherical triangles. Although much information about these topics is already available in Euclid's *Elements*, nevertheless that treatise does not contain the answer to what is the principal question here, how the sides can be obtained from the angles, and the angles from the sides.

[As the heading of I, 12, the first edition introduced "The Length of Straight Lines in a Circle". This caption, for which there is no direct warrant in the autograph, was repeated in the next three editions of the *Revolutions*.

In the autograph, on the other hand, the material preceding the first chapter of Book II, as originally planned, continued as follows].

The measure of a subtended straight line is not the angle, nor is the angle measured by the line. On the contrary, the measure is the arc. Hence a method has been discovered whereby the lines subtending any arc are known. With the help of these lines, the arc corresponding to the angle may be obtained; and conversely the straight line intercepted by the angle may be obtained through the arc. It therefore seems not inappropriate for me to discuss these lines in the following Book, and also the sides and angles of both plane and spherical triangles, which were treated by Ptolemy in scattered examples. I should like to finish these topics once and for all here, thereby clarifying what I have to say later on.

STRAIGHT LINES SUBTENDED IN A CIRCLE *Chapter 12*

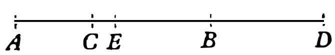
[Book II, Chapter 1, according to Copernicus' original plan]

In accordance with the common practice of mathematicians, I have divided the circle into 360°. With regard to the diameter, however, [a division into] 120 units was adopted by the ancients [for example, Ptolemy, *Syntaxis,* I, 10]. But later writers wanted to avoid the complication of fractions in multiplying and dividing the numbers for the lines [subtended in a circle], most of which are incommensurable as lengths, and often even when squared. Some of these later writers resorted to 1,200,000 units; others, 2,000,000; and still others established some other sensible diameter, after the Hindu symbols for numbers came into use. This numerical notation certainly surpasses every other, whether Greek or Latin, in lending itself to computations with exceptional speed. For this reason I too have accepted 200,000 units in a diameter as sufficient to be able to exclude any obvious error. For where quantities are not related to each other as one integer to another, it is enough to obtain an approximation. I shall explain this [subject] in six theorems and one problem, following Ptolemy closely.

THEOREM I

The diameter of a circle being given, the sides of the triangle, square, pentagon, hexagon, and decagon circumscribed by the circle are also given.

For, the radius, as half of the diameter, is equal to the side of the hexagon. But the square on the side of the triangle is three times, and the square on the side of the square is twice, the square on the side of the hexagon, as is demonstrated in Euclid's *Elements*. Therefore the side of the hexagon is given as 100,000 units long; the side of the square as 141,422; and the side of the triangle as 173,205.



Now let the side of the hexagon be *AB*. Let it be divided at the point *C* in mean and extreme ratio, in accordance with Euclid, Book II, Problem 1, or VI, 10. Let the greater segment be *CB*, and let it be extended an equal length, *BD*. Then the whole line *ABD* also will be divided in extreme and mean ratio. As the smaller segment, the extension *BD* is the side of the decagon inscribed in the circle in which *AB* is the side of the hexagon, as is clear from Euclid, XIII, 5 and 9.

Now *BD* will be obtained as follows. Bisect *AB* at *E*. From Euclid, XIII, 3, it is clear that the square of *EBD* equals five times the square of *EB*. But *EB* is given as 50,000 units long. Five times its square gives *EBD* as 111,803 units long. If *EB*'s 50,000 are subtracted, the remainder is *BD*'s 61,803 units, the side of the decagon which we were looking for.

Furthermore, the side of the pentagon, the square on which is equal to the sum of the squares on the sides of the hexagon and decagon, is given as 117,557 units.

Therefore, when the diameter of a circle is given, the sides of the triangle, square, pentagon, hexagon, and decagon which can be inscribed in the circle are given. Q.E.D.

COROLLARY

Consequently it is clear that when the chord subtending any arc is given, the chord subtending the rest of the semicircle is also given.

The angle inscribed in a semicircle is a right angle. Now in right triangles, the square on the diameter, that is, the side subtending the right angle, is equal to the squares on the sides forming the right angle. Now the side of the decagon, which subtends an arc of 36°, has been shown [Theorem I] to consist of 61,803 units, of which the diameter contains 200,000. Hence the chord subtending the remaining 144° of the semicircle is also given as consisting of 190,211 units. And from the side of the pentagon which, with its 117,557 units of the diameter, subtends an arc of 72°, the straight line subtending the remaining 108° of the semicircle is obtained as 161,803 units.

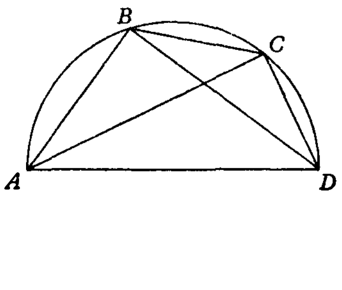
THEOREM II, PRELIMINARY [TO THEOREM III]

If a quadrilateral is inscribed in a circle, the rectangular product of the diagonals is equal to the rectangular products of the opposite sides.

For let the quadrilateral inscribed in a circle be *ABCD*. I say that the product of the diagonals $AC \times DB$ is equal to the products of $AB \times DC$ and $AD \times BC$. For let us make the angle *ABE* equal to the angle at *CBD*. Then the whole angle *ABD* is equal to the whole angle *EBC*, angle *EBD* being taken as common to both. Moreover, the angles at *ACB* and *BDA* are equal to each other, since they intercept the same segment of the circle. Therefore the two similar triangles [*BCE* and *BDA*] will have their sides proportional, $BC : BD = EC : AD$, and the product of $EC \times BD$ is equal to the product of $BC \times AD$. But also the triangles *ABE* and *CBD* are similar, because the angles at *ABE* and *CBD* are equal by construction, and the angles *BAC* and *BDC* are equal because they intercept the same arc of the circle. Consequently, as before, $AB : BD = AE : CD$, and the product of $AB \times CD$ is equal to the product of $AE \times BD$. But it has already been shown that the product of $AD \times BC$ is equal to the product of $BD \times EC$. By addition, then, the product of $BD \times AC$ is equal to the products of $AD \times BC$ and $AB \times CD$. This is what it was useful to prove.

THEOREM III

For it follows from the foregoing that if the straight lines subtending unequal arcs in a semicircle are given, the chord subtending the arc by which the larger arc exceeds the smaller is also given.

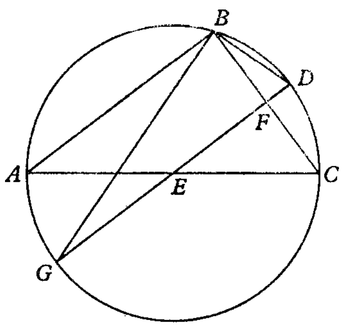


Thus in the semicircle *ABCD*, with diameter *AD*, let the chords subtending unequal arcs be *AB* and *AC*. What we wish to find is the chord subtending *BC*. From what was said above [Theorem I, Corollary], the chords *BD* and *CD*, subtending the arcs remaining in the semicircle, are given. As a result, in the semicircle the quadrilateral *ABCD* is formed. Its diagonals *AC* and *BD* are given, together with the three sides, *AB, AD,* and *CD.* In this quadrilateral, as has been demonstrated already [Theorem II], the product of $AC \times BD$ is equal to the product of $AB \times CD$ and $AD \times BC$. Therefore, if the product $AB \times CD$ is subtracted from the product $AC \times BD$, the remainder is the product $AD \times BC$. Hence, if we divide by *AD,* so far as that is possible, we obtain a number for the chord *BC,* which we were seeking.

From the foregoing, the sides of the pentagon and hexagon, for example, are given. Consequently the chord subtending 12°, the difference between them [72°-60°], is given in this way as 20,905 units of the diameter.

THEOREM IV

If the chord subtending any arc is given, the chord subtending half of the arc is also given.

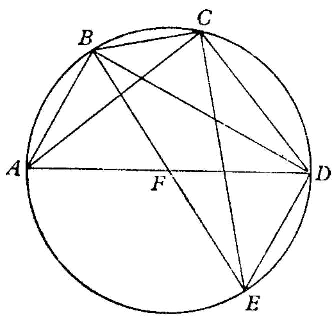


Let us describe the circle *ABC,* and let its diameter be *AC.* Let *BC* be the given arc with its subtending chord. From the center *E,* let the line *EF* intersect *BC* at right angles. Then, according to Euclid III, 3, *EF* will bisect *BC* at *F,* and when *EF* is extended, it will bisect the arc at *D.* Also draw the chords *AB* and *BD. ABC* and *EFC* are right triangles. Moreover, since they have angle *ECF* in common, they are similar triangles. Therefore, just as *CF* is half of *BFC,* so *EF* is half of *AB.* But *AB,* which subtends the remaining arc of the semicircle, is given [Theorem I, Corollary]. Hence *EF* is likewise given, and also *DF,* as the rest of the half diameter. Let the diameter be completed as *DEG.* Join *BG.* Then in the triangle *BDG,* from the right angle *B* the perpendicular *BF* falls on the base. Consequently the product of $GD \times DF$ is equal to the square of *BD.* Accordingly *BD* is given in length as subtending half of the arc *BDC.*

Since the chord subtending 12° has already been given [Theorem III], the chord subtending 6° is also given as 10,467 units; 3°, as 5,235 units; $1\,^{1}/_{2}$°, as 2,618 units; and $^{3}/_{4}$°, as 1,309 units.

THEOREM V

Furthermore, when the chords subtending two arcs are given, the chord subtending the whole arc consisting of the two arcs is also given.

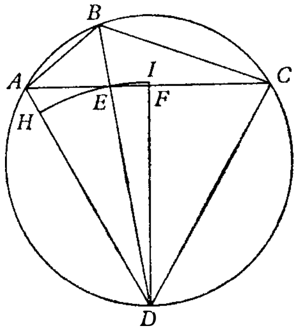


In a circle let the given chords be *AB* and *BC.* I say that the chord subtending the whole arc *ABC* is also given. For, draw the diameters *AFD* and *BFE,* and also the straight lines *BD* and *CE.* These chords are given by what precedes [Theorem I, Corollary], because *AB* and *BC* are given, and *DE* is equal to *AB.* Join *CD,* completing the quadrilateral *BCDE.* Its diagonals *BD* and *CE,* as well as three of its sides, *BC, DE,* and *BE,* are given. The remaining side, *CD,* will also be given by Theorem II. Therefore *CA,* as the chord subtending the rest of the semicircle, is given as the chord subtending the whole arc *ABC.* This is what we were looking for.

Then thus far the straight lines subtending 3°, $1\,^{1}/_{2}$°, and $^{3}/_{4}$° have been found. With these intervals anyone can construct a table with very precise relationships. But when [it comes to] advancing by [a whole] degree and adding one to another, or by half a degree, or in some other way, there will be a not unfounded doubt about the chords subtending these arcs, since we lack the graphical relationships by which they would be demonstrated. Yet nothing prevents us from attaining this result by another method, without any perceptible error and by assuming a number which is very slightly inaccurate. Ptolemy too [*Syntaxis*, I, 10] looked for the chords subtending 1° and $^1/_2$°, after reminding us first [of the following].

THEOREM VI

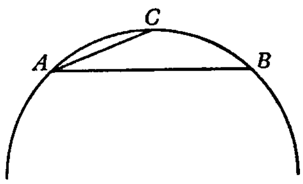
The ratio of a greater arc to a lesser arc is bigger than the ratio of the subtending straight lines.



In a circle, let the two unequal arcs, *AB* and *BC*, be contiguous, and let *BC* be the greater arc. I say that the ratio *BC* : *AB* is bigger than the ratio *BC* : *AB* of the chords forming the angle *B*. Let it be bisected by the line *BD*. Join *AC*. Let it intersect *BD* in the point *E*. Likewise join *AD* and *CD*. They are equal because they subtend equal arcs. Now in the triangle *ABC*, the line which bisects the angle also intersects *AC* at *E*. Hence the ratio of the base's segments *EC* : *AE* is equal to the ratio *BC* : *AB*. Since *BC* is greater than *AB*, *EC* also is greater than *EA*. Erect *DF* perpendicular to *AC*. *DF* will bisect *AC* at the point *F*, which must lie in the greater segment, *EC*. In every triangle the greater angle is opposite the greater side. Hence in triangle *DEF*, the side *DE* is greater than *DF*. *AD* is even greater than *DE*. Therefore an arc drawn with *D* as center, and with *DE* as radius, will intersect *AD*; and pass beyond *DF*. Let the arc intersect *AD* in *H*, and let it be extended to the straight line *DFI*. Then the sector *EDI* is greater than the triangle *EDF*. But the triangle *DEA* is greater than the sector *DEH*. Therefore the ratio of triangle *DEF* to triangle *DEA* is smaller than the ratio of sector *DEI* to sector *DEH*. But sectors are proportional to their arcs or central angles, whereas triangles which have the same vertex are proportional to their bases. Consequently the ratio of the angles *EDF* : *ADE* is bigger than the ratio of the bases *EF* : *AE*. Hence, by addition, the ratio of the angles *FDA* : *ADE* is bigger than the ratio *AF* : *AE*, and in the same way *CDA* : *ADE* is bigger than *AC* : *AE*. And by subtraction, *CDE* : *EDA* also is bigger than *CE* : *EA*. However, the angles *CDE* and *EDA* are to each other as the arcs *CB* : *AB*, but the bases *CE* : *AE* are as the chords *BC* : *AB*. Therefore the ratio of the arcs *CB* : *AB* is bigger than the ratio of the chords *BC* : *AB*. Q.E.D.

PROBLEM

An arc is always greater than the straight line subtending it, while a straight line is the shortest of the lines having the same end points. Yet this inequality, [in descending] from greater to lesser portions of a circle, approaches equality, so that in the end the straight and circular lines are extinguished simultaneously at their last point of contact on the circle. Prior to that, consequently, they must differ from each other by no perceptible distinction.



For example, let arc *AB* be 3°, and arc *AC* 1 $^1/_2$°. The chord subtending *AB* has been shown [Theorem IV] to consist of 5235 units, of which the diameter is assumed to have 200,000, and the chord subtending *AC* has 2618 units. The arc *AB* is double the arc *AC*, whereas the chord *AB* is less than double the chord *AC*, which exceeds 2,617 by only one unit. But if we take *AB* as 1 $^1/_2$° and *AC* as $^3/_4$°, we shall have chord *AB* as 2618 units, and *AC* as 1309 units. Although *AC* ought to be greater than half of the chord *AB*, yet it seems not to differ from half, the ratios of the arcs and straight lines now appearing to be the same. Hence we see that we have reached the level where the difference between the straight and circular lines becomes absolutely imperceptible, as though they had merged into a single line. Hence I have no hesitation in fitting the 1309 units of $^3/_4{}^\circ$ in the same proportion to the chords subtending 1° and the other fractional parts thereof. Thus, by adding $^1/_4{}^\circ$ to $^3/_4{}^\circ$, we establish the chord subtending 1° as 1745 units; $^1/_2{}^\circ$, as $872\,^1/_2$ units; and $^1/_3{}^\circ$, as approximately 582 units.

Yet I believe that it is enough if I put in the Table only half-lines subtending double the arcs. By this shortcut I shall compress in a quadrant what formerly had to be spread out over a semicircle. The main reason for doing so is that in demonstrations and calculations half-lines are used more frequently than whole lines. I have drawn up a Table which progresses by sixths of a degree. It has three columns. In the first column are the degrees, or parts, of a circumference, and sixths of a degree. The second column contains the numerical value for the half-line subtending double the arc. The third column shows, for each degree, the difference intervening between these numerical values. These differences permit the interpolation of the proportional amounts corresponding to individual minutes of degrees. The Table, then, is as follows.

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
Degree	Minute			Degree	Minute		
0	10	291	291	6	10	10742	289
0	20	582		6	20	11031	
0	30	873		6	30	11320	
0	40	1163		6	40	11609	
0	50	1454		6	50	11898	
1	0	1745		7	0	12187	
1	10	2036		7	10	12476	
1	20	2327		7	20	12764	288
1	30	2617		7	30	13053	
1	40	2908		7	40	13341	
1	50	3199		7	50	13629	
2	0	3490		8	0	13917	
2	10	3781		8	10	14205	
2	20	4071		8	20	14493	
2	30	4362		8	30	14781	
2	40	4653		8	40	15069	
2	50	4943	290	8	50	15356	287
3	0	5234		9	0	15643	
3	10	5524		9	10	15931	
3	20	5814		9	20	16218	
3	30	6105		9	30	16505	
3	40	6395		9	40	16792	
3	50	6685		9	50	17078	
4	0	6975		10	0	17365	
4	10	7265		10	10	17651	286
4	20	7555		10	20	17937	
4	30	7845		10	30	18223	
4	40	8135		10	40	18509	
4	50	8425		10	50	18795	
5	0	8715		11	0	19081	
5	10	9005		11	10	19366	285
5	20	9295		11	20	19652	
5	30	9585		11	30	19937	
5	40	9874		11	40	20222	
5	50	10164	289	11	50	20507	
6	0	10453		12	0	20791	

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
12	10	21076	284	18	10	31178	276
12	20	21360		18	20	31454	
12	30	21644		18	30	31730	
12	40	21928		18	40	32006	
12	50	22212		18	50	32282	275
13	0	22495	283	19	0	32557	
13	10	22778		19	10	32832	
13	20	23062		19	20	33106	
13	30	23344		19	30	33381	274
13	40	23627		19	40	33655	
13	50	23910	282	19	50	33929	
14	0	24192		20	0	34202	
14	10	24474		20	10	34475	273
14	20	24756		20	20	34748	
14	30	25038	281	20	30	35021	
14	40	25319		20	40	35293	272
14	50	25601		20	50	35565	
15	0	25882		21	0	35837	
15	10	26163		21	10	36108	271
15	20	26443	280	21	20	36379	
15	30	26724		21	30	36650	
15	40	27004		21	40	36920	270
15	50	27284		21	50	37190	
16	0	27564	279	22	0	37460	
16	10	27843		22	10	37730	269
16	20	28122		22	20	37999	
16	30	28401		22	30	38268	
16	40	28680		22	40	38537	268
16	50	28959	278	22	50	38805	
17	0	29237		23	0	39073	
17	10	29515		23	10	39341	267
17	20	29793		23	20	39608	
17	30	30071	277	23	30	39875	
17	40	30348		23	40	40141	266
17	50	30625		23	50	40408	
18	0	30902		24	0	40674	

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
Degree	Minute			Degree	Minute		
24	10	40939	265	30	10	50252	251
24	20	41204		30	20	50503	
24	30	41469		30	30	50754	250
24	40	41734	264	30	40	51004	
24	50	41998		30	50	51254	
25	0	42262		31	0	51504	249
25	10	42525	263	31	10	51753	
25	20	42788		31	20	52002	248
25	30	43051		31	30	52250	
25	40	43313	262	31	40	52498	247
25	50	43575		31	50	52745	
26	0	43837		32	0	52992	246
26	10	44098	261	32	10	53238	
26	20	44359		32	20	53484	
26	30	44620	260	32	30	53730	245
26	40	44880		32	40	53975	
26	50	45140		32	50	54220	244
27	0	45399	259	33	0	54464	
27	10	45658		33	10	54708	243
27	20	45916	258	33	20	54951	
27	30	46175		33	30	55194	242
27	40	46433		33	40	55436	
27	50	46690	257	33	50	55678	241
28	0	46947		34	0	55919	
28	10	47204	256	34	10	56160	240
28	20	47460		34	20	56400	
28	30	47716	255	34	30	56641	239
28	40	47971		34	40	56880	
28	50	48226		34	50	57119	238
29	0	48481	254	35	0	57358	
29	10	48735		35	10	57596	
29	20	48989	253	35	20	57833	237
29	30	49242		35	30	58070	
29	40	49495	252	35	40	58307	236
29	50	49748		35	50	58543	
30	0	50000		36	0	58779	235

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
36	10	59014	235	42	10	67129	215
36	20	59248	234	42	20	67344	
36	30	59482		42	30	67559	214
36	40	59716	233	42	40	67773	
36	50	59949		42	50	67987	213
37	0	60181	232	43	0	68200	212
37	10	60413		43	10	68412	
37	20	60645	231	43	20	68624	211
37	30	60876		43	30	68835	
37	40	61107	230	43	40	69046	210
37	50	61337		43	50	69256	
38	0	61566	229	44	0	69466	209
38	10	61795		44	10	69675	
38	20	62024		44	20	69883	208
38	30	62251	228	44	30	70091	207
38	40	62479		44	40	70298	
38	50	62706	227	44	50	70505	206
39	0	62932		45	0	70711	205
39	10	63158	226	45	10	70916	
39	20	63383		45	20	71121	204
39	30	63608	225	45	30	71325	
39	40	63832		45	40	71529	203
39	50	64056	224	45	50	71732	202
40	0	64279	223	46	0	71934	
40	10	64501	222	46	10	72136	201
40	20	64723		46	20	72337	200
40	30	64945	221	46	30	72537	
40	40	65166	220	46	40	72737	199
40	50	65386		46	50	72936	
41	0	65606	219	47	0	73135	198
41	10	65825		47	10	73333	197
41	20	66044	218	47	20	73531	
41	30	66262		47	30	73728	196
41	40	66480	217	47	40	73924	195
41	50	66697		47	50	74119	
42	0	66913	216	48	0	74314	194

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
48	10	74508	194	54	10	81072	170
48	20	74702		54	20	81242	169
48	30	74896		54	30	81411	
48	40	75088	192	54	40	81580	168
48	50	75280	191	54	50	81748	167
49	0	75471	190	55	0	81915	
49	10	75661		55	10	82082	166
49	20	75851	189	55	20	82248	165
49	30	76040		55	30	82413	164
49	40	76229	188	55	40	82577	
49	50	76417	187	55	50	82741	163
50	0	76604		56	0	82904	162
50	10	76791	186	56	10	83066	
50	20	76977		56	20	83228	161
50	30	77162	185	56	30	83389	160
50	40	77347	184	56	40	83549	159
50	50	77531		56	50	83708	
51	0	77715	183	57	0	83867	158
51	10	77897	182	57	10	84025	157
51	20	78079		57	20	84182	
51	30	78261	181	57	30	84339	156
51	40	78442	180	57	40	84495	155
51	50	78622		57	50	84650	
52	0	78801	179	58	0	84805	154
52	10	78980	178	58	10	84959	153
52	20	79158		58	20	85112	152
52	30	79335	177	58	30	85264	
52	40	79512	176	58	40	85415	151
52	50	79688		58	50	85566	150
53	0	79864	175	59	0	85717	
53	10	80038	174	59	10	85866	149
53	20	80212		59	20	86015	148
53	30	80386	173	59	30	86163	147
53	40	80558	172	59	40	86310	
53	50	80730		59	50	86457	146
54	0	80902	171	60	0	86602	145

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs: Degree	Arcs: Minute	Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
60	10	86747	144	66	10	91472	118
60	20	86892		66	20	91590	117
60	30	87036	143	66	30	91706	116
60	40	87178	142	66	40	91822	115
60	50	87320		66	50	91936	114
61	0	87462	141	67	0	92050	113
61	10	87603	140	67	10	92164	
61	20	87743	139	67	20	92276	112
61	30	87882		67	30	92388	111
61	40	88020	138	67	40	92499	110
61	50	88158	137	67	50	92609	109
62	0	88295		68	0	92718	
62	10	88431	136	68	10	92827	108
62	20	88566	135	68	20	92935	107
62	30	88701	134	68	30	93042	106
62	40	88835		68	40	93148	105
62	50	88968	133	68	50	93253	
63	0	89101	132	69	0	93358	104
63	10	89232	131	69	10	93462	103
63	20	89363		69	20	93565	102
63	30	89493	130	69	30	93667	
63	40	89622	129	69	40	93769	101
63	50	89751	128	69	50	93870	100
64	0	89879		70	0	93969	99
64	10	90006	127	70	10	94068	98
64	20	90133	126	70	20	94167	
64	30	90258		70	30	94264	97
64	40	90383	125	70	40	94361	96
64	50	90507	124	70	50	94457	95
65	0	90631	123	71	0	94552	94
65	10	90753	122	71	10	94646	93
65	20	90875	121	71	20	94739	
65	30	90996		71	30	94832	92
65	40	91116	120	71	40	94924	91
65	50	91235	119	71	50	95015	90
66	0	91354	118	72	0	95105	

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
Degree	Minute			Degree	Minute		
72	10	95195	89	78	10	97875	59
72	20	95284	88	78	20	97934	58
72	30	95372	87	78	30	97992	
72	40	95459	86	78	40	98050	57
72	50	95545	85	78	50	98107	56
73	0	95630		79	0	98163	55
73	10	95715	84	79	10	98218	54
73	20	95799	83	79	20	98272	
73	30	95882	82	79	30	98325	53
73	40	95964	81	79	40	98378	52
73	50	96045		79	50	98430	51
74	0	96126	80	80	0	98481	50
74	10	96206	79	80	10	98531	49
74	20	96285	78	80	20	98580	
74	30	96363	77	80	30	98629	48
74	40	96440		80	40	98676	47
74	50	96517	76	80	50	98723	46
75	0	96592	75	81	0	98769	45
75	10	96667	74	81	10	98814	44
75	20	96742	73	81	20	98858	43
75	30	96815	72	81	30	98902	42
75	40	96887		81	40	98944	
75	50	96959	71	81	50	98986	41
76	0	97030	70	82	0	99027	40
76	10	97099	69	82	10	99067	39
76	20	97169	68	82	20	99106	38
76	30	97237		82	30	99144	
76	40	97304	67	82	40	99182	37
76	50	97371	66	82	50	99219	36
77	0	97437	65	83	0	99255	35
77	10	97502	64	83	10	99290	34
77	20	97566	63	83	20	99324	33
77	30	97630		83	30	99357	
77	40	97692	62	83	40	99389	32
77	50	97754	61	83	50	99421	31
78	0	97815	60	84	0	99452	30

TABLE OF THE STRAIGHT LINES SUBTENDED IN A CIRCLE

Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree	Arcs		Half-Chords Subtending Double Arcs	Differences for the Fractions of a Degree
Degree	Minute			Degree	Minute		
84	10	99482	29	87	10	99878	14
84	20	99511	28	87	20	99892	13
84	30	99539	27	87	30	99905	12
84	40	99567		87	40	99917	
84	50	99594	26	87	50	99928	11
85	0	99620	25	88	0	99939	10
85	10	99644	24	88	10	99949	9
85	20	99668	23	88	20	99958	8
85	30	99692	22	88	30	99966	7
85	40	99714		88	40	99973	6
85	50	99736	21	88	50	99979	
86	0	99756	20	89	0	99985	5
86	10	99776	19	89	10	99989	4
86	20	99795	18	89	20	99993	3
86	30	99813		89	30	99996	2
86	40	99830	17	89	40	99998	1
86	50	99847	16	89	50	99999	0
87	0	99863	15	90	0	100000	0

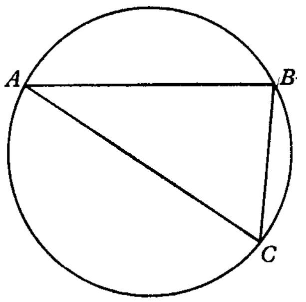
THE SIDES AND ANGLES OF PLANE RECTILINEAR TRIANGLES

Chapter 13

[Book II, Chapter 2, according to Copernicus' original plan]

I

If the angles of a triangle are given, the sides are given.



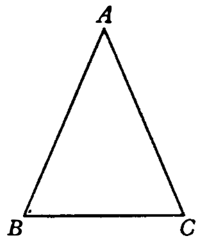
I say, let there be a triangle *ABC*. Circumscribe a circle around it, in accordance with Euclid, Book IV, Problem 5. Then the arcs *AB*, *BC*, and *CA* will likewise be given, according to the system in which 360° are equal to two right angles. But when the arcs are given, the sides of the triangle inscribed in the circle are also given as chords, in the Table set forth above, in units whereof the diameter is assumed to have 200,000.

II

But if an angle and two sides of a triangle are given, the remaining side and the other angles will also be known.

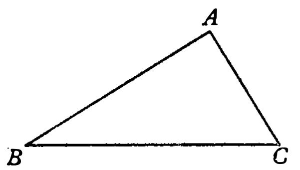
For, the given sides are either equal or unequal, while the given angle is either right or acute or obtuse, and the given sides either include or do not include the given angle.

II A



First, in the triangle *ABC* let the two given sides, *AB* and *AC*, which include the given angle *A*, be equal. Then the other angles, which are at the base *BC*, since they are equal, are also given as halves of the remainder when *A* is subtracted from two right angles. And if originally an angle at the base is given, its equal is thereupon given; and from these, the remainder of two right angles is given. But when the angles of a triangle are given, the sides are given, and the base *BC* is given by the Table, in units whereof *AB* or *AC* as radius has 100,000, or the diameter 200,000.

II B

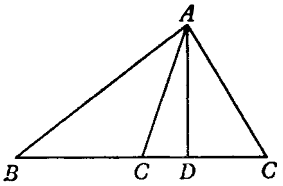


But if *BAC* is a right angle included by sides which are given, the same result will follow.

It is quite obvious that the squares on *AB* and *AC* are equal to the square on the base *BC*. Therefore *BC* is given in length, and so the sides are given in relation to one another. But the segment of the circle which encloses the right triangle is a semicircle, whose diameter is the base *BC*. Therefore, in units whereof *BC* has 200,000, *AB* and *AC* will be given as sides opposite the remaining angles *B* and *C*. Their place in the Table will accordingly make them known in degrees, whereof 180 are equal to two right angles. The same result will follow if *BC* is given with either of the two sides which include the right angle. This is now quite obvious, in my judgement.

II C

Now let the given angle *ABC* be acute, and let it also be included by the given sides *AB* and *BC*. From the point *A* drop a perpendicular to *BC*, extended if necessary, according as the perpendicular falls inside or outside the triangle. Let the perpendicular be *AD*. By means of it two right triangles *ABD* and *ADC* are established. In *ABD* the angles are given, because *D* is a right angle, and *B* is given by hypothesis. Therefore *AD* and *BD*, as sides opposite the angles *A* and *B*, are given by the Table in units whereof *AB*, as the diameter of a circle, has 200,000. And on the same scale on which *AB* was given in length, *AD* and *BD* are given in similar units, and so also is *CD*, by which *BC* exceeds *BD*. In the right triangle *ADC*, therefore, the sides *AD* and *CD* being given, the required side *AC* and the angle *ACD* are likewise given by the preceding proof.

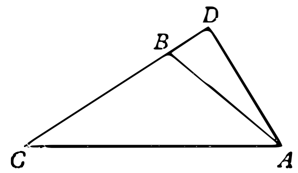


II D

The result will not be different if the angle *B* is obtuse. For from the point *A*, a perpendicular *AD*, dropped on the straight line *BC* extended, makes a triangle *ABD*, whose angles are given. For, *ABD* is given as the supplementary angle of *ABC*, and *D* is a right angle. Therefore *BD* and *AD* are given in units whereof *AB* is 200,000. And since *BA* and *BC* have a given ratio to each other, therefore *BC* is given also in the same units as *BD*, and so is the whole of *CBD*. Likewise in the right triangle *ADC*, therefore, since the two sides *AD* and *CD* are given, the required *AC* also is given, as well as the angle *BAC*, with the remainder *ACB*, which were required.

II E

Now let either one of the given sides be opposite the given angle *B*. Let [this opposite side] be *AC* and [the other given side] *AB*. Then *AC* is given by the Table in units whereof the diameter of the circle circumscribed around the triangle *ABC* has 200,000. Moreover, in accordance with the given ratio of *AC* to *AB*, *AB* is given in similar units. And by the Table the angle at *ACB* is given, together with the remaining angle *BAC*. Through the latter, the chord *CB* also is given. When this ratio is given, [the length of the sides] is given in any units whatsoever.



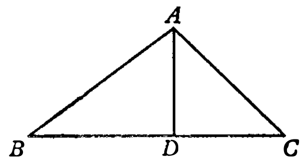
III

If all the sides of a triangle are given, the angles are given.

In the case of the equilateral triangle, the fact that each of its angles is one-third of two right angles is too well known to be mentioned.

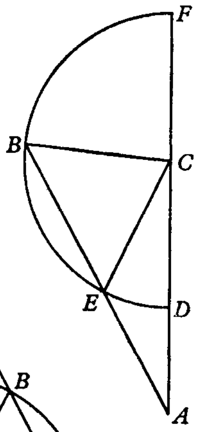
Also in the case of the isosceles triangle the situation is clear. For, the equal sides are to the third side as halves of the diameter are to the chord subtending the arc. Through the arc, the angle included by the equal sides is given by the Table in units whereof a central angle of 360° is equal to four right angles. Then the other angles, which are at the base, are also given as halves [of the remainder when the angle included by the equal sides is subtracted] from two right angles.

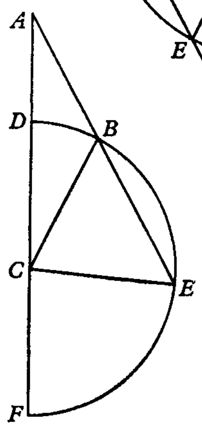
It therefore now remains to give the proof for scalene triangles too. These will similarly be divided into right triangles. Then let *ABC* be a scalene triangle of given sides. On the longest side, for instance, *BC*, drop the perpendicular *AD*. But the square of *AB*, which is opposite an acute angle, as we are told by Euclid, II, 13, is less than the squares on the other two sides, the difference being twice the product $BC \times CD$. For, *C* must be an acute angle; otherwise *AB* would be,



contrary to the hypothesis, the longest side, as may be inferred from Euclid, I, 17, and the next two theorems. Therefore *BD* and *DC* are given; and in a situation to which we have already frequently returned, *ABD* and *ADC* will be right triangles of given sides and angles. From these, the required angles of triangle *ABC* are also known.

Alternatively, the next to the last theorem in Euclid, III, will demonstrate the same result, perhaps more conveniently. Let the shortest side be *BC*. With *C* as center, and with *BC* as radius, let us describe a circle which will intersect both of the remaining sides or either one of them.





First let it intersect both, *AB* at the point *E*, and *AC* at *D*. Also extend the line *ADC* to the point *F* in order to complete the diameter *DCF*. From this construction it is clear, in accordance with that Euclidean theorem, that the product $FA \times AD$ is equal to the product $BA \times AE$, since both products are equal to the square of the line drawn tangent to the circle from *A*. But the whole of *AF* is given, since all of its segments are given. *CF* and *CD*, as radii, are of course equal to *BC*, and *AD* is the excess of *CA* over *CD*. Therefore the product $BA \times AE$ is also given. So is *AE* in length, together with the remainder *BE*, the chord subtending the arc *BE*. By joining *EC*, we shall have *BCE* as an isosceles triangle of given sides. Therefore angle *EBC* is given. Hence in triangle *ABC* the remaining angles *C* and *A* will also be known from what precedes.

Now do not let the circle intersect *AB*, as in the second figure, where *AB* meets the curve of the circumference. Nevertheless *BE* will be given. Moreover, in the isosceles triangle *BCE* the angle *CBE* is given, and so also is its supplement *ABC*. By exactly the same process of reasoning as before, the remaining angles are given.

What has been said, containing as it does a considerable part of surveying, may suffice for rectilinear triangles. Now let us turn to spherical triangles.

SPHERICAL TRIANGLES — *Chapter 14*

[Book II, Chapter 3, according to Copernicus' original plan]

I here regard a convex triangle as the figure which is enclosed on a spherical surface by three arcs of great circles. But the size of an angle, as well as the difference between angles, [is measured] on an arc of the great circle which is drawn with the [angle's] point of intersection as its pole. This arc is intercepted by the quadrants enclosing the angle. For, the arc so intercepted is to the whole circumference as the angle at the intersection is to four right angles. These, as I said, contain 360 equal degrees.

I

If there are three arcs of great circles of a sphere, any two of which, when joined together, are longer than the third, clearly a spherical triangle can be formed from them.

For, this statement about arcs is proved for angles by Euclid, XI, 23. Since the ratio of angles and arcs is the same, and great circles pass through the center of the sphere, evidently the three sectors of the circles, of which these are arcs, form a solid angle at the center of the sphere. The theorem is therefore obvious.

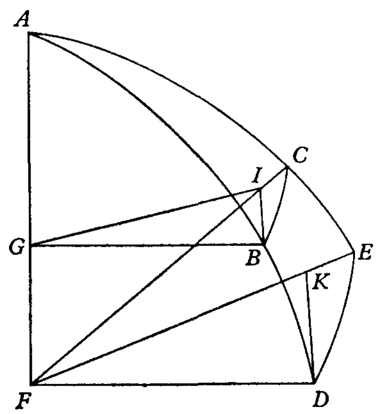
II

Any arc of a triangle must be less than a semicircle.

For, a semicircle does not form an angle at the center, but proceeds through it in a straight line. On the other hand, the two remaining angles, to which arcs belong, cannot enclose a solid angle at the center, and consequently not a spherical triangle. This was the reason, in my opinion, why Ptolemy, in expounding this class of triangles, especially in connection with the shape of the spherical sector, stipulates that the assumed arcs should not be greater than a semicircle [*Syntaxis*, I, 13].

III

In right spherical triangles, the ratio of the chord subtending twice the side opposite the right angle to the chord subtending twice either one of the sides including the right angle is equal to the ratio of the diameter of the sphere to the chord subtending twice the angle included, on a great circle of the sphere, between the remaining side and the hypotenuse.



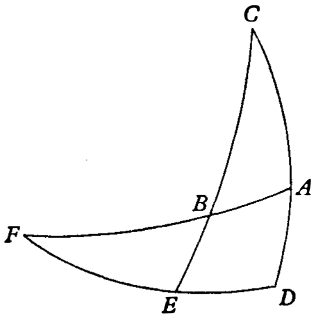
For let there be a spherical triangle *ABC*, in which *C* is a right angle. I say that the ratio of the chord subtending twice *AB* to the chord subtending twice *BC* is equal to the ratio of the diameter of the sphere to the chord subtending twice the angle *BAC* on a great circle.

With *A* as pole, draw *DE* as the arc of a great circle. Complete the quadrants *ABD* and *ACE*. From *F*, the center of the sphere, draw the intersections of the circles: *FA*, of *ABD* and *ACE*; *FE*, of *ACE* and *DE*; *FD*, of *ABD* and *DE*; and also *FC*, of the circles *AC* and *BC*. Then draw *BG* perpendicular to *FA*, *BI* to *FC*, and *DK* to *FE*. Join *GI*.

If a circle intersects another circle while passing through its poles, it intersects it at right angles. Therefore *AED* is a right angle. So is *ACB* by hypothesis. Hence both planes *EDF* and *BCF* are perpendicular to *AEF*. In this last-mentioned plane at point *K* draw a straight line perpendicular to the intersection *FKE*. Then this perpendicular will form with *KD* another right angle, in accordance with the definition of planes perpendicular to each other. Consequently *KD* is perpendicular also to *AEF*, according to Euclid, XI, 4. In the same way *BI* is drawn perpendicular to the same plane, and therefore *DK* and *BI* are parallel to each other, according to Euclid, XI, 6. Likewise *GB* is parallel to *FD*, because *FGB* and *GFD* are right angles. According to Euclid's *Elements*, XI, 10, angle *FDK* will be equal to *GBI*. But *FKD* is a right angle, and so is *GIB* according to the definition of a perpendicular line. The sides of similar triangles being proportional, *DF* is to *BG* as *DK* is to *BI*. But *BI* is half of the chord subtending twice the arc *CB*, since *BI* is perpendicular to the radius *CF*. In the same way *BG* is half of the chord subtending twice the side *BA*; *DK* is half of the chord subtending twice *DE*, or twice angle *A*; and *DF* is half of the diameter of the sphere. Clearly, therefore, the ratio of the chord subtending twice *AB* to the chord subtending twice *BC* is equal to the ratio of the diameter to the chord subtending twice the angle *A*, or twice the intercepted arc *DE*. The demonstration of this Theorem will prove to be useful.

IV

In any triangle having a right angle, if another angle and any side are given, the remaining angle and the remaining sides will also be given.



For let the triangle *ABC* have angle *A* right, and either of the other angles, for instance, *B*, also given. But with regard to the given side, I make a threefold division. For either it is adjacent to the given angles, like *AB*; or only to the right angle, like *AC*; or it is opposite the right angle, like *BC*.

Then first let *AB* be the given side. With *C* as pole, draw *DE* as the arc of a great circle. Complete the quadrants *CAD* and *CBE*. Produce *AB* and *DE* until they intersect at point *F*. Then *F* in turn will be the pole of *CAD*, since *A* and *D* are right angles. If great circles on a sphere intersect each other at right angles, they bisect each other, and pass through each other's poles. Therefore *ABF* and *DEF* are quadrants. Since *AB* is given, *BF*, the remainder of the quadrant, is also given, and angle *EBF* is equal to its vertical angle *ABC*, which was given. But, according to the preceding Theorem, the ratio of the chord subtending twice *BF* to the chord subtending twice *EF* is equal to the ratio of the diameter of the sphere to the chord subtending twice the angle *EBF*. But three of these are given: the diameter of the sphere, twice *BF*, and twice the angle *EBF*, or their halves. Therefore, according to Euclid, VI, 15, half of the chord subtending twice *EF* is also given. By the Table, the arc *EF* is given. So is *DE*, the remainder of the quadrant, or the required angle *C*.

In the same way, in turn, for the chords subtending twice the arcs, *DE* is to *AB* as *EBC* is to *CB*. But three are already given: *DE*, *AB*, and *CBE* as a quadrant. Therefore the fourth, the chord subtending twice *CB*, is also given, and so is the required side *CB*. And for the chords subtending twice the arcs, *CB* is to *CA* as *BF* is to *EF*. For, both of these ratios are equal to the ratio of the diameter of the sphere to the chord subtending twice the angle *CBA*; and ratios equal to the same ratio are equal to each other. Therefore, since the three members *BF*, *EF*, and *CB* are given, the fourth member *CA* is given, and *CA* is the third side of the triangle *ABC*.

Now, let *AC* be the side assumed as given, and let it be required to find sides *AB* and *BC* as well as the remaining angle *C*. Again, if we invert the argument, the ratio of the chord subtending twice *CA* to the chord subtending twice *CB* will be equal to the ratio of the chord subtending twice the angle *ABC* to the diameter. From this, the side *CB* is given, as well as *AD* and *BE* as remainders of the quadrants. Thus we shall again have the ratio of the chord subtending twice *AD* to the chord subtending twice *BE* equal to the ratio of the chord subtending twice *ABF*, and that is the diameter, to the chord subtending twice *BF*. Therefore the arc *BF* is given, and its remainder is the side *AB*. By a process of reasoning similar to the preceding, from the chords subtending twice *BC*, *AB*, and *FBE*, the chord subtending twice *DE*, or the remaining angle *C*, is given.

Furthermore, if *BC* is assumed, once more, as before, *AC* as well as the remainders *AD* and *BE* will be given. From them, through the subtending straight lines and the diameter, as has often been explained, the arc *BF* and the remaining side *AB* are given. Then, according to the previous Theorem, through *BC*, *AB*, and *CBE*, as given, the arc *ED* is obtained, that is to say, the remaining angle C, which we were looking for.

And thus once more in triangle *ABC*, two angles *A* and *B* being given, of which *A* is a right angle, and one of the three sides being given, the third angle is given together with the two remaining sides. Q.E.D.

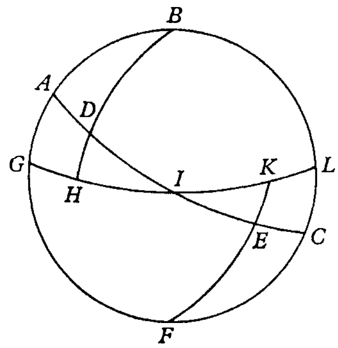
V

If the angles of a triangle are given, one of them being a right angle, the sides are given.

Keep the previous diagram. In it, because angle *C* is given, arc *DE* is given, and so is *EF*, as the remainder of the quadrant. *BEF* is a right angle, because *BE* is drawn from the pole of *DEF*. *EBF* is the vertical angle of a given angle. Therefore triangle *BEF*, having a right angle *E*, and another given angle *B*, and a given side *EF*, has its sides and angles given, in accordance with the preceding Theorem. Therefore *BF* is given, and so is *AB*, the remainder of the quadrant. Likewise in the triangle *ABC*, the remaining sides *AC* and *BC* are shown, by what precedes, to be given.

VI

If on the same sphere two triangles each have a right angle and another corresponding angle and a corresponding side equal, whether that side be adjacent to the equal angles or opposite either of the equal angles, the remaining corresponding sides will also be equal, and so will the remaining angle.



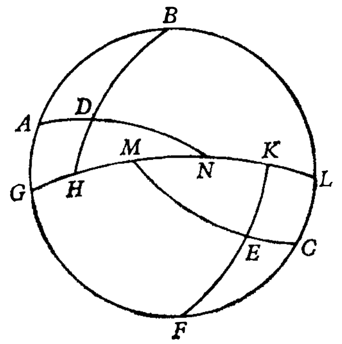
Let there be a hemisphere *ABC*. On it take two triangles *ABD* and *CEF*. Let *A* and *C* be right angles. Furthermore let angle *ADB* be equal to *CEF*, and let one side be equal to one side. First let the equal side be adjacent to the equal angles, that is, let *AD* = *CE*. I say that also side *AB* is equal to side *CF*, *BD* to *EF*, and the remaining angle *ABD* to the remaining angle *CFE*. For with their poles in *B* and *F*, draw *GHI* and *IKL* as quadrants of great circles. Complete *ADI* and *CEI*. These must intersect each other at the hemisphere's pole in the point I, since *A* and *C* are right angles, and *GHI* and *CEI* are drawn through the poles of the circle *ABC*. Therefore, since *AD* and *CE* are assumed to be equal sides, the remaining arcs *DI* and *IE* will be equal, and so will *IDH* and *IEK* as vertical angles of angles assumed equal. *H* and *K* are right angles. Ratios equal to the same ratio are equal to each other. The ratio of the chord subtending twice *ID* to the chord subtending twice *HI* will be equal to the ratio of the chord subtending twice *EI* to the chord subtending twice *IK*. For, each of these ratios, according to Theorem III, above, is equal to the ratio of the diameter of the sphere to the chord subtending twice the angle *IDH*, or the equal chord subtending twice *IEK*. The chord subtending twice the arc *DI* is equal to the chord subtending twice *IE*. Hence, according to Euclid's *Elements*, V, 14, also in the case of twice *IK* and *HI* the chords will be equal. In equal circles, equal straight lines cut off equal arcs, and fractions multiplied by the same factor preserve the same ratio. Therefore, as simple arcs *IH* and *IK* will be equal. So will *GH* and *KL*, the remainders of the quadrants. Hence angles *B* and *F* are clearly equal. Therefore the ratios of the chord subtending twice *AD* to the chord subtending twice *BD*, and of the chord subtending twice *CE* to the chord subtending twice *BD* are equal to the ratio of the chord subtending twice *EC* to the chord subtending twice *EF*. For, both of these ratios are equal to the ratio of the chord subtending twice *HG*, or its equal *KL*, to the chord subtending twice *BDH*, that is, the diameter, according to the converse of Theorem III. *AD* is equal to *CE*. Therefore, according to Euclid's *Elements*, V, 14, *BD* is equal to *EF*, on account of the straight lines subtending twice these arcs.

With *BD* and *EF* equal, I shall prove in the same way that the remaining sides and angles are equal. And if *AB* and *CF* are in turn assumed to be the equal sides, the same conclusions will follow from the equality of the ratios.

VII

The same conclusion will now be proved also if there is no right angle, provided that the side adjacent to the equal angles is equal to the corresponding side.

Thus in the two triangles *ABD* and *CEF*, let any two angles *B* and *D* be equal to the two corresponding angles *E* and *F*. Also let side *BD*, which is adjacent to the equal angles, be equal to side *EF*. I say that again the triangles have their sides and angles equal.



For, once more, with *B* and *F* as poles, draw *GH* and *KL* as arcs of great circles. Let *AD* and *GH*, when extended, intersect each other at *N*, while *EC* and *LK*, when similarly extended, intersect each other at *M*. Then the two triangles *HDN* and *EKM* have angles *HDN* and *KEM* equal, as vertical angles of angles assumed to be equal. *H* and *K* are right angles because they pass through the poles. Moreover, sides *DH* and *EK* are equal. Therefore the triangles have their angles and sides equal, in accordance with the preceding Theorem.

And once again, *GH* and *KL* are equal arcs, since angles *B* and *F* were assumed to be equal. Therefore the whole of *GHN* is equal to the whole of *MKL*, in accordance with the axiom about equals added to equals. Consequently here too the two triangles *AGN* and *MCL* have one side *GN* equal to one side *ML*, angle *ANG* equal to *CML*, and right angles *G* and *L*. For this reason these triangles also will have their sides and angles equal. When equals are subtracted from equals, the remainders will be equal, *AD* to *CE*, *AB* to *CF*, and angle *BAD* to the remaining angle *ECF*. Q.E.D.

VIII

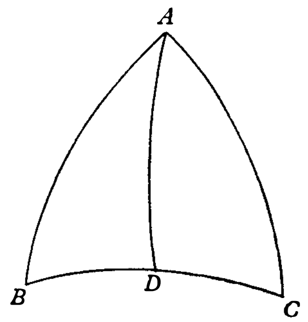
Furthermore, if two triangles have two sides equal to the two corresponding sides, as well as an angle equal to an angle, whether it be the angle included by the equal sides, or an angle at the base, the base will also be equal to the base, and the remaining angles to the remaining angles.

As in the preceding diagram, let side *AB* be equal to side *CF*, and *AD* to *CE*. First, let angle *A*, included by the equal sides, be equal to angle *C*. I say that also the base *BD* is equal to the base *EF*, angle *B* to *F*, and the remaining angle *BDA* to the remaining angle *CEF*. For we shall have two triangles, *AGN* and *CLM*, in which *G* and *L* are right angles; *GAN* and *MCL* are equal as supplementary angles of *BAD* and *ECF*, which are equal; and *GA* is equal to *LC*. Therefore the triangles have their corresponding angles and sides equal. Hence, *AD* and *CE* being equal, the remainders *DN* and *ME* are also equal. But it has already been shown that angle *DNH* is equal to angle *EMK*. *H* and *K* being right angles, the two triangles *DHN* and *EMK* also will have their corresponding angles and sides equal. Hence, as remainders *BD* will also be equal to *EF*. and *GH* to *KL*. Their angles *B* and *F* are equal, and so are the remaining angles *ADB* and *FEC*.

But instead of the sides *AD* and *EC*, let the bases *BD* and *EF* be assumed to be equal. With these bases opposite equal angles, but everything else remaining as before, the proof will proceed in the same way. For, *GAN* and *MCL* are equal, as supplements of equal angles. *G* and *L* are right angles. *AG* is equal to *CL*. Hence, in the same way as before, we shall have two triangles *AGN* and *MCL* with their corresponding angles and sides equal. The same is true also for their sub-triangles, *DHN* and *MEK*. For, *H* and *K* are right angles; *DNH* is equal to *KME*; *DH* and *EK* are equal sides, as remainders of quadrants. From these equalities the same conclusions follow as those which I enunciated.

IX

On a sphere too, the angles at the base of an isosceles triangle are equal to each other.



Let *ABC* be a triangle with *AB* and *AC*, two of its sides, equal. I say that *ABC* and *ACB*, the angles at the base, are also equal. From the vertex *A*, draw a great circle intersecting the base at right angles, that is, passing through the poles [of the base]. Let the great circle be *AD*. In the two triangles *ABD* and *ADC*, then, side *BA* is equal to side *AC*; *AD* is common to both triangles; and the angles at *D* are right angles. It is therefore clear that, in accordance with the preceding Theorem, angles *ABC* and *ACB* are equal. Q.E.D.

COROLLARY

Accordingly it follows that the arc drawn through the vertex of an isosceles triangle at right angles to the base will bisect the base and, at the same time, the angle included by the equal sides, and conversely, as is clear from this Theorem and the preceding one.

X

Any two triangles having their corresponding sides equal will also have their corresponding angles equal, each to each.

For in both cases the three segments of great circles form pyramids, whose vertices are at the center of the sphere. But their bases are the plane triangles bounded by the straight lines subtending the arcs of the convex triangles. These pyramids are similar and equal, according to the definition of equal and similar solid figures. When two figures are similar, however, the rule is that, taken in any order, their corresponding angles are equal. Therefore these triangles will have their corresponding angles equal. In particular, those who define similar figures more generally want them to be whatever figures have similar configurations in which their corresponding angles are equal. From these considerations it is clear, I think, that on a sphere triangles having their corresponding sides equal are similar, as in the case of plane triangles.

XI

Every triangle having two sides and an angle given becomes a triangle of given angles and sides.

For if the given sides are equal, the angles at the base will be equal. Drawing an arc from the vertex at right angles to the base will readily make clear what is required, in accordance with the Corollary of Theorem IX.

But the given sides may be unequal, as in triangle *ABC*. Let its angle *A* be given, together with two sides. These either include or do not include the given angle.

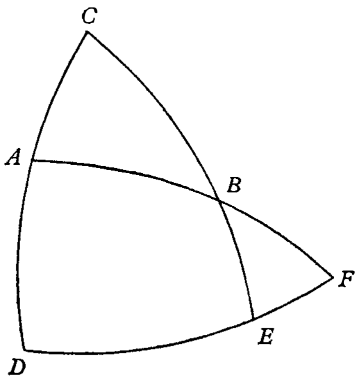
First, let it be included by the given sides, *AB* and *AC*. With *C* as pole, draw *DEF* as the arc of a great circle. Complete the quadrants *CAD* and *CBE*. Produce

AB to intersect *DE* at point *F*. Thus also in the triangle *ADF*, the side *AD* is given as the remainder when *AC* is subtracted from the quadrant. Moreover, angle *BAD* is given as the remainder when *CAB* is subtracted from two right angles. For, the ratio of the angles and their sizes are the same as those which result from the intersection of straight lines and planes. *D* is a right angle. Therefore, in accordance with Theorem IV, *ADF* will be a triangle of given angles and sides. Again, in triangle *BEF*, angle *F* has been found; *E* is right, because its sides pass through the poles; and side *BF* is also known as the quantity by which the whole of *ABF* exceeds *AB*. In accordance with the same theorem, therefore, *BEF* also will be a triangle of given angles and sides. Hence, through *BE*, *BC* is given as the remainder of the quadrant and a required side. Through *EF*, the remainder of the whole of *DEF* is given as *DE*, and this is the angle *C*. Through the angle *EBF*, its vertical angle *ABC* is given, and this was required.

But if, instead of *AB*, *CB*, the side opposite the given angle, is assumed, the same result will follow. For, *AD* and *BE* are given as the remainders of the quadrants. By the same argument the two triangles *ADF* and *BEF*, as before, have their angles and sides given. From them, the sides and angles of the subject triangle *ABC* are given, as was proposed.

XII

Furthermore, if any two angles and a side are given, the same results will follow.



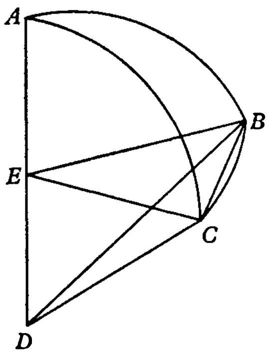
For, keeping the construction in the preceding diagram, in triangle *ABC* let the two angles *ACB* and *BAC* be given, as well as the side *AC*, which is adjacent to both angles. If, in addition, either of the given angles were a right angle, everything else could be deduced by reasoning in accordance with Theorem IV, above. However, I want this to be a different case, in which neither of the given angles is a right angle. Then *AD* will be the remainder of the quadrant *CAD*; angle *BAD* is the remainder when *BAC* is subtracted from two right angles; and *D* is a right angle. Therefore the angles and sides of triangle *AFD* are given, in accordance with Theorem IV, above. But since angle *C* is given, the arc *DE* is given, and so is the remainder *EF*. *BEF* is a right angle, and *F* is an angle common to both triangles. In the same way, in accordance with Theorem IV, above, *BE* and *FB* are given, and from them the required remaining sides *AB* and *BC* will be known.

On the other hand, one of the given angles may be opposite the given side. For example, if angle *ABC* is given instead of *ACB*, while everything else remains unchanged, the same proof as before will make known the whole of *ADF* as a triangle of given angles and sides. The same is true for the sub-triangle *BEF*. For, angle *F* is common to both; *EBF* is the vertical angle of a given angle; and *E* is a right angle. Therefore, as is proved above, all its sides are also given. From them, finally, the same conclusions follow as those which I enunciated. For, all these properties are always interconnected by an invariant mutual relationship, as befits the form of a sphere.

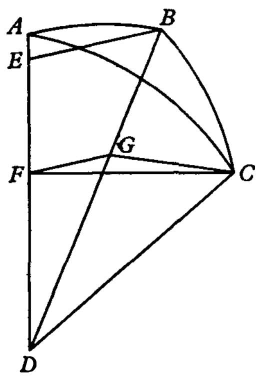
XIII

Finally, if all the sides of a triangle are given, the angles are given.

Let all the sides of triangle *ABC* be given. I say that all the angles also are



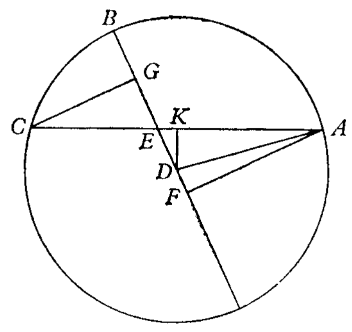
found. For, the triangle will have sides which are either equal or not equal. Then, first, let *AB* and *AC* be equal. Obviously, the halves of the chords subtending twice *AB* and *AC* will also be equal. Let these half-chords be *BE* and *CE*. They will intersect each other in the point *E*, because they are equidistant from the center of the sphere on *DE*, the intersection of their circles. This is clear from Euclid, III, Definition 4, and its converse. But according to Euclid, III, 3, *DEB* is a right angle in plane *ABD*, and so is *DEC* in plane *ACD*. Therefore *BEC* is the angle of inclination of those planes, according to Euclid, XI, Definition 4. We shall find angle *BEC* in the following way. For, it will be subtended by the straight line *BC*. Then we shall have the rectilinear triangle *BEC*. Its sides will be given through their arcs, which are given. Also the angles of *BEC* will be given, and we shall have the required angle *BEC*, that is, the spherical angle *BAC*, and the remaining angles, through what precedes.



But the triangle may be scalene, as in the second diagram. Obviously, the halves of the chords subtending twice the sides will not intersect one another. For let arc *AC* be greater than *AB*, and let *CF* be half of the chord subtending twice *AC*. Then *CF* will pass below. But if the arc is smaller, the half-chord will be higher, according as these lines happen to be nearer to or farther away from the center, in accordance with Euclid, III, 15. Then let *FG* be drawn parallel to *BE*. Let *FG* intersect *BD*, the intersection of the circles, in the point *G*. Join *CG*. Clearly, then, *EFG* is a right angle, being of course equal to *AEB*, and *EFC* is also a right angle, since *CF* is half of the chord subtending twice *AC*. Then *CFG* will be the angle of intersection of the circles *AB* and *AC*. Therefore we obtain *CFG* also. For *DF* is to *FG* as *DE* is to *EB*, since *DFG* and *DEB* are similar triangles. Hence *FG* is given in the same units as those in which *FC* is also given. But the same ratio holds also for *DG* to *DB*. *DG* also will be given in units whereof *DC* is 100,000. What is more, angle *GDC* is given through arc *BC*. Therefore, in accordance with Theorem II on Plane Triangles, side *GC* is given in the same units as the remaining sides of the plane triangle *GFC*. Consequently, in accordance with the last Theorem on Plane Triangles, we shall have angle *GFC*, that is, the required spherical angle *BAC*, and then we shall obtain the remaining angles, in accordance with Theorem XI on Spherical Triangles.

XIV

If a given arc of a circle is divided anywhere so that the sum of both segments is less than a semicircle, the ratio of half the chord subtending twice one segment to half the chord subtending twice the other segment being given, the arcs of the segments will also be given.

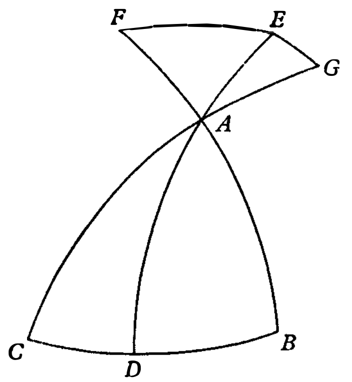


For let arc *ABC* be given, about *D* as center. Let *ABC* be divided at random in the point *B*, yet in such a way that the segments are less than a semicircle. Let the ratio of half the chord subtending twice *AB* to half of the chord subtending twice *BC* be given in some unit of length. I say that the arcs *AB* and *BC* are also given.

For, draw the straight line *AC*, which will be intersected by the diameter at the point *E*. Now from the end-points *A* and *C*, drop perpendiculars to the diameter. Let these perpendiculars be *AF* and *CG*, which must be halves of the chords subtending twice *AB* and *BC*. Then in the right triangles *AEF* and *CEG*, the vertical angles at *E* are equal. Therefore the triangles have their corresponding angles equal. Being similar triangles, they have their sides opposite the equal angles proportional: as *AF* is to *CG*, *AE* is to *EC*. Hence we shall have *AE* and *EC* in the same units as those in which *AF* or *GC* was given. From *AE* and *EC*, the whole of *AEC* will be given in the same units. But *AEC*, as the chord subtending the arc *ABC*, is given in those units in which the radius *DEB* is given. In the same units, *AK*, as half of *AC*, and the remainder *EK*, are also given. Join *DA* and *DK*, which will also be given in the same units as *DB*. For, *DK* is half of the chord subtending the segment remaining when *ABC* is subtracted from a semicircle. This remaining segment is included within angle *DAK*. Therefore angle *ADK* is given as including half of the arc *ABC*. But in the triangle *EDK*, since two sides are given, and *EKD* is a right angle, *EDK* will also be given. Hence the whole angle *EDA* will be given. It includes the arc *AB*, from which the remainder *CB* will also be obtained. This is what we wanted to prove.

XV

If all the angles of a triangle are given, even though none of them is a right angle, all the sides are given.



Let there be the triangle *ABC*, with all of its angles given, but none of them a right angle. I say that all of its sides are also given. For, from any of the angles, for instance *A*, through the poles of *BC* draw the arc *AD*. This will intersect *BC* at right angles. *AD* will fall inside the triangle, unless one of the angles *B* or *C* at the base is obtuse, and the other acute. Should this be the case, the perpendicular would have to be drawn from the obtuse angle to the base. Complete the quadrants *BAF*, *CAG*, and *DAE*. Draw the arcs *EF* and *EG* with their poles in B and C. Therefore *F* and *G* will also be right angles. Then in the right triangles, the ratio of half the chord subtending twice *AE* to half the chord subtending twice *EF* will be equal to the ratio of half the diameter of the sphere to half the chord subtending double the angle *EAF;* similarly in triangle *AEG*, with its right angle at *G*, the ratio of half the chord subtending twice *AE* to half of the chord subtending twice *EG* is equal to the ratio of half the diameter of the sphere to half of the chord subtending double the angle *EAG*. Then since these ratios are equal, the ratio of half the chord subtending twice *EF* to half of the chord subtending twice *EG* will be equal to the ratio of half the chord subtending double the angle *EAF* to half of the chord subtending double the angle *EAG*. *FE* and *EG* are given arcs, being the remainders when angles *B* and *C* are subtracted from right angles. From *FE* and *EG*, then, we shall obtain the ratio of angles *EAF* and *EAG*, that is, of their vertical angles, *BAD* and *CAD*. But the whole of *BAC* is given. Therefore, in accordance with the preceding Theorem, angles *BAD* and *CAD* will also be given. Then, in accordance with Theorem V, we shall obtain sides *AB*, *BD*, *AC*, *CD*, and the whole of *BC*.

For the present let this digression suffice for triangles, so far as they are necessary for our purpose. If they had to be discussed more fully, a special volume would have been required.

Book Two

INTRODUCTION

I have given a general account of the earth's three motions, by which I promised to explain all the phenomena of the heavenly bodies [I, 11]. I shall do so next, to the best of my ability, by analyzing and investigating them, one by one. I shall begin, however, with the most familiar revolution of all, the period of a day and night. This, as I said [I, 4], is called *nuchthemeron* by the Greeks. I have taken it as belonging particularly and directly to the earth's globe, since the month, year, and other intervals of time bearing many names proceed from this rotation, as number does from unity, time being the measure of motion. Hence with regard to the inequality of days and nights, the rising and setting of the sun and of the degrees of the zodiac and its signs, and that sort of consequence of this rotation, I shall make some few remarks, especially because many have written about these topics quite fully, yet in harmony and agreement with my views. It makes no difference that they base their explanations on a motionless earth and rotating universe, while I take the opposite position and accompany them to the same goal. For, mutually interrelated phenomena, it so happens, show a reversible agreement. Yet I shall omit nothing essential. But let nobody be surprised if I still refer simply to the rising and setting of the sun and stars, and similar phenomena. On the contrary, it will be recognized that I use the customary terminology, which can be accepted by everybody. Yet I always bear in mind that

For us who are borne by the earth, the sun and the moon pass by,
And the stars return on their rounds, and again they drop out of sight.

THE CIRCLES AND THEIR NAMES *Chapter 1*

The equator, as I said [I, 11], is the largest of the parallels of latitude described around the poles of the daily rotation of the earth's globe. The ecliptic, on the other hand, is a circle passing through the middle of the signs of the zodiac, and below the ecliptic the center of the earth circles in an annual revolution. But the ecliptic meets the equator obliquely, in agreement with the inclination of the earth's axis to the ecliptic. Hence, as a result of the earth's daily rotation, on either side of the equator a circle is described tangent to the ecliptic as the outermost limit of its obliquity. These two circles are called the "tropics", because in them seem to occur the sun's tropes or reversals in direction, that is to say, in winter and summer. Hence the northern one is usually called the "summer solstice", and the other one in the south, the "winter solstice", as was explained above in the general account of the earth's revolutions [I, 11].

Next comes the "horizon", as it is called, which the Romans term the "boundary", since it separates the part of the universe visible to us from the part which is hidden. [All the bodies that rise] seem to rise at the horizon, [and] all the bodies that set [seem to set at the horizon]. It has its center on the surface of the earth, and its pole at our zenith. But the earth is incommensurable with the immensity of the heavens. Even the entire space intervening, according to my conception, between the sun and the moon cannot be classed with the vastness of the heavens. Hence the horizon seems to bisect the heavens like a circle passing through the center of the universe, as I showed earlier [I, 6]. But the horizon meets the equator obliquely. Hence the horizon too is tangent, on either side of the equator, to a pair of parallels of latitude: in the north, [the circle limiting the stars which are] always visible, and in the south, those which are always hidden. The former is called the "arctic", the latter the "antarctic", by Proclus and most of the Greeks. The arctic and antarctic circles become larger or smaller in proportion to the obliquity of the horizon or the altitude of the pole of the equator.

There remains the meridian, which passes through the poles of the horizon and also through the poles of the equator. Therefore the meridian is perpendicular to both of these circles. When the sun reaches the meridian, it indicates noon and midnight. But these two circles, I mean the horizon and the meridian, which have their centers on the surface of the earth, depend absolutely on the motion of the earth and our sight, wherever it may be. For everywhere the eye acts as the center of the sphere of all the bodies visible in every direction around it. Therefore, as is clearly proved by Eratosthenes, Posidonius, and the other writers on cosmography and the earth's size, all the circles assumed on the earth are also the basis of their counterparts in the heavens and of similar circles. These too are circles having special names, while others may be designated in countless ways.

THE OBLIQUITY OF THE ECLIPTIC, THE DISTANCE BETWEEN THE TROPICS, AND THE METHOD OF DETERMINING THESE QUANTITIES *Chapter 2*

The ecliptic, then, crosses obliquely between the tropics and the equator. Hence it is now necessary, I believe, to investigate the distance between the tropics and, in that connection, the size of the angle at which the equator and ecliptic intersect each other. This [information] must of course be perceived by the senses and with the aid of instruments by which this very valuable result is obtained. Hence make a square out of wood, or preferably out of some more rigid material, [such as] stone or metal, lest perhaps the wood, yielding to a shift in the air, be able to mislead the observer. Let a surface of the square be perfectly smooth, and long enough for the subdivisions which have to be made, so that it would be five or six feet. For in proportion to its size, and with one of the corners as center, a quadrant of a circle is drawn. It is divided into 90 equal degrees. These are subdivided in like manner into 60 minutes, or whatever subdivisions the degrees can accommodate. Then a precisely lathed cylindrical pin is attached to the center. Placed perpendicular to the surface, the pin protrudes a little, perhaps as much as a finger's breadth or less.

After this instrument has been constructed in this way, it is useful to trace the meridian on a floor laid in the horizontal plane and leveled as carefully as possible by means of a hydroscope or water level, lest it sag in any direction. Now on this floor draw a circle, and at its center erect a pointer. Observing where its shadow falls on the circumference of the circle at any time before noon, we shall mark that point. We shall make a similar observation in the afternoon, and bisect the arc of the circle lying between the two points already marked. By this method a straight line drawn from the center through the point of bisection will certainly indicate south and north for us without any error.

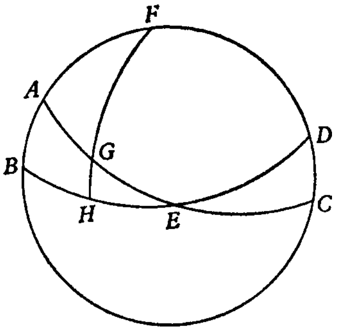
Then on this line as its base, the instrument's plane surface is erected and attached perpendicularly, with its center turned southward. A plumb line dropped from the center meets the meridian line at right angles. The result of this procedure is of course that the surface of the instrument contains the meridian.

Thereafter, on the days of the summer and winter solstices, the sun's shadow at noon must be observed as it is cast at the center by that pin or cylinder. Anything may be used on the aforesaid arc of the quadrant to fix the place of the shadow with greater certainty. We shall note the midpoint of the shadow as accurately as possible in degrees and minutes. For if we do this, the arc found marked off between the two shadows, summer and winter, will show us the distance between the tropics and the entire obliquity of the ecliptic. By taking half of this, we shall have the distance of the tropics from the equator, while the size of the angle of inclination of the equator to the ecliptic will become clear.

Now this interval between the aforementioned limits, north and south, is determined by Ptolemy, in degrees whereof the circle is 360°, as 47° 42′ 40″ [*Syntaxis*, I, 12]. He also finds that before his time the observations of Hipparchus and Eratosthenes were in agreement. This determination is equivalent to 11 units, whereof the entire circle is 83. Half of this interval, which is 23° 51′ 20″, established the distance of the tropics from the equator, in degrees whereof the circle is 360°, and the angle of intersection with the ecliptic. Therefore Ptolemy thought that this was constant, and would always remain so. But from that time these values are found to have decreased continuously down to our own time. For, certain of our contemporaries and I have now discovered that the distance between the tropics is not more than approximately 46° 58′, and the angle of intersection not more than 23° 29′. Hence it is now quite clear that the obliquity of the ecliptic also is variable. I shall say more about this subject below [III, 10], where I shall also show by a quite probable conjecture that the obliquity never was more than 23° 52′, and never will be less than 23° 28′.

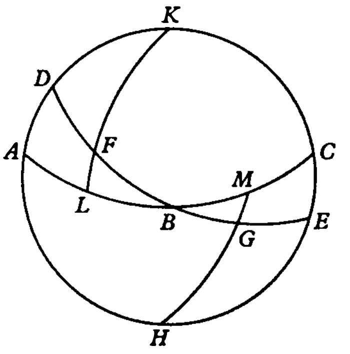
THE ARCS AND ANGLES OF THE INTERSECTIONS OF THE EQUATOR, ECLIPTIC, AND MERIDIAN; THE DERIVATION OF THE DECLINATION AND RIGHT ASCENSION FROM THESE ARCS AND ANGLES, AND THE COMPUTATION OF THEM — *Chapter 3*

Just as I said [II, 1] that the parts of the universe rise and set at the horizon, so I [now] say that the heavens are bisected at the meridian. This also traverses both the ecliptic and the equator in a period of 24 hours. It divides them, by cutting off arcs starting from their vernal or autumnal intersection. It in turn is divided by their interception of an arc [of the meridian]. Since they are all great circles, they form a spherical triangle. This is a right triangle, because there is a right angle where the meridian crosses the equator, through whose poles [the meridian passes,] by definition. The arc of the meridian in this triangle, or an arc so intercepted on any circle passing through the poles of the equator, is called the "declination" of the segment of the ecliptic. But the corresponding arc of the equator, which rises together with its associated arc on the ecliptic, is called the "right ascension".

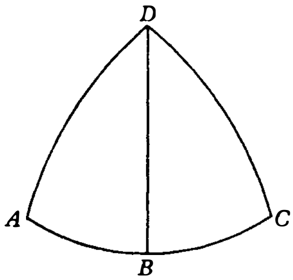


All of this is easily shown in a convex triangle. For, let *ABCD* be the circle, generally called the "colure", which passes through the poles of both the equator and the ecliptic. Let half of the ecliptic be *AEC*; half of the equator, *BED*; the vernal equinox, *E*; the summer solstice, *A*; and the winter solstice, *C*. Assume that *F* is the pole of the daily rotation, and that on the ecliptic *EG* is an arc of, say, 30°. Through its end, draw the quadrant *FGH*. Then in the triangle *EGH*, obviously side *EG* is given as 30°. Angle *GEH* is also given; at its minimum, in degrees whereof 360° = 4 right angles, it will be 23° 28′, in agreement with the minimum declination *AB*. *GHE* is a right angle. Therefore, in accordance with Theorem IV on Spherical Triangles, *EGH* will be a triangle of given angles and sides. The ratio of the chord subtending twice *EG* to the chord subtending twice *GH*, as has of course been shown [Theorem III on Spherical Triangles], is equal to the ratio of the chord subtending twice *AGE*, or of the diameter of the sphere, to the chord subtending twice *AB*. Their half-chords are similarly related. Half of the chord subtending twice *AGE* is 100,000 as a radius; in the same units, the halves of the chords subtending twice *AB* and *EG* are 39,822 and 50,000. If four numbers are proportional, the product of the means is equal to the product of the extremes. Hence we shall have half of the chord subtending twice the arc *GH* as 19,911 units. This half-chord in the Table gives the arc *GH* as 11° 29′, the declination corresponding to the segment *EG*. Therefore in the triangle *AFG* too, sides *FG* and *AG*, as remainders of quadrants, are given as 78° 31′ and 60°, and *FAG* is a right angle. In the same way, the chords subtending twice *FG*, *AG*, *FGH*, and *BH*, or their half-chords will be proportional. Now, since three of these are given, the fourth, *BH*, will also be given as 62° 6′. This is the right ascension as taken from the summer solstice, or from the vernal equinox it will be *HE*, of 27° 54′. Similarly from the given sides *FG* of 78° 31′, *AF* of 66° 32′, and a quadrant, we shall have angle *AGF* of approximately 69° 23 1/2′. Its vertical angle *HGE* is equal. We shall follow this example in all the other cases too.

However, we must not disregard the fact that, at the points where the ecliptic is tangent to the tropics, the meridian intersects the ecliptic at right angles, since at those times the meridian passes through the poles of the ecliptic, as I said. But at the equinoctial points the meridian makes an angle which is as much smaller than a right angle as the ecliptic deviates from a right angle [in intersecting with the equator], so that now [the angle between the meridian and the ecliptic] is 66° 32′. It should also be noticed that equal arcs of the ecliptic, as measured from the equinoctial or solstitial points, are accompanied by equal angles and sides of the triangles. Thus let us draw *ABC* as an arc of the equator, and the ecliptic *DBE*, intersecting each other in *B*. Let this be an equinoctial point. Let us take *FB* and *BG* as equal arcs. Through *K* and *H*, the poles of the daily rotation, draw two quadrants, *KFL* and *HGM*. Then there will be two triangles, *FLB* and *BMG*. Their sides *BF* and *BG* are equal; at *B* there are vertical angles; and at *L* and *M*, right angles. Therefore, in accordance with Theorem VI on Spherical Triangles,



the sides and angles of these triangles are equal. Thus the declinations *FL* and *MG*, as well as the right ascensions *LB* and *BM*, are equal, and the remaining angle *F* is equal to the remaining angle *G*.



In the same way, the situation will be clear when the equal arcs are measured from a solstitial point. Thus let *AB* and *BC* be equal arcs to either side of *B*, where the tropic is tangent to [the ecliptic]. For, draw the quadrants *DA* and *DC* from *D*, the pole of the equator, [and join *DB*]. In like manner there will be two triangles, *ABD* and *DBC*. Their bases *AB* and *BC* are equal; *BD* is a side common to both; and there are right angles at *B*. In accordance with Theorem VIII on Spherical Triangles, these triangles will be shown to have their sides and angles equal. Hence it becomes clear that when these angles and arcs are tabulated for a single quadrant on the ecliptic, they will fit the remaining quadrants of the entire circle.

I shall adduce an example of these relationships in the following description of the Tables. In the first column will be entered the degrees of the ecliptic; in the next place, the declinations corresponding to those degrees; and in the third place, the minutes by which the declinations occurring at the maximum obliquity of the ecliptic differ from, and exceed, these partial declinations; the greatest of these differences is 24′. I shall proceed in the same way in the Tables of [Right Ascensions and Meridian] Angles. For when the obliquity of the ecliptic varies, everything which accompanies it must vary. But in right ascension the variation is found to be extremely small, since it does not exceed $^1/_{10}$ of a "time", and in the course of an hour amounts to only $^1/_{150}$ [thereof]. For, the ancients use the term "times" for the degrees of the equator which rise together with the degrees of the ecliptic. Both of these circles have 360 units, as I have often said [e. g., I, 12]. In order to distinguish between them, however, many have called the ecliptic's units "degrees", but the equator's "times", a nomenclature which I too will follow hereafter. [As I was saying] although this variation is so tiny that it can properly be neglected, I did not mind adding it too. From these variations, then, the same results will be clear in any other obliquity of the ecliptic if, in proportion to the excess of the ecliptic's maximum obliquity over the minimum, to each entry the corresponding fractions are applied. Thus, for example, with the obliquity at 23° 34′, if I wish to know how great a declination belongs to 30° of the ecliptic measured from the equinox, in the Table I find 11° 29′, and under the differences 11′, which would be added as a block when the obliquity of the ecliptic is at its maximum. This was, as I said, 23° 52′. But in the present instance it is assumed to be 23° 34′, which is greater than the minimum by 6′. These 6′ are one-fourth of the 24′ by which the maximum obliquity exceeds [the minimum]. The fraction of 11′ in a similar ratio is about 3′. When I add these 3′ to 11° 29′, I shall have 11° 32′ as the declination at that time of 30° of the ecliptic as measured from the equinoctial point. In [meridian] angles and right ascensions we may proceed in the same way, except that in the latter case we must always add the differences, and in the former case always subtract them, in order to have everything come out more accurate in relation to time.

TABLE OF DECLINATIONS [OF THE DEGREES OF THE ECLIPTIC]

Ecliptic	Declination		Difference	Ecliptic	Declination		Difference	Ecliptic	Declination		Difference
Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute
1	0	24	0	31	11	50	11	61	20	23	20
2	0	48	1	32	12	11	12	62	20	35	21
3	1	12	1	33	12	32	12	63	20	47	21
4	1	36	2	34	12	52	13	64	20	58	21
5	2	0	2	35	13	12	13	65	21	9	21
6	2	23	2	36	13	32	14	66	21	20	22
7	2	47	3	37	13	52	14	67	21	30	22
8	3	11	3	38	14	12	14	68	21	40	22
9	3	35	4	39	14	31	14	69	21	49	22
10	3	58	4	40	14	50	14	70	21	58	22
11	4	22	4	41	15	9	15	71	22	7	22
12	4	45	4	42	15	27	15	72	22	15	23
13	5	9	5	43	15	46	16	73	22	23	23
14	5	32	5	44	16	4	16	74	22	30	23
15	5	55	5	45	16	22	16	75	22	37	23
16	6	19	6	46	16	39	17	76	22	44	23
17	6	41	6	47	16	56	17	77	22	50	23
18	7	4	7	48	17	13	17	78	22	55	23
19	7	27	7	49	17	30	18	79	23	1	24
20	7	49	8	50	17	46	18	80	23	5	24
21	8	12	8	51	18	1	18	81	23	10	24
22	8	34	8	52	18	17	18	82	23	13	24
23	8	57	9	53	18	32	19	83	23	17	24
24	9	19	9	54	18	47	19	84	23	20	24
25	9	41	9	55	19	2	19	85	23	22	24
26	10	3	10	56	19	16	19	86	23	24	24
27	10	25	10	57	19	30	20	87	23	26	24
28	10	46	10	58	19	44	20	88	23	27	24
29	11	8	10	59	19	57	20	89	23	28	24
30	11	29	11	60	20	10	20	90	23	28	24

TABLE OF RIGHT ASCENSIONS

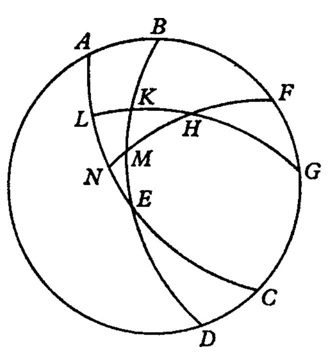
Ecliptic	Equator		Difference	Ecliptic	Equator		Difference	Ecliptic	Equator		Difference
Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute
1	0	55	0	31	28	54	4	61	58	51	4
2	1	50	0	32	29	51	4	62	59	54	4
3	2	45	0	33	30	50	4	63	60	57	4
4	3	40	0	34	31	46	4	64	62	0	4
5	4	35	0	35	32	45	4	65	63	3	4
6	5	30	0	36	33	43	5	66	64	6	3
7	6	25	1	37	34	41	5	67	65	9	3
8	7	20	1	38	35	40	5	68	66	13	3
9	8	15	1	39	36	38	5	69	67	17	3
10	9	11	1	40	37	37	5	70	68	21	3
11	10	6	1	41	38	36	5	71	69	25	3
12	11	0	2	42	39	35	5	72	70	29	3
13	11	57	2	43	40	34	5	73	71	33	3
14	12	52	2	44	41	33	6	74	72	38	2
15	13	48	2	45	42	32	6	75	73	43	2
16	14	43	2	46	43	31	6	76	74	47	2
17	15	39	2	47	44	32	5	77	75	52	2
18	16	34	3	48	45	32	5	78	76	57	2
19	17	31	3	49	46	32	5	79	78	2	2
20	18	27	3	50	47	33	5	80	79	7	2
21	19	23	3	51	48	34	5	81	80	12	1
22	20	19	3	52	49	35	5	82	81	17	1
23	21	15	3	53	50	36	5	83	82	22	1
24	22	10	4	54	51	37	5	84	83	27	1
25	23	9	4	55	52	38	4	85	84	33	1
26	24	6	4	56	53	41	4	86	85	38	0
27	25	3	4	57	54	43	4	87	86	43	0
28	26	0	4	58	55	45	4	88	87	48	0
29	26	57	4	59	56	46	4	89	88	54	0
30	27	54	4	60	57	48	4	90	90	0	0

TABLE OF MERIDIAN ANGLES

Ecliptic	Angle		Difference	Ecliptic	Angle		Difference	Ecliptic	Angle		Difference
Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute	Degree	Degree	Minute	Minute
1	66	32	24	31	69	35	21	61	78	7	12
2	66	33	24	32	69	48	21	62	78	29	12
3	66	34	24	33	70	0	20	63	78	51	11
4	66	35	24	34	70	13	20	64	79	14	11
5	66	37	24	35	70	26	20	65	79	36	11
6	66	39	24	36	70	39	20	66	79	59	10
7	66	42	24	37	70	53	20	67	80	22	10
8	66	44	24	38	71	7	19	68	80	45	10
9	66	47	24	39	71	22	19	69	81	9	9
10	66	51	24	40	71	36	19	70	81	33	9
11	66	55	24	41	71	52	19	71	81	58	8
12	66	59	24	42	72	8	18	72	82	22	8
13	67	4	23	43	72	24	18	73	82	46	7
14	67	10	23	44	72	39	18	74	83	11	7
15	67	15	23	45	72	55	17	75	83	35	6
16	67	21	23	46	73	11	17	76	84	0	6
17	67	27	23	47	73	28	17	77	84	25	6
18	67	34	23	48	73	47	17	78	84	50	5
19	67	41	23	49	74	6	16	79	85	15	5
20	67	49	23	50	74	24	16	80	85	40	4
21	67	56	23	51	74	42	16	81	86	5	4
22	68	4	22	52	75	1	15	82	86	30	3
23	68	13	22	53	75	21	15	83	86	55	3
24	68	22	22	54	75	40	15	84	87	19	3
25	68	32	22	55	76	1	14	85	87	53	2
26	68	41	22	56	76	21	14	86	88	17	2
27	68	51	22	57	76	42	14	87	88	41	1
28	69	2	21	58	77	3	13	88	89	6	1
29	69	13	21	59	77	24	13	89	89	33	0
30	69	24	21	60	77	45	13	90	90	0	0

FOR EVERY HEAVENLY BODY SITUATED OUTSIDE THE ECLIPTIC, PROVIDED THAT THE BODY'S LATITUDE AND LONGITUDE ARE KNOWN, THE METHOD OF DETERMINING ITS DECLINATION, ITS RIGHT ASCENSION, AND THE DEGREE OF THE ECLIPTIC WITH WHICH IT REACHES MID-HEAVEN *Chapter 4*

The foregoing explanations concerned the ecliptic, equator, [meridian], and their intersections. In connection with the daily rotation, however, it is important to know not only those appearances in the ecliptic which reveal the causes of the phenomena of the sun alone. It is important to know also that a similar procedure will show the declination from the equator and the right ascension of those fixed stars and planets which are outside the ecliptic, provided, however, that their longitude and latitude are given.



Accordingly, draw the circle *ABCD* through the poles of the equator and ecliptic. Let *AEC* be a semicircle of the equator with its pole at *F*, and *BED* a semicircle of the ecliptic with its pole at *G*, and its intersection with the equator at point *E*. Now from the pole *G*, draw the arc *GHKL* through a star. Let the place of the star be given as point *H*, through which let the quadrant *FHMN* be drawn from the pole of the daily rotation. Clearly, then, the star at *H* crosses the meridian together with the two points *M* and *N*. The arc *HMN* is the star's declination from the equator, and *EN* is the star's right ascension on the sphere. These are the coordinates which we are looking for.

Now in triangle *KEL*, side *KE* and angle *KEL* are given, and *EKL* is a right angle. Therefore, in accordance with Theorem IV on Spherical Triangles, sides *KL* and *EL* as well as the remaining angle *KLE* are given. Therefore the whole arc *HKL* is given. Consequently in triangle *HLN*, angle *HLN* is given, *LNH* is a right angle, and side *HL* is given. Hence, in accordance with the same Theorem IV on Spherical Triangles, the remaining sides *HN*, the star's declination, and *LN* are given. [When *LN* is subtracted from *EL*], the remainder is *NE*, the right ascension, the arc through which the sphere turns from the equinox to the star.

Alternatively, from the foregoing relationships you may take arc *KE* of the ecliptic as the right ascension of *LE*. Then *LE* in turn will be given by the Table of Right Ascensions. *LK* will be given as the declination corresponding to *LE*. Angle *KLE* will be given by the Table of Meridian Angles. From these quantities, the rest will be determined, as has already been shown. Then, through the right ascension *EN*, we obtain *EM* as the degree of the ecliptic at which the star reaches mid-heaven together with the point *M*.

THE INTERSECTIONS OF THE HORIZON *Chapter 5*

In the right sphere the horizon is a different circle from the horizon in the oblique sphere. For in the right sphere that circle is called the horizon to which the equator is perpendicular, or which passes through the poles of the equator. But in the oblique sphere the equator is inclined to the circle which we call the horizon. Therefore at the horizon in the right sphere all bodies rise and set, and the days are always equal to the nights. For, the horizon bisects all the parallels of latitude described by the daily rotation; it passes through their poles, of course, and under those circumstances the phenomena occur which I have already explained with regard to the meridian [II, 1, 3]. But in this instance we regard the day as extending from sunrise to sunset, and not in some way from daylight to darkness, as it is commonly understood, that is, from dawn to the first artificial light. But I shall say more about this subject in connection with the rising and setting of the zodiacal signs [II, 13].

On the other hand, where the earth's axis is perpendicular to the horizon, nothing rises and sets. On the contrary, everything revolves in a circle, perpetually visible or hidden. The exception is what is produced by another motion, such as the annual revolution around the sun. As a result of this it follows that under those conditions day lasts continuously for a period of six months, and night for the rest of the time. Nor is there any other difference than that between winter and summer, since in that situation the equator coincides with the horizon.

In the oblique sphere, however, certain bodies rise and set, while certain others are always visible or hidden. Meanwhile the days and nights become unequal. Under these circumstances the horizon, being oblique, is tangent to two parallels of latitude, according to the amount of its inclination. Of these two parallels, the one toward the visible pole is the boundary of the bodies which are perpetually visible; and the opposite parallel, the one toward the hidden pole, is the boundary of the bodies which are perpetually hidden. Extending throughout the entire latitude between these limits, therefore, the horizon divides all the intervening parallels of latitude into unequal arcs. The equator is an exception, since it is the greatest of the parallels of latitude, and great circles bisect each other. In the upper hemisphere, then, the horizon obliquely cuts off from the parallels of latitude greater arcs toward the visible pole than toward the southern and hidden pole. The converse is true in the hidden hemisphere. The apparent daily motion of the sun in these arcs produces the inequality of the days and nights.

THE DIFFERENCES IN NOON SHADOWS *Chapter 6*

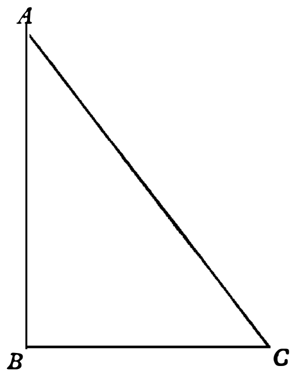
In noon shadows too there are differences, on account of which some people are called periscian, others amphiscian, and still others heteroscian. Now the periscians are the people whom we may label "circumumbratile", since they receive the sun's shadow in all directions. And they are the people whose zenith, or pole of the horizon, is at a distance from the earth's pole which is smaller, or not greater, than the distance of a tropic from the equator. For in those regions the parallels of latitude to which the horizon is tangent are the boundaries of the perpetually visible or hidden stars, and are greater than the tropics, or equal to them. And therefore in the summer time the sun, high up among the perpetually visible stars, in that season casts the shadows of the sundials in all directions. But where the horizon is tangent to the tropics, these themselves become the boundaries of the perpetually visible and perpetually hidden stars. Therefore at the time of the solstice the sun is seen to graze the earth at midnight. At that moment the entire ecliptic coincides with the horizon, six zodiacal signs rise swiftly and simultaneously, the opposite signs in equal number set at the same time, and the pole of the ecliptic coincides with the pole of the horizon.

The amphiscians, whose noon shadows fall on both sides, are the people who live between the two tropics, in the region which the ancients call the middle zone. Throughout that whole area the ecliptic passes directly overhead twice [daily], as is demonstrated in Theorem II of Euclid's *Phenomena*. Hence in the same area the sundials' shadows vanish twice, and as the sun moves to either side, the sundials cast their shadows sometimes to the south, and at other times to the north.

We, the rest of the earth's inhabitants, who live between the amphiscians and the periscians, are the heteroscians, because we cast our noon shadows in only one of these directions, that is, the north.

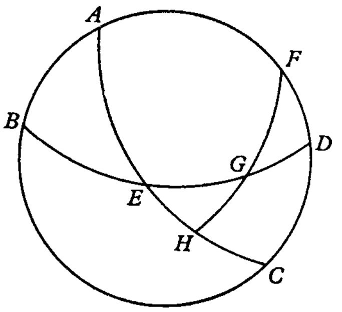
Now the ancient mathematicians used to divide the earth into seven climes by means of the several parallels of latitude passing, for example, through Meroe, Syene, Alexandria, Rhodes, the Hellespont, the middle of the Black Sea, the Dnieper, Constantinople and so on. [These parallels were selected on a threefold basis:] the difference and increase in the length of the longest day [in the specified localities during the course of a year]; the length of the shadows observed by means of sundials at noon on the equinoctial days and the two solstices of the sun; and the altitude of the pole or the width of each clime. These quantities, having partly changed with time, are not exactly the same as they once were. The reason is, as I mentioned [II, 2], the variable obliquity of the ecliptic, which was overlooked by previous astronomers. Or, to speak more precisely, the reason is the variable inclination of the equator to the plane of the ecliptic. Those quantities depend on this inclination. But the altitudes of the pole, or the latitudes of the places, and the shadows on the equinoctial days agree with the recorded ancient observations. This had to happen, because the equator follows the pole of the terrestrial globe. Therefore those climes likewise are not drawn and bounded with sufficient precision by means of any impermanent properties of shadows and days. On the other hand, they are delimited more correctly by their distances from the equator, which remain the same forever. But that variation in the tropics, although it is quite small, in southern localities allows a slight difference of days and shadows, which becomes more perceptible to those who travel north.

Now so far as the shadows of sundials are concerned, then, for any given altitude of the sun obviously the length of the shadow is obtained, and conversely. Thus, let there be a sundial *AB*, which casts a shadow *BC*. Since the pointer is perpendicular to the plane of the horizon, it must always make *ABC* a right angle, in accordance with the definition of lines perpendicular to a plane. Hence, if *AC* is joined, we shall have the right triangle *ABC*, and for a given altitude of the sun, we shall have also angle *ACB* given. In accordance with Theorem I on Plane Triangles, the ratio of the pointer *AB* to its shadow *BC* will be given, and *BC* will be given as a length. In turn, when *AB* and *BC* are given, in accordance with Theorem III on Plane Triangles, angle *ACB* and the altitude of the sun casting that shadow at the time will also be known. In this way, in their description of those climes of the terrestrial globe, the ancients assigned to each clime its own length of noon shadow, not only on the equinoctial days, but also on both solstitial days.



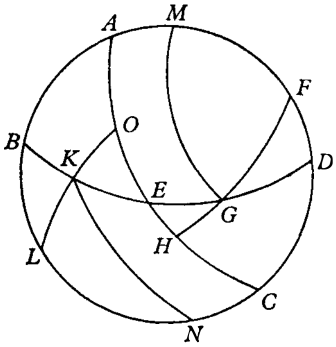
HOW TO DERIVE FROM ONE ANOTHER THE LONGEST DAY, THE DISTANCE BETWEEN SUNRISES, AND THE INCLINATION OF THE SPHERE; THE REMAINING DIFFERENCES BETWEEN DAYS *Chapter 7*

Thus also for any obliquity of the sphere or inclination of the horizon, I shall simultaneously demonstrate the longest and shortest day as well as the distance between sunrises, and the remaining difference between the days. Now the distance between sunrises is the arc of the horizon intercepted between the sunrises at the solstices, summer and winter, or the distance of both of them from the sunrise at the equinox.

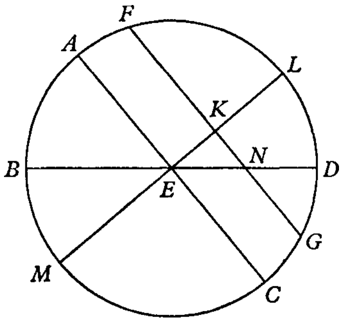


Then let *ABCD* be the meridian. In the eastern hemisphere let *BED* be the semicircle of the horizon, and *AEC* the semicircle of the equator. Let the equator's north pole be *F*. Assume that the sunrise at the summer solstice is in the point *G*. Draw *FGH* as an arc of a great circle. Now since the rotation of the terrestrial globe is accomplished around *F*, the pole of the equator, points *G* and *H* must reach the meridian *ABCD* together. For, their parallels of latitude are drawn around the same poles, and all great circles passing through these poles cut off similar arcs of those parallels. Therefore the time elapsing from the rising at *G* until noon is equally the measure of arc *AEH*, and of *CH*, the rest of the semicircle below the horizon, the time from midnight until sunrise. Now *AEC* is a semicircle, while *AE* and *EC* are quadrants, being drawn from the pole of *ABCD*. Consequently *EH* will be half of the difference between the longest day and the equinoctial day, while *EG* will be the distance between the equinoctial and solstitial sunrises. In triangle *EGH*, therefore, *GEH*, the angle of the obliquity of the sphere, is known through the arc *AB*. *GHE* is a right angle. Side *GH* also is known as the distance of the summer solstice from the equator. Therefore, in accordance with Theorem IV on Spherical Triangles, the remaining sides are also given: *EH*, half of the difference between the equinoctial day and the longest day, as well as *GE*, the distance between the sunrises. Furthermore if, together with side *GH*, side *EH*, [half] the difference between the longest day and the equinoctial day, or *EG* is given, *E*, the angle of the inclination of the sphere, is given, and therefore so is *FD*, the altitude of the pole above the horizon.

Next, assume that *G* on the ecliptic is not the solstice, but any other point. Nevertheless, both of the arcs *EG* and *EH* will be known. For from the Table of Declinations exhibited above, *GH* is obtained as the arc of declination corresponding to that degree of the ecliptic, and all the other quantities are found by the same method of proof. Hence it also follows that the degrees of the ecliptic which are equidistant from the solstice cut off the same arcs of the horizon from the equinoctial sunrise, and in the same direction. They also make the days and nights equal in length. This happens because the same parallel of latitude contains both degrees of the ecliptic, since their declination is equal and in the same direction. However, when equal arcs are taken in both directions from the intersection with the equator, the distances between the risings come out equal again, but in opposite directions, and in the inverse order the lengths of the days and nights are equal too, because on both sides they describe equal arcs of the parallels of latitude, just as the points [on the ecliptic] equidistant from the equinox have equal declinations from the equator.



Now in the same diagram, draw arcs of parallels of latitude. Let them be *GM* and *KN*, intersecting the horizon *BED* in points *G* and *K*. From *L*, the south pole, also draw *LKO* as a quadrant of a great circle. Then the declination *HG* is equal to *KO*. Hence there will be two triangles, *DFG* and *BLK*, in which two sides are equal to two corresponding sides: *FG* to *LK*, and *FD*, the altitude of the pole, to *LB*. *B* and *D* are right angles. Therefore the third side, *DG*, is equal to the third side, *BK*. Their remainders, *GE* and *EK*, the distances between the risings, are also equal. Here too, then, two sides, *EG* and *GH*, are equal to two sides, *EK* and *KO*. The vertical angles at *E* are equal. Hence the remaining sides, *EH* and *EO*, are equal. When these equals are added to equals, as a sum the whole arc *OEC* is equal to the whole arc *AEH*. But since great circles drawn through the poles cut off similar arcs of parallel circles on spheres, *GM* and *KN* will also be similar and equal. Q.E.D.



However, all this can be demonstrated also in another way. Draw the meridian *ABCD* in the same way. Let its center be *E*. Let the diameter of the equator and its intersection with the meridian be *AEC*. Let the diameter of the horizon and the meridian line be *BED*; the axis of the sphere, *LEM*; the visible pole, *L*; and the hidden pole, *M*. Assume that the distance of the summer solstice or that any other declination is *AF*. At this declination draw *FG* as the diameter of a parallel of latitude and also as the parallel's intersection with the meridian. *FG* will intersect the axis at *K*, and the meridian line at *N*. Now according to Posidonius' definition, parallels neither converge nor diverge, but make the perpendicular lines between them everywhere equal. Therefore the straight line *KE* will be equal to half of the chord subtending twice the arc *AF*. Similarly, with reference to the parallel of latitude whose radius is *FK*, *KN* will be half of the chord subtending the arc marking the difference between the equinoctial day and the unequal day. The reason for this is that all the semicircles, of which these lines are the intersections, that is, of which they are the diameters, namely, *BED* of the oblique horizon, *LEM* of the right horizon, *AEC* of the equator, and *FKG* of the parallel of latitude, are perpendicular to the plane of the circle *ABCD*. And, in accordance with Euclid's *Elements*, XI, 19, the lines in which these semicircles intersect one another are perpendicular to the same plane at points *E*, *K*, and *N*. In accordance with Theorem 6 of the same Book, these perpendiculars are parallel to one another. *K* is the center of the parallel of latitude, and *E* is the center of the sphere. Therefore *EN* is half of the chord subtending twice the horizon arc marking the difference between sunrise on the parallel of latitude and the equinoctial sunrise. *AF*, the declination, is given, together with *FL*, the remainder of the quadrant. Hence *KE* and *FK*, as halves of the chords subtending twice the arcs *AF* and *FL*, will be known in units whereof *AE* is 100,000. But in the right triangle *EKN*, angle *KEN* is given through *DL*, the altitude of the pole; and *KNE*, the complementary angle, is equal to *AEB*, because as parallels of latitude on the oblique sphere they are equally inclined to the horizon. Therefore the sides are given in the same units whereof the radius of the sphere is 100,000. Now in units whereof *FK*, the radius of the parallel of latitude, is 100,000, *KN* also will be given. And as half of the chord subtending the entire difference between the equinoctial day and [the day pertaining to] the parallel of latitude, *KN* will be given in units whereof in like manner the parallel as a circle is 360. Hence the ratio of *FK* to *KN* clearly consists of two ratios, namely, the ratio of the chord subtending twice *FL* to the chord subtending twice *AF*, that is, *FK* : *KE*, and the ratio of the chord subtending twice *AB* to the chord subtending twice *DL*. The latter ratio is equal to *EK* : *KN*, with *EK* of course taken as the mean proportional between *FK* and *KN*. Similarly the ratio of *BE* to *EN* is likewise formed by the ratios *BE* : *EK* and *KE* : *EN*, as Ptolemy shows in greater detail by means of spherical segments [*Syntaxis*, I, 13]. In this way, I believe, the inequality of the days and nights is found. But also in the case of the moon and of whatever stars the declination is given, the segments of the parallels of latitude described by them in the daily rotation above the horizon are distinguished from the segments which are below the horizon. From these segments their risings and settings can easily be learned.

TABLE OF THE DIFFERENCE IN THE ASCENSIONS ON AN OBLIQUE SPHERE

Declination	Elevation of the Pole											
	31		32		33		34		35		36	
Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
1	0	36	0	37	0	39	0	40	0	42	0	44
2	1	12	1	15	1	18	1	21	1	24	1	27
3	1	48	1	53	1	57	2	2	2	6	2	11
4	2	24	2	30	2	36	2	42	2	48	2	55
5	3	1	3	8	3	15	3	23	3	31	3	39
6	3	37	3	46	3	55	4	4	4	13	4	23
7	4	14	4	24	4	34	4	45	4	56	5	7
8	4	51	5	2	5	14	5	26	5	39	5	52
9	5	28	5	41	5	54	6	8	6	22	6	36
10	6	5	6	20	6	35	6	50	7	6	7	22
11	6	42	6	59	7	15	7	32	7	49	8	7
12	7	20	7	38	7	56	8	15	8	34	8	53
13	7	58	8	18	8	37	8	58	9	18	9	39
14	8	37	8	58	9	19	9	41	10	3	10	26
15	9	16	9	38	10	1	10	25	10	49	11	14
16	9	55	10	19	10	44	11	9	11	35	12	2
17	10	35	11	1	11	27	11	54	12	22	12	50
18	11	16	11	43	12	11	12	40	13	9	13	39
19	11	56	12	25	12	55	13	26	13	57	14	29
20	12	38	13	9	13	40	14	13	14	46	15	20
21	13	20	13	53	14	26	15	0	15	36	16	12
22	14	3	14	37	15	13	15	49	16	27	17	5
23	14	47	15	23	16	0	16	38	17	17	17	58
24	15	31	16	9	16	48	17	29	18	10	18	52
25	16	16	16	56	17	38	18	20	19	3	19	48
26	17	2	17	45	18	28	19	12	19	58	20	45
27	17	50	18	34	19	19	20	6	20	54	21	44
28	18	38	19	24	20	12	21	1	21	51	22	43
29	19	27	20	16	21	6	21	57	22	50	23	45
30	20	18	21	9	22	1	22	55	23	51	24	48
31	21	10	22	3	22	58	23	55	24	53	25	53
32	22	3	22	59	23	56	24	56	25	57	27	0
33	22	57	23	54	24	19	25	59	27	3	28	9
34	23	55	24	56	25	59	27	4	28	10	29	21
35	24	53	25	57	27	3	28	10	29	21	30	35
36	25	53	27	0	28	9	29	21	30	35	31	52

TABLE OF THE DIFFERENCE IN THE ASCENSIONS ON AN OBLIQUE SPHERE

Declination	Elevation of the Pole: 37		38		39		40		41		42	
Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
1	0	45	0	47	0	49	0	50	0	52	0	54
2	1	31	1	34	1	37	1	41	1	44	1	48
3	2	16	2	21	2	26	2	31	2	37	2	42
4	3	1	3	8	3	15	3	22	3	29	3	37
5	3	47	3	55	4	4	4	13	4	22	4	31
6	4	33	4	43	4	53	5	4	5	15	5	26
7	5	19	5	30	5	42	5	55	6	8	6	21
8	6	5	6	18	6	32	6	46	7	1	7	16
9	6	51	7	6	7	22	7	38	7	55	8	12
10	7	38	7	55	8	13	8	30	8	49	9	8
11	8	25	8	44	9	3	9	23	9	44	10	5
12	9	13	9	34	9	55	10	16	10	39	11	2
13	10	1	10	24	10	46	11	10	11	35	12	0
14	10	50	11	14	11	39	12	5	12	31	12	58
15	11	39	12	5	12	32	13	0	13	28	13	58
16	12	29	12	57	13	26	13	55	14	26	14	58
17	13	19	13	49	14	20	14	52	15	25	15	59
18	14	10	14	42	15	15	15	49	16	24	17	1
19	15	2	15	36	16	11	16	48	17	25	18	4
20	15	55	16	31	17	8	17	47	18	27	19	8
21	16	49	17	27	18	7	18	47	19	30	20	13
22	17	44	18	24	19	6	19	49	20	34	21	20
23	18	39	19	22	20	6	20	52	21	39	22	28
24	19	36	20	21	21	8	21	56	22	46	23	38
25	20	34	21	21	22	11	23	2	23	55	24	50
26	21	34	22	24	23	16	24	10	25	5	26	3
27	22	35	23	28	24	22	25	19	26	17	27	18
28	23	37	24	33	25	30	26	30	27	31	28	36
29	24	41	25	40	26	40	27	43	28	48	29	57
30	25	47	26	49	27	52	28	59	30	7	31	19
31	26	55	28	0	29	7	30	17	31	29	32	45
32	28	5	29	13	30	54	31	31	32	54	34	14
33	29	18	30	29	31	44	33	1	34	22	35	47
34	30	32	31	48	33	6	34	27	35	54	37	24
35	31	51	33	10	34	33	35	59	37	30	39	5
36	33	12	34	35	36	2	37	34	39	10	40	51

TABLE OF THE DIFFERENCE IN THE ASCENSIONS ON AN OBLIQUE SPHERE

Declination	Elevation of the Pole											
	43		44		45		46		47		48	
Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
1	0	56	0	58	1	0	1	2	1	4	1	7
2	1	52	1	56	2	0	2	4	2	9	2	13
3	2	48	2	54	3	0	3	7	3	13	3	20
4	3	44	3	52	4	1	4	9	4	18	4	27
5	4	41	4	51	5	1	5	12	5	23	5	35
6	5	37	5	50	6	2	6	15	6	28	6	42
7	6	34	6	49	7	3	7	18	7	34	7	50
8	7	32	7	48	8	5	8	22	8	40	8	59
9	8	30	8	48	9	7	9	26	9	47	10	8
10	9	28	9	48	10	9	10	31	10	54	11	18
11	10	27	10	49	11	13	11	37	12	2	12	28
12	11	26	11	51	12	16	12	43	13	11	13	39
13	12	26	12	53	13	21	13	50	14	20	14	51
14	13	27	13	56	14	26	14	58	15	30	16	5
15	14	28	15	0	15	32	16	7	16	42	17	19
16	15	31	16	5	16	40	17	16	17	54	18	34
17	16	34	17	10	17	48	18	27	19	8	19	51
18	17	38	18	17	18	58	19	40	20	23	21	9
19	18	44	19	25	20	9	20	53	21	40	22	29
20	19	50	20	35	21	21	22	8	22	58	23	51
21	20	59	21	46	22	34	23	25	24	18	25	14
22	22	8	22	58	23	50	24	44	25	40	26	40
23	23	19	24	12	25	7	26	5	27	5	28	8
24	24	32	25	28	26	26	27	27	28	31	29	38
25	25	47	26	46	27	48	28	52	30	0	31	12
26	27	3	28	6	29	11	30	20	31	32	32	48
27	28	22	29	29	30	38	31	51	33	7	34	28
28	29	44	30	54	32	7	33	25	34	46	36	12
29	31	8	32	22	33	40	35	2	36	28	38	0
30	32	35	33	53	35	16	36	43	38	15	39	53
31	34	5	35	28	36	56	38	29	40	7	41	52
32	35	38	37	7	38	40	40	19	42	4	43	57
33	37	16	38	50	40	30	42	15	44	8	46	9
34	38	58	40	39	42	25	44	18	46	20	48	31
35	40	46	42	33	44	27	46	23	48	36	51	3
36	42	39	44	33	46	36	48	47	51	11	53	47

TABLE OF THE DIFFERENCE IN THE ASCENSIONS ON AN OBLIQUE SPHERE

Declination	Elevation of the Pole											
	49		50		51		52		53		54	
Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
1	1	9	1	12	1	14	1	17	1	20	1	23
2	2	18	2	23	2	28	2	34	2	39	2	45
3	3	27	3	35	3	43	3	51	3	59	4	8
4	4	37	4	47	4	57	5	8	5	19	5	31
5	5	47	5	50	6	12	6	26	6	40	6	55
6	6	57	7	12	7	27	7	44	8	1	8	19
7	8	7	8	25	8	43	9	2	9	23	9	44
8	9	18	9	38	10	0	10	22	10	45	11	9
9	10	30	10	53	11	17	11	42	12	8	12	35
10	11	42	12	8	12	35	13	3	13	32	14	3
11	12	55	13	24	13	53	14	24	14	57	15	31
12	14	9	14	40	15	13	15	47	16	23	17	0
13	15	24	15	58	16	34	17	11	17	50	18	32
14	16	40	17	17	17	56	18	37	19	19	20	4
15	17	57	18	39	19	19	20	4	20	50	21	38
16	19	16	19	59	20	44	21	32	22	22	23	15
17	20	36	21	22	22	11	23	2	23	56	24	53
18	21	57	22	47	23	39	24	34	25	33	26	34
19	23	20	24	14	25	10	26	9	27	11	28	17
20	24	45	25	42	26	43	27	46	28	53	30	4
21	26	12	27	14	28	18	29	26	30	37	31	54
22	27	42	28	47	29	56	31	8	32	25	33	47
23	29	14	30	23	31	37	32	54	34	17	35	45
24	31	4	32	3	33	21	34	44	36	13	37	48
25	32	26	33	46	35	10	36	39	38	14	39	59
26	34	8	35	32	37	2	38	38	40	20	42	10
27	35	53	37	23	39	0	40	42	42	33	44	32
28	37	43	39	19	41	2	42	53	44	53	47	2
29	39	37	41	21	43	12	45	12	47	21	49	44
30	41	37	43	29	45	29	47	39	50	1	52	37
31	43	44	45	44	47	54	50	16	52	53	55	48
32	45	57	48	8	50	30	53	7	56	1	59	19
33	48	19	50	44	53	20	56	13	59	28	63	21
34	50	54	53	30	56	20	59	42	63	31	68	11
35	53	40	56	34	59	58	63	40	68	18	74	32
36	56	42	59	59	63	47	68	26	74	36	90	0

TABLE OF THE DIFFERENCE IN THE ASCENSIONS ON AN OBLIQUE SPHERE

Declination	Elevation of the Pole											
	55		56		57		58		59		60	
Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
1	1	26	1	29	1	32	1	36	1	40	1	44
2	2	52	2	58	3	5	3	12	3	20	3	28
3	4	17	4	27	4	38	4	49	5	0	5	12
4	5	44	5	57	6	11	6	25	6	41	6	57
5	7	11	7	27	7	44	8	3	8	22	8	43
6	8	38	8	58	9	19	9	41	10	4	10	29
7	10	6	10	29	10	54	11	20	11	47	12	17
8	11	35	12	1	12	30	13	0	13	32	14	5
9	13	4	13	35	14	7	14	41	15	17	15	55
10	14	35	15	9	15	45	16	23	17	4	17	47
11	16	7	16	45	17	25	18	8	18	53	19	41
12	17	40	18	22	19	6	19	53	20	43	21	36
13	19	15	20	1	20	50	21	41	22	36	23	34
14	20	52	21	42	22	35	23	31	24	31	25	35
15	22	30	23	24	24	22	25	23	26	29	27	39
16	24	10	25	9	26	12	27	19	28	30	29	47
17	25	53	26	57	28	5	29	18	30	35	31	59
18	27	39	28	48	30	1	31	20	32	44	34	19
19	29	27	30	41	32	1	33	26	34	58	36	37
20	31	19	32	39	34	5	35	37	37	17	39	5
21	33	15	34	41	36	14	37	54	39	42	41	40
22	35	14	36	48	38	28	40	17	42	15	44	25
23	37	19	39	0	40	49	42	47	44	57	47	20
24	39	29	41	18	43	17	45	26	47	49	50	27
25	41	45	43	44	45	54	48	16	50	54	53	52
26	44	9	46	18	48	41	51	19	54	16	57	39
27	46	41	49	4	51	41	54	38	58	0	61	57
28	49	24	52	1	54	58	58	19	62	14	67	4
29	52	20	55	16	58	36	62	31	67	18	73	46
30	55	32	58	52	62	45	67	31	73	55	90	0
31	59	6	62	58	67	42	74	4	90	0		
32	63	10	67	53	74	12	90	0				
33	68	1	74	19	90	0						
34	74	33	90	0								
35	90	0										
36												

The vacant spaces here belong to stars which neither rise nor set.

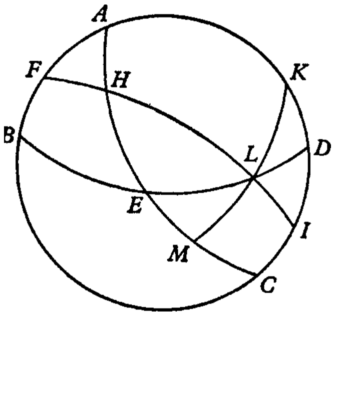
THE HOURS AND PARTS OF THE DAY AND NIGHT *Chapter 8*

From the foregoing, therefore, it is clear that, for a stated altitude of the pole, we may take the difference in the days as indicated for a declination of the sun in the Table. This difference may be added to a quadrant in the case of a northern declination, or subtracted from it in the case of a southern declination. If the result is doubled, we shall have the length of that day, and the duration of the night, which is the rest of the circle.

If either of these two is divided by 15 degrees of the equator, the quotient will show how many equal hours it contains. But if we take the twelfth part, we shall have the duration of a seasonal hour. Now these hours take the name of their day, of which they are always the twelfth part. Hence the terms "summer solstitial, equinoctial, and winter solstitial hours" are found employed by the ancients. Nor were any other hours originally in use than the twelve hours from dawn to dusk. But they used to divide the night into four vigils or watches. This regulation of the hours lasted for a long time by the unspoken agreement of all nations. For the purpose of this regulation, water-clocks were invented. By the subtraction from and addition to the water dripped from these clocks, the hours were adjusted to the difference in the days, so that the subdivision of time would not be obscured even by a cloudy sky. Afterwards, equal hours, common to daytime and nighttime, were generally adopted. Since these equal hours are easier to observe, the seasonal hours became obsolete. Hence, if you ask any ordinary person which is the first, third, sixth, ninth or eleventh hour of the day, he has no answer or at any rate his answer has no relevance to the subject. Also with regard to the numbering of the equal hours, some now take it from noon, others from sunset, others from midnight, and still others from sunrise, in accordance with the decision of each society.

THE OBLIQUE ASCENSION OF THE DEGREES OF THE ECLIPTIC; HOW TO DETERMINE WHAT DEGREE IS AT MID-HEAVEN WHEN ANY DEGREE IS RISING *Chapter 9*

Now that I have thus explained the lengths of the days and nights as well as the difference in those lengths, the next topic in proper order is the oblique ascensions. I refer to the times during which the dodecatemories, that is, the twelve zodiacal signs, or any other arcs of the zodiac, rise. For, between right ascensions and oblique ascensions, there are no differences other than those which I set forth between the equinoctial day and a day which is unequal to its night in length. Now the names of living things have been borrowed for the zodiacal signs, which consist of immovable stars. Starting from the vernal equinox, the signs have been called Ram, Bull, Twins, Crab, and so on, as they follow in order.



For the sake of greater clarity, then, again draw the meridian *ABCD*. Let *AEC*, the semicircle of the equator, and the horizon *BED* intersect each other in the point *E*. Put the equinox in *H*. Let the ecliptic *FHI*, passing through *H*, intersect the horizon in *L*. Through this intersection draw *KLM*, the quadrant of a great circle, from *K*, the pole of the equator. Thus it is certainly clear that arc

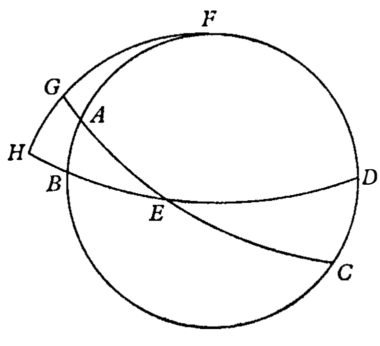
HL of the ecliptic rises with *HE* of the equator. But in the right sphere, *HL* rose with *HEM*. The difference between them is *EM* which, as I showed above [II, 7], is half of [the difference between] the equinoctial day and the unequal day. But what was added there in a northern declination, is subtracted here. In a southern declination, on the other hand, it is added to the right ascension in order to obtain the oblique ascension. Accordingly, how long a whole sign, or other arc of the ecliptic, takes to rise will be made clear by the ascensions computed from the beginning [of the sign or arc] to its end.

Hence it follows that when any degree of the ecliptic, measured from the equinox, is given as rising, the degree which is at mid-heaven is also given. For, *L* [being the point which is] rising [on the ecliptic], given its declination through *HL*, its distance from the equinox, its right ascension *HEM*, and the whole of *AHEM* as the arc of the half-day, then the remainder, *AH*, is given. This is the right ascension of *FH*, which is given by the Table, or also because *AHF*, the angle of the obliquity, is given, together with the side *AH*, while *FAH* is a right angle. Therefore the whole arc *FHL* of the ecliptic is given between the degree of rising and the degree at mid-heaven.

Conversely, if the degree at mid-heaven, for instance, the arc *FH*, is given first, we shall also know the degree which is rising. For, the declination *AF* will be obtained, and so will *AFB*, through the angle of obliquity of the sphere, and the remainder *FB*. Now in triangle *BFL*, angle *BFL* is given by what precedes; so is side *FB*; and *FBL* is a right angle. Therefore the required side *FHL* is given. An alternative method of obtaining it will appear below [II, 10].

THE ANGLE AT WHICH THE ECLIPTIC INTERSECTS THE HORIZON *Chapter 10*

Furthermore, since the ecliptic is a circle oblique to the axis of the sphere, it makes various angles with the horizon. It is perpendicular to the horizon twice for those who live between the tropics, as I have already said with regard to the differences in the shadows [II, 6]. However, I think that it is enough for us to demonstrate only those angles which concern us who live in the heteroscian region. From these angles, the entire theory of the angles will be easily understood. Now in the oblique sphere, when the equinox or first point of the Ram is rising, the ecliptic is lower and turns toward the horizon to the extent added by the greatest southward declination, which occurs when the first point of the Goat is at mid-heaven. Conversely, at a higher altitude the ecliptic makes the angle of rising greater when the first point of the Balance rises, and the first point of the Crab is at mid-heaven. The foregoing statements are quite obvious, I believe. For, these three circles, the equator, ecliptic, and horizon, by passing through the same intersection, meet in the poles of the meridian. The arcs of the meridian intercepted by these circles show how great the angle of rising is judged to be.



But a way of measuring it also for the other degrees of the ecliptic may be explained. Again let the meridian be *ABCD*, half of the horizon *BED*, and half of the ecliptic *AEC*. Let any degree of the ecliptic rise at *E*. We are required to find how great the angle *AEB* is in units whereof 4 right angles = 360°. Since *E* is given as the rising degree, the degree at mid-heaven is also given by the previous discussion, as is also the arc *AE* together with the meridian altitude *AB*.

Because *ABE* is a right angle, the ratio of the chord subtending twice *AE* to the chord subtending twice *AB* is given as equal to the ratio of the diameter of the sphere to the chord subtending twice the arc which measures the angle *AEB*. Therefore the angle *AEB* also is given.

However, the given degree may be, not at the rising, but at mid-heaven. Let it be *A*. Nevertheless the angle of rising will be measured. For, with its pole at *E*, draw *FGH* as the quadrant of a great circle. Complete the quadrants *EAG* and *EBH*. Now *AB*, the altitude of the meridian, is given, and so is *AF*, the remainder of the quadrant. Angle *FAG* is also given by the foregoing, and *FGA* is a right angle. Therefore the arc *FG* is given. So is the remainder *GH*, which measures the required angle of rising. Here too, then, it is clear how, given the degree at mid-heaven, the degree at the rising is given. For, the ratio of the chord subtending twice *GH* to the chord subtending twice *AB* is equal to the ratio of the diameter to the chord subtending twice *AE*, as in Spherical Triangles [I, 14, Theorem III].

For these relations too I have subjoined three kinds of Tables. The first will give the ascensions in the right sphere, beginning with the Ram, and advancing by 6° of the ecliptic. The second will give the ascensions in the oblique sphere, likewise in steps of 6°, from the parallel of latitude whose pole's altitude is 39°, by half-steps of 3°, to the parallel with its pole at 57°. The remaining Table will give the angles made with the horizon, also by steps of 6°, in the same 7 columns. All these computations are based on the minimum obliquity of the ecliptic, 23° 28′, which is approximately correct for our age.

TABLE OF THE ASCENSIONS OF THE ZODIACAL SIGNS IN THE REVOLUTION OF THE RIGHT SPHERE

Ecliptic		Ascension		For a Single Degree		Ecliptic		Ascension		For a Single Degree	
Sign	Degree	Degree	Minute	Degree	Minute	Sign	Degree	Degree	Minute	Degree	Minute
♈	6	5	30	0	55	♎	6	185	30	0	55
	12	11	0	0	55		12	191	0	0	55
	18	16	34	0	56		18	196	34	0	56
	24	22	10	0	56		24	202	10	0	56
	30	27	54	0	57		30	207	54	0	57
♉	6	33	43	0	58	♏	6	213	43	0	58
	12	39	35	0	59		12	219	35	0	59
	18	45	32	1	0		18	225	32	1	0
	24	51	37	1	1		24	231	37	1	1
	30	57	48	1	2		30	237	48	1	2
♊	6	64	6	1	3	♐	6	244	6	1	3
	12	70	29	1	4		12	250	29	1	4
	18	76	57	1	5		18	256	57	1	5
	24	83	27	1	5		24	263	27	1	5
	30	90	0	1	5		30	270	0	1	5
♋	6	96	33	1	5	♑	6	276	33	1	5
	12	103	3	1	5		12	283	3	1	5
	18	109	31	1	5		18	289	31	1	5
	24	115	54	1	4		24	295	54	1	4
	30	122	12	1	3		30	302	12	1	3
♌	6	128	23	1	2	♒	6	308	23	1	2
	12	134	28	1	1		12	314	28	1	1
	18	140	25	1	0		18	320	25	1	0
	24	146	17	0	59		24	326	17	0	59
	30	152	6	0	58		30	332	6	0	58
♍	6	157	50	0	57	♓	6	337	50	0	57
	12	163	26	0	56		12	343	26	0	56
	18	169	0	0	56		18	349	0	0	56
	24	174	30	0	55		24	354	30	0	55
	30	180	0	0	55		30	360	0	0	55

TABLE OF THE ASCENSIONS IN THE OBLIQUE SPHERE

Ecliptic		Altitude of the Pole													
		39		42		45		48		51		54		57	
		Ascension		Ascension		Ascension		Ascension		Ascension		Ascension		Ascension	
Sign	Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
♈	6	3	34	3	20	3	6	2	50	2	32	2	12	1	49
	12	7	10	6	44	6	15	5	44	5	8	4	27	3	40
	18	10	50	10	10	9	27	8	39	7	47	6	44	5	34
	24	14	32	13	39	12	43	11	40	10	28	9	7	7	32
	30	18	26	17	21	16	11	14	51	13	26	11	40	9	40
♉	6	22	30	21	12	19	46	18	14	16	25	14	22	11	57
	12	26	39	25	10	23	32	21	42	19	38	17	13	14	23
	18	31	0	29	20	27	29	25	24	23	2	20	17	17	2
	24	35	38	33	47	31	43	29	25	26	47	23	42	20	2
	30	40	30	38	30	36	15	33	41	30	49	27	26	23	22
♊	6	45	39	43	31	41	7	38	23	35	15	31	34	27	7
	12	51	8	48	52	46	20	43	27	40	8	36	13	31	26
	18	56	56	54	35	51	56	48	56	45	28	41	22	36	20
	24	63	0	60	36	57	54	54	49	51	15	47	1	41	49
	30	69	25	66	59	64	16	61	10	57	34	53	28	48	2
♋	6	76	6	73	42	71	0	67	55	64	21	60	7	54	55
	12	83	2	80	41	78	2	75	2	71	34	67	28	62	26
	18	90	10	87	54	85	22	82	29	79	10	75	15	70	28
	24	97	27	95	19	92	55	90	11	87	3	83	22	78	55
	30	104	54	102	54	100	39	98	5	95	13	91	50	87	46
♌	6	112	24	110	33	108	30	106	11	103	33	100	28	96	48
	12	119	56	118	16	116	25	114	20	111	58	109	13	105	58
	18	127	29	126	0	124	23	122	32	120	28	118	3	115	13
	24	135	4	133	46	132	21	130	48	128	59	126	56	124	31
	30	142	38	141	33	140	23	139	3	137	38	135	52	133	52
♍	6	150	11	149	19	148	23	147	20	146	8	144	47	143	12
	12	157	41	157	1	156	19	155	29	154	38	153	36	153	24
	18	165	7	164	40	164	12	163	41	163	5	162	24	162	47
	24	172	34	172	21	172	6	171	51	171	33	171	12	170	49
	30	180	0	180	0	180	0	180	0	180	0	180	0	180	0

TABLE OF THE ASCENSIONS IN THE OBLIQUE SPHERE

Ecliptic		Altitude of the Pole													
		39		42		45		48		51		54		57	
		Ascension		Ascension		Ascension		Ascension		Ascension		Ascension		Ascension	
Sign	Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute
♎	6	187	26	187	39	187	54	188	9	188	27	188	48	189	11
	12	194	53	195	19	195	48	196	19	196	55	197	36	198	23
	18	202	21	203	0	203	41	204	30	205	24	206	25	207	36
	24	209	49	210	41	211	37	212	40	213	52	215	13	216	48
	30	217	22	218	27	219	37	220	57	222	22	224	8	226	8
♏	6	224	56	226	14	227	38	229	12	231	1	233	4	235	29
	12	232	31	234	0	235	37	237	28	239	32	241	57	244	47
	18	240	4	241	44	243	35	245	40	248	2	250	47	254	2
	24	247	36	249	27	251	30	253	49	256	27	259	32	263	12
	30	255	6	257	6	259	21	261	52	264	47	268	10	272	14
♐	6	262	33	264	41	267	5	269	49	272	57	276	38	281	5
	12	269	50	272	6	274	38	277	31	280	50	284	45	289	32
	18	276	58	279	19	281	58	284	58	288	26	292	32	297	34
	24	283	54	286	18	289	0	292	5	295	39	299	53	305	5
	30	290	35	293	1	295	45	298	50	302	26	306	42	311	58
♑	6	297	0	299	24	302	6	305	11	308	45	312	59	318	11
	12	303	4	305	25	308	4	311	4	314	32	318	38	323	40
	18	308	52	311	8	313	40	316	33	319	52	323	47	328	34
	24	314	21	316	29	318	53	321	37	324	45	328	26	332	53
	30	319	30	321	30	323	45	326	19	329	11	332	34	336	38
♒	6	324	21	326	13	328	16	330	35	333	13	336	18	339	58
	12	329	0	330	40	332	31	334	36	336	58	339	43	342	58
	18	333	21	334	50	336	27	338	18	340	22	342	47	345	37
	24	337	30	338	48	340	3	341	46	343	35	345	38	348	3
	30	341	34	342	39	343	49	345	9	346	34	348	20	350	20
♓	6	345	29	346	21	347	17	348	20	349	32	350	53	352	28
	12	349	11	349	51	350	33	351	21	352	14	353	16	354	26
	18	352	50	353	16	353	45	354	16	354	52	355	33	356	20
	24	356	26	356	40	356	23	357	10	357	53	357	48	358	11
	30	360	0	360	0	360	0	360	0	360	0	360	0	360	0

TABLE OF THE ANGLES MADE BY THE ECLIPTIC WITH THE HORIZON

Ecliptic		Altitude of the Pole														Ecliptic	
		39		42		45		48		51		54		57			
		Angle		Angle		Angle		Angle		Angle		Angle		Angle			
Sign	Degree	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Minute	Degree	Sign
♈	0	27	32	24	32	21	32	18	32	15	32	12	32	9	32	30	
	6	27	37	24	36	21	36	18	36	15	35	12	35	9	35	24	
	12	27	49	24	49	21	48	18	47	15	45	12	43	9	41	18	
	18	28	13	25	9	22	6	19	3	15	59	12	56	9	53	12	
	24	28	45	25	40	22	34	19	29	16	23	13	18	10	13	6	♓
	30	29	27	26	15	23	11	20	5	16	56	13	45	10	31	30	
♉	6	30	19	27	9	23	59	20	48	17	35	14	20	11	2	24	
	12	31	21	28	9	24	56	21	41	18	23	15	3	11	40	18	
	18	32	35	29	20	26	3	22	43	19	21	15	56	12	26	12	
	24	34	5	30	43	27	23	24	2	20	41	16	59	13	20	6	♒
	30	35	40	32	17	28	52	25	26	21	52	18	14	14	26	30	
♊	6	37	29	34	1	30	37	27	5	23	11	19	42	15	48	24	
	12	39	32	36	4	32	32	28	56	25	15	21	25	17	23	18	
	18	41	44	38	14	34	41	31	3	27	18	23	25	19	16	12	
	24	44	8	40	32	37	2	33	22	29	35	25	37	21	26	6	♑
	30	46	41	43	11	39	33	35	53	32	5	28	6	23	52	30	
♋	6	49	18	45	51	42	15	38	35	34	44	30	50	26	36	24	
	12	52	3	48	34	45	0	41	8	37	55	33	43	29	34	18	
	18	54	44	51	20	47	48	44	13	40	31	36	40	32	39	12	
	24	57	30	54	5	50	38	47	6	43	33	39	43	35	50	6	♐
	30	60	4	56	42	53	22	49	54	46	21	42	43	38	56	30	
♌	6	62	40	59	27	56	0	52	34	49	9	45	37	41	57	24	
	12	64	59	61	44	58	26	55	7	51	46	48	19	44	48	18	
	18	67	7	63	56	60	20	57	26	54	6	50	47	47	24	12	
	24	68	59	65	52	62	42	59	30	56	17	53	7	49	47	6	♏
	30	70	38	67	27	64	18	61	17	58	9	54	58	52	38	30	
♍	6	72	0	68	53	65	51	62	46	59	37	56	27	53	16	24	
	12	73	4	70	2	66	59	63	56	60	53	57	50	54	46	18	
	18	73	51	70	50	67	49	64	48	61	46	58	45	55	44	12	
	24	74	19	71	20	68	20	65	19	62	18	59	17	56	16	6	
	30	74	28	71	28	68	28	65	28	62	28	59	28	56	28	0	♎

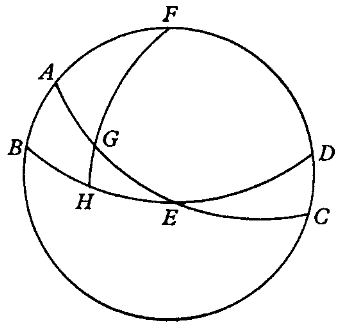
THE USE OF THESE TABLES *Chapter 11*

The use of the Tables is already clear from what has been established. For when the degree of the sun is known, we have received the right ascension. To it, for any equal hour, we add 15° of the equator. If the total exceeds the 360° of a whole circle, they are cast out. The remainder of the right ascension will show the related degree of the ecliptic at mid-heaven at the hour in question, starting from noon. If you perform the same operation for the oblique ascension of your region, in like manner you will have the rising degree of the ecliptic at an hour counted from sunrise. Moreover, for any stars which are outside the zodiac and whose right ascension is known, as I showed above [II, 9], these Tables give the degrees of the ecliptic which are at mid-heaven with these stars, through the same right ascension, starting from the first point of the Ram. The oblique ascension of those stars gives the degree of the ecliptic which rises with them, as the ascensions and degrees of the ecliptic are revealed directly by the Tables. You will proceed in the same way with regard to the setting, but always through the opposite place. Furthermore, if a quadrant is added to the right ascension which is at mid-heaven, the resulting sum is the oblique ascension of the rising degree. Therefore, through the degree at mid-heaven, the degree at the rising is also given, and conversely. The next Table gives the angles made by the ecliptic with the horizon. These angles are determined by the degree of the ecliptic at the rising. From them it is also learned how great the altitude of the ninetieth degree of the ecliptic is from the horizon. A knowledge of this altitude is absolutely necessary in eclipses of the sun.

THE ANGLES AND ARCS OF THOSE CIRCLES WHICH ARE DRAWN THROUGH THE POLES OF THE HORIZON TO THE ECLIPTIC *Chapter 12*

I may next explain the theory of the angles and arcs occurring at the intersections of the ecliptic with those circles which pass through the zenith of the horizon and on which the altitude above the horizon is taken. But the noon altitude of the sun or of any degree of the ecliptic at mid-heaven, and the angle of the ecliptic's intersection with the meridian were set forth above [II, 10]. For, the meridian too is one of the circles which pass through the zenith of the horizon. The angle at the rising has also been discussed already. When this angle is subtracted from a right angle, the remainder is the angle formed with the rising ecliptic by a quadrant passing through the zenith of the horizon.

It remains, then, by repeating the previous diagram [II, 10], to look at the intervening intersections, I mean, of the meridian with the semicircles of the ecliptic and horizon. Take any point on the ecliptic between noon and rising or setting. Let this point be *G*. Through it draw the quadrant *FGH* from *F*, the pole of the horizon. Through the designated hour, the whole arc *AGE* of the ecliptic between the meridian and the horizon is given. *AG* is given by hypothesis. In like manner *AF* is also given, because the noon altitude *AB* is given. The meridian angle *FAG* is likewise given. Therefore *FG* is also given, by what was proved with regard to spherical triangles. The complement *GH*, which is the altitude of *G*, is given, together with angle *FGA*. These we were required to find.



This treatment of the angles and intersections connected with the ecliptic, I excerpted compactly from Ptolemy while I was reviewing the discussion of spherical triangles in general. If anybody wishes to work on this subject, he will be able by himself to find more applications than those which I discussed only as examples.

[An earlier version of the latter part of II, 12 survives in the autograph on folio 46^r, without any indication that it was superseded. It begins in the middle of the second sentence of the second paragraph above, with the choice of any point on the ecliptic]
between the rising and noon. Let it be η, with its quadrant $\zeta\eta\vartheta$. Through the designated hour, the arc $\alpha\eta\varepsilon$ is given, and likewise $\alpha\eta$ as well as $\alpha\zeta$ with the meridian angle $\zeta\alpha\eta$. Therefore, in accordance with Theorem XI on Spherical Triangles, the arc $\zeta\eta$ is given as well as the angle $\zeta\eta\alpha$. These we were required to find. Now the ratios of the chord subtending twice $\varepsilon\eta$ to the chord subtending twice $\eta\vartheta$, and of the chords subtending twice the arcs $\varepsilon\alpha$ and $\alpha\beta$ are both equal to the ratio of the radius to the intercept of angle $\eta\varepsilon\vartheta$. Therefore the altitude $\eta\vartheta$ of the given point η is given. But in triangle $\eta\vartheta\varepsilon$, sides $\eta\varepsilon$ and $\eta\vartheta$ are given, as is also angle ε, while ϑ is a right angle. From these quantities we shall also uncover the measure of the remaining angle $\varepsilon\eta\vartheta$. This treatment of the angles and segments of circles, I excerpted compactly from Ptolemy and others, while I was reviewing the discussion of triangles in general. If anybody wishes to work on this subject, he will be able by himself to find many more applications than those which I discussed only as examples.

THE RISING AND SETTING OF THE HEAVENLY BODIES — *Chapter 13*

The risings and settings of the heavenly bodies also belong with the daily rotation, as is evident. This is true not only for those simple risings and settings which I just discussed, but also for the ways in which the bodies become morning and evening stars. Although the latter phenomena occur in conjunction with the annual revolution, they will nevertheless be treated more appropriately in this place.

The ancient mathematicians distinguish the true [risings and settings] from the visible. The true are as follows. The morning rising of a heavenly body occurs when it appears at the same time as the sun. On the other hand, the morning setting of the body occurs when it sets at sunrise. Throughout this entire interval the body was called a "morning star". But the evening rising occurs when the body appears at sunset. On the other hand, the evening setting occurs when the body sets at the same time as the sun. In the intervening period it is called an "evening star", because it is obscured by day and comes forth at night.

By contrast, the visible risings and settings are as follows. The morning rising of the body occurs when it first emerges and begins to appear at dawn and before sunrise. On the other hand, the morning setting occurs when the body is seen to have set just as the sun is about to rise. The body's evening rising occurs when it first appears to rise at twilight. But its evening setting occurs when it ceases any longer to be visible after sunset. Thereafter the presence of the sun blots the body out, until at their morning rising [the heavenly bodies] emerge in the order described above.

In the same way as these phenomena occur in the fixed stars, they occur also in the planets Saturn, Jupiter, and Mars. But the risings and settings of Venus and Mercury are different. For they are not blotted out by the approach of the sun, as the other planets are, nor are they made visible by its departure. On the contrary, when they precede the sun, they immerse themselves in its brilliance and extricate themselves. When the other planets have their evening rising and morning setting, they are not obscured at any time, but shine throughout almost the entire night. On the other hand, Venus and Mercury disappear completely from [evening] setting to [morning] rising, and cannot be seen anywhere. There is also another difference. In Saturn, Jupiter, and Mars, the true risings and settings are earlier than the visible in the morning, and later in the evening, to the extent that they precede sunrise in the first case, and follow sunset in the second case. On the other hand, in the lower planets the visible morning and evening risings are later than the true, whereas the settings are earlier.

Now the way in which the [risings and settings] may be determined can be understood from what was said above, where I explained the oblique ascension of any star having a known position, and the degree of the ecliptic with which it rises or sets [II, 9]. If at that time the sun appears in that degree or the opposite degree, the star will have its true morning or evening rising or setting.

From these, the visible risings and settings differ according to the brilliance and size of each body. Thus, those which have a more powerful light are obscured by the sun's rays for a shorter time than those which are less bright. Moreover, the limits of disappearance and appearance are determined by the subhorizontal arcs, between the horizon and the sun, on the circles which pass through the poles of the horizon. For fixed stars of the first magnitude, these limits are almost 12°; for Saturn, 11°; for Jupiter, 10°; for Mars, 11 $^1/_2$°; for Venus, 5°; and for Mercury, 10°. But the whole belt in which the remnant of daylight yields to night, the belt which embraces twilight or dawn, contains 18° of the aforesaid circle. When the sun has descended by these 18°, the smaller stars also begin to appear. Now this is the distance at which some people put a plane parallel to the horizon and below it. When the sun reaches this plane, they say that the day is beginning or the night is ending. We may know with what degree of the ecliptic a body rises or sets. We may also discover the angle at which the ecliptic intersects the horizon at that same degree. We may also find at that time as many degrees of the ecliptic between the rising degree and the sun as are enough and as are associated with the sun's depth below the horizon in accordance with the aforementioned limits of the body in question. If so, we shall assert that its first appearance or disappearance is occurring. However, what I explained in the preceding demonstration with regard to the sun's altitude above the earth also fits in all respects its descent below the earth, since there is no difference in anything but position. Thus, the bodies which set so far as the visible hemisphere is concerned, rise so far as the hidden hemisphere is concerned, and everything occurs conversely and is readily understood. Therefore, what has been said about the rising and setting of the heavenly bodies, and, to that extent, about the daily rotation of the terrestrial globe may be enough.

THE INVESTIGATION OF THE PLACES OF THE STARS, AND THE ARRANGEMENT OF THE FIXED STARS IN A CATALOGUE

Chapter 14

[The beginning of a new book, according to Copernicus' original plan; an earlier draft of the first two-thirds of this Chapter survives in the autograph, folio 46^v – 47^v, without any indication that it was superseded; where this earlier draft is somewhat more explicit than the printed text, it too is translated here].

[Earlier draft:
Now that I have expounded the terrestrial globe's daily rotation and its consequences with respect to the days and nights and their parts and variations, the explanations of the annual revolution ought to have come next. Not a few astronomers, however, agree with the traditional practice of giving precedence to the phenomena of the fixed stars as the foundations of this science. Hence I thought that I in particular should adhere to this judgement. For among my principles and fundamental propositions I have assumed that the sphere of the fixed stars is absolutely immovable; and that the wanderings of all the planets are rightly compared with it, since motion requires that something should be at rest. Yet someone may wonder why I adopted this order, whereas in his *Syntaxis* [III, 1, introduction] Ptolemy considered that an explanation of the fixed stars could not be given unless the knowledge of the sun and moon came first, and for this reason he deemed it necessary to postpone his discussion of the fixed stars until then].

[Printed text:

This opinion, I believe, should be opposed. If, on the other hand, you interpret it as referring to the calculations for computing the apparent motion of the sun and moon, perhaps Ptolemy's opinion will hold good. For, the geometer Menelaus likewise kept track of most of the stars and their places through computations based on their conjunctions with the moon.

[Earlier draft:
Of course I acknowledge that the stars' places cannot be determined apart from the moon's, nor in turn can the moon's apart from the sun's. But these are problems which require the help of instruments, and I believed that this topic must not be investigated in any other way. On the other hand, anybody who wants the theory of the [solar and lunar] motions and revolutions in precise tables will accomplish nothing, I maintain, if he disregards the fixed stars. Hence Ptolemy and others before him and after him, who derived the length of the solar year only from the equinoxes or solstices, in striving to establish fundamental propositions for us could never agree about this length. In no topic, consequently, was there greater discord. This so disturbed most [specialists] that they almost abandoned the hope of mastering astronomy and declared the motions in the heavens to be beyond the comprehension of the human mind. Being aware of this attitude, Ptolemy [*Syntaxis,* III, 1] computed the solar year in his own age not without suspecting that an error could appear in the course of time, and advised posterity to seek finer precision in this matter subsequently. Hence it seemed to me worth while in this book, first, to show how much instruments help to determine the places of the sun, moon, and stars, that is, their distances from an equinoctial or solstitial point, and then to explain the sphere of the fixed stars studded with constellations].

[Printed text:

But we shall do much better if we locate any star with the help of instruments through a careful examination of the positions of the sun and moon, as I shall soon show. I am also warned by the ineffectual attempt of those who thought that the length of the solar year should be delimited simply by the equinoxes or solstices, and not also by the fixed stars. In this effort down to our own time they have never been able to agree, so that nowhere has there been greater dissension. This was noticed by Ptolemy. When he computed the solar year in his own age, not without suspecting that an error could appear in the course of time, he advised posterity to seek finer precision in this matter subsequently. Hence it seemed to me worth while in this book to show how skill with instruments may establish the positions of the sun and moon, that is, the amount of their distance from the vernal equinox or other cardinal points of the universe. These places will then facilitate our investigation of other heavenly bodies, by means of which we may also set before the eyes the sphere of the fixed stars studded with constellations, and its representation.

Now I have already explained the instruments by which we may determine the distance between the tropics, the obliquity of the ecliptic, and the inclination of the sphere or the altitude of the pole of the equator [II, 2]. In the same way we can obtain any other altitude of the sun at noon. Through its difference from the inclination of the sphere, this altitude will show us the amount of the sun's declination from the equator. Then through this declination its position at noon, as measured from an equinox or solstice, will also become clear. Now in a period of 24 hours the sun seems to pass through almost $1°$; the hourly fraction thereof amounts to $2\,^1/_2{}'$. Hence for any designated hour other than noon, its position will be easily inferred.

But for observing the positions of the moon and of the stars, another instrument is constructed, which Ptolemy calls the "astrolabe" [*Syntaxis*, V, 1]. Now two rings, or quadrilateral frames of rings, are made in such a way that their flat sides, or members, are set at right angles to their concave – convex surface. These rings are equal and similar in all respects, and of a convenient size. That is, if they are too big, they become less manageable. Yet otherwise, generous dimensions are better than skimpy, for the purpose of division into parts. Thus let [the rings'] width and thickness be at least one-thirtieth of the diameter. Then they will be joined and connected with each other at right angles along the diameter, with the concave – convex surfaces fitting together as though in the roundness of a single sphere. In fact, let one of them take the place of the ecliptic; and the other, of the circle which passes through the poles of both (I mean, of the equator and the ecliptic). Then the sides of the ecliptic ring should be divided into equal parts, which are usually 360, and these may be further subdivided according to the size of the instrument. Also on the other ring, by measuring quadrants from the ecliptic, indicate the poles of the ecliptic. Take a distance from these poles in proportion to the obliquity of the ecliptic, and mark the poles of the equator too.

After these rings have been arranged in this way, two other rings are made. They are fastened at the ecliptic's poles, on which they will move, [one on the] outside and [the other on the] inside. Make these rings equal to the others in thickness between the two flat surfaces, while the width of their rims is similar. Fit them together so that there is contact everywhere between the larger ring's concave surface and the ecliptic's convex surface, as well as between the smaller one's convex surface and the ecliptic's concave surface. However, let there be no obstacle to their being turned about, but let them permit the ecliptic with its meridian freely and easily to slide over them, and conversely. Hence we will neatly perforate these rings at the diametrically opposite poles of the ecliptic, and insert axles to attach and support them. Divide the inner ring also into 360 equal degrees, so that in each quadrant there are $90°$ to the poles.

Furthermore, on the concave surface of this ring, another ring, the fifth, should be placed, and be able to turn in the same plane. To the rims of this ring, attach diametrically opposite brackets with apertures and peepholes or eyepieces. Here the light of the star can impinge and leave along the diameter of the ring, as is the practice in the dioptra. Moreover, mount certain blocks on both sides of the ring, as pointers toward the numbers on the containing ring, for the purpose of observing the latitudes.

Finally, a sixth ring must be attached, to receive the whole astrolabe and support it as it hangs from fastenings at the poles of the equator. Place this sixth ring on a stand, sustained by which it will be perpendicular to the plane of the horizon. Furthermore, when its poles have been adjusted to the inclination of the sphere, let the astrolabe keep its meridian's position similar to that of the meridian in nature, without the slightest swerving away from it.

Then with the instrument fashioned in this way, we may wish to obtain the place of a star. In the evening, or when the sun is about to set, at a time when we also have the moon in view, we will line up the outer ring with the degree of the ecliptic in which we have found by what precedes that the sun is known to be then. We will also turn the intersection of the rings towards the sun, until both of them, I mean, the ecliptic and that outer ring which passes through the poles [of the ecliptic] cast equal shadows on each other. Then we also turn the inner ring toward the moon. Placing our eye in the plane of the inner ring where we will see the moon opposite, as though it were bisected by the same plane, we will mark the spot on the instrument's ecliptic. For, that will be the observed place of the moon in longitude at that time. In fact, without the moon there was no way of understanding the positions of the stars, since of all the heavenly bodies it alone participates in the day and night. Then, as night descends, the star whose place we are seeking can now be seen. We fit the outer ring to the position of the moon. By means of this ring we adjust the position of the astrolabe to the moon, as we did in the case of the sun. Then we also turn the inner ring toward the star, until it seems to touch the plane of the ring, and is visible through the eyepieces which are on the smaller ring within. For in this way we shall find the longitude of the star together with its latitude. While these operations are being performed, the degree of the ecliptic at mid-heaven will be placed before our eyes, and therefore it will be clear as crystal at what hour the observation was carried out.

[Earlier draft:

After these rings have been arranged in this way, two other rings are made. These are not equal [to the first two rings] in diameter, but they resemble them in thickness and width. Attach the latter pair at the ecliptic's poles, fitting [one on the] outside and [the other on the] inside. Perforate them neatly, and install axles on which they may turn. But they are put together so that the outside ring's convex [surface] and the inside ring's concave [surface] touch [the ecliptic], yet without any friction which could interfere with their being rotated. On the inside ring too, divide the quadrants into degrees like those into which the ecliptic was divided. Furthermore, on the inside ring's concave surface another ring should be placed in the same plane, in which it can turn without interference in relation to the inside ring. To this [fifth ring] attach diametrically opposite brackets with apertures, as is the practice in the dioptra, for the purpose of observing the latitudes. Finally, a sixth ring must be attached which is capable of supporting the whole astrolabe, fastened to and swinging on the equator's [poles], as I said. Put [this sixth ring] on a stand or some other somewhat elevated place, where it is sustained perpendicular to the plane of the horizon. Furthermore, when its poles have been adjusted to the inclination of the sphere, let the ring keep the meridian as placed in nature aligned with itself, and do not let the ring swerve away from the meridian at all.

Then with the instrument fashioned in this way, we may wish to obtain the place of a star.

In the evening, or when the sun is about to set, at a time when the moon too can be seen, we line up the outside ring with the degree of the instrument's ecliptic in which the sun will be thought to appear at that time. We also turn the intersection of the rings toward the sun until both of them, the ecliptic and the outside ring which passes through the [ecliptic's] poles, cast shadows on each other that are equal and bisect each other. Then we also turn the inner ring toward the moon. Placing our eye on one side, where we will see the moon on the opposite side as though bisected by the same plane, we mark the spot on the instrument's ecliptic, since that will be the moon's place in longitude at that time. For without the moon there was no way of arriving at the positions of the stars, because it alone is the intermediary between light and darkness. Then, as night descends, the star whose place we want has now become visible. We put the outside ring on the place of the moon. By means of this ring we adjust the position of the astrolabe to the moon, as we did in the case of the sun. Then we also turn the inside ring toward the star, until ... [The earlier draft ends abruptly here].

For example, in the 2nd year of the emperor Antoninus Pius, on the 9th day of Pharmuthi, the 8th Egyptian month, about sunset, Ptolemy in Alexandria wanted to observe the place of the star in the chest of the Lion which is called Basiliscus or Regulus [*Syntaxis* VII, 2]. Training his astrolabe on the sun, which was already setting, 5 [$^1/_2$] equinoctial hours after noon, he found the sun at $3\,^1/_{24}{}^\circ$ within the Fishes. By moving the inner ring, he observed the moon following $92\,^1/_8{}^\circ$ after the sun. Therefore the place of the moon was then seen at $5\,^1/_6{}^\circ$ within the Twins. Half an hour later, when the 6th hour after noon was being completed, the star had already begun to appear, as 4° within the Twins was at mid-heaven. Ptolemy turned the outer ring of the instrument to the place where the moon had already been found. By proceeding with the inner ring, he determined the distance of the star from the moon in the order of the zodiacal signs as $57\,^1/_{10}{}^\circ$. Now the moon was found $92\,^1/_8{}^\circ$ away from the setting sun, as was mentioned, and this fixed the moon at $5\,^1/_6{}^\circ$ within the Twins. But in the interval of half an hour the moon should have moved $^1/_4{}^\circ$, since the fraction per hour of the moon's motion amounts to $^1/_2{}^\circ$, more or less. However, on account of the lunar parallax, which had to be subtracted at that time, the moon must have moved a little less than $^1/_4{}^\circ$, and he determined the difference as about $^1/_{12}{}^\circ$. Accordingly the moon must have been at $5\,^1/_3{}^\circ$ within the Twins. But when I discuss the lunar parallaxes, it will be evident that the difference was not so great [IV, 16]. Hence it can be quite clear that the observed place of the moon exceeded 5° within the Twins by more than $^1/_3{}^\circ$ and by hardly less than $^2/_5{}^\circ$. To this position, the addition of $57\,^1/_{10}{}^\circ$ establishes the place of the star at $2\,^1/_2{}^\circ$ within the Lion, at a distance from the sun's summer solstice of about $32\,^1/_2{}^\circ$, with a north latitude of $^1/_6{}^\circ$. This was the place of Basiliscus, through which the approach to all the other fixed stars lay open. Now this observation was performed by Ptolemy, according to the Roman [calendar] on 23 February 139 C.E., the first year of the 229th Olympiad.

In this way that most outstanding of astronomers noted the distance of each of the stars from the vernal equinox at that time, and he set forth the constellations of the celestial creatures. By these achievements he gave no small assistance to this study of mine, and relieved me of quite an arduous task. I believed that the places of the stars should not be located with reference to the equinoxes, whicfi shift in the course of time, but that the equinoxes should be located with reference to the sphere of the fixed stars. Hence I can easily start the cataloguing of the stars at some other unchangeable beginning. I have decided to commence with the Ram, as the first zodiacal sign, and with its first star, which is in its head.

My purpose is that in this way always the same definitive appearance will remain for those bodies which shine as a team, as though fixed and linked together, once they have taken their permanent place. Now through the wonderful zeal and skill of the ancients they were grouped into 48 figures. The exceptions are those stars which the circle of the perpetually hidden stars kept from the fourth clime, which passes near Rhodes, so that these stars, as unknown to the ancients, remained unattached to a constellation. Nor were the stars formed into figures for any other reason, according to the opinion of the younger Theon in his commentary on Aratus, than that their vast number should be separated into parts, which could be known one by one under certain designations. This practice is quite old, since we read that even Job, Hesiod, and Homer mentioned the Pleiades, Hyades, Arcturus, and Orion. Therefore in tabulating the stars according to their longitude, I shall not use the twelve zodiacal signs, which are derived from the equinoxes and solstices, but the simple and familiar number of degrees. In all other respects I shall follow Ptolemy, with a few exceptions, which I find either corrupt or distorted in some way. But the method of determining the distance of the stars from those cardinal points will be explained by me in the next Book.

DESCRIPTIVE CATALOGUE OF THE SIGNS AND STARS
I: THOSE WHICH ARE IN THE NORTHERN REGION

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
LITTLE BEAR OR DOG'S TAIL						
At the tip of the tail	53	30	N.	66	0	3
To the east in the tail	55	50	N.	70	0	4
At the beginning of the tail	69	20	N.	74	0	4
The more southerly [star] on the western side of the quadrangle	83	0	N.	75	20	4
The northern [star] on the same side	87	0	N.	77	40	4
The more southerly [star] on the [quadrangle's] eastern side	100	30	N.	72	40	2
The northern [star] on the same side	109	30	N.	74	50	2
7 stars: 2 of the 2nd magnitude, 1 of the 3rd, 4 of the 4th						
Near the Dog's Tail, outside the constellation, on a straight line with the [quadrangle's] eastern side, quite far to the south	103	20	N.	71	10	4
GREAT BEAR, [ALSO] CALLED THE DIPPER						
On the muzzle	78	40	N.	39	50	4
[Of the stars] in the two eyes, the one to the west	79	10	N.	43	0	5
East of the foregoing	79	40	N.	43	0	5
[Of the two stars] in the forehead, the one to the west	79	30	N.	47	10	5
The eastern [star] in the forehead	81	0	N.	47	0	5
At the edge of the western ear	81	30	N.	50	30	5
Of the two [stars] in the neck, the one to the west	85	50	N.	43	50	4
The one to the east	92	50	N.	44	20	4
Of the two [stars] in the chest, the one to the north	94	20	N.	44	0	4
The one farther south	93	20	N.	42	0	4
In the knee of the left foreleg	89	0	N.	35	0	3
Of the two [stars] in the left front paw, the one to the north	89	50	N.	29	0	3
The one farther south	88	40	N.	28	30	3
In the knee of the right foreleg	89	0	N.	36	0	4
Below that knee	101	10	N.	33	30	4
In the shoulder	104	0	N.	49	0	2
In the groin	105	30	N.	44	30	2
At the beginning of the tail	116	30	N.	51	0	3
In the left hind leg	117	20	N.	46	30	2
Of the two [stars] in the left hind paw, the one to the west	106	0	N.	29	38	3
East of the foregoing	107	30	N.	28	15	3
In the joint of the left [hind leg]	115	0	N.	35	15	4
Of the two [stars] in the right hind paw, the one to the north	123	10	N.	25	50	3

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
The one farther south	123	40	N.	25	0	3
Of the three [stars] in the tail, the first one east of the beginning [of the tail]	125	30	N.	53	30	2
The one in the middle of these [three]	131	20	N.	55	40	2
The last one, at the tip of the tail	143	10	N.	54	0	2
27 stars: 6 of the 2nd magnitude, 8 of the 3rd, 8 of the 4th, 5 of the 5th						
NEAR THE DIPPER, OUTSIDE THE CONSTELLATION						
South of the tail	141	10	N.	39	45	3
The dimmer [star] to the west of the foregoing	133	30	N.	41	20	5
Between the Bear's front paws and the Lion's head	98	20	N.	17	15	4
[The star] farther north than the foregoing	96	40	N.	19	10	4
The last of the three dim [stars]	99	30	N.	20	0	dim
To the west of the foregoing	95	30	N.	22	45	dim
Farther west	94	30	N.	23	15	dim
Between the front paws and the Twins	100	20	N.	22	15	dim
8 stars outside the constellation: 1 of the 3rd magnitude, 2 of the 4th, 1 of the 5th, 4 dim						
DRAGON						
In the tongue	200	0	N.	76	30	4
In the mouth	215	10	N.	78	30	4 brighter
Above the eye	216	30	N.	75	40	3
In the cheek	229	40	N.	75	20	4
Above the head	223	30	N.	75	30	3
In the first twisting of the neck, the one to the north	258	40	N.	82	20	4
Of these [stars], the one to the south	295	50	N.	78	15	4
The middle one of these same [stars]	262	10	N.	80	20	4
East of the foregoing, in the second twisting [of the neck]	282	50	N.	81	10	4
The southern [star] on the western side of the quadrilateral	331	20	N.	81	40	4
The northern [star] on the same side	343	50	N.	83	0	4
The northern [star] on the eastern side	1	0	N.	78	50	4
The southern [star] on the same side	346	10	N.	77	50	4
In the third twisting [of the neck], the southern [star] of the triangle	4	0	N.	80	30	4
Of the remaining [stars] of the triangle, the one to the west	15	0	N.	81	40	5
The one to the east	19	30	N.	80	15	5
Of the three [stars] in the triangle to the west, [the star to the east]	66	20	N.	83	30	4
Of the remaining [stars] in the same triangle, the one to the south	43	40	N.	83	30	4
The one to the north of the two preceding [stars]	35	10	N.	84	50	4
Of the two small [stars west] of the triangle, the one to the east	110	0	N.	87	30	6

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
Of these [two stars] the one to the west	105	0	N.	86	50	6
Of the three [stars] which follow in a straight line, the one to the south	152	30	N.	81	15	5
The middle one of the three	152	50	N.	83	0	5
The one farther north	151	0	N.	84	50	3
Of the two [stars] to the west of the foregoing, the one farther north	153	20	N.	78	0	3
[The one] farther south	156	30	N.	74	40	4 brighter
To the west of the foregoing, in the coil of the tail	156	0	N.	70	0	3
Of the two [stars] at a very great distance, the one to the west	120	40	N.	64	40	4
East of the foregoing	124	30	N.	65	30	3
To the east, on the tail	102	30	N.	61	15	3
At the tip of the tail	96	30	N.	56	15	3
Therefore, 31 stars: 8 of the 3rd magnitude, 17 of the 4th, 4 of the 5th, 2 of the 6th						
CEPHEUS						
In the right foot	28	40	N.	75	40	4
In the left foot	26	20	N.	64	15	4
On the right side below the belt	0	40	N.	71	10	4
Above the right shoulder and touching it	340	0	N.	69	0	3
Touching the right hip joint	332	40	N.	72	0	4
East of the same hip and touching it	333	20	N.	74	0	4
In the chest	352	0	N.	65	30	5
In the left arm	1	0	N.	62	30	4 brighter
Of the three [stars] in the tiara, the one to the south	339	40	N.	60	15	5
The one in the middle of these [three]	340	40	N.	61	15	4
Of the three, the one to the north	342	20	N.	61	30	5
11 stars: 1 of the 3rd magnitude, 7 of the 4th, 3 of the 5th						
Of the two [stars] outside the constellation, the one to the west of the tiara	337	0	N.	64	0	5
The one to the east of it	344	40	N.	59	30	4
HERDSMAN OR BEAR-KEEPER						
Of the three [stars] in the left hand, the one to the west	145	40	N.	58	40	5
The middle one of the three, farther south	147	30	N.	58	20	5
Of the three, the one to the east	149	0	N.	60	10	5
In the left hip joint	143	0	N.	54	40	5
In the left shoulder	163	0	N.	49	0	3
In the head	170	0	N.	53	50	4 brighter
In the right shoulder	179	0	N.	48	40	4
Of the two [stars] in the staff, the one farther south	179	0	N.	53	15	4

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
The one farther north, at the tip of the staff	178	20	N.	57	30	4
Of the two [stars] on the spear below the shoulder, the one to the north	181	0	N.	46	10	4 brighter
Of these [two], the one farther south	181	50	N.	45	30	5
At the tip of the right hand	181	35	N.	41	20	5
Of the two [stars] in the palm, the one to the west	180	0	N.	41	40	5
East of the foregoing	180	20	N.	42	30	5
At the tip of the handle of the staff	181	0	N.	40	20	5
In the right leg	173	20	N.	40	15	3
Of the two [stars] in the belt, the one to the east	169	0	N.	41	40	4
The one to the west	168	20	N.	42	10	4 brighter
In the right heel	178	40	N.	28	0	3
Of the three [stars] in the left leg, the one to the north	164	40	N.	28	0	3
The middle one of the three	163	50	N.	26	30	4
The one farther south	164	50	N.	25	0	4
22 stars: 4 of the 3rd magnitude, 9 of the 4th, 9 of the 5th						
Outside the constellation, between the legs, called "Arcturus"	170	20	N.	31	30	1
NORTHERN CROWN						
The bright [star] in the crown	188	0	N.	44	30	2 brighter
The [most] westerly of all	185	0	N.	46	10	4 brighter
East [of the foregoing], to the north	185	10	N.	48	0	5
East [of the foregoing] farther north	193	0	N.	50	30	6
East of the bright [star], to the south	191	30	N.	44	45	4
Immediately to the east [of the foregoing]	190	30	N.	44	50	4
Somewhat farther to the east of the foregoing	194	40	N.	46	10	4
The most easterly of all [the stars] in the crown	195	0	N.	49	20	4
8 stars: 1 of the 2nd magnitude, 5 of the 4th, 1 of the 5th, 1 of the 6th						
KNEELER						
In the head	221	0	N.	37	30	3
In the right armpit	207	0	N.	43	0	3
In the right arm	205	0	N.	40	10	3
In the right [side of the] groin	201	20	N.	37	10	4
In the left shoulder	220	0	N.	48	0	3
In the left arm	225	20	N.	49	30	4 brighter
In the left [side of the] groin	231	0	N.	42	0	4
Of the three [stars] in the left palm, [the one to the east]	238	50	N.	52	50	4 brighter
Of the other two, the one to the north	235	0	N.	54	0	4 brighter
The one farther south	234	50	N.	53	0	4
In the right side	207	10	N.	56	10	3
In the left side	213	30	N.	53	30	4

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
In the left buttock	213	20	N.	56	10	5
At the top of the same leg	214	30	N.	58	30	5
Of the three [stars] in the left leg, the one to the west	217	20	N.	59	50	3
East of the foregoing	218	40	N.	60	20	4
The third one, east [of the foregoing]	219	40	N.	61	15	4
In the left knee	237	10	N.	61	0	4
In the left thigh	225	30	N.	69	20	4
Of the three [stars] in the left foot, the one to the west	188	40	N.	70	15	6
The middle one of these [three]	220	10	N.	71	15	6
Of the three, the one to the east	223	0	N.	72	0	6
At the top of the right leg	207	0	N.	60	15	4 brighter
[The star] farther north in the same leg	198	50	N.	63	0	4
In the right knee	189	0	N.	65	30	4 brighter
Of the two [stars] below the same knee, the one farther south	186	40	N.	63	40	4
The one farther north	183	30	N.	64	15	4
In the right shin	184	30	N.	60	0	4
At the tip of the right foot; identical with [the star] at the tip of the Herdsman's staff	178	20	N.	57	30	4
Not including the foregoing, 28 stars: 6 of the 3rd magnitude, 17 of the 4th, 2 of the 5th, 3 of the 6th						
Outside the constellation, to the south of the right arm	206	0	N.	38	10	5
LYRE						
The bright [star] called "Lyre" or "Little Lute"	250	40	N.	62	0	1
Of the two adjacent [stars], the one to the north	253	40	N.	62	40	4 brighter
The one farther south	253	40	N.	61	0	4 brighter
Between the curvature of the arms	262	0	N.	60	0	4
Of the two [stars] close together in the east, the one to the north	265	20	N.	61	20	4
The one farther south	265	0	N.	60	20	4
Of the two [stars] to the west on the crosspiece, the one to the north	254	20	N.	56	10	3
The one farther south	254	10	N.	55	0	4 dimmer
Of the two [stars] to the east on the same crosspiece, the one to the north	257	30	N.	55	20	3
The one farther south	258	20	N.	54	45	4 dimmer
10 stars: 1 of the 1st magnitude, 2 of the 3rd, 7 of the 4th						
SWAN OR BIRD						
In the mouth	267	50	N.	49	20	3
In the head	272	20	N.	50	30	5
In the middle of the neck	279	20	N.	54	30	4 brighter

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
In the breast	291	50	N.	56	20	3
The bright [star] in the tail	302	30	N.	60	0	2
In the bend of the right wing	282	40	N.	64	40	3
Of the three [stars] in the spread of the right [wing], the one farther south	285	50	N.	69	40	4
The one in the middle	284	30	N.	71	30	4 brighter
The last of the three, at the tip of the wing	280	0	N.	74	0	4 brighter
In the bend of the left wing	294	10	N.	49	30	3
In the middle of that wing	298	10	N.	52	10	4 brighter
At the tip of the same [wing]	300	0	N.	74	0	3
In the left foot	303	20	N.	55	10	4 brighter
In the left knee	307	50	N.	57	0	4
Of the two [stars] in the right foot, the one to the west	294	30	N.	64	0	4
The one to the east	296	0	N.	64	30	4
The cloudy [star] in the right knee	305	30	N.	63	45	5
17 stars: 1 of the 2nd magnitude, 5 of the 3rd, 9 of the 4th, 2 of the 5th						
TWO ADDITIONAL [STARS] NEAR THE SWAN, OUTSIDE THE CONSTELLATION						
Of the two [stars] below the left wing, the one farther south	306	0	N.	49	40	4
The one farther north	307	10	N.	51	40	4
CASSIOPEA						
In the head	1	10	N.	45	20	4
In the breast	4	10	N.	46	45	3 brighter
In the girdle	6	20	N.	47	50	4
Above the seat, at the hips	10	0	N.	49	0	3 brighter
At the knees	13	40	N.	45	30	3
In the leg	20	20	N.	47	45	4
At the tip of the foot	355	0	N.	48	20	4
In the left arm	8	0	N.	44	20	4
In the left elbow	7	40	N.	45	0	5
In the right elbow	357	40	N.	50	0	6
In the foot of the chair	8	20	N.	52	40	4
In the middle of the back [of the chair]	1	10	N.	51	40	3 dimmer
At the edge [of the back of the chair]	357	10	N.	51	40	6
13 stars: 4 of the 3rd magnitude, 6 of the 4th, 1 of the 5th, 2 of the 6th						
PERSEUS						
At the tip of the right hand, in a cloudy wrapping	21	0	N.	40	30	cloudy
In the right elbow	24	30	N.	37	30	4
In the right shoulder	26	0	N.	34	30	4 dimmer
In the left shoulder	20	50	N.	32	20	4
In the head or cloud	24	0	N.	34	30	4
In the shoulder blades	24	50	N.	31	10	4
The bright [star] on the right side	28	10	N.	30	0	2

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
Of the three [stars] on the same side, the one to the west	28	40	N.	27	30	4
The one in the middle	30	20	N.	27	40	4
The remaining [one] of the three	31	0	N.	27	30	3
In the left elbow	24	0	N.	27	0	4
The bright [star] in the left hand, and in the head of Medusa	23	0	N.	23	0	2
In the same head, the one to the east	22	30	N.	21	0	4
In the same head, the one to the west	21	0	N.	21	0	4
Still farther west of the foregoing	20	10	N.	22	15	4
In the right knee	38	10	N.	28	15	4
In the knee, to the west of the foregoing	37	10	N.	28	10	4
Of the two [stars] in the belly, the one to the west	35	40	N.	25	10	4
The one to the east	37	20	N.	26	15	4
In the right hip	37	30	N.	24	30	5
In the right calf	39	40	N.	28	45	5
In the left hip	30	10	N.	21	40	4 brighter
In the left knee	32	0	N.	19	50	3
In the left leg	31	40	N.	14	45	3 brighter
In the left heel	24	30	N.	12	0	3 dimmer
At the top of the foot, on the left side	29	40	N.	11	0	3 brighter

26 stars: 2 of the 2nd magnitude, 5 of the 3rd, 16 of the 4th, 2 of the 5th, 1 cloudy

NEAR PERSEUS, OUTSIDE THE CONSTELLATION

East of the left knee	34	10	N.	31	0	5
North of the right knee	38	20	N.	31	0	5
West of the head of Medusa	18	0	N.	20	40	dim

3 stars: 2 of the 5th magnitude, 1 dim

REINSMAN OR CHARIOTEER

Of the two [stars] in the head, the one farther south	55	50	N.	30	0	4
The one farther north	55	40	N.	30	50	4
The bright [star] in the left shoulder, called "Capella"	78	20	N.	22	30	1
In the right shoulder	56	10	N.	20	0	2
In the right elbow	54	30	N.	15	15	4
In the right palm	56	10	N.	13	30	4 brighter
In the left elbow	45	20	N.	20	40	4 brighter
Of the goats, the one to the west	45	30	N.	18	0	4 dimmer
Of the goats in the left palm, the one to the east	46	0	N.	18	0	4 brighter
In the left calf	53	10	N.	10	10	3 dimmer
In the right calf, and at the tip of the northern horn of the Bull	49	0	N.	5	0	3 brighter
In the ankle	49	20	N.	8	30	5
In the buttock	49	40	N.	12	20	5

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
The small [star] in the left foot	24	0	N.	10	20	6
14 stars: 1 of the first magnitude, 1 of the 2nd, 2 of the 3rd, 7 of the 4th, 2 of the 5th, 1 of the 6th						
SERPENT CARRIER OR SNAKE HOLDER						
In the head	228	10	N.	36	0	3
Of the two [stars] in the right shoulder, the one to the west	231	20	N.	27	15	4 brighter
The one to the east	232	20	N.	26	45	4
Of the two [stars] in the left shoulder, the one to the west	216	40	N.	33	0	4
The one to the east	218	0	N.	31	50	4
In the left elbow	211	40	N.	34	30	4
Of the two [stars] in the left hand, the one to the west	208	20	N.	17	0	4
The one to the east	209	20	N.	12	30	3
In the right elbow	220	0	N.	15	0	4
In the right hand, the one to the west	205	40	N.	18	40	4 dimmer
The one to the east	207	40	N.	14	20	4
In the right knee	224	30	N.	4	30	3
In the right shin	227	0	N.	2	15	3 brighter
Of the four [stars] in the right foot, the one to the west	226	20	S.	2	15	4 brighter
The one to the east	227	40	S.	1	30	4 brighter
The third one, to the east	228	20	S.	0	20	4 brighter
The remaining one, to the east	229	10	S.	0	45	5 brighter
Touching the heel	229	30	S.	1	0	5
In the left knee	215	30	N.	11	50	3
Of the three [stars] in the left leg, in a straight line, the one to the north	215	0	N.	5	20	5 brighter
The middle one of these [three]	214	0	N.	3	10	5
Of the three, the one farther south	213	10	N.	1	40	5 brighter
In the left heel	215	40	N.	0	40	5
Touching the instep of the left foot	214	0	S.	0	45	4
24 stars: 5 of the 3rd magnitude, 13 of the 4th, 6 of the 5th						
NEAR THE SERPENT CARRIER, OUTSIDE THE CONSTELLATION						
Of the three [stars] to the east of the right shoulder, the one farthest north	235	20	N.	28	10	4
The middle one of the three	236	0	N.	26	20	4
The southern one of the three	233	40	N.	25	0	4
Farther east of the three	237	0	N.	27	0	4
At a distance from the four, to the north	238	0	N.	33	0	4
Therefore, 5 [stars] outside the constellation, all of the 4th magnitude						
THE SERPENT OF THE SERPENT CARRIER						
In the quadrilateral, in the cheek	192	10	N.	38	0	4
Touching the nostrils	201	0	N.	40	0	4

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
In the temple	197	40	N.	35	0	3
At the beginning of the neck	195	20	N.	34	15	3
In the middle of the quadrilateral and in the mouth	194	40	N.	37	15	4
North of the head	201	30	N.	42	30	4
In the first curve of the neck	195	0	N.	29	15	3
Of the three [stars] to the east, the one to the north	198	10	N.	26	30	4
The middle one of these	197	40	N.	25	20	3
The most southerly of the three	199	40	N.	24	0	3
Of the two [stars] in the Snake Holder's left [hand], the one to the west	202	0	N.	16	30	4
East of the foregoing in the same hand	211	30	N.	16	15	5
East of the right hip	227	0	N.	10	30	4
Of the two [stars] east [of the foregoing], the one to the south	230	20	N.	8	30	4 brighter
The one to the north	231	10	N.	10	30	4
East of the right hand in the coil of the tail	237	0	N.	20	0	4
East [of the foregoing] in the tail	242	0	N.	21	10	4 brighter
At the tip of the tail	251	40	N.	27	0	4
18 stars: 5 of the 3rd magnitude, 12 of the 4th, 1 of the 5th						
ARROW						
At the tip	273	30	N.	39	20	4
Of the three [stars] in the shaft, the one to the east	270	0	N.	39	10	6
The middle one of these [three]	269	10	N.	39	50	5
The western one of the three	268	0	N.	39	0	5
In the notch	266	40	N.	38	45	5
5 stars: 1 of the 4th magnitude, 3 of the 5th, 1 of the 6th						
EAGLE						
In the middle of the head	270	30	N.	26	50	4
In the neck	268	10	N.	27	10	3
In the shoulder blades, the bright [star] called the "Eagle"	267	10	N.	29	10	2 brighter
Very near the foregoing, farther north	268	0	N.	30	0	3 dimmer
In the left shoulder, the one to the west	266	30	N.	31	30	3
The one to the east	269	20	N.	31	30	5
In the right shoulder, the one to the west	263	0	N.	28	40	5
The one to the east	264	30	N.	26	40	5 brighter
In the tail, touching the Milky Way	255	30	N.	26	30	3
9 stars: 1 of the 2nd magnitude, 4 of the 3rd, 1 of the 4th, 3 of the 5th						
NEAR THE EAGLE, OUTSIDE THE CONSTELLATION						
South of the head, the one to the west	272	0	N.	21	40	3
The one to the east	272	10	N.	29	10	3

Constellations of the stars	Longitude: Degrees	Longitude: Minutes	Latitude	Latitude: Degrees	Latitude: Minutes	Magnitude
To the southwest of the right shoulder	259	20	N.	25	0	4 brighter
To the south [of the foregoing]	261	30	N.	20	0	3
Farther south	263	0	N.	15	30	5
The westernmost of all [six stars outside the constellation]	254	30	N.	18	10	3
6 stars outside the constellation: 4 of the 3rd magnitude, 1 of the 4th, and 1 of the 5th						
DOLPHIN						
Of the three [stars] in the tail, the one to the west	281	0	N.	29	10	3 dimmer
Of the other two, the one farther north	282	0	N.	29	0	4 dimmer
The one farther south	282	0	N.	26	40	4
In the western side of the rhomboid, the one farther south	281	50	N.	32	0	3 dimmer
In the same side, the one to the north	283	30	N.	33	50	3 dimmer
In the eastern side, the one to the south	284	40	N.	32	0	3 dimmer
In the same side, the one to the north	286	50	N.	33	10	3 dimmer
Of the three [stars] between the tail and the rhombus, the one farther south	280	50	N.	34	15	6
Of the other two toward the north, the one to the west	280	50	N.	31	50	6
The one to the east	282	20	N.	31	30	6
10 stars, namely, 5 of the 3rd magnitude, 2 of the 4th, 3 of the 6th						
HORSE SEGMENT						
Of the two [stars] in the head, the one to the west	289	40	N.	20	30	dim
The one to the east	292	20	N.	20	40	dim
Of the two [stars] in the mouth, the one to the west	289	40	N.	25	30	dim
The one to the east	291	0	N.	25	0	dim
4 stars, all dim						
WINGED HORSE OR PEGASUS						
In the open mouth	298	40	N.	21	30	3 brighter
Of the two [stars] close together in the head, the one to the north	302	40	N.	16	50	3
The one farther south	301	20	N.	16	0	4
Of the two [stars] in the mane, the one farther south	314	40	N.	15	0	5
The one farther north	313	50	N.	16	0	5
Of the two [stars] in the neck, the one to the west	312	10	N.	18	0	3
The one to the east	313	50	N.	19	0	4
In the left hock	305	40	N.	36	30	4 brighter
In the left knee	311	0	N.	34	15	4 brighter

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
In the right hock	317	0	N.	41	10	4 brighter
Of the two [stars] close together in the chest, the one to the west	319	30	N.	29	0	4
The one to the east	320	20	N.	29	30	4
Of the two [stars] in the right knee, the one to the north	322	20	N.	35	0	3
The one farther south	321	50	N.	24	30	5
Of the two [stars] in the body below the wing, the one to the north	327	50	N.	25	40	4
The one farther south	328	20	N.	25	0	4
In the shoulder blades and attachment of the wing	350	0	N.	19	40	2 dimmer
In the right shoulder and top of the leg	325	30	N.	31	0	2 dimmer
At the tip of the wing	335	30	N.	12	30	2 dimmer
In the midriff; also in the head of Andromeda	341	10	N.	26	0	2 dimmer
20 stars, namely, 4 of the 2nd magnitude, 4 of the 3rd, 9 of the 4th, 3 of the 5th						
ANDROMEDA						
In the shoulder blades	348	40	N.	24	30	3
In the right shoulder	349	40	N.	27	0	4
In the left shoulder	347	40	N.	23	0	4
Of the three [stars] in the right arm, the one farther south	347	0	N.	32	0	4
The one farther north	348	0	N.	33	30	4
The middle one of the three	348	20	N.	32	20	5
Of the three [stars] at the tip of the right hand, the one farther south	343	0	N.	41	0	4
The middle one of these [three]	344	0	N.	42	0	4
The northern one of the three	345	30	N.	44	0	4
In the left arm	347	30	N.	17	30	4
In the left elbow	349	0	N.	15	50	3
Of the three [stars] in the girdle, the one to the south	357	10	N.	25	20	3
The one in the middle	355	10	N.	30	0	3
The northern one of the three	355	20	N.	32	30	3
In the left foot	10	10	N.	23	0	3
In the right foot	10	30	N.	37	20	4 brighter
To the south of these	8	30	N.	35	20	4 brighter
Of the two [stars] below the back of the knee, the one to the north	5	40	N.	29	0	4
The one to the south	5	20	N.	28	0	4
In the right knee	5	30	N.	35	30	5
Of the two [stars] in the robe or its train, the one to the north	6	0	N.	34	30	5
The one to the south	7	30	N.	32	30	5
At a distance from the right hand and outside the constellation	5	0	N.	44	0	3
23 stars, since [there are] 7 of the 3rd magnitude, 12 of the 4th, 4 of the 5th						

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
TRIANGLE						
In the vertex of the triangle	4	20	N.	16	30	3
Of the three [stars] in the base, the one to the west	9	20	N.	20	40	3
The one in the middle	9	30	N.	20	20	4
Of the three, the one to the east	10	10	N.	19	0	3
4 stars: 3 of the 3rd magnitude, 1 of the 4th						
Accordingly, in the northern region [there are] altogether 360 stars: 3 of the 1st magnitude, 18 of the 2nd, 81 of the 3rd, 177 of the 4th, 58 of the 5th, 13 of the 6th, 1 cloudy, 9 dim.						

[II:] THOSE WHICH ARE IN THE MIDDLE AND NEAR THE ZODIAC

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
RAM						
Of the two [stars] in the horn, the one to the west, and the first of all [the stars]	0	0	N.	7	20	3 dimmer
In the horn, the one to the east	1	0	N.	8	20	3
Of the two [stars] in the open mouth, the one to the north	4	20	N.	7	40	5
The one farther south	4	50	N.	6	0	5
In the neck	9	50	N.	5	30	5
In the loins	10	50	N.	6	0	6
At the beginning of the tail	14	40	N.	4	50	5
Of the three [stars] in the tail, the one to the west	17	10	N.	1	40	4
The one in the middle	18	40	N.	2	30	4
Of the three, the one to the east	20	20	N.	1	50	4
In the hip	13	0	N.	1	10	5
Behind the knee	11	20	S.	1	30	5
At the tip of the hind foot	8	10	S.	5	15	4 brighter
13 stars: 2 of the 3rd magnitude, 4 of the 4th, 6 of the 5th, 1 of the 6th						
NEAR THE RAM, OUTSIDE THE CONSTELLATION						
The bright [star] above the head	3	50	N.	10	0	3 brighter
Above the back, the farthest to the north	15	0	N.	10	10	4
Of the remaining three dim [stars], the one to the north	14	40	N.	12	40	5
The one in the middle	13	0	N.	10	40	5
Of these [three], the one to the south	12	30	N.	10	40	5
5 stars: 1 of the 3rd magnitude, 1 of the 4th, 3 of the 5th						
BULL						
Of the four [stars] at the cut, the one farthest north	19	40	S.	6	0	4
The second one, after the foregoing	19	20	S.	7	15	4
The third one	18	0	S.	8	30	4
The fourth one, the farthest south	17	50	S.	9	15	4
In the right shoulder	23	0	S.	9	30	5
In the chest	27	0	S.	8	0	3
In the right knee	30	0	S.	12	40	4
In the right hock	26	20	S.	14	50	4
In the left knee	35	30	S.	10	0	4
In the left hock	36	20	S.	13	30	4
Of the Hyades, the 5 [stars] in the face which are called the "Piglets", the one in the nostrils	32	0	S.	5	45	3 dimmer
Between the foregoing and the northern eye	33	40	S.	4	15	3 dimmer
Between the same [star] and the southern eye	34	10	S.	0	50	3 dimmer
In that eye, the bright [star] called "Palilicium" by the Romans	36	0	S.	5	10	1

Venus' apogee 48°20′

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
In the northern eye	35	10	S.	3	0	3 dimmer
Between the beginning of the southern horn and the ear	40	30	S.	4	0	4
Of the two [stars] in the same horn, the one farther south	43	40	S.	5	0	4
The one farther north	43	20	S.	3	30	5
At the tip of the same [horn]	50	30	S.	2	30	3
At the beginning of the northern horn	49	0	S.	4	0	4
At the tip of the same [horn], and also in the right foot of the Reinsman	49	0	N.	5	0	3
Of the two [stars] in the northern ear, the one to the north	35	20	N.	4	30	5
Of these [two], the one to the south	35	0	N.	4	0	5
Of the two small [stars] in the neck, the one to the west	30	20	N.	0	40	5
The one to the east	32	20	N.	1	0	6
Of the western [stars] of the quadrilateral in the neck, the one to the south	31	20	N.	5	0	5
The one to the north on the same side	32	10	N.	7	10	5
The one to the south on the eastern side	35	20	N.	3	0	5
The one to the north on this side	35	0	N.	5	0	5
Of the western side of the Pleiades, the northern end [called] "Vergiliae"	25	30	N.	4	30	5
The southern end of the same side	25	50	N.	4	40	5
The eastern, very narrow end of the Pleiades	27	0	N.	5	20	5
The small [star] of the Pleiades, at a distance from the outermost	26	0	N.	3	0	5
32 stars, not including the one at the tip of the northern horn: 1 of the 1st magnitude, 6 of the 3rd, 11 of the 4th, 13 of the 5th, 1 of the 6th						
NEAR THE BULL, OUTSIDE THE CONSTELLATION						
Below, between the foot and the shoulder	18	20	S.	17	30	4
Of the three [stars] near the southern horn, the one to the west	43	20	S.	2	0	5
The middle one of the three	47	20	S.	1	45	5
The eastern one of the three	49	20	S.	2	0	5
Of the two [stars] below the tip of the same horn, the one to the north	52	20	S.	6	20	5
The one to the south	52	20	S.	7	40	5
Of the five [stars] below the northern horn, the one to the west	50	20	N.	2	40	5
The second one to the east	52	20	N.	1	0	5
The third one to the east	54	20	N.	1	20	5
Of the remaining two, the one to the north	55	40	N.	3	20	5
The one to the south	56	40	N.	1	15	5
11 stars outside the constellation: 1 of the 4th magnitude, 10 of the 5th						
TWINS						
In the head of the western Twin, Castor	76	40	N.	9	30	2

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
The yellowish [star] in the head of the eastern Twin, Pollux	79	50	N.	6	15	2
In the left elbow of the western Twin	70	0	N.	10	0	4
In the same arm	72	0	N.	7	20	4
In the shoulder blades of the same Twin	75	20	N.	5	30	4
In the right shoulder of the same [Twin]	77	20	N.	4	50	4
In the left shoulder of the eastern Twin	80	0	N.	2	40	4
In the right side of the western Twin	75	0	N.	2	40	5
In the left side of the eastern Twin	76	30	N.	3	0	5
In the left knee of the western Twin	66	30	N.	1	30	3
In the left knee of the eastern [Twin]	71	35	S.	2	30	3
In the left groin of the same [Twin]	75	0	S.	0	30	3
In the right joint of the same [Twin]	74	40	S.	0	40	3
The western [star] in the foot of the western Twin	60	0	S.	1	30	4 brighter
The eastern [star] in the same foot	61	30	S.	1	15	4
At the end of the foot of the western Twin	63	30	S.	3	30	4
At the top of the foot of the eastern [Twin]	65	20	S.	7	30	3
At the bottom of the same foot	68	0	S.	10	30	4
18 stars: 2 of the 2nd magnitude, 5 of the 3rd, 9 of the 4th, 2 of the 5th						
NEAR THE TWINS, OUTSIDE THE CONSTELLATION						
The western [star] at the top of the foot of the western Twin	57	30	S.	0	40	4
The bright [star] west of the knee of the same [Twin]	59	50	N.	5	50	4 brighter
West of the left knee of the eastern Twin	68	30	S.	2	15	5
Of the three [stars] east of the right hand of the eastern [Twin], the one to the north	81	40	S.	1	20	5
The one in the middle	79	40	S.	3	20	5
Of the three [stars] near the right arm, the one to the south	79	20	S.	4	30	5
The bright [star] east of the three	84	0	S.	2	40	4
7 stars outside the constellation: 3 of the 4th magnitude, 4 of the 5th						
CRAB						
The middle [star] in the cloud in the chest; called "Praesepe"	93	40	N.	0	40	cloudy
Of the two western [stars] of the quadrilateral, the one to the north	91	0	N.	1	15	4 dimmer
The one to the south	91	20	S.	1	10	4 dimmer
Of the two eastern [stars] called the "Asses", the one to the north	93	40	N.	2	40	4 brighter
The southern Ass	94	40	S.	0	10	4 brighter
In the southern claw or arm	99	50	S.	5	30	4
In the northern arm	91	40	N.	11	50	4
At the tip of the northern foot	86	0	N.	1	0	5
At the tip of the southern foot	90	30	S.	7	30	4 brighter
9 stars: 7 of the 4th magnitude, 1 of the 5th, 1 cloudy						

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
NEAR THE CRAB, OUTSIDE THE CONSTELLATION						
Above the elbow of the southern claw	103	0	S.	2	40	4 dimmer
East of the tip of the same claw	105	0	S.	5	40	4 dimmer
Of the two [stars] above the small cloud, the one to the west	97	20	N.	4	50	5
East of the foregoing	100	20	N.	7	15	5
4 [stars] outside the constellation: 2 of the 4th magnitude, 2 of the 5th						
LION						
In the nostrils	101	40	N.	10	0	4
In the open mouth	104	30	N.	7	30	4
Of the two [stars] in the head, the one to the north	107	40	N.	12	0	3
The one to the south	107	30	N.	9	30	3 brighter
Of the three [stars] in the neck, the one to the north	113	30	N.	11	0	3
The one in the middle	115	30	N.	8	30	2
Of the three, the one to the south	114	0	N.	4	30	3
In the heart; called "Little King" or "Regulus"	115	50	N.	0	10	1
Of the two [stars] in the chest, the one to the south	116	50	S.	1	50	4
Slightly to the west of the star in the heart	113	20	S.	0	15	5
In the knee of the right foreleg	110	40		0	0	5
In the right paw	117	30	S.	3	40	6
In the knee of the left foreleg	122	30	S.	4	10	4
In the left paw	115	50	S.	4	15	4
In the left armpit	122	30	S.	0	10	4
Of the three [stars] in the belly, the one to the west	120	20	N.	4	0	6
Of the two to the east, the one to the north	126	20	N.	5	20	6
The one to the south	125	40	N.	2	20	6
Of the two [stars] in the loins, the one to the west	124	40	N.	12	15	5
The one to the east	127	30	N.	13	40	2
Of the two [stars] in the buttock, the one to the north	127	40	N.	11	30	5
The one to the south	129	40	N.	9	40	3
In the hind hip	133	40	N.	5	50	3
In the bend [of the leg]	135	0	N.	1	15	4
In the joint of the hind [leg]	135	0	S.	0	50	4
In the hind foot	134	0	S.	3	0	5
At the tip of the tail	137	50	N.	11	50	1 dimmer
27 stars: 2 of the 1st magnitude, 2 of the 2nd, 6 of the 3rd, 8 of the 4th, 5 of the 5th, 4 of the 6th						
NEAR THE LION, OUTSIDE THE CONSTELLATION						
Of the two [stars] above the back, the one to the west	119	20	N.	13	20	5

Mars' apogee 109°50′

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
The one to the east	121	30	N.	15	30	5
Of the three [stars] below the belly, the one to the north	129	50	N.	1	10	4 dimmer
The one in the middle	130	30	S.	0	30	5
Of the three, the one to the south	132	20	S.	2	40	5
In the cloudy formation between the outermost [stars] of the Lion and the Bear, the star farthest to the north, called "Berenice's Hair"	138	10	N.	30	0	brilliant
Of the two [stars] to the south, the one to the west	133	50	N.	25	0	dim
The one to the east, in the shape of an ivy leaf	141	50	N.	25	30	dim
Outside the constellation, 8 [stars]: 1 of the 4th magnitude, 4 of the 5th, 1 brilliant, 2 dim						
VIRGIN						
Of the two [stars] at the top of the head, the one to the west and south	139	40	N.	4	15	5
The one to the east and farther north	140	20	N.	5	40	5
Of the two [stars] in the face, the one to the north	144	0	N.	8	0	5
The one to the south	143	30	N.	5	30	5
At the tip of the left, southern wing	142	20	N.	6	0	3
Of the four [stars] in the left wing, the one to the west	151	35	N.	1	10	3
The second one, to the east	156	30	N.	2	50	3
The third	160	30	N.	2	50	5
The last of the four, to the east	164	20	N.	1	40	4
In the right side below the girdle	157	40	N.	8	30	3
Of the three [stars] in the right, northern wing, the one to the west	151	30	N.	13	50	5
Of the other two, the one to the south	153	30	N.	11	40	6
Of these [two], the one to the north, called "Vindemiator"	155	30	N.	15	10	3 brighter
In the left hand; called the "Spike"	170	0	S.	2	0	1
Below the girdle and in the right buttock	168	10	N.	8	40	3
Of the western [stars] in the quadrilateral in the left hip, the one to the north	169	40	N.	2	20	5
The one to the south	170	20	N.	0	10	6
Of the two eastern [stars], the one to the north	173	20	N.	1	30	4
The one to the south	171	20	N.	0	20	5
In the left knee	175	0	N.	1	30	5
On the eastern [side] of the right hip	171	20	N.	8	30	5
In the gown, the one in the middle	180	0	N.	7	30	4
The one to the south	180	40	N.	2	40	4
The one to the north	181	40	N.	11	40	4
In the left, southern foot	183	20	N.	0	30	4
In the right, northern foot	186	0	N.	9	50	3
26 stars: 1 of the 1st magnitude, 7 of the 3rd, 6 of the 4th, 10 of the 5th, 2 of the 6th						

Jupiter's apogee 154°20′

Mercury's apogee 183°20′

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
NEAR THE VIRGIN, OUTSIDE THE CONSTELLATION						
Of the three [stars] in a straight line below the left arm, the one to the west	158	0	S.	3	30	5
The one in the middle	162	20	S.	3	30	5
The one to the east	165	35	S.	3	20	5
Of the three [stars] in a straight line below the Spike, the one to the west	170	30	S.	7	20	6
The one in the middle, a double [star]	171	30	S.	8	20	5
Of the three, the one to the east	173	20	S.	7	50	6
6 [stars] outside the constellation: 4 of the 5th magnitude, 2 of the 6th						
CLAWS						
Of the two [stars] at the tip of the southern claw, the bright one	191	20	N.	0	40	2 brighter
The dimmer one to the north	190	20	N.	2	30	5
Of the two [stars] at the tip of the northern claw, the bright one	195	30	N.	8	30	2
The dimmer one, west of the foregoing	191	0	N.	8	30	5
In the middle of the southern claw	197	20	N.	1	40	4
In the same [claw], the one to the west	194	40	N.	1	15	4
In the middle of the northern claw	200	50	N.	3	45	4
In the same [claw, the star] to the east	206	20	N.	4	30	4
8 stars: 2 of the 2nd magnitude, 4 of the 4th, 2 of the 5th						
NEAR THE CLAWS, OUTSIDE THE CONSTELLATION						
Of the three [stars] north of the northern claw, the one to the west	199	30	N.	9	0	5
Of the two to the east, the one to the south	207	0	N.	6	40	4
Of these [two], the one to the north	207	40	N.	9	15	4
Of the three [stars] between the claws, the one to the east	205	50	N.	5	30	6
Of the other two to the west, the one to the north	203	40	N.	2	0	4
The one to the south	204	30	N.	1	30	5
Of the three [stars] below the southern claw, the one to the west	196	20	S.	7	30	3
Of the other two to the east, the one to the north	204	30	S.	8	10	4
The one to the south	205	20	S.	9	40	4
9 [stars] outside the constellation: 1 of the 3rd magnitude, 5 of the 4th, 2 of the 5th, 1 of the 6th						
SCORPION						
Of the three bright [stars] in the forehead, the one to the north	209	40	N.	1	20	3 brighter
The one in the middle	209	0	S.	1	40	3
Of the three, the one to the south	209	0	S.	5	0	3

Constellations of the stars	Longitude		Latitude			Magnitude	
	Degrees	Minutes		Degrees	Minutes		
Farther south and in the foot	209	20	S.	7	50	3	
Of the two [stars] close together, the bright one to the north	210	20	N.	1	40	4	
The one to the south	210	40	N.	0	30	4	
Of the three bright [stars] in the body, the one to the west	214	0	S.	3	45	3	
The reddish [star] in the middle, called "Antares"	216	0	S.	4	0	2 brighter	
Of the three, the one to the east	217	50	S.	5	30	3	
Of the two [stars] in the last claw, the one to the west	212	40	S.	6	10	5	
The one to the east	213	50	S.	6	40	5	
In the first segment of the body	221	50	S.	11	0	3	
In the second segment	222	10	S.	15	0	4	
Of the double [star] in the third [segment], the one to the north	223	20	S.	18	40	4	
Of the double [star], the one to the south	223	30	S.	18	0	3	
In the fourth segment	226	30	S.	19	30	3	Saturn's apogee 226°30′
In the fifth [segment]	231	30	S.	18	50	3	
In the sixth segment	233	50	S.	16	40	3	
In the seventh [segment], the star next to the sting	232	20	S.	15	10	3	
Of the two [stars] in the sting, the one to the east	230	50	S.	13	20	3	
The one to the west	230	20	S.	13	30	4	
21 stars: 1 of the 2nd magnitude, 13 of the 3rd, 5 of the 4th, 2 of the 5th							
NEAR THE SCORPION, OUTSIDE THE CONSTELLATION							
The cloudy [star], east of the sting	234	30	S.	13	15	cloudy	
Of the two [stars] north of the sting, the one to the west	228	50	S.	6	10	5	
The one to the east	232	50	S.	4	10	5	
3 [stars] outside the constellation: 2 of the 5th magnitude, 1 cloudy							
ARCHER							
At the tip of the arrow	237	50	S.	6	30	3	
In the grip of the left hand	241	0	S.	6	30	3	
In the southern part of the bow	241	20	S.	10	50	3	
Of the two [stars] in the northern [part of the bow], the one to the south	242	20	S.	1	30	3	
Farther north at the tip of the bow	240	0	N.	2	50	4	
In the left shoulder	248	40	S.	3	10	3	
To the west of the foregoing, in the arrow	246	20	S.	3	50	4	
The double, cloudy [star] in the eye	248	30	N.	0	45	cloudy	
Of the three [stars] in the head, the one to the west	249	0	N.	2	10	4	
The one in the middle	251	0	N.	1	30	4 brighter	
The one to the east	252	30	N.	2	0	4	

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
Of the three [stars] in the northern [part of the] garment, the one farther south	254	40	N.	2	50	4
The one in the middle	255	40	N.	4	30	4
Of the three, the one to the north	256	10	N.	6	30	4
The dim [star] east of the three [foregoing]	259	0	N.	5	30	6
Of the two [stars] in the southern [part of the] garment, the one to the north	262	50	N.	5	50	5
The one to the south	261	0	N.	2	0	6
In the right shoulder	255	40	S.	1	50	5
In the right elbow	258	10	S.	2	50	5
In the shoulder blades	253	20	S.	2	30	5
In the broad of the back	251	0	S.	4	30	4 brighter
Below the armpit	249	40	S.	6	45	3
In the hock of the left front [leg]	251	0	S.	23	0	2
In the knee of the same leg	250	20	S.	18	0	2
In the hock of the right front [leg]	240	0	S.	13	0	3
In the left shoulder blade	260	40	S.	13	30	3
In the knee of the right front [leg]	260	0	S.	20	10	3
Of the four [stars] on the northern side at the beginning of the tail, the one to the west	261	0	S.	4	50	5
On the same side, the one to the east	261	10	S.	4	50	5
On the southern side, the one to the west	261	50	S.	5	50	5
On the same side, the one to the east	263	0	S.	6	30	5
31 stars: 2 of the 2nd magnitude, 9 of the 3rd, 9 of the 4th, 8 of the 5th, 2 of the 6th, 1 cloudy						
GOAT						
Of the three [stars] in the western horn, the one to the north	270	40	N.	7	30	3
The one in the middle	271	0	N.	6	40	6
Of the three, the one to the south	270	40	N.	5	0	3
At the tip of the eastern horn	272	20	N.	8	0	6
Of the three [stars] in the open mouth, the one to the south	272	20	N.	0	45	6
Of the other two, the one to the west	272	0	N.	1	45	6
The one to the east	272	10	N.	1	30	6
Below the right eye	270	30	N.	0	40	5
Of the two [stars] in the neck, the one to the north	275	0	N.	4	50	6
The one to the south	275	10	S.	0	50	5
In the right knee	274	10	S.	6	30	4
In the left, bent knee	275	0	S.	8	40	4
In the left shoulder	280	0	S.	7	40	4
Of the two [stars] close together below the belly, the one to the west	283	30	S.	6	50	4
The one to the east	283	40	S.	6	0	5
Of the three [stars] in the middle of the body, the one to the east	282	0	S.	4	15	5
Of the two others to the west, the one to the south	280	0	S.	4	0	5

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
Of these [two], the one to the north	280	0	S.	2	50	5
Of the two [stars] in the back, the one to the west	280	0	S.	0	0	4
The one to the east	284	20	S.	0	50	4
Of the two [stars] in the southern [part of the] rib cage, the one to the west	286	40	S.	4	45	4
The one to the east	288	20	S.	4	30	4
Of the two [stars] at the beginning of the tail, the one to the west	288	10	S.	2	10	3
The one to the east	289	40	S.	2	0	3
Of the four [stars] in the northern part of the tail, the one to the west	290	10	S.	2	20	4
Of the other three, the one to the south	292	0	S.	5	0	5
The one in the middle	291	0	S.	2	50	5
The one to the north, at the tip of the tail	292	0	N.	4	20	5
28 stars: 4 of the 3rd magnitude, 9 of the 4th, 9 of the 5th, 6 of the 6th						
WATER BEARER						
In the head	293	40	N.	15	45	5
In the right shoulder, the brighter one	299	44	N.	11	0	3
The dimmer one	298	30	N.	9	40	5
In the left shoulder	290	0	N.	8	50	3
Below the armpit	290	40	N.	6	15	5
Of the three [stars] in the garment below the left hand, the one to the east	280	0	N.	5	30	3
The one in the middle	279	30	N.	8	0	4
Of the three, the one to the west	278	0	N.	8	30	3
In the right elbow	302	50	N.	8	45	3
In the right hand, the one to the north	303	0	N.	10	45	3
Of the other two to the south, the one to the west	305	20	N.	9	0	3
The one to the east	306	40	N.	8	30	3
Of the two [stars] close together in the right hip, the one to the west	299	30	N.	3	0	4
The one to the east	300	20	N.	2	10	5
In the right buttock	302	0	S.	0	50	4
Of the two [stars] in the left buttock, the one to the south	295	0	S.	1	40	4
The one farther north	295	30	N.	4	0	6
In the right shin, the one to the south	305	0	S.	7	30	3
The one to the north	304	40	S.	5	0	4
In the left hip	301	0	S.	5	40	5
Of the two [stars] in the left shin, the one to the south	300	40	S.	10	0	5
The one to the north, below the knee	302	10	S.	9	0	5
In the water poured by the hand, the first [star]	303	20	N.	2	0	4
To the east, farther south	308	10	N.	0	10	4
To the east, in the first curve of the water	311	0	S.	1	10	4
To the east of the foregoing	313	20	S.	0	30	4
In the second curve, the one to the south	313	50	S.	1	40	4

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
Of the two [stars] to the east, the one to the north	312	30	S.	3	30	4
The one to the south	312	50	S.	4	10	4
At a distance to the south	314	10	S.	8	15	5
Of the two [stars] close together east of the foregoing, the one to the west	316	0	S.	11	0	5
The one to the east	316	30	S.	10	50	5
Of the three [stars] in the third curve of the water, the one to the north	315	0	S.	14	0	5
The one in the middle	316	0	S.	14	45	5
Of the three, the one to the east	316	30	S.	15	40	5
Of the three [stars] to the east in a similar formation, the one to the north	310	20	S.	14	10	4
The one in the middle	310	50	S.	15	0	4
Of the three, the one to the south	311	40	S.	15	45	4
Of the three [stars] in the last curve, the one to the west	305	10	S.	14	50	4
Of the two [stars] to the east, the one to the south	306	0	S.	15	20	4
The one to the north	306	30	S.	14	0	4
The last [star] in the water; also in the mouth of the Southern Fish	300	20	S.	23	0	1
42 stars: 1 of the 1st magnitude, 9 of the 3rd, 18 of the 4th, 13 of the 5th, 1 of the 6th						
NEAR THE WATER BEARER, OUTSIDE THE CONSTELLATION						
Of the three [stars] east of the curve in the water, the one to the west	320	0	S.	15	30	4
Of the other two, the one to the north	323	0	S.	14	20	4
Of these [two], the one to the south	322	20	S.	18	15	4
3 stars: brighter than the 4th magnitude						
FISHES						
The western fish:						
In the mouth	315	0	N.	9	15	4
Of the two [stars] in the back of the head, the one to the south	317	30	N.	7	30	4 brighter
The one to the north	321	30	N.	9	30	4
Of the two [stars] in the back, the one to the west	319	20	N.	9	20	4
The one to the east	324	0	N.	7	30	4
In the belly, the one to the west	319	20	N.	4	30	4
The one to the east	323	0	N.	2	30	4
In the tail of the same fish	329	20	N.	6	20	4
On its line, the first [star] from the tail	334	20	N.	5	45	6
The one to the east	336	20	N.	2	45	6
Of the three bright [stars] east of these [two foregoing], the one to the west	340	30	N.	2	15	4
The one in the middle	343	50	N.	1	10	4
The one to the east	346	20	S.	1	20	4

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
Of the two small [stars] in the bend, the one to the north	345	40	S.	2	0	6
The one to the south	346	20	S.	5	0	6
Of the three [stars] east of the bend, the one to the west	350	20	S.	2	20	4
The one in the middle	352	0	S.	4	40	4
The one to the east	354	0	S.	7	45	4
In the interwining of both lines	356	0	S.	8	30	3
On the northern line, to the west of the interwining	354	0	S.	4	20	4
Of the three [stars] east of the foregoing, the one to the south	353	30	N.	1	30	5
The one in the middle	353	40	N.	5	20	3
Of the three, the one to the north and the last on the line	353	50	N.	9	0	4
The eastern fish:						
Of the two [stars] in the mouth, the one to the north	355	20	N.	21	45	5
The one to the south	355	0	N.	21	30	5
Of the three small [stars] in the head, the one to the east	352	0	N.	20	0	6
The one in the middle	351	0	N.	19	50	6
Of the three, the one to the west	350	20	N.	23	0	6
Of the three [stars] in the southern fin, the one to the west, near the left elbow of Andromeda	349	0	N.	14	20	4
The one in the middle	349	40	N.	13	0	4
Of the three, the one to the east	351	0	N.	12	0	4
Of the two [stars] in the belly, the one to the north	355	30	N.	17	0	4
The one farther south	352	40	N.	15	20	4
In the eastern fin, near the tail	353	20	N.	11	45	4

34 stars: 2 of the 3rd magnitude, 22 of the 4th, 3 of the 5th, 7 of the 6th

NEAR THE FISHES, OUTSIDE THE CONSTELLATION

On the northern side of the quadrilateral below the western fish, the one to the west	324	30	S.	2	40	4
The one to the east	325	35	S.	2	30	4
On the southern side, the one to the west	324	0	S.	5	50	4
The one to the east	325	40	S.	5	30	4

4 [stars] outside the constellation, of the 4th magnitude

Accordingly, in the zodiac there are altogether 346 stars, namely, 5 of the 1st magnitude, 9 of the 2nd, 64 of the 3rd, 133 of the 4th, 105 of the 5th, 27 of the 6th, 3 cloudy. In addition to [this] number, there is also the Hair, which, as I remarked above, was called "Berenice's Hair" by the astronomer Conon.

[III:] THOSE WHICH ARE IN THE SOUTHERN REGION

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
WHALE						
At the tip of the nostril	11	0	S.	7	45	4
Of the three [stars] in the jaw, the one to the east	11	0	S.	11	20	3
The middle one, in the middle of the mouth	6	0	S.	11	30	3
The western one of the three, in the cheek	3	50	S.	14	0	3
In the eye	4	0	S.	8	10	4
In the hair, to the north	5	30	S.	6	20	4
In the mane, to the west	1	0	S.	4	10	4
Of the four [stars] in the chest, the northern one of those to the west	355	20	S.	24	30	4
The southern one	356	40	S.	28	0	4
Of those to the east, the one to the north	0	0	S.	25	10	4
The one to the south	0	20	S.	27	30	3
Of the three [stars] in the body, the one in the middle	345	20	S.	25	20	3
The one to the south	346	20	S.	30	30	4
Of the three, the one to the north	348	20	S.	20	0	3
Of the two [stars] near the tail, the one to the east	343	0	S.	15	20	3
The one to the west	338	20	S.	15	40	3
Of the quadrilateral in the tail, of the [stars] to the east, the one to the north	335	0	S.	11	40	5
The one to the south	334	0	S.	13	40	5
Of the remaining [stars] to the west, the one to the north	332	40	S.	13	0	5
The one to the south	332	20	S.	14	0	5
At the northern tip of the tail	327	40	S.	9	30	3
At the southern tip of the tail	329	0	S.	20	20	3
22 stars: 10 of the 3rd magnitude, 8 of the 4th, 4 of the 5th						
ORION						
The cloudy [star] in the head	50	20	S.	16	30	cloudy
The bright reddish [star] in the right shoulder	55	20	S.	17	0	1
In the left shoulder	43	40	S.	17	30	2 brighter
East of the foregoing	48	20	S.	18	0	4 dimmer
In the right elbow	57	40	S.	14	30	4
In the right forearm	59	40	S.	11	50	6
Of the four [stars] in the right hand, of those to the south, the one to the east	59	50	S.	10	40	4
The one to the west	59	20	S.	9	45	4
On the northern side, the one to the east	60	40	S.	8	15	6
On the same side, the one to the west	59	0	S.	8	15	6
Of the two [stars] in the club, the one to the west	55	0	S.	3	45	5
The one to the east	57	40	S.	3	15	5
Of the four [stars] in a straight line in the back, the one to the east	50	50	S.	19	40	4

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
The second, to the west	49	40	S.	20	0	6
The third, to the west	48	40	S.	20	20	6
In the fourth place, to the west	47	30	S.	20	30	5
Of the nine [stars] in the shield, the farthest north	43	50	S.	8	0	4
The second	42	40	S.	8	10	4
The third	41	20	S.	10	15	4
The fourth	39	40	S.	12	50	4
The fifth	38	30	S.	14	15	4
The sixth	37	50	S.	15	50	3
The seventh	38	10	S.	17	10	3
The eighth	38	40	S.	20	20	3
The remaining one of these, the farthest south	39	40	S.	21	30	3
Of the three bright [stars] in the belt, the one to the west	48	40	S.	24	10	2
The one in the middle	50	40	S.	24	50	2
Of the three [stars] in a straight line, the one to the east	52	40	S.	25	30	2
In the hilt of the sword	47	10	S.	25	50	3
Of the three [stars] in the sword, the one to the north	50	10	S.	28	40	4
The one in the middle	50	0	S.	29	30	3
The one to the south	50	20	S.	29	50	3 dimmer
Of the two [stars] at the tip of the sword, the one to the east	51	0	S.	30	30	4
The one to the west	49	30	S.	30	50	4
The bright [star] in the left foot; also in the River	42	30	S.	31	30	1
In the left shin	44	20	S.	30	15	4 brighter
In the left heel	46	40	S.	31	10	4
In the right knee	53	30	S.	33	30	3

38 stars: 2 of the 1st magnitude, 4 of the 2nd, 8 of the 3rd, 15 of the 4th, 3 of the 5th, 5 of the 6th, and 1 cloudy

RIVER

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
Beyond the left foot of Orion, at the beginning of the River	41	40	S.	31	50	4
In the bend at Orion's leg, the one farthest north	42	10	S.	28	15	4
Of the two [stars] east of the foregoing, the one to the east	41	20	S.	29	50	4
The one to the west	38	0	S.	28	15	4
Of the next two, the one to the east	36	30	S.	25	15	4
The one to the west	33	30	S.	25	20	4
Of the three after the foregoing, the one to the east	29	40	S.	26	0	4
The one in the middle	29	0	S.	27	0	4
Of the three, the one to the west	26	10	S.	27	50	4
Of the four at a distance, the one to the east	20	20	S.	32	50	3

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
West of the foregoing	18	0	S.	31	0	4
The third one, to the west	17	30	S.	28	50	3
Of all four, the [farthest] west	15	30	S.	28	0	3
Of four [other stars], once more in like manner, the one to the east	10	30	S.	25	30	3
West of the foregoing	8	10	S.	23	50	4
Still farther west than the foregoing	5	30	S.	23	10	3
Of these four, the farthest west	3	50	S.	23	15	4
In the bend of the River, touching the chest of the Whale	358	30	S.	32	10	4
East of the foregoing	359	10	S.	34	50	4
Of the three [stars] to the east, the one to the west	2	10	S.	38	30	4
The one in the middle	7	10	S.	38	10	4
Of the three, the one to the east	10	50	S.	39	0	5
Of the two western [stars] in the quadrilateral, the one to the north	14	40	S.	41	30	4
The one to the south	14	50	S.	42	30	4
On the eastern side, the one to the west	15	30	S.	43	20	4
Of these four, the one to the east	18	0	S.	43	20	4
Toward the east, of the two [stars] close together, the one to the north	27	30	S.	50	20	4
The one farther south	28	20	S.	51	45	4
Of the two [stars] in the bend, the one to the east	21	30	S.	53	50	4
The one to the west	19	10	S.	53	10	4
Of the three [stars] in the remaining distance, the one to the east	11	10	S.	53	0	4
The one in the middle	8	10	S.	53	30	4
Of the three, the one to the west	5	10	S.	52	0	4
The bright [star] at the end of the River	353	30	S.	53	30	1
34 stars: 1 of the 1st magnitude, 5 of the 3rd, 27 of the 4th, 1 of the 5th						
HARE						
Of the quadrilateral in the ears, of the western [stars] the one to the north	43	0	S.	35	0	5
The one to the south	43	10	S.	36	30	5
On the eastern side, the one to the north	44	40	S.	35	30	5
The one to the south	44	40	S.	36	40	5
In the chin	42	30	S.	39	40	4 brighter
At the end of the left forefoot	39	30	S.	45	15	4 brighter
In the middle of the body	48	50	S.	41	30	3
Below the belly	48	10	S.	44	20	3
Of the two [stars] in the hind feet, the one to the north	54	20	S.	44	0	4
The one farther south	52	20	S.	45	50	4
In the loins	53	20	S.	38	20	4
At the tip of the tail	56	0	S.	38	10	4
12 stars: 2 of the 3rd magnitude, 6 of the 4th, 4 of the 5th						

Constellations of the stars	Longitude Degrees	Longitude Minutes		Latitude Degrees	Latitude Minutes	Magnitude
DOG						
The most brilliant [star], in the mouth, called the "Dog Star"	71	0	S.	39	10	1 brightest
In the ears	73	0	S.	35	0	4
In the head	74	40	S.	36	30	5
Of the two [stars] in the neck, the one to the north	76	40	S.	37	45	4
The one to the south	78	40	S.	40	0	4
In the chest	73	50	S.	42	30	5
Of the two [stars] in the right knee, the one to the north	69	30	S.	41	15	5
The one to the south	69	20	S.	42	30	5
At the tip of the forefoot	64	20	S.	41	20	3
Of the two [stars] in the left knee, the one to the west	68	0	S.	46	30	5
The one to the east	69	30	S.	45	50	5
Of the two [stars] in the left shoulder, the one to the east	78	0	S.	46	0	4
The one to the west	75	0	S.	47	0	5
In the left hip	80	0	S.	48	45	3 dimmer
Below the belly, between the thighs	77	0	S.	51	30	3
In the instep of the right foot	76	20	S.	55	10	4
At the tip of that foot	77	0	S.	55	40	3
At the tip of the tail	85	30	S.	50	30	3 dimmer
18 stars: 1 of the 1st magnitude, 5 of the 3rd, 5 of the 4th, 7 of the 5th						
NEAR THE DOG, OUTSIDE THE CONSTELLATION						
North of the Dog's head	72	50	S.	25	15	4
In a straight line below the hind feet, [the star] to the south	63	20	S.	60	30	4
The one farther north	64	40	S.	58	45	4
Still farther north than the foregoing	66	20	S.	57	0	4
Of these four, the last [star], farthest north	67	30	S.	56	0	4
Of the three [stars] almost in a straight line to the west, the one to the west	50	20	S.	55	30	4
The one in the middle	53	40	S.	57	40	4
Of the three, the one in the east	55	40	S.	59	30	4
Of the two bright [stars] below the foregoing, the one to the east	52	20	S.	59	40	2
To the west	49	20	S.	57	40	2
The last one, farther south than the aforementioned	45	30	S.	59	30	4
11 stars: 2 of the 2nd magnitude, 9 of the 4th						
LITTLE DOG OR PROCYON						
In the neck	78	20	S.	14	0	4

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
The bright star in the thigh: Procyon or the Little Dog	82	30	S.	16	10	1
2 [stars]: 1 of the 1st magnitude, 1 of the 4th						
ARGO OR SHIP						
Of the two [stars] at the end of the ship, the one to the west	93	40	S.	42	40	5
The one to the east	97	40	S.	43	20	3
Of the two [stars] in the stern, the one to the north	92	10	S.	45	0	4
The one farther south	92	10	S.	46	0	4
West of the two [foregoing]	88	40	S.	45	30	4
The bright [star] in the middle of the shield	89	40	S.	47	15	4
Of the three [stars] below the shield, the one to the west	88	40	S.	49	45	4
The one to the east	92	40	S.	49	50	4
Of the three, the one in the middle	91	50	S.	49	15	4
At the end of the rudder	97	20	S.	49	50	4
Of the two [stars] in the keel of the stern, the one to the north	87	20	S.	53	0	4
The one to the south	87	20	S.	58	30	3
In the deck of the stern, the one to the north	93	30	S.	55	30	5
Of the three [stars] in the same deck, the one to the west	95	30	S.	58	30	5
The one in the middle	96	40	S.	57	15	4
The one to the east	99	50	S.	57	45	4
The bright [star] to the east in the crossbank	104	30	S.	58	20	2
Of the two dim [stars] below the foregoing, the one to the west	101	30	S.	60	0	5
The one to the east	104	20	S.	59	20	5
Of the two [stars] above the aforementioned bright [star], the one to the west	106	30	S.	56	40	5
The one to the east	107	40	S.	57	0	5
Of the three [stars] in the small shields and the foot of the mast, the one to the north	119	0	S.	51	30	4 brighter
The one in the middle	119	30	S.	55	30	4 brighter
Of the three, the one to the south	117	20	S.	57	10	4
Of the two [stars] close together below the foregoing, the one to the north	122	30	S.	60	0	4
The one farther south	122	20	S.	61	15	4
Of the two [stars] in the middle of the mast, the one to the south	113	30	S.	51	30	4
The one in the north	112	40	S.	49	0	4
Of the two [stars] at the top of the sail, the one to the west	111	20	S.	43	20	4
The one to the east	112	20	S.	43	30	4
Below the third [star], east of the shield	98	30	S.	54	30	2 dimmer
In the juncture of the deck	100	50	S.	51	15	2
Between the oars in the keel	95	0	S.	63	0	4
The dim [star] east of the foregoing	102	20	S.	64	30	6
The bright [star] east of the foregoing, in the deck	113	20	S.	63	50	2

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
The bright [star] farther south, below the keel	121	50	S.	69	40	2
Of the three [stars] east of the foregoing, the one to the west	128	30	S.	65	40	3
The one in the middle	134	40	S.	65	50	3
The one to the east	139	20	S.	65	50	2
Of the two [stars] to the east, at the juncture, the one to the west	144	20	S.	62	50	3
The one to the east	151	20	S.	62	15	3
In the northern, western oar, the star to the west	57	20	S.	65	50	4 brighter
The one to the east	73	30	S.	65	40	3 brighter
In the remaining oar, [the star] to the west: Canopus	70	30	S.	75	0	1
The remaining [star], east of the foregoing	82	20	S.	71	50	3 brighter
45 stars: 1 of the 1st magnitude, 6 of the 2nd, 8 of the 3rd, 22 of the 4th, 7 of the 5th, 1 of the 6th						
HYDRA						
Of the five [stars] in the head, [and] of the two to the west, the one to the south, in the nostrils	97	20	S.	15	0	4
Of the two, the one to the north, in the eye	98	40	S.	13	40	4
Of the two to the east, the one to the north, in the back of the head	99	0	S.	11	30	4
Of these, the one to the south, in the open mouth	98	50	S.	14	45	4
East of all the foregoing, in the cheek	100	50	S.	12	15	4
Of the two [stars] in the beginning of the neck, the one to the west	103	40	S.	11	50	5
The one to the east	106	40	S.	13	30	4
Of the three [stars] in the bend of the neck, the one in the middle	111	40	S.	15	20	4
East of the foregoing	114	0	S.	14	50	4
The farthest south	111	40	S.	17	10	4
To the south, of the two [stars] close together, the dim one to the north	112	30	S.	19	45	6
The bright one of these, to the east and to the south	113	20	S.	20	30	2
Of the three [stars] east of the bend in the neck, the one to the west	119	20	S.	26	30	4
The one to the east	124	30	S.	23	15	4
The one in the middle of these [three]	122	0	S.	26	0	4
Of the three [stars] in a straight line, the one to the west	131	20	S.	24	30	3
The one in the middle	133	20	S.	23	0	4
The one to the east	136	20	S.	22	10	3
Of the two [stars] below the bottom of the Cup, the one to the north	144	50	S.	25	45	4
The one to the south	145	40	S.	30	10	4

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
In the triangle east of the foregoing, the one to the west	155	30	S.	31	20	4
Of these, the one to the south	157	50	S.	34	10	4
Of the same three [stars], the one to the east	159	30	S.	31	40	3
East of the Crow, next to the tail	173	20	S.	13	30	4
At the tip of the tail	186	50	S.	17	30	4
25 stars: 1 of the 2nd magnitude, 3 of the 3rd, 19 of the 4th, 1 of the 5th, 1 of the 6th						
NEAR HYDRA, OUTSIDE THE CONSTELLATION						
South of the head	96	0	S.	23	15	3
East of the [stars] in the neck	124	20	S.	26	0	3
2 [stars] outside the constellation, of the 3rd magnitude						
CUP						
In the bottom of the cup; also in Hydra	139	40	S.	23	0	4
Of the two [stars] in the middle of the cup, the one to the south	146	0	S.	19	30	4
Of these, the one to the north	143	30	S.	18	0	4
In the southern edge of the lip	150	20	S.	18	30	4 brighter
In the northern edge	142	40	S.	13	40	4
In the southern handle	152	30	S.	16	30	4 dimmer
In the northern handle	145	0	S.	11	50	4
7 stars of the 4th magnitude						
CROW						
In the beak; also in Hydra	158	40	S.	21	30	3
In the neck	157	40	S.	19	40	3
In the breast	160	0	S.	18	10	5
In the right, western wing	160	50	S.	14	50	3
Of the two [stars] in the eastern wing, the one to the west	160	0	S.	12	30	3
The one to the east	161	20	S.	11	45	4
At the tip of the foot; also in Hydra	163	50	S.	18	10	3
7 stars: 5 of the 3rd magnitude, 1 of the 4th, 1 of the 5th						
CENTAUR						
Of the four [stars] in the head, the one farthest south	183	50	S.	21	20	5
Farther north	183	20	S.	13	50	5
Of the two in the middle, the one to the west	182	30	S.	20	30	5
The one to the east; the last of the four	183	20	S.	20	0	5
In the left, western shoulder	179	30	S.	25	30	3
In the right shoulder	189	0	S.	22	30	3
In the left side of the back	182	30	S.	17	30	4
Of the four [stars] in the shield, the northern one of the two to the west	191	30	S.	22	30	4
The southern one	192	30	S.	23	45	4

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
Of the remaining two, the one at the top of the shield	195	20	S.	18	15	4
The one farther south	196	50	S.	20	50	4
Of the three [stars] in the right side, the one to the west	186	40	S.	28	20	4
The one in the middle	187	20	S.	29	20	4
The one to the east	188	30	S.	28	0	4
In the right arm	189	40	S.	26	30	4
In the right elbow	196	10	S.	25	15	3
At the tip of the right hand	200	50	S.	24	0	4
The bright [star] at the beginning of the human body	191	20	S.	33	30	3
Of the two dim [stars], the one to the east	191	0	S.	31	0	5
The one to the west	189	50	S.	30	20	5
In the juncture of the back	185	30	S.	33	50	5
West of the foregoing, in the back of the horse	182	20	S.	37	30	5
Of the three [stars] in the groin, the one to the east	179	10	S.	40	0	3
The one in the middle	178	20	S.	40	20	4
Of the three, the one to the west	176	0	S.	41	0	5
Of the two [stars] close together in the right hip, the one to the west	176	0	S.	46	10	2
The one to the east	176	40	S.	46	45	4
In the chest below the wing of the horse	191	40	S.	40	45	4
Of the two [stars] in the belly, the one to the west	179	50	S.	43	0	2
The one to the east	181	0	S.	43	45	3
In the instep of the right foot	183	20	S.	51	10	2
In the calf of the same [leg]	188	40	S.	51	40	2
In the instep of the left foot	188	40	S.	55	10	4
Below the muscle of the same [leg]	184	30	S.	55	40	4
At the top of the right forefoot	181	40	S.	41	10	1
In the left knee	197	30	S.	45	20	2
Outside [the constellation] below the right thigh	188	0	S.	49	10	3
37 stars: 1 of the 1st magnitude, 5 of the 2nd, 7 of the 3rd, 15 of the 4th, 9 of the 5th						
BEAST HELD BY THE CENTAUR						
At the top of the hind foot near the Centaur's hand	201	20	S.	24	50	3
In the instep of the same foot	199	10	S.	20	10	3
Of the two [stars] in the shoulder, the one to the west	204	20	S.	21	15	4
The one to the east	207	30	S.	21	0	4
In the middle of the body	206	20	S.	25	10	4
In the belly	203	30	S.	27	0	5
In the hip	204	10	S.	29	0	5
Of the two [stars] in the joint of the hip, the one to the north	208	0	S.	28	30	5
The one to the south	207	0	S.	30	0	5
At the top of the loins	208	40	S.	33	10	5

Constellations of the stars	Longitude			Latitude		Magnitude
	Degrees	Minutes		Degrees	Minutes	
Of the three [stars] at the tip of the tail, the one to the south	195	20	S.	31	20	5
The one in the middle	195	10	S.	30	0	4
Of the three, the one to the north	196	20	S.	29	20	4
Of the two [stars] in the throat, the one to the south	212	10	S.	17	0	4
The one to the north	212	40	S.	15	20	4
Of the two [stars] in the open mouth, the one to the west	209	0	S.	13	30	4
The one to the east	210	0	S.	12	50	4
Of the two [stars] in the forefoot, the one to the south	240	40	S.	11	30	4
Farther north	239	50	S.	10	0	4
19 stars: 2 of the 3rd magnitude, 11 of the 4th, 6 of the 5th						
HEARTH OR CENSER						
Of the two [stars] in the base, the one to the north	231	0	S.	22	40	5
The one to the south	233	40	S.	25	45	4
In the middle of the small altar	229	30	S.	26	30	4
Of the three [stars] in the brazier, the one to the north	224	0	S.	30	20	5
Of the two [stars] close together, the one to the south	228	30	S.	34	10	4
The one to the north	228	20	S.	33	20	4
In the middle of the flame	224	10	S.	34	10	4
7 stars: 5 of the 4th magnitude, 2 of the 5th						
SOUTHERN CROWN						
Outside the southern edge, to the west	242	30	S.	21	30	4
East of the foregoing, in the crown	245	0	S.	21	0	5
East of the foregoing	246	30	S.	20	20	5
Farther east	248	10	S.	20	0	4
East of the foregoing, west of the Archer's knee	249	30	S.	18	30	5
The bright [star] in the knee, to the north	250	40	S.	17	10	4
Farther north	250	10	S.	16	0	4
Still farther north	249	50	S.	15	20	4
Of the two [stars] in the northern edge, the one to the east	248	30	S.	15	50	6
The one to the west	248	0	S.	14	50	6
At some distance to the west of these [two foregoing]	245	10	S.	14	40	5
Still farther west	343	0	S.	15	50	5
The remaining [star], farther south	242	30	S.	18	30	5
13 stars: 5 of the 4th magnitude, 6 of the 5th, 2 of the 6th						

Constellations of the stars	Longitude		Latitude			Magnitude
	Degrees	Minutes		Degrees	Minutes	
SOUTHERN FISH						
In the mouth; also at the edge of the River	300	20	S.	23	0	1
Of the three [stars] in the head, the one to the west	294	0	S.	21	20	4
The one in the middle	297	30	S.	22	15	4
The one to the east	299	0	S.	22	30	4
At the gill	297	40	S.	16	15	4
In the southern fin and back	288	30	S.	19	30	5
Of the two [stars] in the belly, the one to the east	294	30	S.	15	10	5
The one to the west	292	10	S.	14	30	4
Of the three [stars] in the northern fin, the one to the east	288	30	S.	15	15	4
The one in the middle	285	10	S.	16	30	4
Of the three, the one to the west	284	20	S.	18	10	4
At the tip of the tail	289	20	S.	22	15	4
Not including the first [star], 11 stars: 9 of the 4th magnitude, 2 of the 5th						
NEAR THE SOUTHERN FISH, OUTSIDE THE CONSTELLATION						
Of the bright [stars] west of the Fish, the one to the west	271	20	S.	22	20	3
The one in the middle	274	30	S.	22	10	3
Of the three, the one to the east	277	20	S.	21	0	3
The dim [star] west of the foregoing	275	20	S.	20	50	5
Of the others toward the north, the one farther south	277	10	S.	16	0	4
The one farther north	277	10	S.	14	50	4
6 stars: 3 of the 3rd magnitude, 2 of the 4th, 1 of the 5th						
In the southern region [there are] 316 stars: 7 of the 1st magnitude, 18 of the 2nd, 60 of the 3rd, 167 of the 4th, 54 of the 5th, 9 of the 6th, 1 cloudy. Therefore, [there are] altogether 1022 stars: 15 of the 1st magnitude, 45 of the 2nd, 208 of the 3rd, 474 of the 4th, 216 of the 5th, 50 of the 6th, 9 dim, 5 cloudy.						

Book Three

THE PRECESSION OF THE EQUINOXES AND SOLSTICES

Chapter 1

Having portrayed the appearance of the fixed stars, I must pass on to the topics connected with the annual revolution. Hence I shall first discuss the shift in the equinoxes, on account of which the fixed stars too are believed to move. (I always bear in mind that the circles and poles generated by the earth's motion appear in like form and in the same way in the heavens, as I have often remarked [II, 1], and these are the subjects treated here) [these words were written by Copernicus in the margin of fol. 71^r, and later deleted].

Now, the ancient astronomers, I have found, made no distinction between the tropical or natural year, which is measured from an equinox or solstice, and the year which is completed with reference to one of the fixed stars. Hence they thought that the Olympic years, which they started with the rising of Procyon, were the same as the years measured from a solstice (since the difference between the one and the other had not yet been discovered).

Now Hipparchus of Rhodes, a man of marvelous acumen, was the first to notice that these years are different from each other. When he was scrutinizing the length of the year more intently, he found that as measured with reference to the fixed stars, it was longer than when measured with reference to the equinoxes or solstices. Hence he thought that the fixed stars too had a motion in the order of the zodiacal signs, but a very slow motion which could not be perceived immediately [Ptolemy, *Syntaxis*, III, 1]. Now however, with the passage of time it has become absolutely clear. Because of it we see that at present the signs and stars rise and set quite differently from the indications of the ancients, and that the twelve signs of the zodiac have shifted a considerable distance away from those constellations of the fixed stars which originally agreed with them in name and position.

Moreover, the motion is found to be nonuniform. The desire to account for this nonuniformity has elicited various beliefs. In the opinion of some people, the universe, being in suspension, has a certain oscillation, a motion such as we find in the latitudes of the planets [VI, 2]; within fixed limits on either side, the advance will be matched at some time by a return; and the deviation from the mean in both directions is not greater than 8°. But this idea, which is already obsolete, could not survive. The principal reason is, as is now quite clear, that the first point of the constellation Ram is more than three times 8° away from the vernal equinox. The same is true of other stars, while in the meantime throughout so many centuries no trace of a return has been perceived. Others have indeed held that the sphere of the fixed stars moves forward, but with unequal strides; yet they have laid down no definite pattern. Besides, another marvel of nature supervened: the obliquity of the ecliptic does not appear as great to us as it did before Ptolemy, as I said above.

As the explanation of these observations, some people excogitated a ninth sphere, and others a tenth, by which they thought that these phenomena are brought to pass in this way. Yet they could not furnish what they promised. An eleventh sphere too has already begun to emerge into the light of day, as though so large a number of circles were not enough. By invoking the motion of the earth, I shall easily refute this number of circles as superfluous by showing that they have no connection with the sphere of the fixed stars. For, as I have already explained in part in Book I [Chapter 11], the two revolutions, I mean, the annual inclination and the revolution of the earth's center, are not exactly equal, the inclination being of course completed a little ahead of the period of the center. Hence, as must follow, the equinoxes and solstices seem to move forward. The reason is not that the sphere of the fixed stars moves eastward, but rather that the equator moves westward, the equator being oblique to the plane of the ecliptic in proportion to the inclination of the axis of the terrestrial globe. For it would be more appropriate to say that the equator is oblique to the ecliptic than that the ecliptic is oblique to the equator (since a smaller thing is being compared with something bigger). Indeed, the ecliptic, being described by the annual revolution at the distance between the sun and the earth, is much bigger than the equator, which is produced, as I said [I, 11], by the earth's daily motion around its axis. And in this way those intersections at the equinoxes, together with the entire obliquity of the ecliptic, are seen to move ahead in the course of time, whereas the stars lag behind. Now the measurement of this motion and the explanation of its variation were not known to earlier [astronomers]. The reason is that the period of its revolution is still undiscovered on account of its unforeseeable slowness. For in so many centuries, since it was first discovered by mortal man, it has completed barely $^1/_{15}$ of a circle. Nevertheless, so far as I can, I shall clarify this matter by means of what I have learned about it from the history of the observations down to our own time.

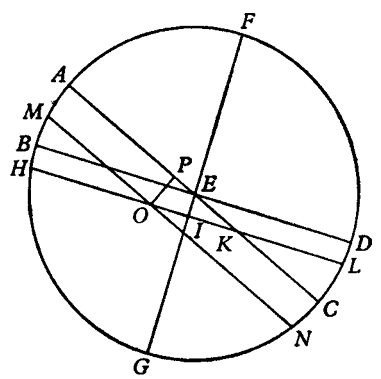
HISTORY OF THE OBSERVATIONS PROVING THAT THE PRECESSION OF THE EQUINOXES AND SOLSTICES IS NOT UNIFORM — *Chapter 2*

Now in the first period of 76 years according to Callippus, and in the 36th year thereof, which was the 30th year after the death of Alexander the Great, Timocharis of Alexandria, the first man to be concerned about the places of the fixed stars, reported that the Spike, which the Virgin holds, was at a distance of $82\,^1/_3{}^\circ$ from the [summer] solstitial point, with a latitude of 2° souht. The northernmost [star] of the three in the forehead of the Scorpion, and the first in order as that zodiacal sign is formed, had a [north] latitude of $1\,^1/_3{}^\circ$, with a distance of 32° from the autumnal equinox. Again, in the 48th year of the same period he found the Spike in the Virgin at a distance of $82\,^1/_2{}^\circ$ from the summer solstice, while its latitude remained the same. But in the 50th year of the 3rd Callippic period, the 196th year of Alexander, the star called Regulus, which is in the Lion's chest, was found by Hipparchus to be following the summer solstice by 29°50′. Then in the first year of the emperor Trajan, which was the 99th year after the birth of Christ, and the 422nd year after the death of Alexander, the Roman geometer Menelaus reported that the longitudinal distance of the Spike in the Virgin from the [summer] solstice was $86\,^1/_4{}^\circ$, while the [star] in the forehead of the Scorpion was $35\,^{11}/_{12}{}^\circ$ away from the autumnal equinox. Following them, in the aforementioned 2nd year of Antoninus Pius [II, 14], which was the 462nd year after the death of Alexander, Ptolemy learned that Regulus in the Lion had acquired a longitudinal distance of $32\,^1/_2°$ from the [summer] solstice, the Spike $86\,^1/_2°$, and the aforesaid [star] in the forehead of the Scorpion $36\,^1/_3°$ from the autumnal equinox. In latitude there was no change at all, as was indicated above in the Catalogue. I have reviewed these determinations just as they were reported by those [astronomers].

But a long time later, namely, 1202 years after Alexander's death, Al-Battani of Raqqa made the next observation, in which we may have the utmost confidence. In that year Regulus or Basiliscus in the Lion was seen to have reached 44° 5′ from the [summer] solstice; and the star in the forehead of the Scorpion, 47° 50′ from the autumnal equinox. In all these observations the latitude of each star always remained the same, so that in this regard [astronomers] no longer have any doubt.

Hence in the year 1525 C.E., the first year after a leap year according to the Roman [calendar], and the 1849th Egyptian year since Alexander's death, at Frombork in Prussia I too observed the Spike, which has been mentioned frequently. Its maximum altitude on the meridian was seen to be approximately 27°. But I found the latitude of Frombork to be $54°\ 19\,^1/_2{}'$. Therefore the Spike's declination from the equator evidently was 8°40′. Hence its place was established as follows.

Through the poles of both the ecliptic and the equator I drew the meridian *ABCD*. Let it intersect the equator in the diameter *AEC*, and the ecliptic in the diameter *BED*. Let the ecliptic's north pole be *F*, and its axis *FEG*. Let *B* be the first point of the Goat, and *D* of the Crab. Now take the arc *BH* equal to the star's south latitude of 2°. From the point *H*, draw *HL* parallel to *BD*. Let *HL* intersect the axis of the ecliptic in *I*, and the equator in *K*. Also take *MA*, an arc of 8°40′, in agreement with the star's southern declination. From the point *M*, draw *MN* parallel to *AC*. *MN* will intersect *HIL*, which is parallel to the ecliptic. Then let *MN* intersect *HIL* in the point *O*. *OP*, the straight line at right angles [to *MN*], will be equal to half of the chord subtending twice the declination *AM*. But the circles whose diameters are *FG*, *HL*, and *MN* are perpendicular to the plane *ABCD*. Their intersections, according to Euclid's *Elements*, XI, 19, are perpendicular to the same plane at points *O* and *I*. These intersections are parallel to each other, according to Proposition 6 of the same Book. Moreover, *I* is the center of the circle whose diameter is *HL*. Therefore *OI* will be equal to half of the chord subtending, on the circle whose diameter is *HL*, twice the arc which is similar to the star's longitudinal distance from the first point of the Balance. This is the arc which we are seeking.



Now it is found in the following way. The angles at *OKP* and *AEB* are equal, being alternate interior angles, and *OPK* is a rigth angle. Therefore the ratio of *OP* to *OK* is the same as the ratio of half the chord subtending twice *AB* to *BE*, and of half the chord subtending twice *AH* to *HIK*, since the triangles involved are similar to *OPK*. But *AB* is $23°\ 28\,^1/_2{}'$; and half of the chord subtending twice *AB* is 39,832 units, whereof *BE* is 100,000. *ABH* is $25°\ 28\,^1/_2{}'$; half of the chord subtending twice *ABH* is 43,010. *MA*, half of the chord subtending twice the declination, is 15,069 units. Hence it follows that the whole of *HIK* is 107,978 units; *OK* is 37,831 units; and the remainder *HO* is 70,147. But twice *HOI* subtends the circular segment *HGL* of 176°. *HOI* will be 99,939 units, whereof *BE* was 100,000.

The remainder OI will therefore be 29,792. But with $HOI = 100{,}000$ units as half of a diameter, OI will be 29,810 units, corresponding to an arc of approximately 17° 21′. This was the distance of the Spike in the Virgin from the first point of the Balance, and this was the place of the star.

Also a decade earlier, namely, in the year 1515, I found its declination 8° 36′, and its place at 17° 14′ [from the first point] of the Balance. But Ptolemy reported its declination as only $^1/_2$° [*Syntaxis*, VII, 3]. Therefore its place would have been at 26° 40′ within the Virgin, which appears to be more accurate in comparison with the earlier observations.

Hence it seems quite clear that virtually throughout the whole interval from Timocharis to Ptolemy in 432 years the equinoxes and solstices shifted in precedence 1° regularly every 100 years, as there was always a constant ratio between the time and the extent of their movement, which in its entirety amounted to 4 $^1/_3$°. For also when the distance between the summer solstice and Basiliscus in the Lion is compared for the interval from Hipparchus to Ptolemy, in 266 years the equinoxes shifted 2 $^2/_3$°. Here too, then, by being compared with the time they are found to have moved forward 1° in 100 years. On the other hand, the [star] at the top of the forehead of the Scorpion in the 782 years intervening between Al-Battani and Menelaus traversed 11° 55′. To 1° there will have to be assigned, as will be seen, not 100 years at all, but 66 years. Moreover, in the 741 years from Ptolemy [to Al-Battani], only 65 years are to be assigned to 1°. Finally, if the remaining period of 645 years is compared with the difference of 9° 11′ of my observation, 1° will receive 71 years. Hence in those 400 years before Ptolemy, clearly the precession of the equinoxes was slower than from Ptolemy to Al-Battani, when it was also quicker than from Al-Battani to our times.

Likewise in the motion of the obliquity a difference is discovered. For, Aristarchus of Samos found the obliquity of the ecliptic and equator to be 23° 51′ 20″, the same as Ptolemy; Al-Battani, 23° 36′; Al-Zarkali the Spaniard, 190 years after him, 23° 34′; and in the same way 230 years later, Profatius the Jew, about 2′ less. But in our time it is found not greater than 23° 28 $^1/_2$′. Hence it is also clear that from Aristarchus to Ptolemy, the motion was a minimum, but from Ptolemy to Al-Battani a maximum.

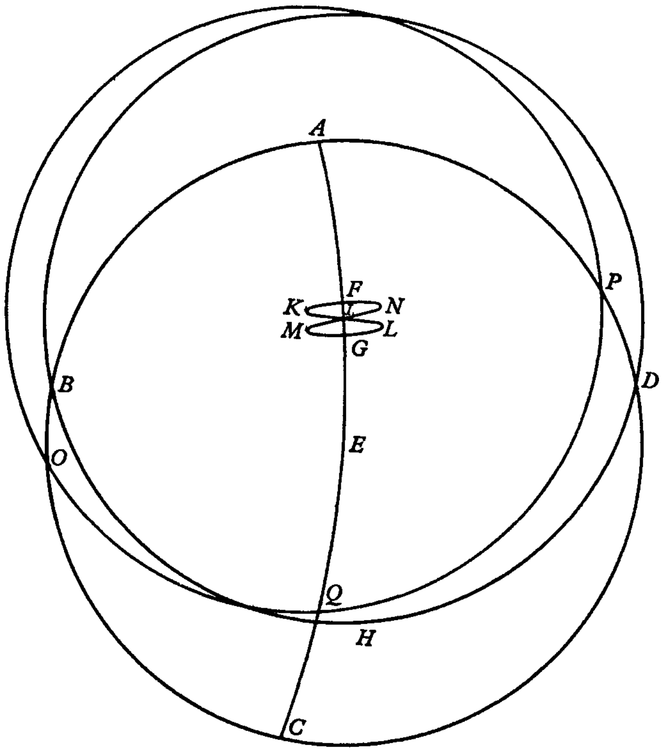
HYPOTHESES BY WHICH THE SHIFT IN THE EQUINOXES AS WELL AS IN THE OBLIQUITY OF THE ECLIPTIC AND EQUATOR MAY BE DEMONSTRATED — *Chapter 3*

From the foregoing it seems to be clear, then, that the equinoxes and solstices shift with a nonuniform motion. Nobody will adduce a better explanation of this, perhaps, than by a certain divagation of the earth's axis and of the poles of its equator. For this seems to follow from the hypothesis that the earth moves. For obviously the ecliptic remains forever unchangeable, as is attested by the constant latitudes of the fixed stars, whereas the equator shifts. For if the motion of the earth's axis agreed simply and precisely with the motion of its center, as I said [I, 11], absolutely no precession of the equinoxes and solstices would appear. However, since these motions differ from each other, but with a variable difference, the solstices and equinoxes also had to move ahead of the places of the stars in a nonuniform motion. The same thing happens in the motion of inclination. This motion likewise nonuniformly alters the obliquity of the ecliptic, an obliquity which would nevertheless be more properly assigned to the equator.

For this reason, since the poles and circles on a sphere are interconnected and fit together, it is necessary to posit two interacting motions performed entirely by the poles and similar to swinging librations. Now one motion will be that which alters the inclination of those circles to each other by deflecting the poles in that manner up and down around the angle of intersection. The other [will be the motion] which increases and decreases the solstitial and equinoctial precessions by producing a crosswise motion in both directions. Now I call these motions "librations", because like objects swinging along the same path between two limits, they become faster in the middle and slowest at the extremes, as generally happens in the latitudes of the planets, as we shall see in the proper place [VI, 2]. Moreover, [these motions] differ in period, since two cycles of the nonuniformity of the equinoxes are completed in one cycle of the obliquity. Now in every apparent nonuniform motion something must be posited as a mean, through which the pattern of the nonuniformity can be grasped. Similarly, here too of course mean poles and a mean equator as well as mean equinoctial intersections and solstitial points had to be posited. Turning to either side of these means, but within fixed limits, the poles and the circle of the earth's equator make those uniform motions appear nonuniform. Thus those two librations running in conjunction with each other make the poles of the earth in the course of time describe certain lines resembling a twisted little crown.

But these matters are not easily explained adequately with words. Hence they will not be understood when heard, I am afraid, unless they are also seen with the eyes. Therefore let us draw on a sphere the ecliptic *ABCD*. Let its north pole be *E*, the first point of the Goat *A*, of the Crab *C*, of the Ram *B*, and of the Balance *D*. Through the points *A* and *C* as well as the pole *E*, draw the circle *AEC*. Let the greatest distance between the north poles of the ecliptic and of the equator be *EF*, the least distance *EG*, and similarly the mean position of the pole, *I*. About *I*, describe *BHD* as the equator. Let this be called the mean equator, with *B* and *D* as the mean equinoxes. Let all these things be carried around the pole *E* in a constantly uniform motion in precedence, that is, in the contrary order of the zodiacal signs in the sphere of the fixed stars, in a slow motion, as I said [III, 1]. Now posit, for the terrestrial poles, two interacting motions, like [those of] swinging objects. [Of these two motions] one [occurs] between the limits *F* and *G*; it will be called the "motion of anomaly", that is, of the nonuniformity of the inclination. The other, [which runs] crosswise from precedence to consequence, and from consequence to precedence, I shall call the "anomaly of the equinoxes". It is twice as fast as the first one. Both of these motions, meeting in the poles of the earth, deflect them in a wonderful way.

For in the first place, put the north pole of the earth at *F*. The equator drawn around it will pass through the same intersections *B* and *D*, namely, through the poles of the circle *AFEC*. But this equator will make the angles of the obliquity greater, in proportion to the arc *FI*. As the pole of the earth is about to proceed from this assumed beginning toward the mean obliquity in *I*, the other motion intervenes and does not permit the pole to go directly along *FI*. On the contrary, the second motion deflects the pole through a roundabout course and extreme

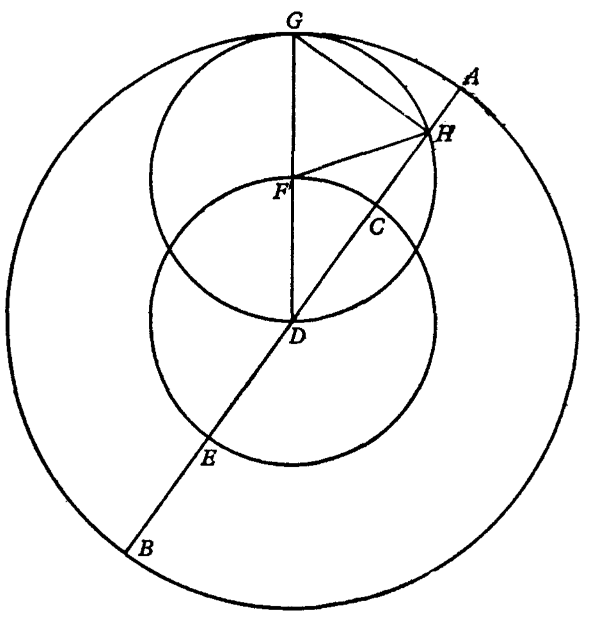


divergence in consequence. Let this be K. When the apparent equator OQP is described around this point, its intersection will be, not in B, but behind it in O, and the precession of the equinoxes is diminished in proportion to the amount of BO. Turning at this point and proceeding in precedence, the pole is taken to the mean position I by both motions acting conjointly and simultaneously. The apparent equator coincides throughout with the uniform or mean equator. As the pole of the earth passes through this point, it presses on in precedence. It separates the apparent equator from the mean equator, and increases the precession of the equinoxes up to the other limit, L. As the pole turns away from this position, it subtracts what it had just added to the equinoxes, until it reaches the point G. There it makes the obliquity a minimum at the same intersection B, where the motion of the equinoxes and solstices will again appear very slow, in almost exactly the same way as at F. At this time their nonuniformity has clearly completed its revolution, since it has passed from the mean through both of the extremes. But the motion of the obliquity [has passed] through only half of its circuit, from the greatest inclination to the least. Then as the pole proceeds in consequence, it presses on to the outermost limit in M. When it returns therefrom, it coincides again with the mean position I. As it presses on once more in precedence, it passes through the limit N, and finally completes what I called the twisted line $FKILGMINF$. Thus it is clear that in one cycle of the obliquity, the pole of the earth reaches the limit in precedence twice, and the limit in consequence twice.

HOW AN OSCILLATING MOTION OR MOTION IN LIBRATION IS CONSTRUCTED OUT OF CIRCULAR [MOTIONS] *Chapter 4*

Now I shall hereafter show that this motion is in agreement with the phenomena [III, 6]. But meanwhile someone will ask in what way these librations can be understood to be uniform, since it was stated in the beginning [I, 4] that a motion in the heavens is uniform or composed of uniform and circular [motions]. In this instance, however, both of the two motions appear as a single motion within the limits of both, so that a cessation [of motion] must intervene. I will indeed admit that they are paired, but [that oscillating motions are formed] from uniform [motions] is proved in the following way.

Let there be a straight line *AB*. Let it be divided into four equal parts at points *C, D,* and *E.* Around *D,* draw the circles *ADB* and *CDE,* with the same center and in the same plane. On the circumference of the inner circle, take any point *F* at random. With *F* as center, and with radius *FD,* draw the circle *GHD.* Let this intersect the straight line *AB* at the point *H.* Draw the diameter *DFG.* It must be shown that the movable point *H* slides back and forth in both directions along the same straight line *AB,* on account of the paired motions of the circles *GHD* and *CFE* acting conjointly. This will happen if *H* is understood to move in the opposite direction from *F* and twice as far. For, the same angle *CDF,* being located at the center of the circle *CFE* and at the circumference of *GHD,* intercepts as arcs of equal circles both *FC* and *GH,* which is twice *FC.* Assume that at some time when the straight lines *ACD* and *DFG* coincide, the movable point *H* coincides at *G* with *A,* while *F* is at *C.* Now, however, the center *F* moves to the right along *FC,* and *H* moves along the arc *GH* to the left twice as far as *CF,* or these directions may be reversed. Then the line *AB* will be the track for *H.* Otherwise, it would happen that a part is greater than its whole. This is easily understood, I believe. Now, having been drawn along by the broken line *DFH,*



which is equal to *AD, H* has moved away from its previous position *A* by the length of *AH*, this distance being the excess of the diameter *DFG* over the chord *DH*. In this way *H* will be taken to the center *D*. This will happen when the circle *DHG* is tangent to the straight line *AB*, while *GD* is of course perpendicular to *AB*. Then *H* will reach the other limit *B*, from which it will return again for the same reason.

[In the autograph, fol. 75r, Chapter 4 originally ended with the following passage, which Copernicus subsequently deleted:
Some people call this the "motion along the width of a circle", that is, along the diameter. But they derive its period and magnitude from the circle's circumference, as I shall show a little later on [III, 5]. Moreover, it should be noted here in passing that if the circles *HG* and *CF* are unequal, with all the other conditions remaining the same, they will describe, not a straight line, but a conic or cylindric section, called an "ellipse" by the mathematicians. However, [I shall discuss] these matters elsewhere].

[Printed text:
Therefore it is clear that from two circular motions acting conjointly in this way, a rectilinear motion is compounded, as well as an oscillating and nonuniform motion from uniform [motions]. Q. E. D.

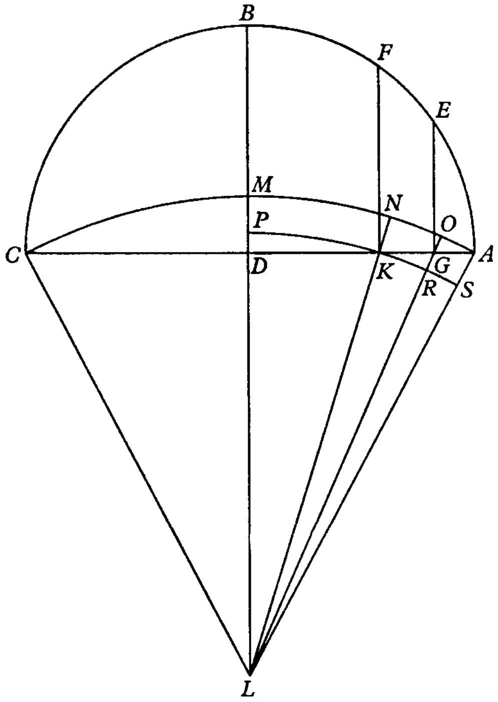
From this demonstration it also follows that the straight line *GH* will always be perpendicular to *AB*, since the lines *DH* and *HG* will subtend a right angle in a semicircle. Therefore *GH* will be half of the chord subtending twice the arc *AG*. The other line *DH* will be half of the chord subtending twice the arc which remains when *AG* is subtracted from a quadrant, since the circle *AGB* is twice *HGD* in diameter.

PROOF OF THE NONUNIFORMITY IN THE PRECESSION OF THE EQUINOXES AND IN THE OBLIQUITY — *Chapter 5*

Accordingly some people call this the "motion along the width of a circle", that is, along the diameter. Yet they treat its period and uniformity in terms of the circumference, but its magnitude in terms of chords. Hence it appears nonuniform, faster around the center and slower near the circumference, as is easily demonstrated.

Now let there be a semicircle *ABC*, with its center at *D*, and diameter *ADC*. Bisect the semicircle at the point *B*. Take equal arcs *AE* and *BF*, and from points *F* and *E* drop the perpendiculars *EG* and *FK* on *ADC*. Now twice *DK* subtends twice *BF*, and twice *EG* subtends twice *AE*. Therefore *DK* and *EG* are equal. But in accordance with Euclid's *Elements*, III, 7, *AG* is less than *GE*, and will also be less than *DK*. But *GA* and *KD* were traversed in equal times, because the arcs *AE* and *BF* are equal. Therefore near the circumference *A* the motion is slower than near the center *D*.

Now that this has been demonstrated, put the center of the earth at *L*, so that the straight line *LD* is perpendicular to *ABC*, the plane of the semicircle. Through the points *A* and *C*, with its center at *L*, draw *AMC* as the arc of a circle. Extend *LDM* as a straight line. Therefore the pole of the semicircle *ABC* will be at *M*, and *ADC* will be the intersection of the circles. Join *LA* and *LC*. In like manner join *LK* and *LG*; when these are extended as straight lines, let them intersect the

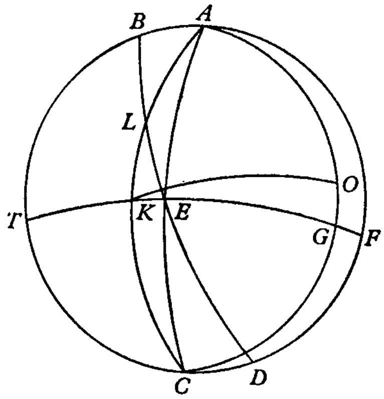


arc *AMC* in *N* and *O*. Now at *LDK* there is a right angle. Therefore the angle at *LKD* is acute. Hence it is also true that the line *LK* is longer than *LD*. Even more so, in the obtuse triangles side *LG* is longer than side *LK*, and *LA* than *LG*.

Now a circle drawn with its center at *L*, and with radius *LK*, will fall beyond *LD*, but will intersect the remaining lines *LG* and *LA*. Let the circle be drawn, and let it be *PKRS*. Triangle *LDK* is smaller than the sector *LPK*. But triangle *LGA* is bigger than the sector *LRS*. Therefore the ratio of the triangle *LDK* to the sector *LPK* is less than the ratio of the triangle *LGA* to the sector *LRS*. In turn, the ratio of the triangle *LDK* to the triangle *LGA* will also be less than the ratio of the sector *LPK* to the sector *LRS*. In accordance with Euclid's *Elements*, VI, 1, base *DK* is to base *AG* as triangle *LKD* is to triangle *LGA*. The ratio of the sector to the sector, however, is as angle *DLK* is to angle *RLS*, or as arc *MN* is to arc *OA*. Therefore the ratio of *DK* to *GA* is less than the ratio of *MN* to *OA*. But I have already shown that *DK* is bigger than *GA*. All the more, then, will *MN* be greater than *OA*. These are known to be described in equal periods of time by the poles of the earth along the equal arcs *AE* and *BF* of the anomaly. Q. E. D.

However, the difference between the maximum and minimum obliquity is quite small, and does not exceed $^{2}/_{5}{}^{\circ}$. Therefore also between the curve *AMC* and the straight line *ADC* the difference will be imperceptible. Hence no error will occur if we operate simply with the line *ADC* and the semicircle *ABC*. Just about the same thing happens with regard to the other motion of the poles which affects the equinoxes, since it does not reach $^{1}/_{2}{}^{\circ}$, as will be made clear below.

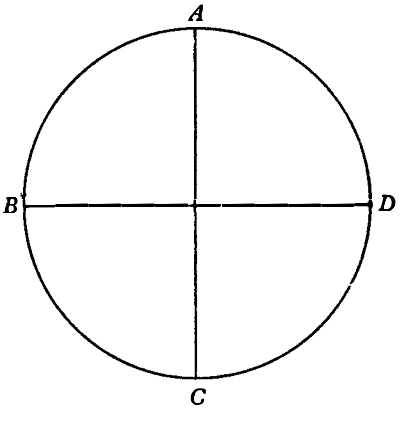
Again let there be the circle *ABCD* through the poles of the ecliptic and mean equator. We may call this circle the "mean colure of the Crab". Let half of the ecliptic be *DEB*. Let the mean equator be *AEC*. Let them intersect each other in the point *E*, where the mean equinox will be. Let the pole of the equator be *F*,



through which draw the great circle *FET*. This will therefore be the colure of the mean or uniform equinoxes. Now to make the proof easier, let us separate the libration of the equinoxes from the [libration of the] obliquity of the ecliptic. On the colure *EF,* take the arc *FG*. Let *G,* the apparent pole of the equator, be understood to move through *FG* from *F,* the mean pole. With *G* as pole, draw *ALKC* as the semicircle of the apparent equator. This will intersect the ecliptic in *L*. Therefore the point *L* will be the apparent equinox. Its distance from the mean equinox will be the arc *LE,* governed by the equality of *EK* with *FG*. But we may make *K* a pole, and describe the circle *AGC*. We may also posit that during the time in which the libration *FG* occurs, the pole of the equator does not remain the true pole in the point *G*; on the contrary, under the influence of the second libration it diverges toward the obliquity of the ecliptic along the arc *GO*. Therefore, while the ecliptic *BED* remains stationary, the true apparent equator will shift in accordance with the dislocation of the pole *O*. And in the same way the motion of *L,* the intersection of the apparent equator, will be faster around *E,* the mean equinox, and slowest at the extremes, approximately in proportion to the libration of the poles, which has already been demonstrated [III, 3]. To have perceived this was worth while.

THE UNIFORM MOTIONS OF THE PRECESSION OF THE EQUINOXES AND OF THE INCLINATION OF THE ECLIPTIC *Chapter 6*

Now every circular motion which appears nonuniform occupies four boundary zones. There is [a zone] where it appears slow, and [one] where it is fast, as extremes; and midway between, it is average. For at the end of the deceleration and beginning of the acceleration it changes in the direction of the average [velocity]; from the average it increases to [the highest] speed; from high speed it tends again toward the average; then the remainder returns from the uniform [speed] to the previous slowness. These considerations make known in what part of the circle the place of the nonuniformity or anomaly was at a [given] time. From these properties the cycle of the anomaly is also understood.



For example, in a circle divided into four equal parts let *A* be the place of the greatest slowness, *B* the average velocity on the increase, *C* the end of the increase and the beginning of the decrease, and *D* the average velocity on the decrease. Now from Timocharis to Ptolemy, as was indicated above [III, 2], the apparent motion of the precession of the equinoxes has been found slower than at all other times. For a while it appeared regular and uniform, as is shown by the observations of Aristyllus, Hipparchus, Agrippa, and Menelaus in the middle of the period. This proves, therefore, that the apparent motion of the equinoxes was at its very slowest. In the middle of the period it was at the beginning of the acceleration. At that time the cessation of the deceleration, combined with the beginning of the acceleration, by counteracting each other made the motion seem uniform in the meantime. Hence Timocharis' observation must be placed in the last part of the circle within *DA*. But Ptolemy's observation will fall in the first quadrant within *AB*. Furthermore, in the second period from Ptolemy to Al-Battani of Raqqa the motion is found to be faster than in the third period. Hence this indicates that the highest velocity, that is, the point *C,* passed by in the second period of time. The anomaly has now reached the third quadrant of the circle within *CD*. In the third period down to our time the cycle of the anomaly is nearly completed and is returning to where it began with Timocharis. For we may incorporate the entire cycle of 1819 years from Timocharis to us in the customary 360°. In proportion to 432 years, we shall have an arc of $85\,^1/_2{}^\circ$; but for 742 years, 146° 51′; and for the remaining 645 years, the remaining arc of 127° 39′. I obtained these results offhand and by a simple conjecture. But I reexamined them in a more precise computation of the extent to which they would agree more exactly with the observations. I found that in 1819 Egyptian years the motion of the anomaly had already completed its revolution, and exceeded it by 21° 24′. The time of a period contains only 1717 Egyptian years. By this calculation the first segment of the circle is determined to be 90° 35′; the second, 155° 34′; but the third in 543 years will contain the remaining 113° 51′ of the circle.

After these results had been established in this way, the mean motion of the precession of the equinoxes also became clear. It is 23° 57′ in the same 1717 years in which the entire nonuniformity is restored to its original state. For in 1819 years we had an apparent motion of about 25° 1′. But, the difference between 1717 years and 1819 being 102, in 102 years after Timocharis the apparent motion must have been about 1° 4′. For it probably was a little greater than the completion of 1° in 100 years at that time when it was decreasing but had not yet reached the end of the deceleration. Accordingly, if we subtract $1\,^1/_{15}{}^\circ$ from 25° 1′, the remainder will be, as I mentioned, in 1717 Egyptian years the mean and uniform motion, which was then equal to the nonuniform and apparent motion of 23° 57′. Hence the entire uniform revolution of the precession of the equinoxes mounts up to 25,816 years. During that time about $15\,^1/_{28}$ cycles of the anomaly are completed.

This computation is also in conformity with the motion of the obliquity, whose cycle I said is twice as slow as the precession of the equinoxes [III,3]. Ptolemy reported that the obliquity of 23° 51′ 20″ had not changed at all in the 400 years before him since Aristarchus of Samos. Hence this shows that it then stayed nearly steady around the limit of maximum obliquity, when of course the precession of the equinoxes was also having its slowest motion. At present the same restoration of the slow motion is also approaching. However, the inclination of the axis is not crossing over in like manner to the maximum, but to the minimum. In the intervening period the inclination was found, as I said [III, 2], by Al-Battani to be 23° 35′; by Al-Zarkali the Spaniard, 190 years after him, 23° 34′; and in the same way 230 years later, by Profatius the Jew, about 2′ less. Finally, so far as our own times are concerned, in frequent observations over the past 30 years, I have found it to be about $23^\circ\,28\,^2/_5{}'$. From this determination George Peurbach and Johannes Regiomontanus, who were my immediate predecessors, differ very little.

[Earlier draft:
In the year 1460 C.E. George Peurbach reported the inclination to be 23°, in agreement with the previously mentioned astronomers, plus only 28′, however; in addition to the whole degrees, a fraction more than 29′ was reported in the year 1491 C.E. by Domenico Maria da Novara; according to Johannes Regiomontanus, $23^\circ\,28\,^1/_2{}'$. (Copernicus originally incorporated the references to Peurbach and Novara in his text, and then added the remark about Regiomontanus in the margin. Later he deleted the Peurbach-Novara passage, but forgot to strike out Regiomontanus)].

Here again it is absolutely clear that the shift in the obliquity in the 900 years after Ptolemy happened to be greater than in any other period of time. Therefore, since we already have the cycle of the anomaly of precession in 1717 years, we shall also have half a period of the obliquity in that time, and its complete cycle in 3434 years. Hence if we divide 360° by the same number of 3434 years, or 180° by 1717, the annual motion of the simple anomaly will come out as 6′ 17″ 24‴ 9⁗. When this quantity is again divided by 365 days, the daily motion becomes 1″ 2‴ 2⁗. Similarly when the mean motion of the precession of the equinoxes – and this was 23° 57′ – is divided by 1717 years, the annual motion will come out as 50″ 12‴ 5⁗, and when this quantity is divided by 365 days, the daily motion will be 8‴ 15⁗.

Now to make the motions clearer and to have them handy when occasion requires, I shall exhibit them in Tables or Catalogues. The annual motion will be added continuously and equally. If a number exceeds 60, a unit will always be moved over to the higher fraction of a degree or to the degrees. I have extended the Tables as far as the 60-year line (for the sake of convenience). For in 60 years the same set of numbers appears (only the designations of degrees and fractions of degrees being transposed). Thus what was previously a second becomes a minute, and so on. By this shortcut with these brief Tables and with only two entries we may obtain and infer the uniform motions for the years in question up to 3600 years. The same holds true also for the number of the days.

In computing the heavenly motions, however, I shall use Egyptian years everywhere. Among the civil [years], they alone are found to be uniform. For the measuring unit had to agree with what was measured. Harmony to this extent does not occur in the years of the Romans, Greeks, and Persians. With them an intercalation is made, not in any one way, but as each of the nations preferred. The Egyptian year, however, presents no ambiguity with its definite number of 365 days. [They comprise] 12 equal months, which are called in order by their own names: Thoth, Phaophi, Athyr, Choiach, Tybi, Mechyr, Phamenoth, Pharmuthi, Pachon, Pauni, Ephiphi, and Mesori. These in like manner contain 6 groups of 60 days, and the 5 remaining days are termed intercalary. For this reason in the computation of the uniform motions, Egyptian years are most convenient. Any other years are easily reduced to them by a transposition of days.

THE UNIFORM MOTION OF THE PRECESSION OF THE EQUINOXES IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 5°32′

Years	Longitude					Years	Longitude				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	0	50	12	31	0	0	25	56	14
2	0	0	1	40	24	32	0	0	26	46	26
3	0	0	2	30	36	33	0	0	27	36	38
4	0	0	3	20	48	34	0	0	28	26	50
5	0	0	4	11	0	35	0	0	29	17	2
6	0	0	5	1	12	36	0	0	30	7	15
7	0	0	5	51	24	37	0	0	30	57	27
8	0	0	6	41	36	38	0	0	31	47	39
9	0	0	7	31	48	39	0	0	32	37	51
10	0	0	8	22	0	40	0	0	33	28	3
11	0	0	9	12	12	41	0	0	34	18	15
12	0	0	10	2	25	42	0	0	35	8	27
13	0	0	10	52	37	43	0	0	35	58	39
14	0	0	11	42	49	44	0	0	36	48	51
15	0	0	12	33	1	45	0	0	37	39	3
16	0	0	13	23	13	46	0	0	38	29	15
17	0	0	14	13	25	47	0	0	39	19	27
18	0	0	15	3	37	48	0	0	40	9	40
19	0	0	15	53	49	49	0	0	40	59	52
20	0	0	16	44	1	50	0	0	41	50	4
21	0	0	17	34	13	51	0	0	42	40	16
22	0	0	18	24	25	52	0	0	43	30	28
23	0	0	19	14	37	53	0	0	44	20	40
24	0	0	20	4	50	54	0	0	45	10	52
25	0	0	20	55	2	55	0	0	46	1	4
26	0	0	21	45	14	56	0	0	46	51	16
27	0	0	22	35	26	57	0	0	47	41	28
28	0	0	23	25	38	58	0	0	48	31	40
29	0	0	24	15	50	59	0	0	49	21	52
30	0	0	25	6	2	60	0	0	50	12	5

THE UNIFORM MOTION OF THE PRECESSION OF THE EQUINOXES IN DAYS AND PERIODS OF SIXTY DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	0	0	8	31	0	0	0	4	15
2	0	0	0	0	16	32	0	0	0	4	24
3	0	0	0	0	24	33	0	0	0	4	32
4	0	0	0	0	33	34	0	0	0	4	40
5	0	0	0	0	41	35	0	0	0	4	48
6	0	0	0	0	49	36	0	0	0	4	57
7	0	0	0	0	57	37	0	0	0	5	5
8	0	0	0	1	6	38	0	0	0	5	13
9	0	0	0	1	14	39	0	0	0	5	21
10	0	0	0	1	22	40	0	0	0	5	30
11	0	0	0	1	30	41	0	0	0	5	38
12	0	0	0	1	39	42	0	0	0	5	46
13	0	0	0	1	47	43	0	0	0	5	54
14	0	0	0	1	55	44	0	0	0	6	3
15	0	0	0	2	3	45	0	0	0	6	11
16	0	0	0	2	12	46	0	0	0	6	19
17	0	0	0	2	20	47	0	0	0	6	27
18	0	0	0	2	28	48	0	0	0	6	36
19	0	0	0	2	36	49	0	0	0	6	44
20	0	0	0	2	45	50	0	0	0	6	52
21	0	0	0	2	53	51	0	0	0	7	0
22	0	0	0	3	1	52	0	0	0	7	9
23	0	0	0	3	9	53	0	0	0	7	17
24	0	0	0	3	18	54	0	0	0	7	25
25	0	0	0	3	26	55	0	0	0	7	33
26	0	0	0	3	34	56	0	0	0	7	42
27	0	0	0	3	42	57	0	0	0	7	50
28	0	0	0	3	51	58	0	0	0	7	58
29	0	0	0	3	59	59	0	0	0	8	6
30	0	0	0	4	7	60	0	0	0	8	15

THE NONUNIFORM MOTION OF THE EQUINOXES IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 6° 45′

Years	Motion					Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	6	17	24	31	0	3	14	59	28
2	0	0	12	34	48	32	0	3	21	16	52
3	0	0	18	52	12	33	0	3	27	34	16
4	0	0	25	9	36	34	0	3	33	51	41
5	0	0	31	27	0	35	0	3	40	9	5
6	0	0	37	44	24	36	0	3	46	26	29
7	0	0	44	1	49	37	0	3	52	43	53
8	0	0	50	19	13	38	0	3	59	1	17
9	0	0	56	36	37	39	0	4	5	18	42
10	0	1	2	54	1	40	0	4	11	36	6
11	0	1	9	11	25	41	0	4	17	53	30
12	0	1	15	28	49	42	0	4	24	10	54
13	0	1	21	46	13	43	0	4	30	28	18
14	0	1	28	3	38	44	0	4	36	45	42
15	0	1	34	21	2	45	0	4	43	3	6
16	0	1	40	38	26	46	0	4	49	20	31
17	0	1	46	55	50	47	0	4	55	37	55
18	0	1	53	13	14	48	0	5	1	55	19
19	0	1	59	30	38	49	0	5	8	12	43
20	0	2	5	48	3	50	0	5	14	30	7
21	0	2	12	5	27	51	0	5	20	47	31
22	0	2	18	22	51	52	0	5	27	4	55
23	0	2	24	40	15	53	0	5	33	22	20
24	0	2	30	57	39	54	0	5	39	39	44
25	0	2	37	15	3	55	0	5	45	57	8
26	0	2	43	32	27	56	0	5	52	14	32
27	0	2	49	49	52	57	0	5	58	31	56
28	0	2	56	7	16	58	0	6	4	49	20
29	0	3	2	24	40	59	0	6	11	6	45
30	0	3	8	42	4	60	0	6	17	24	9

THE NONUNIFORM MOTION OF THE EQUINOXES IN DAYS AND PERIODS OF SIXTY DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	0	1	2	31	0	0	0	32	3
2	0	0	0	2	4	32	0	0	0	33	5
3	0	0	0	3	6	33	0	0	0	34	7
4	0	0	0	4	8	34	0	0	0	35	9
5	0	0	0	5	10	35	0	0	0	36	11
6	0	0	0	6	12	36	0	0	0	37	13
7	0	0	0	7	14	37	0	0	0	38	15
8	0	0	0	8	16	38	0	0	0	39	17
9	0	0	0	9	18	39	0	0	0	40	19
10	0	0	0	10	20	40	0	0	0	41	21
11	0	0	0	11	22	41	0	0	0	42	23
12	0	0	0	12	24	42	0	0	0	43	25
13	0	0	0	13	26	43	0	0	0	44	27
14	0	0	0	14	28	44	0	0	0	45	29
15	0	0	0	15	30	45	0	0	0	46	31
16	0	0	0	16	32	46	0	0	0	47	33
17	0	0	0	17	34	47	0	0	0	48	35
18	0	0	0	18	36	48	0	0	0	49	37
19	0	0	0	19	38	49	0	0	0	50	39
20	0	0	0	20	40	50	0	0	0	51	41
21	0	0	0	21	42	51	0	0	0	52	43
22	0	0	0	22	44	52	0	0	0	53	45
23	0	0	0	23	46	53	0	0	0	54	47
24	0	0	0	24	48	54	0	0	0	55	49
25	0	0	0	25	50	55	0	0	0	56	51
26	0	0	0	26	52	56	0	0	0	57	53
27	0	0	0	27	54	57	0	0	0	58	55
28	0	0	0	28	56	58	0	0	0	59	57
29	0	0	0	29	58	59	0	0	1	0	59
30	0	0	0	31	1	60	0	0	1	2	2

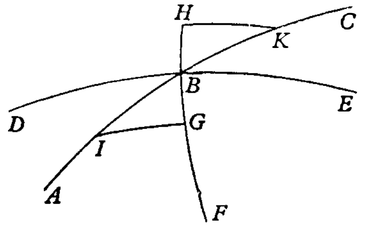
WHAT IS THE GREATEST DIFFERENCE BETWEEN THE UNIFORM AND THE APPARENT PRECESSION OF THE EQUINOXES? *Chapter 7*

[Earlier draft: Originally Copernicus began III, 7 with the following passage, which he later deleted:

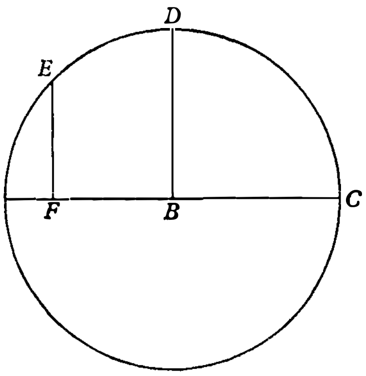
Since I have explained as well as I could the uniform and mean motion of the precession of the equinoxes, I must ask how great the maximum difference is between it and the apparent motion. Through this [maximum difference] I shall easily obtain the individual [differences] also. Now the motion of the double anomaly, that is, [the anomaly] of the equinoxes in the 432 years from Timocharis to Ptolemy evidently was 90° 35′ [III, 6]. But the mean motion of precession was 6°, and the apparent 4° 20′, the difference between them being 1° 40′. I have located the final phase of the slow motion and the beginning of the acceleration in the middle of this period. In it, therefore, the mean motion had to coincide with the apparent, and the apparent equinoxes with the mean. On both sides of that terminus, consequently, there were halves and equal distances, I mean, 45° 17 $^1/_2$′ and in like manner differences of 50′ between the apparent equinoxes and the mean [equinoxes].

[Printed text:

The mean motions having been set forth in this way, we must now ask how great the maximum difference is between the uniform and the apparent motion of the equinoxes, or [how great] the diameter of the small circle is through which the motion in anomaly revolves. For when this is known, it will be easy to determine any other differences between these motions. Now, as was indicated above [III, 2], between Timocharis' first [observation] and Ptolemy's [observation] in the 2nd year of Antoninus there were 432 years. In that time the mean motion is 6°. But the apparent [motion] was 4° 20′. The difference between them is 1° 40′. Furthermore, the motion of the double anomaly was 90° 35′. Moreover, in the middle of this period or about that time, as has been seen [III, 6], the apparent motion reached the extreme of greatest slowness. In this [period] it must agree with the mean motion, while the true and mean equinoxes must have been at the same intersection of the circles. Therefore when the motion and time are divided in half, on both sides the differences between the nonuniform and uniform motions will be $^5/_6$°. These differences are enclosed on either side below 45° 17 $^1/_2$′ arcs of the circle of anomaly.



Now that these things have been established in this way, let *ABC* be an arc of the ecliptic, *DBE* the mean equator, and *B* the mean intersection of the apparent equinoxes, whether the Ram or the Balance. Through the poles of *DBE*, draw *FB*. Now to either side on *ABC* take equal arcs *BI* and *BK* of $^5/_6$°, so that the whole of *IBK* is 1° 40′. Also draw two arcs *IG* and *HK* of the apparent equators at right angles to *FB*, extended to *FBH*. Now I say "at right angles", although the poles of *IG* and *HK* are very often outside the circle *BF*, since the motion in inclination intermingles itself, as was seen in the hypothesis [III, 3]. But because the distance is quite small, not exceeding at its maximum $^1/_{450}$ of a right angle [=12′], I treat those angles as though they were right angles, so far as perception is concerned. For, no error will appear on that account. Now in triangle *IBG*, angle *IBG* is given as 66° 20′. For, the complementary angle *DBA* was 23° 40′, the mean obliquity of the ecliptic. *BGI* is a right angle. Moreover, angle *BIG* is almost exactly equal to its alternate interior angle *IBD*. Side *IB* is given as 50′. Therefore *BG*, the distance between the poles of the mean and apparent [equator], is equal to 20′. Similarly, in triangle *BHK*, two angles *BHK* and *HBK* are equal to the two angles *IBG* and *IGB*, and side *BK* is equal to side *BI*. *BH* will also be equal to *BG*'s 20′. But all of this is concerned with very small quantities, which do not amount to 1 $^1/_2$° of the ecliptic. In these quantities the straight lines are virtually equal to the arcs subtended by them, any divergence being barely found in 60ths of a second. I am satisfied with minutes, however, and will commit no error if I use straight lines instead of arcs. For, *GB* and *BH* will be proportional to *IB* and *BK*, and the same ratio will hold true for the motions in both poles as well as in both intersections.



Let a part of the ecliptic be *ABC*. On it let the mean equinox be *B*. With this as pole, draw a semicircle *ADC*, intersecting the ecliptic in points *A* and *C*. From the pole of the ecliptic also draw *DB*, which will bisect at *D* the semicircle we have drawn. Let *D* be understood to be the utmost limit of the deceleration and the beginning of the acceleration. In the quadrant *AD*, take the arc *DE* of 45° 17 $^1/_2$′. Through the point *E*, drop *EF* from the pole of the ecliptic, and let *BF* be 50′. From these [particulars] it is proposed to find the whole of *BFA*. Now it is evident that twice *BF* subtends twice the segment *DE*. But as *BF*'s 7107 units are to *AFB*'s 10,000, so *BF*'s 50′ are to *AFB*'s 70′. Therefore *AB* is given as 1° 10′. This is the greatest difference between the mean and apparent motions of the equinoxes. This is what we were looking for, and what is followed by the poles' greatest divergence of 28′. These [28′] correspond, in the intersections of the equator, to the 70′ in the anomaly of the equinoxes, which I call the "double [anomaly]" in contrast to the other "simple [anomaly]" of the obliquity.

THE INDIVIDUAL DIFFERENCES BETWEEN THESE MOTIONS, AND A TABLE EXHIBITING THOSE DIFFERENCES *Chapter 8*

Now *AB* is given as 70′, an arc which seems not to differ in length from the straight line subtending it. Hence it will not be difficult to exhibit any other individual differences between the mean and the apparent motions. These differences, the subtraction or addition of which confers order upon the appearances, are called "prosthaphaereses" by the Greeks, and "equations" by the moderns. I prefer to use the Greek word as more appropriate.

Now if *ED* is 3°, according to the ratio of *AB* to the subtending chord *BF*, we shall have *BF* as a prosthaphaeresis of 4′; for 6°, there will be 7′; for 9°, 11′; and so on. We must operate in the same way, I believe, also with regard to the shift in the obliquity, where 24′ have been found, as I said [III,5], between the maximum and the minimum. In a semicircle of the simple anomaly these 24′ are traversed in 1717 years. Half of the duration in a quadrant of the circle will be 12′. There the pole of the small circle of this anomaly will be, with the obliquity at 23° 40′. And in this way, as I said, we shall infer the remaining parts of the difference almost exactly in proportion to what was said above, as contained in the appended Table.

Through these demonstrations the apparent motions can be put together in various ways. Nevertheless the most satisfactory procedure was that in which each individual prosthaphaeresis is taken separately. As a result the computation of the motions becomes easier to understand, and conforms more closely to the explanations of what has been demonstrated. Hence I drew up a Table of 60 lines, advancing 3° at a time. For in this arrangement it will not take up a lot of space, nor will it seem too compact and brief; in the other similar cases, I shall do the same. The present Table will have only 4 columns. The first 2 of them contain the degrees of both semicircles. I call these degrees the "common number", because the number itself yields the obliquity of the ecliptic, while twice the number will serve as the prosthaphaeresis of the equinoxes, the beginning of which is taken from the start of the acceleration. The 3rd column will contain the prosthaphaereses of the equinoxes corresponding to every 3rd degree. These prosthaphaereses must be added to or subtracted from the mean motion, which I initiate from the first star in the head of the Ram at the vernal equinox. The subtractive prosthaphaereses [pertain to] the anomaly in the smaller semicircle or first column, while the additive prosthaphaereses [pertain to] the second [column] and the following semicircle. Finally, the last column contains the minutes, called "the differences between the proportions of the obliquity", mounting to 60 as the maximum. For in place of 24′, the surplus by which the greatest obliquity exceeds the smallest, I put 60. In proportion thereto I adjust the fractions of the remaining surpluses in a similar ratio. Therefore at the beginning and end of the anomaly I put 60. But where the surplus reaches 22′, as in an anomaly of 33°, I put 55 in place of 22′. Thus for 20′, I put 50, as in an anomaly of 48°, and so on for the rest, as in the appended Table.

TABLE OF THE PROSTHAPHAERESES OF THE EQUINOXES AND OF THE OBLIQUITY OF THE ECLIPTIC

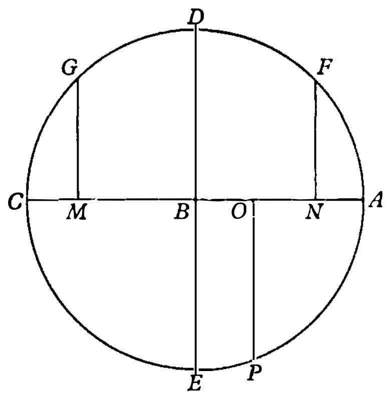
Common Numbers		Prosthaphaereses of the Equinoxes		Proportional Minutes of the Obliquity	Common Numbers		Prosthaphaereses of the Equinoxes		Proportional Minutes of the Obliquity
Degree	Degree	Degree	Minute		Degree	Degree	Degree	Minute	
3	357	0	4	60	93	267	1	10	28
6	354	0	7	60	96	264	1	10	27
9	351	0	11	60	99	261	1	9	25
12	348	0	14	59	102	258	1	9	24
15	345	0	18	59	105	255	1	8	22
18	342	0	21	59	108	252	1	7	21
21	339	0	25	58	111	249	1	5	19
24	336	0	28	57	114	246	1	4	18
27	333	0	32	56	117	243	1	2	16
30	330	0	35	56	120	240	1	1	15
33	327	0	38	55	123	237	0	59	14
36	324	0	41	54	126	234	0	56	12
39	321	0	44	53	129	231	0	54	11
42	318	0	47	52	132	228	0	52	10
45	315	0	49	51	135	225	0	49	9
48	312	0	52	50	138	222	0	47	8
51	309	0	54	49	141	219	0	44	7
54	306	0	56	48	144	216	0	41	6
57	303	0	59	46	147	213	0	38	5
60	300	1	1	45	150	210	0	35	4
63	297	1	2	44	153	207	0	32	3
66	294	1	4	42	156	204	0	28	3
69	291	1	5	41	159	201	0	25	2
72	288	1	7	39	162	198	0	21	1
75	285	1	8	38	165	195	0	18	1
78	282	1	9	36	168	192	0	14	1
81	279	1	9	35	171	189	0	11	0
84	276	1	10	33	174	186	0	7	0
87	273	1	10	32	177	183	0	4	0
90	270	1	10	30	180	180	0	0	0

REVIEW AND CORRECTION OF THE DISCUSSION OF THE PRECESSION OF THE EQUINOXES

Chapter 9

The nonuniform motion began to accelerate (this is the start of the anomalous motion, as I arrange it) halfway between the 36th year of the first Callippic period and the second year of Antoninus [Pius], according to my conjectural assumption. I must therefore still investigate whether my guess was right and in agreement with the observations.

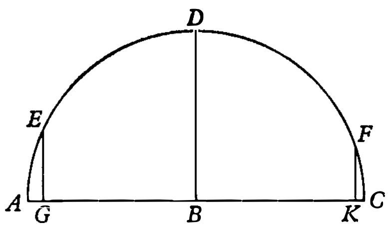
Let us recall those three stars observed by Timocharis, Ptolemy, and Al-Battani of Raqqa. In the first interval [between Timocharis and Ptolemy], clearly, there were 432 Egyptian years, and 742 in the second [interval between Ptolemy and Al-Battani]. In the first period the uniform motion was 6°, the nonuniform motion 4° 20′, and the double anomaly 90° 35′, with 1° 40′ being subtracted from the uniform motion. In the second period the uniform motion was 10° 21′, the nonuniform motion $11\frac{1}{2}°$, and the double anomaly 155° 34′, with 1° 9′ being added to the uniform motion.



As before, let *ABC* be an arc of the ecliptic. Let *B* be the mean vernal equinox. With *B* as its pole, describe the circlet *ADCE*, the arc *AB* being 1° 10′. Regard *B* as moving uniformly toward *A*, that is, in precedence. Let *A* be the western limit, where *B* reaches its greatest divergence in precedence from the variable equinox, and let *C* be the eastern limit of *B*'s divergence in consequence from the variable equinox. Furthermore, drop *DBE* from the pole of the ecliptic through the point *B*. Together with the ecliptic, *DBE* will divide the circlet *ADCE* into four equal parts, since the two circles intersect each other at right angles through their poles. In the semicirclet *ADC* the motion is in consequence, whereas it is in precedence in the other semicirclet *CEA*. Therefore the middle of the apparent equinox's retardation will be at *D* because of the counteraction to *B*'s motion. On the other hand, the greatest speed will occur at *E*, since the motions in the same direction reinforce each other. Moreover, in front of and behind *D* take the arcs *FD* and *DG*, each being 45° $17\frac{1}{2}′$. Let *F* be the anomaly's first terminus, that is, Timocharis'; *G*, the second, Ptolemy's; and *P*, the third, Al-Battani's. Through these points [*F*, *G*, *P*] and through the poles of the ecliptic drop the great circles *FN*, *GM*, and *OP*, all of which within the circlet are very much like straight lines. Then, the circlet *ADCE* being 360°, the arc *FDG* will be 90° 35′, reducing the mean motion by *MN*'s 1° 40′, *ABC* being 2° 20′. *GCEP* will be 155° 34′, increasing [the mean motion] by *MO*'s 1° 9′. Consequently the remaining 113° 51′ [=360°−(90°35′+155°34′)] of *PAF* will also enhance [the mean motion] by the remainder *ON*'s 31′ [=MN−MO=1°40′−1°9′], of which *AB* is similarly 70′. The whole arc *DGCEP* will be 200° $51\frac{1}{2}′$ [=45°$17\frac{1}{2}′$+155°34′] and *EP*, the excess over a semicircle, will be 20° $51\frac{1}{2}′$. Hence, according to the Table of the Straight Lines Subtended in a Circle, as a straight line *BO* will have 356 units, of which *AB* is 1000. But if *AB* is 70′, *BO* will be about 24′, and *BM* was taken as 50′. Therefore *MBO* as a whole is 74′ and the remainder *NO* is 26′. But previously *MBO* was 1° 9′; and the remainder *NO*, 31′. In the latter case [31′−26′] there is a shortage of 5′, which are in excess in the former case [74′−69′]. Therefore the circlet *ADCE* must be rotated until both cases are adjusted. This will happen if we make the arc *DG* $42\frac{1}{2}°$, so that the other arc *DF* is 48° 5′. For in this way, it will be seen, both errors are straightened out, and so are all the other data. Starting from *D*, the extreme limit of the retardation, the nonuniform motion in the first interval will comprise the whole arc *DGCEPAF* of 311° 55′; in the second interval, *DG* of 42 $^1/_2$°; and in the third interval, *DGCEP* of 198° 4′. And in the first interval, according to the foregoing demonstration, *BN* will be an additive prosthaphaeresis of 52′, of which *AB* is 70′; in the second interval *MB* will be a subtractive prosthaphaeresis of 47 $^1/_2$′; and in the third interval *BO* will again be an additive prosthaphaeresis of about 21′. Therefore in the first interval *MN* as a whole amounts to 1° 40′, and in the second interval *MBO* as a whole amounts to 1° 9′, in quite exact agreement with the observations. Hence the simple anomaly in the first interval is clearly 155° 57 $^1/_2$′; in the second interval, 21° 15′; and in the third interval, 99° 2′. Q. E. D.

WHAT IS THE GREATEST VARIATION IN THE INTERSECTIONS OF THE EQUATOR AND ECLIPTIC? *Chapter 10*

My discussion of the variation in the obliquity of the ecliptic and equator will be confirmed in like manner and found to be accurate. For in Ptolemy we had for the second year of Antoninus [Pius] the corrected simple anomaly as 21 $^1/_4$°, with which the greatest obliquity of 23° 51′ 20″ was found. From this situation to my observation there are about 1387 years, during which the extent of the simple anomaly is computed as 144° 4′, and at this time the obliquity is found to be about 23° 28 $^2/_5$′.



On this basis reproduce *ABC* as an arc of the ecliptic, or instead as a straight line on account of its small size. On *ABC* repeat the semicirclet of the simple anomaly with its pole at *B*, as before. Let *A* be the limit of the greatest, and *C* of the smallest, inclination, the difference between them being the object of our inquiry. Therefore take *AE* as an arc of 21° 15′ on the circlet. *ED*, the rest of the quadrant, will be 68° 45′. *EDF* as a whole will be computed as 144° 4′ and by subtraction *DF* will be 75° 19′. Drop *EG* and *FK* perpendicular to the diameter *ABC*. On account of the variation in the obliquity from Ptolemy to us, *GK* will be recognized as a great circle arc of 22′ 56″. But *GB*, being similar to a straight line, is half of the chord subtending twice *ED* or its equal, and is 932 units, of which *AC* as a diameter is 2000. Furthermore, *KB*, being half of the chord subtending twice *DF*, would be 967 of the same units. The sum *GK* becomes 1899 units, of which *AC* is 2000. But when *GK* is reckoned as 22′ 56″, *AC* will be approximately 24′, the difference between the greatest and smallest obliquity, the difference which we have been seeking. Clearly, therefore, the obliquity was greatest between Timocharis and Ptolemy, when it was fully 23° 52′, and now it is approaching its minimum of 23° 28′. From this scheme there are also obtained any intermediate obliquities of these circles by the same method as was explained with regard to precession [III, 8].

DETERMINING THE EPOCHS OF THE UNIFORM MOTIONS OF THE EQUINOXES AND ANOMALY

Chapter 11

Now that I have explained all these topics in this manner, it remains for me to determine, with regard to the motions of the vernal equinox, the places which some [scientists] call the "epochs", from which are taken the computations for any given time whatever. The absolute beginning of this calculation was established by Ptolemy [*Syntaxis*, III, 7] as the start of the reign of Nabonassar of the Babylonians. Most [scholars], misled by the similarity of the name, have thought that he was Nebuchadnezzar, who lived much later, as is shown by an examination of the chronology and by Ptolemy's computation. According to historians, Nabonassar as the ruler was followed by Shalmaneser, king of the Chaldeans. Preferring a better known period, however, I thought it suitable to commence with the first Olympiad, which is found to have preceded Nabonassar by 28 years. It began with the summer solstice, when Sirius rose for the Greeks and the Olympic games were celebrated, as Censorinus and other recognized authorities have stated. Hence, according to a more precise chronological calculation, which is necessary for computing the heavenly motions, there are 27 years and 247 days from the first Olympiad at noon on the first day of the Greek month Hecatombaeon until Nabonassar and noon on the first day of the Egyptian month Thoth. From that time to the death of Alexander there are 424 Egyptian years. From the death of Alexander there are 278 Egyptian years, 118 $^1/_2$ days, to the beginning of the years of Julius Caesar at midnight preceding 1 January, when Julius Caesar commenced the year which he instituted. As high priest, he established this year when he was consul for the third time, his colleague being Marcus Aemilius Lepidus. Following this year, so ordained by Julius Caesar, the subsequent years are called "Julian". From Caesar's fourth consulship to Octavian Augustus, the Romans reckon 18 such years up to 1 January, although it was on 17 January that the son of the deified Julius Caesar, on the motion of Munatius Plancus, was granted the title Emperor Augustus by the senate and the other citizens during his seventh consulship, his colleague being Marcus Vipsanius [Agrippa]. The Egyptians, however, because they passed under Roman rule after the death of Antony and Cleopatra two years earlier, count 15 years, 246 $^1/_2$ days, to noon of the first day of the month Thoth, which was 30 August for the Romans. Accordingly there are 27 years according to the Romans, but according to the Egyptians 29 of their years, 130 $^1/_2$ days, from Augustus to the years of Christ, which likewise begin in January. From that time to the second year of Antoninus [Pius] when Claudius Ptolemy catalogued the positions of the stars observed by himself, there are 138 Roman years, 55 days; the Egyptians add 34 days for these years. To this time from the first Olympiad there is a total of 913 years, 101 days. In this period the uniform precession of the equinoxes is 12° 44′, and the simple anomaly is 95° 44′. But in the second year of Antoninus [Pius] as is known [Ptolemy, *Syntaxis*, VII, 5], the vernal equinox preceded the first of the stars in the head of the Ram by 6° 40′. Since the double anomaly was 42 $^1/_2$° [III, 9], the subtractive difference between the uniform and the apparent motion was 48′. When this difference is restored to the apparent motion of 6° 40′, the mean place of the vernal equinox is established as 7° 28′. If we add the 360° of a circle to this place and subtract 12° 44′ from the sum, we shall have for the first Olympiad, which began at noon on the first day of the Athenian month Hecatombaeon, the mean place of the vernal equinox at 354° 44′, so that it then followed the first star of the Ram by 5° 16′ [= 360°—354° 44′]. Similarly, if 95° 45′ are subtracted from 21° 15′ of the simple anomaly, the remainder for the same beginning of the Olympiads will be 285° 30′ as the position of the simple anomaly. Again, by adding the motions accomplished during the various intervals, and always eliminating 360° as often as they accumulate, we shall have as the positions or epochs, for Alexander, 1° 2′ for the uniform motion, and 332° 52′ for the simple anomaly; for Caesar, 4° 55′ for the mean motion, and 2° 2′ for the anomaly; for Christ, 5° 32′ as the position of the mean motion, and 6° 45′ for the anomaly; and so on for the others we shall take the epochs of the motions for whatever beginning is chosen for an era.

COMPUTING THE PRECESSION OF THE VERNAL EQUINOX AND THE OBLIQUITY — *Chapter 12*

Then, whenever we want to obtain the position of the vernal equinox, if the years from the chosen starting point to the given time are nonuniform, like the Roman years which we commonly use, we shall convert them into uniform or Egyptian years. For in computing uniform motions, I shall use no years other than the Egyptian, for the reason which I mentioned [near the end of III, 6].

In case the number of years exceeds 60, we shall divide it into periods of 60 years. When we start to consult the Tables of the Motions [of the Equinoxes, following III, 6] for these periods of 60 years, we shall at that time bypass as extraneous the first column occurring in the Motions. Beginning with the second column, that of the degrees, if there are any [entries], we shall take them as well as the remaining degrees and accompanying minutes sixtyfold. Then, entering the Tables a second time, for the years remaining [after the elimination of whole periods of 60 years] we shall take the clusters of 60° plus the degrees and minutes as they are recorded from the first column on. We shall do likewise with regard to the days and periods of 60 days when we wish to add to them uniform motions in accordance with the Tables of Days and Minutes. Nevertheless in this operation minutes of days, or even whole days, would be disregarded without any harm on account of the slowness of these motions, since it is a question in the daily motion only of seconds or sixtieths of seconds. When we have collected all these entries together with their epoch, by adding separately those of each kind and eliminating every group of six clusters of 60°, if there are more than 360°, for the given time we shall have the mean place of the vernal equinox as well as the distance by which it precedes the first star of the Ram, or by which the star follows the equinox.

We shall obtain the anomaly too in the same way. With the simple anomaly, we shall find located in the last column of the Table of Prosthaphaereses [following III, 8] the proportional minutes, which we shall keep to one side. Then, with the double anomaly we shall find in the third column of the same Table the prosthaphaeresis, that is, the degrees and minutes by which the true motion differs from the mean motion. If the double anomaly is less than a semicircle, we shall subtract the prosthaphaeresis from the mean motion. But if the double anomaly has more than 180° and exceeds a semicircle, we shall add it to the mean motion.

This sum or difference will contain the true and apparent precession of the vernal equinox, or conversely the distance at that time of the first star of the Ram from the vernal equinox. But if you are looking for the position of any other star, add the longitude assigned to it in the Catalogue of the Stars.

Since operations usually become clearer through examples, let us undertake to find the true place of the vernal equinox, the distance of the Spike of the Virgin from it, and the obliquity of the ecliptic for 16 April 1525 C.E. In 1524 Roman years, 106 days, from the beginning of the years of Christ until this time, obviously there are 381 leap days, that is, 1 year, 16 days. In uniform years, the total becomes 1525 years, 122 days, equal to 25 periods of 60 years plus 25 years, and two periods of 60 days plus 2 days. In the Table of the Uniform Motion [following III, 6] 25 periods of 60 years correspond to 20° 55′ 2″; 25 years, to 20′ 55″; 2 periods of 60 days, to 16″; and the remaining 2 days, to sixtieths of seconds. All these values, together with the epoch, which was 5° 32′ [end of III, 11], amount to 26° 48′ as the mean precession of the vernal equinox.

Similarly, in 25 periods of 60 years the motion of the simple anomaly has two clusters of 60° plus 37° 15′ 3″; in 25 years, 2° 37′ 15″; in two periods of 60 days, 2′ 4″; and in 2 days, 2″. These values, together with the epoch, which is 6° 45′ [end of III, 11], amount to two clusters of 60° plus 46° 40′ as the simple anomaly. The proportional minutes corresponding to it in the last column of the Table of Prosthaphaereses [following III, 8] will be kept for the purpose of investigating the obliquity, and in this instance only 1′ is found. Then with the double anomaly, which has 5 clusters of 60° plus 33° 20′, I find a prosthaphaeresis of 32′, which is additive because the double anomaly is greater than a semicircle. When this prosthaphaeresis is added to the mean motion, the true and apparent precession of the vernal equinox comes out as 27° 21′. To this, finally, if I add 170°, the distance of the Spike of the Virgin from the first star of the Ram, its position [197° 21′] with reference to the vernal equinox will be to the east, at 17° 21′ within the Balance, where it was found at about the time of my observation [reported in III, 2].

The ecliptic's obliquity and declinations are subject to the following rule. When the proportional minutes amount to 60, the increases recorded in the Table of Declinations [following II, 3], I mean, the differences between the maximum and minimum obliquity, are added as a block to the individual degrees of declination. But in this instance one of those [proportional] minutes adds only 24″ to the obliquity. Therefore the declinations of the degrees of the ecliptic, as entered in the Table, remain unchanged at this time because the minimum obliquity is now approaching us, whereas at other times the declinations are more perceptibly variable.

Thus, for example, if the simple anomaly is 99°, as it was 880 Egyptian years after Christ, it is linked with 25 proportional minutes. But 60′ : 24′ (24′ being the difference between the greatest and smallest obliquity) = 25′ : 10′. When these 10′ are added to 28′, the sum is 23° 38′, the obliquity as it existed at that time. Then if I also want to know the declination of any degree on the ecliptic, for example, 3° within the Bull, which is 33° from the equinox, in the Table [of Declinations of the Degrees of the Ecliptic, after II, 3], I find 12° 32′, with a difference of 12′. But 60 : 25 = 12 : 5. When these 5′ are added to the degrees of declination, the total is 12° 37′ for 33° of the ecliptic. We can use the same method employed for the angles of intersection between the ecliptic and equator also for the right ascensions (unless we prefer the ratios of spherical triangles), except that we must always subtract from the right ascensions what is added to the angles of intersection, in order that all the results may come out chronologically more precise.

THE LENGTH AND NONUNIFORMITY OF THE SOLAR YEAR — *Chapter 13*

The statement that the equinoctial and solstitial precession (which, as I said [beginning of III, 3], results from the deflection of the earth's axis) proceeds in this manner will be confirmed also by the annual motion of the earth's center, as this motion appears in the sun, the topic which I must now discuss. When computed from either of the equinoxes or solstices, the length of the year becomes a variable, as must of course follow, on account of the nonuniform shift in those cardinal points, these phenomena being interconnected.

We must therefore distinguish the seasonal year from the sidereal year, and define them. I term that year "natural" or "seasonal" which marks the four annual seasons for us, but that year "sidereal" which returns to one of the fixed stars. The natural year, which is also called "tropical", is nonuniform, as the observations of the ancients make abundantly clear. For it contains a quarter of a day [$^1/_4{}^d$] more than 365 whole days, according to the determinations made by Callippus, Aristarchus of Samos, and Archimedes of Syracuse, who in the Athenian manner put the beginning of the year at the summer solstice. Claudius Ptolemy, however, being aware that the pinpointing of a solstice is difficult and uncertain, did not have enough confidence in their observations, and preferred to rely on Hipparchus. The latter left records not only of solar solstices but also of equinoxes in Rhodes, and declared that the $^1/_4{}^d$ lacked a small fraction. This was later established as $^1/_{300}{}^d$ by Ptolemy in the following way [*Syntaxis*, III, 1].

He takes the autumnal equinox which Hipparchus observed very carefully at Alexandria in the 177th year after the death of Alexander the Great on the third intercalary day at midnight, which was followed by the fourth intercalary day according to the Egyptians. Then Ptolemy adduces an autumnal equinox observed by himself at Alexandria in the third year of Antoninus [Pius], which was the 463rd year after Alexander's death, on the ninth day of Athyr, the third Egyptian month, about one hour after sunrise. Between this observation and Hipparchus', accordingly, there were 285 Egyptian years, 70 days, 7 $^1/_5$ hours. On the other hand, there should have been 71 days, 6 hours, if the tropical year had been $^1/_4{}^d$ more than [365] whole days. In 285 years, therefore, $^{19}/_{20}{}^d$ were lacking. Hence it follows that a whole day drops out in 300 years.

Ptolemy derives the like conclusion also from the vernal equinox. For he recalls the one reported by Hipparchus in the 178th year after Alexander, on the 27th day of Mechir, the sixth Egyptian month, at sunrise. Ptolemy himself finds the vernal equinox in the 463rd year after Alexander, on the 7th day of Pachon, the ninth Egyptian month, a little more than an hour after noon. In 285 years, $^{19}/_{20}{}^d$ are similarly lacking. Aided by this information, Ptolemy measured the tropical year as 365 days, 14 minutes of a day, 48 seconds of a day.

Subsequently at Raqqa in Syria with no less diligence Al-Battani observed the autumnal equinox in the 1206th year after the death of Alexander. He found that it occurred at about 7 $^2/_5$ hours during the night following the seventh day of the month Pachon, that is, $4\,{}^{3}/_{5}$ hours before daylight on the eighth day [of Pachon]. Then he compared his own observation with the one made by Ptolemy in the third year of Antoninus [Pius] one hour after sunrise at Alexandria, which is 10° [$= {}^{2}/_{3}{}^{h}$] west of Raqqa. He reduced Ptolemy's observation to his own Raqqa meridian, where Ptolemy's equinox would have had to occur $1\,{}^{2}/_{3}$ hours [$1^{h} + {}^{2}/_{3}{}^{h}$] after sunrise. Therefore, in the interval of 743 [1206−463] uniform years there was a surplus of 178 days, $17\,{}^{3}/_{5}$ hours, instead of an accumulation of quarter-days totaling $185\,{}^{3}/_{4}$ days. Since 7 days, ${}^{2}/_{5}$ of an hour [$185^{d}18^{h} - 178^{d}17\,{}^{3}/_{5}{}^{h}$] were missing it was apparent that the ${}^{1}/_{4}{}^{d}$ lacked ${}^{1}/_{106}{}^{d}$. He accordingly divided 7 days, ${}^{2}/_{5}$ of an hour, by 743, in agreement with the number of years, the quotient being 13 minutes, 36 seconds. Subtracting this quantity from ${}^{1}/_{4}{}^{d}$, he asserted that the natural year contains 365 days, 5 hours, 46 minutes, 24 seconds [$+13^{m}36^{s} = 6^{h}$].

I too observed the autumnal equinox at Frombork, which we may call "Gynopolis", in 1515 C.E. on 14 September. This was in the 1840th Egyptian year after the death of Alexander, on the sixth day of the month Phaophi, ${}^{1}/_{2}$ hour after sunrise. However, Raqqa lies east of my area by about 25°, equal to $1\,{}^{2}/_{3}$ hours. Therefore, in the interval between my equinox and Al-Battani's, over and above 633 Egyptian years there were 153 days, $6\,{}^{3}/_{4}$ hours, instead of 158 days, 6 hours. From that observation by Ptolemy at Alexandria to the time of my observation, reduced to the same place, there are 1376 Egyptian years, 332 days, ${}^{1}/_{2}$ hour, since the difference between Alexandria and us is about an hour. Therefore, in 633 years from the time of Al-Battani to us, 4 days, $22\,{}^{3}/_{4}$ hours, would have been lacking, and one day in 128 years. On the other hand, in 1376 years since Ptolemy, about 12 days would have been missing, and one day in 115 years. In both instances the year has again turned out to be nonuniform.

I also observed the vernal equinox which occurred in the following year, 1516 C.E., $4\,{}^{1}/_{3}$ hours after the midnight preceding 11 March. From that vernal equinox of Ptolemy (the meridian of Alexandria being compared with ours) there are 1376 Egyptian years, 332 days, $16\,{}^{1}/_{3}$ hours. Hence it is also clear that the intervals of the vernal and autumnal equinoxes are unequal. The solar year, taken in this way, is very far from being uniform.

For in the case of the autumnal equinoxes, between Ptolemy and us (as was pointed out) by comparison with the uniform distribution of the years the ${}^{1}/_{4}{}^{d}$ lacked ${}^{1}/_{115}{}^{d}$. This deficiency disagrees with Al-Battani's equinox by half a day. On the other hand, what holds true for the period from Al-Battani to us (when the ${}^{1}/_{4}{}^{d}$ must have lacked ${}^{1}/_{128}{}^{d}$) does not fit Ptolemy, for whom the computed result precedes his observed equinox by more than a whole day, and Hipparchus' by more than two days. In like manner a computation based on the period from Ptolemy to Al-Battani exceeds Hipparchus' equinox by two days.

The uniform length of the solar year, therefore, is more correctly derived from the sphere of the fixed stars, as was first discovered by Thabit ibn Qurra. He found its length to be 365 days, plus 15 minutes of a day and 23 seconds of a day, or approximately 6 hours, 9 minutes, 12 seconds. He probably based his reasoning on the fact that when the equinoxes and solstices recurred more slowly, the year appeared longer than when they recurred more swiftly, in accordance with a definite ratio, moreover. This could not happen unless a uniform length were available by comparison with the sphere of the fixed stars. Consequently we must not heed Ptolemy in this regard. He thought that it was ridiculous and outlandish for the annual uniform motion of the sun to be measured by its return to any of the fixed stars, and that this was no more appropriate than if it were done by someone with reference to Jupiter or Saturn [*Syntaxis*, III, 1]. Therefore the explanation is at hand why the tropical year was longer before Ptolemy, whereas after him it became shorter in a variable diminution.

But also in connection with the starry or sidereal year a variation can occur. Nevertheless, it is limited and far smaller than the one which I just explained. The reason is that this same motion of the earth's center, which appears in the sun, is also nonuniform, with another twofold variation. The first of these variations is simple, having an annual period. The second, which by its alternations produces an inequality in the first, is perceived not at once but after a long passage of time. Therefore the computation of the uniform year is neither elementary nor easy to understand. For suppose that somebody wished to derive the uniform year merely from the definite distance of a star having a known position. This can be done by using the astrolabe with the moon as intermediary, the procedure I explained in connection with Regulus in the Lion [II, 14]. Variation will not be completely avoided, unless at that time on account of the earth's motion the sun either has no prosthaphaeresis or undergoes a similar and equal prosthaphaeresis at both cardinal points. If this does not happen, and if there is some variation in the nonuniformity of the cardinal points, it will be evident that a uniform revolution certainly does not occur in equal times. On the other hand, if at both cardinal points the entire variation is subtracted or added proportionally, the process will be perfect.

Furthermore, an understanding of the nonuniformity requires prior knowledge of the mean motion, which we seek for that reason, being engaged therein like Archimedes in squaring the circle. Nevertheless, for the purpose of eventually reaching the solution of this problem, I find that there are altogether four causes of the apparent nonuniformity. The first is the nonuniformity in the precession of the equinoxes, which I have explained [III, 3]. The second is the inequality in the arcs of the ecliptic which the sun is seen to traverse, a nearly annual inequality. This is also subject to a variation by the third cause, which I shall call the "second inequality". The last is the fourth, which shifts the higher and lower apsides of the earth's center, as will be made clear below [III, 20]. Of all these [four causes], Ptolemy [*Syntaxis*, III, 4] knew only the second, which by itself could not have produced the annual nonuniformity, but does so, rather, when intermingled with the others. However, in order to demonstrate the difference between uniformity and appearance in the sun, an absolutely precise measurement of the year seems unnecessary. On the contrary, for this demonstration it would be satisfactory to take as the length of the year $365\,^1/_4$ days, in which the motion of the first inequality is completed. For, what falls so little short of a complete circle, disappears entirely when absorbed in a smaller magnitude. But for the sake of orderly procedure and ease of comprehension I now set forth the uniform motions of the annual revolution of the earth's center. Later I shall add to them by distinguishing between the uniform and apparent motions on the basis of the required proofs [III, 15].

THE UNIFORM AND MEAN MOTIONS IN THE REVOLUTIONS OF THE EARTH'S CENTER *Chapter 14*

The length of the uniform year, I have found, is only $1^{10}/_{60}$ day-seconds longer than Thabit ibn Qurra's value [III, 13]. Thus it is 365 days plus 15 day-minutes, 24 day-seconds, and 10 sixtieths of a day-second, equal to 6 uniform hours, 9 minutes, 40 seconds, and the precise uniformity of the year is clearly linked with the sphere of the fixed stars. Therefore, by multiplying the 360° of a circle by 365 days, and dividing the product by 365 days, 15 day-minutes, $24\,^{10}/_{60}$ day-seconds, we shall have the motion in an Egyptian year as $5\times60°+59°$ 44′ 49″ 7‴ 4⁗. In 60 similar years the motion is, after the elimination of whole circles, $5\times60°+44°$ 49′ 7″ 4‴. Furthermore, if we divide the annual motion by 365 days, we shall have the daily motion as 59′ 8″ 11‴ 22⁗. By adding to this value the mean and uniform precession of the equinoxes [III, 6], we shall obtain also the uniform annual motion in a tropical year as $5\times60°+$ 59° 45′ 39″ 19‴ 9⁗, and the daily motion as 59′ 8″ 19‴ 37⁗. For this reason we may call the former solar motion "simple uniform", to use the familiar expression, and the latter motion "composite uniform". I shall also set them out in Tables, as I have done for the precession of the equinoxes [following III, 6]. Appended to these Tables is the uniform solar motion in anomaly, a topic I shall discuss later on [III, 18].

TABLE OF THE SUN'S SIMPLE UNIFORM MOTION IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 272°31′

Years	Motion					Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	5	59	44	49	7	31	5	52	9	22	39
2	5	59	29	38	14	32	5	51	54	11	46
3	5	59	14	27	21	33	5	51	39	0	53
4	5	58	59	16	28	34	5	51	23	50	0
5	5	58	44	5	35	35	5	51	8	39	7
6	5	58	28	54	42	36	5	50	53	28	14
7	5	58	13	43	49	37	5	50	38	17	21
8	5	57	58	32	56	38	5	50	23	6	28
9	5	57	43	22	3	39	5	50	7	55	35
10	5	57	28	11	10	40	5	49	52	44	42
11	5	57	13	0	17	41	5	49	37	33	49
12	5	56	57	49	24	42	5	49	22	22	56
13	5	56	42	38	31	43	5	49	7	12	3
14	5	56	27	27	38	44	5	48	52	1	10
15	5	56	12	16	46	45	5	48	36	50	18
16	5	55	57	5	53	46	5	48	21	39	25
17	5	55	41	55	0	47	5	48	6	28	32
18	5	55	26	44	7	48	5	47	51	17	39
19	5	55	11	33	14	49	5	47	36	6	46
20	5	54	56	22	21	50	5	47	20	55	53
21	5	54	41	11	28	51	5	47	5	45	0
22	5	54	26	0	35	52	5	46	50	34	7
23	5	54	10	49	42	53	5	46	35	23	14
24	5	53	55	38	49	54	5	46	20	12	21
25	5	53	40	27	56	55	5	46	5	1	28
26	5	53	25	17	3	56	5	45	49	50	35
27	5	53	10	6	10	57	5	45	34	39	42
28	5	52	54	55	17	58	5	45	19	28	49
29	5	52	39	44	24	59	5	45	4	17	56
30	5	52	24	33	32	60	5	44	49	7	4

TABLE OF THE SUN'S SIMPLE UNIFORM MOTION IN DAYS, PERIODS OF SIXTY DAYS AND MINUTES OF A DAY

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	59	8	11	31	0	30	33	13	52
2	0	1	58	16	22	32	0	31	32	22	3
3	0	2	57	24	34	33	0	32	31	30	15
4	0	3	56	32	45	34	0	33	30	38	26
5	0	4	55	40	56	35	0	34	29	46	37
6	0	5	54	49	8	36	0	35	28	54	49
7	0	6	53	57	19	37	0	36	28	3	0
8	0	7	53	5	30	38	0	37	27	11	11
9	0	8	52	13	42	39	0	38	26	19	23
10	0	9	51	21	53	40	0	39	25	27	34
11	0	10	50	30	5	41	0	40	24	35	45
12	0	11	49	38	16	42	0	41	23	43	57
13	0	12	48	46	27	43	0	42	22	52	8
14	0	13	47	54	39	44	0	43	22	0	20
15	0	14	47	2	50	45	0	44	21	8	31
16	0	15	46	11	1	46	0	45	20	16	42
17	0	16	45	19	13	47	0	46	19	24	54
18	0	17	44	27	24	48	0	47	18	33	5
19	0	18	43	35	35	49	0	48	17	41	16
20	0	19	42	43	47	50	0	49	16	49	28
21	0	20	41	51	58	51	0	50	15	57	39
22	0	21	41	0	9	52	0	51	15	5	50
23	0	22	40	8	21	53	0	52	14	14	2
24	0	23	39	16	32	54	0	53	13	22	13
25	0	24	38	24	44	55	0	54	12	30	25
26	0	25	37	32	55	56	0	55	11	38	36
27	0	26	36	41	6	57	0	56	10	46	47
28	0	27	35	49	18	58	0	57	9	54	59
29	0	28	34	57	29	59	0	58	9	3	10
30	0	29	34	5	41	60	0	59	8	11	22

TABLE OF THE SUN'S UNIFORM COMPOSITE MOTION IN YEARS AND PERIODS OF SIXTY YEARS

Egyptian Years	Motion					Egyptian Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	5	59	45	39	19	31	5	52	35	18	53
2	5	59	31	18	38	32	5	52	20	58	12
3	5	59	16	57	57	33	5	52	6	37	31
4	5	59	2	37	16	34	5	51	52	16	51
5	5	58	48	16	35	35	5	51	37	56	10
6	5	58	33	55	54	36	5	51	23	35	29
7	5	58	19	35	14	37	5	51	9	14	48
8	5	58	5	14	33	38	5	50	54	54	7
9	5	57	50	53	52	39	5	50	40	33	26
10	5	57	36	33	11	40	5	50	26	12	46
11	5	57	22	12	30	41	5	50	11	52	5
12	5	57	7	51	49	42	5	49	57	31	24
13	5	56	53	31	8	43	5	49	43	10	43
14	5	56	39	10	28	44	5	49	28	50	2
15	5	56	24	49	47	45	5	49	14	29	21
16	5	56	10	29	6	46	5	49	0	8	40
17	5	55	56	8	25	47	5	48	45	48	0
18	5	55	41	47	44	48	5	48	31	27	19
19	5	55	27	27	3	49	5	48	17	6	38
20	5	55	13	6	23	50	5	48	2	45	57
21	5	54	58	45	42	51	5	47	48	25	16
22	5	54	44	25	1	52	5	47	34	4	35
23	5	54	30	4	20	53	5	47	19	43	54
24	5	54	15	43	39	54	5	47	5	23	14
25	5	54	1	22	58	55	5	46	51	2	33
26	5	53	47	2	17	56	5	46	36	41	52
27	5	53	32	41	37	57	5	46	22	21	11
28	5	53	18	20	56	58	5	46	8	0	30
29	5	53	4	0	15	59	5	45	53	39	49
30	5	52	49	39	34	60	5	45	39	19	9

TABLE OF THE SUN'S UNIFORM COMPOSITE MOTION IN DAYS, PERIODS OF SIXTY DAYS, AND MINUTES OF A DAY

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	59	8	19	31	0	30	33	18	8
2	0	1	58	16	39	32	0	31	32	26	27
3	0	2	57	24	58	33	0	32	31	34	47
4	0	3	56	33	18	34	0	33	30	43	6
5	0	4	55	41	38	35	0	34	29	51	26
6	0	5	54	49	57	36	0	35	28	59	46
7	0	6	53	58	17	37	0	36	28	8	5
8	0	7	53	6	36	38	0	37	27	16	25
9	0	8	52	14	56	39	0	38	26	24	45
10	0	9	51	23	16	40	0	39	25	33	4
11	0	10	50	31	35	41	0	40	24	41	24
12	0	11	49	39	55	42	0	41	23	49	43
13	0	12	48	48	15	43	0	42	22	58	3
14	0	13	47	56	34	44	0	43	22	6	23
15	0	14	47	4	54	45	0	44	21	14	42
16	0	15	46	13	13	46	0	45	20	23	2
17	0	16	45	21	33	47	0	46	19	31	21
18	0	17	44	29	53	48	0	47	18	39	41
19	0	18	43	38	12	49	0	48	17	48	1
20	0	19	42	46	32	50	0	49	16	56	20
21	0	20	41	54	51	51	0	50	16	4	40
22	0	21	41	3	11	52	0	51	15	13	0
23	0	22	40	11	31	53	0	52	14	21	19
24	0	23	39	19	50	54	0	53	13	29	39
25	0	24	38	28	10	55	0	54	12	37	58
26	0	25	37	36	30	56	0	55	11	46	18
27	0	26	36	44	49	57	0	56	10	54	38
28	0	27	35	53	9	58	0	57	10	2	57
29	0	28	35	1	28	59	0	58	9	11	17
30	0	29	34	9	48	60	0	59	8	19	37

TABLE OF THE SUN'S UNIFORM MOTION IN ANOMALY IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 211°19′

Egyptian Years	Motion					Egyptian Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	5	59	44	24	46	31	5	51	56	48	11
2	5	59	28	49	33	32	5	51	41	12	58
3	5	59	13	14	20	33	5	51	25	37	45
4	5	58	57	39	7	34	5	51	10	2	32
5	5	58	42	3	54	35	5	50	54	27	19
6	5	58	26	28	41	36	5	50	38	52	6
7	5	58	10	53	27	37	5	50	23	16	52
8	5	57	55	18	14	38	5	50	7	41	39
9	5	57	39	43	1	39	5	49	52	6	26
10	5	57	24	7	48	40	5	49	36	31	13
11	5	57	8	32	35	41	5	49	20	56	0
12	5	56	52	57	22	42	5	49	5	20	47
13	5	56	37	22	8	43	5	48	49	45	33
14	5	56	21	46	55	44	5	48	34	10	20
15	5	56	6	11	42	45	5	48	18	35	7
16	5	55	50	36	29	46	5	48	2	59	54
17	5	55	35	1	16	47	5	47	47	24	41
18	5	55	19	26	3	48	5	47	31	49	28
19	5	55	3	50	49	49	5	47	16	14	14
20	5	54	48	15	36	50	5	47	0	39	1
21	5	54	32	40	23	51	5	46	45	3	48
22	5	54	17	5	10	52	5	46	29	28	35
23	5	54	1	29	57	53	5	46	13	53	22
24	5	53	45	54	44	54	5	45	58	18	9
25	5	53	30	19	30	55	5	45	42	42	55
26	5	53	14	44	17	56	5	45	27	7	42
27	5	52	59	9	4	57	5	45	11	32	29
28	5	52	43	33	51	58	5	44	55	57	16
29	5	52	27	58	38	59	5	44	40	22	3
30	5	52	12	23	25	60	5	44	24	46	50

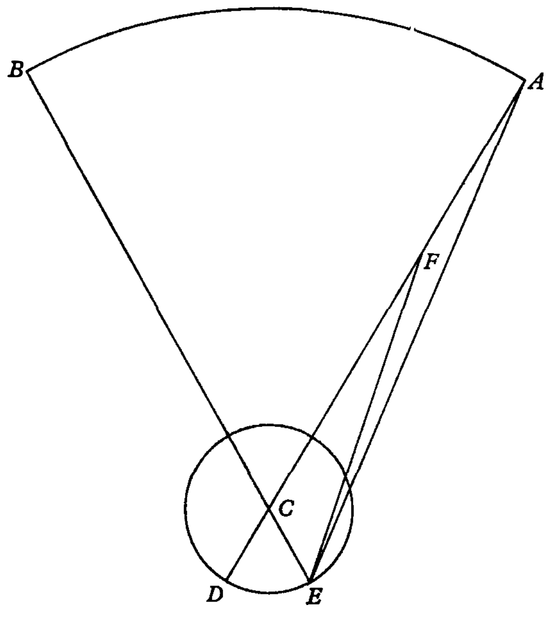
THE SUN'S ANOMALY IN DAYS AND PERIODS OF SIXTY DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	59	8	7	31	0	30	33	11	48
2	0	1	58	16	14	32	0	31	32	19	55
3	0	2	57	24	22	33	0	32	31	28	3
4	0	3	56	32	29	34	0	33	30	36	10
5	0	4	55	40	36	35	0	34	29	44	17
6	0	5	54	48	44	36	0	35	28	52	25
7	0	6	53	56	51	37	0	36	28	0	32
8	0	7	53	4	58	38	0	37	27	8	39
9	0	8	52	13	6	39	0	38	26	16	47
10	0	9	51	21	13	40	0	39	25	24	54
11	0	10	50	29	21	41	0	40	24	33	2
12	0	11	49	37	28	42	0	41	23	41	9
13	0	12	48	45	35	43	0	42	22	49	16
14	0	13	47	53	43	44	0	43	21	57	24
15	0	14	47	1	50	45	0	44	21	5	31
16	0	15	46	9	57	46	0	45	20	13	38
17	0	16	45	18	5	47	0	46	19	21	46
18	0	17	44	26	12	48	0	47	18	29	53
19	0	18	43	34	19	49	0	48	17	38	0
20	0	19	42	42	27	50	0	49	16	46	8
21	0	20	41	50	34	51	0	50	15	54	15
22	0	21	40	58	42	52	0	51	15	2	23
23	0	22	40	6	49	53	0	52	14	10	30
24	0	23	39	14	56	54	0	53	13	18	37
25	0	24	38	23	4	55	0	54	12	26	45
26	0	25	37	31	11	56	0	55	11	34	52
27	0	26	36	39	18	57	0	56	10	42	59
28	0	27	35	47	26	58	0	57	9	51	7
29	0	28	34	55	33	59	0	58	8	59	14
30	0	29	34	3	41	60	0	59	8	7	22

PRELIMINARY THEOREMS FOR PROVING THE NONUNIFORMITY OF THE SUN'S APPARENT MOTION *Chapter 15*

For the sake of better comprehension of the sun's apparent nonuniformity, however, I shall show even more clearly that with the sun at the universe's midpoint, about which as center the earth revolves, if the distance between the sun and the earth is, as I have said [I, 5, 10], imperceptible in comparison with the immensity of the sphere of the fixed stars, the sun will appear to move uniformly with respect to any given point or star on that sphere.

Let *AB* be a great circle of the universe in the place of the ecliptic. Let *C* be its center, where the sun is located. With radius *CD*, the distance sun-earth, in comparison with which the height of the universe is immense, in that same plane of the ecliptic describe *DE* as the circle in which the annual revolution of the earth's center is performed. I say that the sun will appear to move uniformly with respect to any given point or star on the circle *AB*. Let the given point be *A*, where the sun is seen from the earth. Let the earth be at *D*. Draw *ACD*. Now let the earth move through any arc *DE*. Draw *AE* and *BE* from *E*, the endpoint of the earth's [motion]. Therefore the sun will now be seen from *E* at point *B*. Since *AC* is immense in comparison with *CD* or its equivalent *CE*, *AE* also will be immense as compared with *CE*. For, on *AC* take any point *F*, and join *EF*. Then from *C* and *E*, the endpoints of the base, two straight lines fall outside triangle *EFC* on point *A*. Therefore, by the converse of Euclid's *Elements*, I, 21, angle *FAE* will be smaller than angle *EFC*. Consequently, when the straight lines are immensely extended, they will ultimately comprise *CAE* as an angle so acute that it can no longer be perceived. *CAE* constitutes the difference by which angle *BCA* exceeds angle *AEC*. These angles even seem equal because the difference [between them] is so slight. The lines *AC* and *AE* seem parallel, and the sun seems to move uniformly with respect to any point on the sphere of the stars, just as if it revolved around *E* as center. Q. E. D.



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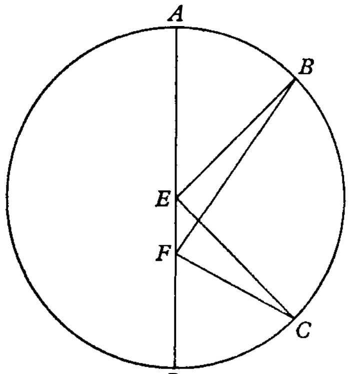
Its nonuniformity, however, is explained in two ways. Either the circular path of the earth's center is not concentric with the sun or universe...]

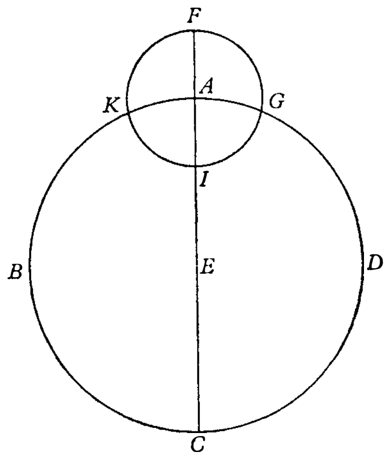
The sun's [motion], however, is demonstrably nonuniform, because the motion of the earth's center in its annual revolution does not occur precisely around the center of the sun. This can of course be explained in two ways, either by an eccentric circle, that is, a circle whose center is not identical with the sun's center, or by an epicycle on a concentric circle [that is, a circle whose center is identical with the sun's center, and functions as the epicycle's deferent].



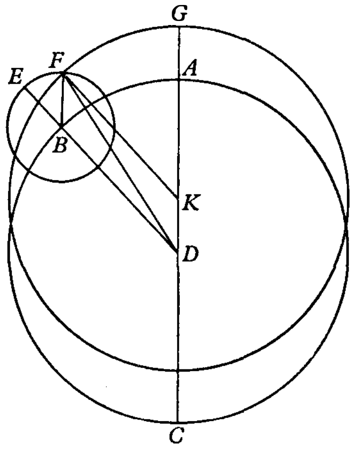
The explanation by means of an eccentric proceeds as follows. In the plane of the ecliptic let *ABCD* be an eccentric. Let its center *E* lie at no negligible distance from *F*, the center of the sun or of the universe. Let *AEFD*, the diameter of the eccentric, pass through both centers. Let *A* be the apogee, which in Latin is called the "higher apse", the position farthest from the center of the universe. On the other hand, let *D* be the perigee, which is the "lower apse", the position nearest [to the center of the universe]. Then, while the earth moves uniformly on its circle *ABCD* about center *E*, from *F* (as I just said) its motion will appear nonuniform. For, if we take *AB* and *CD* as equal arcs, and draw the straight lines *BE, CE, BF,* and *CF,* angles *AEB* and *CED* will be equal, intercepting equal arcs around center *E*. However, the observed angle *CFD*, being an exterior angle, is greater than the interior angle *CED*. Therefore, angle *CFD* is also greater than angle *AEB*, which is equal to angle *CED*. But angle *AEB*, as an exterior angle, is likewise greater than the interior angle *AFB*. So much the more is angle *CFD* greater than angle *AFB*. But both were produced in equal times, since *AB* and *CD* are equal arcs. Therefore, the uniform motion around *E* will appear nonuniform around *F*.

The same result may be seen more simply, because arc *AB* lies farther from *F* than does arc *CD*. For, according to Euclid's *Elements*, III, 7, with reference to the lines intercepting these arcs, *AF* and *BF* are longer than *CF* and *DF*. In optics it is proved that equal magnitudes appear larger when nearer than when farther away. Therefore, the proposition concerning the eccentric is established.

[Marginal note, inserted in the wrong place, and later deleted, but restored by the editors: The proof is exactly the same, if the earth were at rest in *F*, and the sun moved in the circumference *ABC*, as in Ptolemy and other authors.]

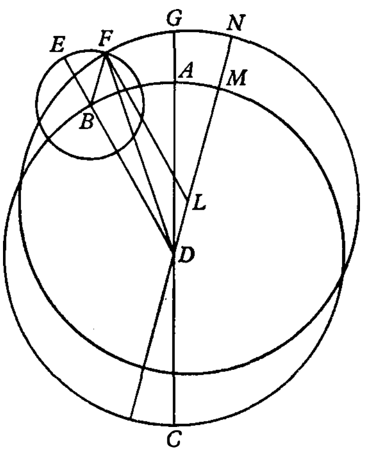


The same result will be accomplished also by an epicycle on a concentric. Let *E*, the center of the universe, where the sun is situated, also be the center of the concentric *ABCD*. In the same plane let *A* be the center of the epicycle *FG*. Through both centers draw the straight line *CEAF*, with the epicycle's apogee at *F* and perigee at *I*. Clearly, then, uniform motion occurs in *A*, but the apparent nonuniformity in the epicycle *FG*. For, suppose that *A* moves in the direction of *B*, that is, in consequence, whereas the center of the earth moves from the apogee *F* in precedence. The motion of *E* will appear faster at the perigee, which is *I*, because the motions of both *A* and *I* are in the same direction. On the other hand, at the apogee, which is *F*, *E* will seem to be slower, because it is moved only by the overbalancing motion of two contrary [motions]. When the earth is situated at *G*, it will surpass the uniform motion, behind which it will lag when it is situated at *K*. In either case the difference will be the arc *AG* or *AK*, by which therefore the sun likewise will seem to move nonuniformly.

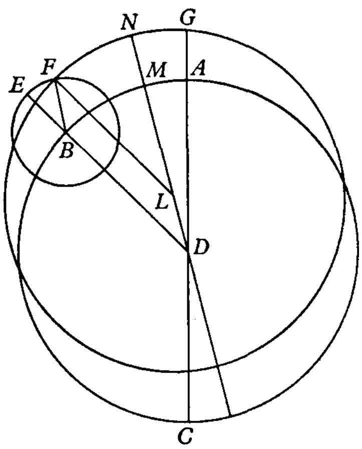


Whatever is done by an epicycle, however, can be accomplished in the same way by an eccentric. This is described equal to the concentric and in the same plane by the planet as it travels on the epicycle, the distance from the eccentric's center to the concentric's center being the length of the epicycle's radius. This happens, moreover, in three ways.

Suppose that the epicycle on the concentric and the planet on the epicycle execute revolutions that are equal but opposite in direction. Then the planet's motion will trace a fixed eccentric, whose apogee and perigee have unchangeable positions. Thus, let *ABC* be a concentric; *D*, the center of the universe; and *ADC*, a diameter. Assume that when the epicycle is in *A*, the planet is in the epicycle's apogee. Let this be *G*, and let the epicycle's radius fall on the straight line *DAG*. Take *AB* as an arc of the concentric. With *B* as center, and with radius equal to *AG*, describe the epicycle *EF*. Draw *DB* and *EB* as a straight line. Take the arc *EF* similar to *AB* and in the opposite direction. Place the planet or the earth at *F*, and join *BF*. On *AD* take the line segment *DK* equal to *BF*. Then the angles at *EBF* and *BDA* are equal, and therefore *BF* and *DK* are parallel and equal. But if straight lines are joined to equal and parallel straight lines, they also are parallel and equal, according to Euclid, I, 33. Since *DK* and *AG* are taken to be equal, and *AK* is their common annex, *GAK* will be equal to *AKD*, and therefore equal also to *KF*. Hence, the circle described with *K* as center, and radius *KAG*, will pass through *F*. By the composite motion of *AB* and *EF*, *F* describes an eccentric equal to the concentric, and therefore also fixed. For while the epicycle executes revolutions equally with the concentric, the apsides of the eccentric so described must remain in the same place (since *BF* and *AD* are always parallel on account of the equality of the angles *EBF* and *BDK* [these words were later deleted]).



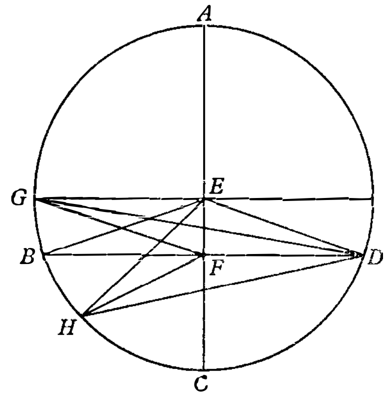
But if the revolutions executed by the epicycle's center and circumference are unequal, the planet's motion will no longer trace a fixed eccentric. Instead, the eccentric's center and apsides move in precedence or in consequence according as the planet's motion is swifter or slower than the center of its epicycle. Thus, suppose that angle *EBF* is bigger than angle *BDA*, but angle *BDM* is constructed equal to angle *EBF*. It will likewise be shown that, if *DL* on line *DM* is taken equal to *BF*, the circle described with *L* as center and radius *LMN*, equal to *AD*, will pass through the planet at *F*. Hence, the planet's composite motion obviously describes *NF* as the arc of an eccentric circle, whose apogee has meanwhile moved in precedence from point *G* through the arc *GN*. On the other hand, if the planet's motion on the epicycle is slower [than the motion of the epicycle's center], then the eccentric's center will move in consequence as far as the epicycle's center moves. For example, if angle *EBF* is smaller than angle *BDA* but equal to angle *BDM*, what I have said obviously happens.



From all these analyses it is clear that the same apparent nonuniformity always occurs either through an epicycle on a concentric or through an eccentric equal to the concentric. There is no difference between them provided that the distance between their centers is equal to the epicycle's radius.

Hence it is not easy to decide which of them exists in the heavens. For his part Ptolemy believed that the model of the eccentric was adequate where he understood there was a simple inequality, and the positions of the apsides were fixed and unchangeable, as in the case of the sun, according to his thinking [*Syntaxis*, III, 4]. But for the moon and the other five planets, which travel with a twofold or manifold nonuniformity, he adopted eccentrepicycles. By means of these models, furthermore, it is easily shown that the greatest difference between the uniform and apparent motions is seen when the planet appears midway between the higher and lower apsides according to the eccentric model, but according to the epicyclic model when the planet touches the deferent, as Ptolemy makes clear [*Syntaxis,* III, 3].

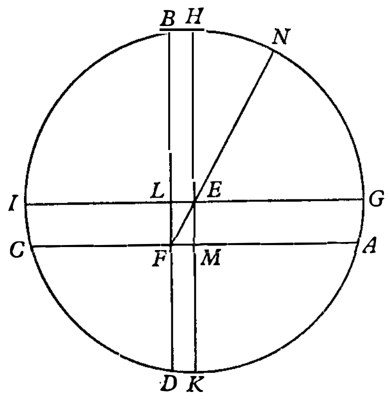
The proof proceeds as follows in the case of the eccentric. Let it be *ABCD,* with *E* as center, and *AEC* as the diameter passing through the sun at *F* outside the center. Through *F* draw *BFD* perpendicular [to diameter *AEC*]; join *BE* and *ED*. Let *A* be the apogee; *C,* the perigee; *B* and *D,* the apparent midpoints between them. The exterior angle *AEB* [of triangle *BEF*], it is clear, comprises the uniform motion, while the interior angle *EFB* comprises the apparent motion. The difference between them is the angle *EBF*. I say that no angle greater than angle *B* or angle *D* can be drawn from the circumference to the line *EF*. For, take points *G* and *H* before and after *B*. Join *GD, GE, GF,* and *HE, HF, HD*. Then *FG,* which is nearer to the center, is longer than *DF*. Therefore, angle *GDF* will be bigger than angle *DGF*. But the angles *EDG* and *EGD* are equal (since the sides *EG* and *ED* falling on the base [*DG*] are equal). Therefore angle *EDF,* which is equal to angle *EBF,* is greater than angle *EGF*. In like manner *DF* also is longer than *FH,* and angle *FHD* is greater than angle *FDH*. But the whole angle *EHD* is equal to the whole angle *EDH,* since *EH* is equal to *ED*. Therefore the remainder, angle *EDF,* which is equal to angle *EBF,* is also greater than the remainder *EHF*. Hence nowhere will a greater angle be drawn to the line *EF* than from points *B* and *D*. Consequently, the greatest difference between the uniform motion and the apparent motion occurs at the apparent midpoint between the apogee and the perigee.



THE SUN'S APPARENT NONUNIFORMITY *Chapter 16*

The foregoing are general proofs applicable not only to the solar phenomena but also to the nonuniformity of other bodies. For the present I shall take up the phenomena of the sun and the earth. Within that topic I shall first discuss what we have received from Ptolemy and other ancient authors, and then what we have learned from the more recent period and experience.

Ptolemy found that there were $94\,{}^1/_2$ days from the vernal equinox to the [summer] solstice, and $92\,{}^1/_2$ days from the [summer] solstice to the autumnal equinox [*Syntaxis,* III, 4]. On the basis of the [elapsed] time, the mean and uniform motion was 93° 9′ in the first interval; and in the second interval, 91° 11′. With these figures divide up the circle of the year. Let it be *ABCD,* with its center at *E, AB* = 93° 9′ for the first interval of time, and *BC* = 91° 11′ for the second. Let the vernal equinox be observed from *A*; the summer solstice, from *B*; the autumnal equinox, from *C*; and the remaining cardinal point, the winter solstice, from *D*. Join *AC* and *BD,* which intersect each other at right angles in *F,* where we place the sun. Then the arc *ABC* is greater than a semicircle; also, *AB* is greater than *BC*. Hence Ptolemy inferred [*Syntaxis,* III, 4] that *E,* the center of the circle, lies between lines *BF* and *FA*; and the apogee, between the vernal equinox and the solar summer solstice. Now through the



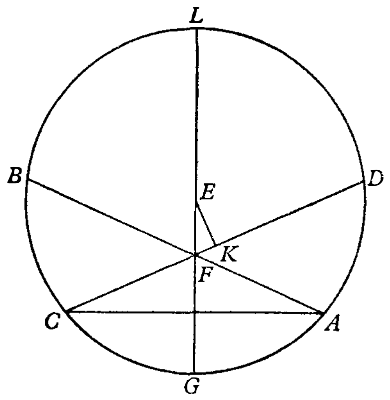
center *E* [and parallel] to *AFC,* draw *IEG,* which will intersect *BFD* in *L.* [Parallel] to *BFD,* draw *HEK,* which will cross *AF* at *M.* In this way there will be constructed the rectangular parallelogram *LEMF.* Its diagonal *FE,* when extended in the straight line *FEN,* will mark the earth's greatest distance from the sun, and the position of the apogee, in *N.* Then, since arc *ABC* is 184° 20′ [=93° 9′+91° 11′], *AH,* which is half of it, is 92° 10′. If this is subtracted from *AGB,* it leaves a remainder *HB* of 59′ [= 93° 9′−92° 10′]. Furthermore, when the [90] degrees of *HG,* a quadrant of the circle, are subtracted from *AH* [= 92° 10′], the remainder *AG* has 2° 10′. But half of the chord subtending twice the arc *AG* has 378 units, of which the radius has 10,000, and is equal to *LF.* Half of the chord subtending twice the arc *BH* is *LE,* which has 172 of the same units. Therefore, two sides of the triangle *ELF* being given, the hypotenuse *EF* will have 414 of the same units of which the radius has 10,000, or approximately $^1/_{24}$ of the radius *NE.* But *EF* : *EL* is the ratio of the radius *NE* to half of the chord subtending twice the arc *NH.* Therefore *NH* is given as 24 $^1/_2$°, and so is angle *NEH,* to which *LFE,* the angle of the apparent [motion], is equal. Consequently, this was the distance by which the higher apse preceded the summer solstice before Ptolemy.

On the other hand, *IK* is a quadrant of a circle. From it, subtract *IC* and *DK,* equal to *AG* [=2° 10′] and *HB* [=59′]. The remainder *CD* has 86° 51′ [=90° − 3° 9′]. When this is subtracted from *CDA* [=175°40′= 360°−184° 20′], the remainder *DA* has 88° 49′ [=175° 40′− 86° 51′]. But 88 $^1/_8$ days correspond to 86°51′; and to 88°49′, 90 days, plus $^1/_8$ day = 3 hours. In these periods, in terms of the earth's uniform motion, the sun seemed to pass from the autumnal equinox to the winter solstice, and to return from the winter solstice to the vernal equinox in what is left of the year.

Ptolemy states [*Syntaxis,* III, 4] that he too found these values no different from what Hipparchus had reported before him. Accordingly, he thought that for the rest of time the higher apse would remain 24 $^1/_2$° before the summer solstice, and that the eccentricity I mentioned, $^1/_{24}$ of the radius, would abide forever. Both values are now found to have changed with a perceptible difference.

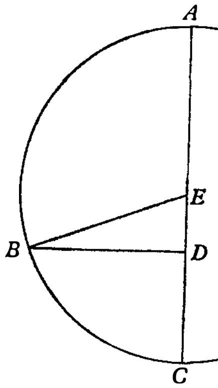
Al-Battani recorded 93^d 35^{dm} from the vernal equinox to the summer solstice, and to the autumnal equinox, 186^d 37^{dm}. From these figures he deduced by Ptolemy's method an eccentricity no greater than 346 units, of which the radius is 10,000. Al-Zarkali the Spaniard agrees with Al-Battani in regard to the eccentricity, but reported the apogee 12° 10′ before the solstice, whereas to Al-Battani it seemed to be 7° 43′ before the same solstice. From these results the inference was drawn that another nonuniformity in the motion of the earth's center still remains, as is confirmed also by the observations of our age.

For during the ten or more years since I have devoted my attention to investigating these topics, and in particular in 1515 C.E., I have found that 186^d 5 $^1/_2$^{dm} are completed between the vernal and autumnal equinoxes. To avoid an error in determining the solstices, which my predecessors are suspected by some scholars of having occasionally committed, in my research I added certain other solar positions which, in addition to the equinoxes, were not at all difficult to observe, such as the middle of the signs of the Bull, Virgin, Lion, Scorpion, and Water Bearer. Thus from the autumnal equinox to the middle of the Scorpion I found 45^d 16^{dm}, and 178^d 53 $^1/_2$^{dm} to the vernal equinox.

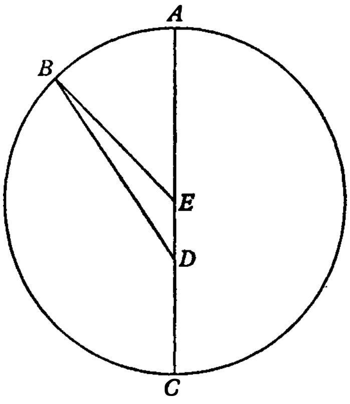


Now in the first interval the uniform motion is 44° 37′, and 176° 19′ in the second interval. With this information as a basis, reproduce the circle *ABCD*. Let *A* be the point from which the sun appeared at the vernal equinox; *B*, the point from which the autumnal equinox was observed; and *C*, the middle of the Scorpion. Join *AB* and *CD*, which intersect each other in *F*, the center of the sun. Draw *AC*. Then arc *CB* is known, since it is 44° 37′. Therefore angle *BAC* is given in terms of 360° = 2 right angles. *BFC*, the angle of the apparent motion, is 45° in terms of 360° = 4 right angles; but on the basis of 360° = 2 right angles, angle *BFC* = 90°. Hence the remainder, angle *ACD* [= *BFC*—*BAC*], which intercepts the arc *AD*, is 45° 23′ [= 90° — 44° 37′]. But the entire portion *ACB* = 176° 19′. When *BC* is subtracted [from *ACB*], the remainder *AC* = 131° 42′ [= 176° 19′ — 44° 37′]. When this figure is added to *AD* [= 45° 23′], the sum, arc *CAD*, = 177° $5\frac{1}{2}'$. Therefore, since each segment *ACB* [= 176°19′] and *CAD* is less than a semicircle, the center is clearly contained in *BD*, the rest of the circle. Let the center be *E*, and through *F* draw the diameter *LEFG*. Let *L* be the apogee, and *G* the perigee. Drop *EK* perpendicular to *CFD*. Now the chords subtending the given arcs are derived from the Table: *AC* = 182,494, and *CFD* = 199,934 units, of which the diameter = 200,000. Then the angles of triangle *ACF* are given. According to Theorem I on Plane Triangles [I, 13], the ratio of the sides will also be given: *CF* = 97,967 of the units of which *AC* = 182,494. Therefore *FD* [= *CFD*—*CF* = 199,934—97,967 = 101,967] exceeds half [of *CFD* = 199,934 ÷ 2 or 99,967], the excess being *FK* = 2,000 of the same units [101,967—99,967]. The segment *CAD* [≅ 177°6′] is less than a semicircle by 2° 54′. Half of the chord subtending this arc is equal to *EK* and has 2534 units. Therefore, in triangle *EFK* the two sides *FK* and *KE* which form the right angle are given. Of the given sides and angles, *EF* will have 323 units, of which *EL* has 10,000; and angle *EFK* has $51\frac{2}{3}$°, when 360° = 4 right angles. Therefore, the whole angle *AFL* [= *EFK*+(*AFD* = *BFC* = 45°)] has $96\frac{2}{3}$°[= $51\frac{2}{3}$° + 45°], and the remainder, angle *BFL* [= 180°—*AFL*] has $83\frac{1}{3}$°. If *EL* has 60 units, *EF* will be approximately one unit, 56 minutes of a unit. This was the sun's distance from the center of the circle, having now become barely $\frac{1}{31}$, whereas to Ptolemy it seemed to be $\frac{1}{24}$. Furthermore, the apogee, which then preceded the summer solstice by $24\frac{1}{2}$°, now follows it by $6\frac{2}{3}$°.

EXPLANATION OF THE FIRST AND ANNUAL SOLAR INEQUALITY, TOGETHER WITH ITS PARTICULAR VARIATIONS *Chapter 17*



Hence, since several variations are found in the solar inequality, I think that I should first set forth the annual variation, which is better known than the others. For this purpose, reproduce the circle *ABC*, with its center *E*, diameter *AEC*, apogee *A*, perigee *C*, and the sun at *D*. Now the greatest difference between the uniform [motion] and the apparent [motion] has been shown [III, 15] to occur at the apparent midpoint between the two apsides. For this reason, on *AEC* construct the perpendicular *BD*, intersecting the circumference in point *B*. Join *BE*. In the right triangle *BDE* two sides are given, namely, *BE*, the radius of the circle, and *DE*, the distance from the sun to the center. Therefore the angles of the triangle will be given, among them angle *DBE*, the difference between *BEA*, the angle of the uniform [motion], and the right angle *EDB*, [which is the angle of the] apparent [motion].



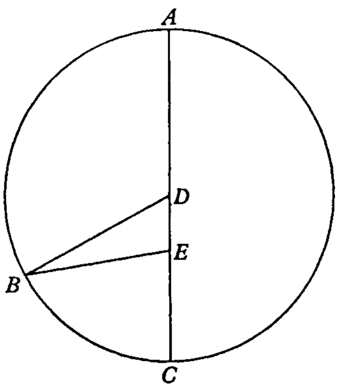
However, to the extent that *DE* has increased and decreased, the whole shape of the triangle has changed. Thus, the angle *B* was 2° 23′ before Ptolemy; 1° 59′ at the time of Al-Battani and Al-Zarkali; and at present it is 1° 51′. According to Ptolemy [*Syntaxis*, III, 4], the arc *AB*, intercepted by the angle *AEB*, was 92° 23′, and *BC* 87° 37′; *AB* was 91° 59′, and *BC* 88° 1′, according to Al-Battani; at present, *AB* is 91° 51′, *BC* 88° 9′.

From these facts the remaining variations are clear. For, as in the second diagram, take any other arc *AB*, such that angle *AEB*, the supplementary angle *BED*, and the two sides *BE* and *ED* are given. By the Theorems on Plane Triangles, angle *EBD* of the prosthaphaeresis and the difference between the uniform and apparent [motions] will be given. These differences also must change on account of the variation in the side *ED*, which was just mentioned.

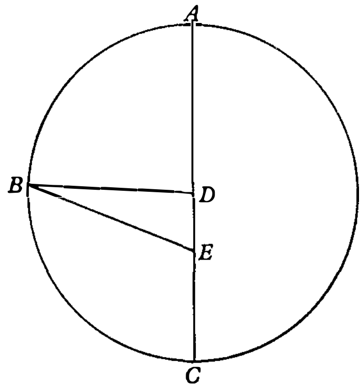
ANALYSIS OF THE UNIFORM MOTION IN LONGITUDE *Chapter 18*

The foregoing explanation of the annual solar inequality was based, not on the simple variation (as was made clear), but on a variation disclosed through the long passage of time to be intermingled with the simple variation. Later on [III, 20] I shall separate these variations from each other. Meanwhile, the mean and uniform motion of the earth's center will be established with greater numerical accuracy, the better it is distinguished from the nonuniform variations, and the longer the period of time over which it extends. Now this investigation will proceed as follows.

I took the autumnal equinox observed by Hipparchus at Alexandria in the 32nd year of the 3rd Callippic period, which was the 177th year after the death of Alexander, as was mentioned above [III, 13], on the third of the five intercalary days at midnight, followed by the fourth day. But since Alexandria lies about one hour east of Cracow in longitude, the time [at Cracow] was about an hour before midnight. Therefore, according to the computation reported above, the position of the autumnal equinox in the sphere of the fixed stars was 176° 10′ from the beginning of the Ram, and this was the apparent place of the sun, its distance from the higher apse being 114 $^1/_2$° [=24° 30′+90°]. To depict this situation, about center *D* draw *ABC*, the circle described by the center of the earth. Let *ADC* be the diameter, in which the sun is placed at *E*, with the apogee at *A* and the perigee at *C*. Let *B* be the point where the sun appeared to be at the autumnal equinox. Draw the straight lines *BD* and *BE*. Then angle *DEB*, the sun's apparent distance from the apogee, is 144 $^1/_2$°. At that time *DE* was 416 units, of which *BD* = 10,000. Therefore, according to Theorem IV [II, E] on Plane Triangles, in triangle *BDE* the angles are given. Angle *DBE*, the difference between angle *BED* and angle *BDA*, is 2°10′. But since angle *BED* = 114°30′, angle *BDA* will be 116° 40′ [=114° 30′+2° 10′]. Therefore, the mean or uniform place of the sun is 178° 20′ [=176° 10′+2° 10′] from the beginning of the Ram in the sphere of the fixed stars.



With this observation I compared the autumnal equinox which I observed in Frombork on the same Cracow meridian in the year 1515 C.E. on 14 September, in the 1840th Egyptian year after the death of Alexander on the 6th day of Phaophi, the second Egyptian month, half an hour after sunrise [III, 13]. At that time the place of the autumnal equinox, according to computation and observation, was 152° 45′ in the sphere of the fixed stars, at a distance of 83° 20′ from the higher apse, according to the foregoing analysis [III, 16, end]. Construct the angle *BEA* = 83° 20′, with 180° = 2 right angles. In triangle [*BDE*], two sides are given: *BD* = 10,000 units, and *DE* = 323 units. According to Theorem IV [II, E] on Plane Triangles, angle *DBE* will be about 1° 50′. If a circle circumscribes triangle *BDE*, angle *BED* will intercept an arc of 166° 40′, when 360° = 2 right angles. Side *BD* will be 19,864 units, of which the diameter = 20,000. In accordance with the given ratio of *BD* to *DE*, about 640 of the same units will be established as the length of *DE*, which subtends angle *DBE* = 3° 40′ at the circumference, but 1° 50′ as a central angle [=3° 40′ ÷ 2]. This was the prosthaphaeresis and the difference between the uniform and apparent [motions]. By adding it to angle *BED* = 83° 20′, we shall have angle *BDA* and arc *AB* = 85° 10′ [=83° 20′+1° 50′] as the distance of the uniform [motion] from the apogee. Hence the mean place of the sun in the sphere of the fixed stars is 154° 35′ [=152° 45′+1° 50′]. Between the two observations there are 1662 Egyptian years, 37 days, 18 minutes of a day, 45 seconds of a day. In addition to the complete revolutions, 1660 in number, the mean and uniform motion was about 336° 15′, in agreement with the number which I set down in the Table of Uniform Motion [following III, 14].



ESTABLISHING THE POSITIONS AND EPOCHS FOR THE SUN'S UNIFORM MOTION *Chapter 19*

From the death of Alexander the Great to Hipparchus' observation the elapsed time is 176 years, 362 days, 27 ½ minutes of a day, in which the mean motion is computed as 312° 43′. This figure is subtracted from the 178° 20′ for Hipparchus' observation [III, 18], supplemented by the 360° of a circle. The remainder, 225° 37′ [360°+178° 20′ = 538° 20′− 312° 43′ = 225° 37′], will be the position for the meridian of Cracow and Frombork, the place of my observation, at noon on the first day of Thoth, the first Egyptian month, for the epoch of the era commencing with the death of Alexander the Great. From that time to the epoch of the Roman era of Julius Caesar, in 278 years, 118 ½ days, the mean motion, after [the elimination of] complete revolutions, is 46° 27′. When this figure is added to the number for Alexander's position [225° 37′+ 46° 27′], the sum is 272° 4′ for Caesar's position at midnight preceding 1 January, the customary start of the Roman years and days. Then in 45 years, 12 days, or in 323 years, 130 ½ days after Alexander the Great [$278^y\ 118\ {}^1/_2{}^d + 45^y 12^d$], comes Christ's position at 272° 31′. Christ was born in the 3rd year of the 194th Olympiad [193×4 = 772+3]. This amounts to 775 years, 12 ½ days, from the beginning of the first Olympiad to midnight preceding 1 January [in the year of Christ's birth]. This likewise puts the position of the first Olympiad at 96° 16′ at noon on the first day of the month Hecatombaeon, the present equivalent of this day being 1 July in the Roman calendar. In this way the epochs of the simple solar motion are related to the sphere of the fixed stars. Furthermore, the positions of the composite [motion] are obtained by applying the precession [of the equinoxes]. Corresponding to the simple positions, the composite positions are, for the Olympiads, 90° 59′ [=96°

16′− 5° 16′; III, 11, end]; Alexander, 226° 38′ [=225° 37′+1° 2′]; Caesar, 276° 59′ [=272° 4′+4° 55′]; and Christ, 278° 2′ [=272°31′+5° 32′]. All these positions are reduced (as I mentioned) to the meridian of Cracow.

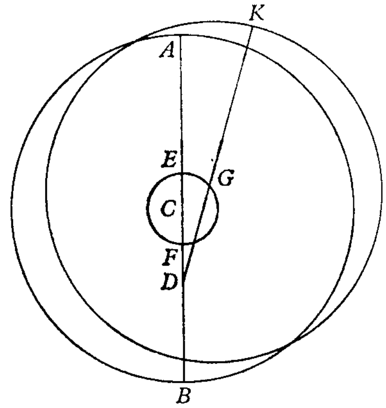
THE SECOND AND TWOFOLD INEQUALITY IMPOSED ON THE SUN BY THE SHIFT OF THE APSIDES *Chapter 20*

The shift in the solar apse now presents a problem which is more acute because, whereas Ptolemy regarded the apse as fixed, others thought that it accompanied the motion of the sphere of the stars, in conformity with their doctrine that the fixed stars move too. Al-Zarkali believed in the nonuniformity of this [motion], which even happened to regress. He relied on the following evidence. Al-Battani had found the apogee, as was mentioned above [III, 16], 7° 43′ ahead of the solstice: in the 740 years since Ptolemy it had advanced nearly 17° [≅24° 30′−7° 43′]. In the 193 years thereafter it seemed to Al-Zarkali to have retrogressed about 4 $^1/_2$° [≅12° 10′−7° 43′]. He therefore believed that the center of the annual orbit had an additional motion on a circlet. As a result, the apogee was deflected back and forth, while the distance from the center of the orbit to the center of the universe varied.

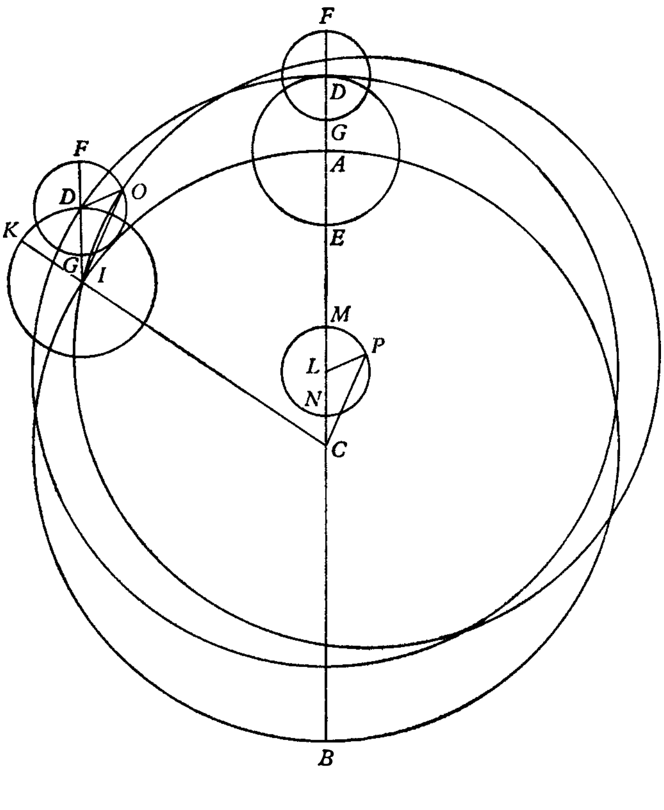
[Al-Zarkali's] idea was quite ingenious, but it has not been accepted because it is inconsistent with the other findings taken as a whole. Thus, consider the successive stages of that motion. For some time before Ptolemy it stood still. In 740 years or thereabouts it progressed through 17°. Then in 200 years it retrogressed 4° or 5°. Thereafter until our age it moved forward. The entire period has witnessed no other retrogression nor the several stationary points which must intervene at both limits when motions reverse their direction. [The absence of] these [retrogressions and stationary points] cannot possibly be understood in a regular and circular motion. Therefore many [specialists] believe that some error occurred in the observations of those [astronomers, that is, Al-Battani and Al-Zarkali]. Both [were] equally skillful and careful practitioners so that it is doubtful which one we should prefer to follow.

For my part I confess that nowhere is there a greater difficulty than in understanding the solar apogee, where we infer large [quantities] from certain minute and barely perceptible [magnitudes]. For near the perigee and apogee an entire degree produces a change of only 2′, more or less, in the prosthaphaeresis. On the other hand, near the intermediate distances 5° or 6° are traversed for 1′. Hence a slight error can develop into a very large one. Accordingly, even in putting the apogee at 6 $^2/_3$° within the Crab [III, 16], I was not satisfied to trust the time-measuring instruments, unless my results were also confirmed by solar and lunar eclipses. For any error lurking in the instruments is undoubtedly disclosed by the eclipses. It is highly probable, therefore, as we can deduce from the general structure of the motion, that it is direct, yet nonuniform. For after that stationary [interval] from Hipparchus to Ptolemy the apogee appeared in a continuous, regular, and progressive advance until the present time. An exception occurred between Al-Battani and Al-Zarkali through a mistake (it is believed), since everything else seems to fit. For in a similar way the solar prosthaphaeresis has likewise not yet stopped diminishing. Hence it seems to follow the same circular pattern, and both nonuniformities are in phase with that first and simple anomaly of the obliquity of the ecliptic, or with a similar irregularity.

To make this situation clearer, in the plane of the ecliptic draw the circle *AB*, with its center at *C*, and its diameter *ACB*, on which put the solar globe at *D* as the center of the universe. With *C* as center, describe *EF* as another circle of small dimensions which does not contain the sun. On this circlet let the center of the annual revolution of the earth's center be understood to move in a certain very slow advance. Together with the line *AD*, the circlet *EF* advances in consequence, whereas the center of the annual revolution moves along the circlet *EF* in precedence, both motions being quite slow. Therefore the center of the annual orbit will at one time be found at its greatest distance [from the sun], *DE*, and at another time at its least [distance], *DF*. Its motion will be slower at *E*, and faster at *F*. In the circlet's intervening arcs [the center of the annual orbit] makes that distance between the centers increase and decrease with time, and [it makes] the higher apse alternately precede and follow that apse or apogee which lies on line *ACD* and serves as the mean apogee. Thus, take the arc *EG*. With *G* as center, draw a circle equal to *AB*. Then the higher apse will lie on line *DGK*, and the distance *DG* will be shorter than *DE*, in accordance with Euclid, III, 8. These relations are demonstrated in this way by an eccentreccentric, and also by an epicyclepicyclet, as follows.



Let *AB* be concentric with the universe and with the sun. Let *ACB* be the diameter on which the higher apse lies. With *A* as center, describe the epicycle *DE*. Again, with *D* as center, draw the epicyclet *FG*, on which the earth revolves. Let all [these constructions] lie in the same plane, that of the ecliptic. Let the first



epicycle move in consequence in about a year. Let the second epicycle, that is, *D,* likewise move in a year, but in precedence. Let the revolutions of both epicycles be equal with respect to line *AC*. Furthermore, let the center of the earth [by moving] away from *F* in precedence add a little to *D*. Hence, when the earth is at *F,* clearly it will make the solar apogee a maximum, and a minimum when it is at *G*. In the intervening arcs of the epicyclet *FG,* moreover, it will make the apogee precede or follow [the mean apogee], accelerate or decelerate, increase or decrease. Thus the motion appears nonuniform, as was previously demonstrated by the epicycleccentric.

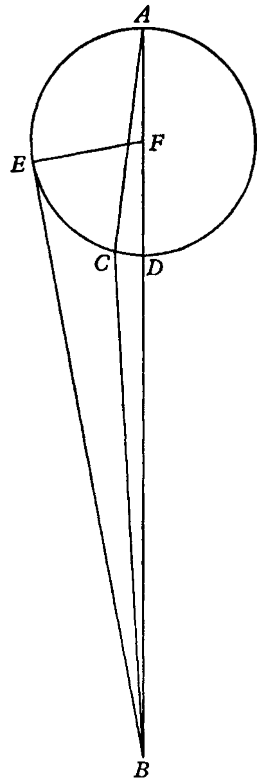
Now take the arc *AI*. With *I* as center, reconstruct the epicyclepicycle. Join *CI,* and prolong it along the straight line *CIK*. Angle *KID* will be equal to *ACI* on account of the equality of the revolutions. Therefore, as I showed above [III, 15], point *D* will describe around *L* as center and with eccentricity $CL = DI$ an eccentric equal to the concentric *AB*. *F* will also trace its own eccentric, with eccentricity $CLM = IDF$; and *G* likewise, with eccentricity $IG = CN$. Suppose that meanwhile the center of the earth has already traversed any arc *FO* on its own epicycle, the second one. *O* will now describe an eccentric whose center lies not on line *AC,* but on a line, such as *LP,* parallel to *DO*. Furthermore, if *OI* and *CP* are joined, they will be equal [to each other], but smaller than *IF* and *CM*; and angle *DIO* [will be] equal to angle *LCP,* in accordance with Euclid, I, 8. To that extent the solar apogee on line *CP* will be seen to precede *A*.

Hence it is also clear that the same thing happens with an eccentrepicycle. For from the previous [arrangement take] only that eccentric which is described by the epicyclet *D* around *L* as center. Let the center of the earth revolve along the arc *FO* under the aforementioned conditions, that is, a little beyond an annual revolution. Around *P* as center, it will trace a second circle, eccentric with respect to the first eccentric, and thereafter the same phenomena will recur. Since so many arrangements lead to the same result, I would not readily say which one is real, except that the perpetual agreement of the computations and phenomena compels the belief that it is one of them.

HOW LARGE IS THE SECOND VARIATION IN THE SOLAR INEQUALITY? *Chapter 21*

We have already seen [III, 20] that the second inequality follows the first and simple anomaly of the obliquity of the ecliptic, or something like it. Hence, unless impeded by some error of previous observers, we shall obtain its variations with precision. For by computation we have the simple anomaly as about 165° 39′ in 1515 C.E., and its beginning, by calculating backwards, in about 64 B. C. From that time until ours the total is 1580 years. When the anomaly began then, the eccentricity, I found, was at its maximum = 417 units, of which the radius = 10,000. Our eccentricity, on the other hand, was shown to be 323 [units].

Now let *AB* be a straight line, on which *B* is the sun and the center of the universe. Let the greatest eccentricity be *AB*; and the smallest, *DB*. With diameter *AD*, describe a circlet. On it take arc *AC* as the measure of the first, simple anomaly, which was 165° 39′. *AB* is given as 417 units, found at the beginning of the simple anomaly, that is, at *A*. On the other hand, at present *BC* is 323 units. Hence we shall have triangle *ABC,* with sides *AB* and *BC* given. One angle, *CAD,* also



[is given], because arc $CD = 14°\,21'$, being the remainder [when arc $AC = 165°\,39'$ is subtracted] from the semicircle. Therefore, in accordance with the Theorems on Plane Triangles the remaining side AC will be given, and also angle ABC, which is the difference between the apogee's mean and nonuniform motions. Since AC subtends a given arc, diameter AD of circle ACD will also be given. For, from angle $CAD = 14°\,21'$, we shall have $CB = 2486$ units, of which the diameter of the circle circumscribing the triangle is 100,000. The ratio $BC : AB$ gives $AB = 3225$ of the same units. AB intercepts the angle $ACB = 341°\,26'$. The remainder, with $360° = 2$ right angles, is the angle $CBD = 4°\,13'$, [$= 360° - (341°26' + 14°21' = 355°47')$], which is intercepted by $AC = 735$ units. Therefore, in units of which $AB = 417$, AC has been found to be about 95 units. Since AC subtends a given arc, it will have a ratio to AD as the diameter. Therefore AD is given as 96 units, of which $ADB = 417$. DB, the remainder [$= ADB - AD = 417 - 96$] $= 321$ units, the minimum extent of the eccentricity. Angle CBD, which was found to be $4°\,13'$ at the circumference, but $2°\,6\,{}^1/_2{}'$ at the center, is the prosthaphaeresis to be subtracted from the uniform motion of AB around B as center.

Now draw straight line BE tangent to the circle at point E. Take F as center, and join EF. In right triangle BEF, side EF is given as 48 units [$= {}^1/_2 \times 96 =$ diameter AD], and BDF as 369 units [$FD = 48 + 321 = DB$]. In units of which FDB as radius $= 10{,}000$, $EF = 1300$. This is half of the chord subtending twice the angle EBF and, with $360° = 4$ right angles, is $7°\,28'$, the greatest prosthaphaeresis between the uniform motion F and the apparent motion E.

Hence all the other individual differences can be obtained. Thus, assume that angle $AFE = 6°$. We shall have a triangle with sides EF and FB given, as well as angle EFB. From this information the prosthaphaeresis EBF will emerge as $41'$. But if angle $AFE = 12°$, we shall have the prosthaphaeresis $= 1°\,23'$; if $18°$, then $2°\,3'$; and so on for the rest by this method, as was stated above in connection with the annual prosthaphaereses [III, 17].

HOW THE SOLAR APOGEE'S UNIFORM AND NONUNIFORM MOTIONS ARE DERIVED — *Chapter 22*

The time when the greatest eccentricity coincided with the beginning of the first and simple anomaly was the 3rd year of the 178th Olympiad and the 259th year after Alexander the Great, according to the Egyptians [64 B. C.; III, 21]. Hence the apogee's true and mean positions were both at $5\,{}^1/_2°$ within the Twins, that is, $65\,{}^1/_2°$ from the vernal equinox. The true equinoctial precession, which also coincided with the mean [precession] at that time, was $4°\,38'$. When this figure is subtracted from $65\,{}^1/_2°$, the remainder, $60°\,52'$ from the beginning of the Ram in the fixed stars, was the place of the apogee. Furthermore, the apogee's place was found to be $6\,{}^2/_3°$ within the Crab in the 2nd year of the 573rd Olympiad or 1515 C.E. The precession of the vernal equinox by computation was $27{}^1/_4°$. If this figure is subtracted from $96\,{}^2/_3°$, the remainder is $69°\,25'$. The first anomaly at that time was $165°\,39'$. The prosthaphaeresis, by which the true place preceded the mean [place], was shown to have been $2°\,7'$ [$\cong 2°6{}^1/_2{}'$; III, 21]. Therefore the mean place of the solar apogee was known to be $71°\,32'$ [$= 69°\,25' + 2°\,7'$]. Hence in 1580 uniform Egyptian years the apogee's mean and uniform motion was 10° 41′ [≅ 71° 32′–60° 52′]. When this figure is divided by the number of years, we shall have the annual rate as 24″ 20‴ 14⁗.

DETERMINING THE SOLAR ANOMALY AND ESTABLISHING ITS POSITIONS *Chapter 23*

If the foregoing figures are subtracted from the simple, annual motion, which was 359° 44′ 49″ 7‴ 4⁗ [III, 14], the remainder, 359° 44′ 24″ 46‴ 50⁗, will be the annual uniform motion of the anomaly. Furthermore, when this [remainder] is divided by 365, the daily rate will emerge as 59′ 8″ 7‴ 2⁗, in agreement with what was set out in the Tables above [following III, 14]. Hence we shall also have the positions of the recognized epochs, beginning with the first Olympiad. For, the mean solar apogee half an hour after sunrise on 14 September in the 2nd year of the 573rd Olympiad was shown to be at 71° 37′, from which the mean solar distance was 83° 3′ [71° 32+83° 3′ = 154° 35′; III, 18]. From the first Olympiad there are 2290 Egyptian years, 281 days, 46 day-minutes. In this time the motion in anomaly, after the elimination of whole circles, was 42° 49′. When this figure is subtracted from 83° 3′, the remainder is 40° 14′ as the position of the anomaly at the first Olympiad. In the same way as before, the place for the epoch of Alexander is 166° 38′; for Caesar, 211° 11′; and for Christ, 211° 19′.

TABULAR PRESENTATION OF THE VARIATIONS IN THE UNIFORM AND APPARENT [SOLAR MOTIONS] *Chapter 24*

In order to enhance the usefulness of what has been proved concerning the variations in the sun's uniform and apparent [motions], I shall also set them out in a Table having sixty lines and six columns or rows. The first two columns will contain the number [of degrees of the annual anomaly] in both semicircles, I mean, the ascending [from 0° to 180°] and descending [from 360° to 180°] semicircles, arranged at intervals of 3°, as I did above for the [prosthaphaereses of the] motions of the equinoxes [following III, 8]. The third column will record the degrees [and minutes] of the variation in the motion of the solar apogee or in the anomaly; as correlated with every third degree, this variation rises to a maximum of about 7 $^1/_2$°. The fourth column will be reserved for the proportional minutes, which are 60 at the maximum. They enter the reckoning in conjunction with [the sixth column's] increase in the annual anomaly's prosthaphaereses, when these are greater [than the prosthaphaereses arising from the minimum distance between the sun and the center of the universe]. Since the largest increase for these [prosthaphaereses] is 32′, a sixtieth part [thereof] will be 32″. Then, in accordance with the size of the increases, which I shall derive from the eccentricity by the method explained above [III, 21], I shall put down the number of the sixtieths alongside every third degree. The fifth column will carry the annual and first variation's individual prosthaphaereses, based on the sun's least distance from the center [of the universe]. The sixth and last column will show the increases in those prosthaphaereses which occur at the greatest eccentricity. The Table follows.

TABLE OF THE SOLAR PROSTHAPHAERESES

Common Numbers		Central Prosthaphaereses		Proportional Minutes	Orbital Prosthaphaereses		Increases
Degree	Degree	Degree	Minute		Degree	Minute	Minute
3	357	0	21	60	0	6	1
6	354	0	41	60	0	11	3
9	351	1	2	60	0	17	4
12	348	1	23	60	0	22	6
15	345	1	44	60	0	27	7
18	342	2	5	59	0	33	9
21	339	2	25	59	0	38	11
24	336	2	46	59	0	43	13
27	333	3	5	58	0	48	14
30	330	3	24	57	0	53	16
33	327	3	43	57	0	58	17
36	324	4	2	56	1	3	18
39	321	4	20	55	1	7	20
42	318	4	37	54	1	12	21
45	315	4	53	53	1	16	22
48	312	5	8	51	1	20	23
51	309	5	23	50	1	24	24
54	306	5	36	49	1	28	25
57	303	5	50	47	1	31	27
60	300	6	3	46	1	34	28
63	297	6	15	44	1	37	29
66	294	6	27	42	1	39	29
69	291	6	37	41	1	42	30
72	288	6	46	40	1	44	30
75	285	6	53	39	1	46	30
78	282	7	1	38	1	48	31
81	279	7	8	36	1	49	31
84	276	7	14	35	1	49	31
87	273	7	20	33	1	50	31
90	270	7	25	32	1	50	32

TABLE OF THE SOLAR PROSTHAPHAERESES

Common Numbers		Central Prosthaphaereses		Proportional Minutes	Orbital Prosthaphaereses		Increases
Degree	Degree	Degree	Minute		Degree	Minute	Minute
93	267	7	28	30	1	50	32
96	264	7	28	29	1	50	33
99	261	7	28	27	1	50	32
102	258	7	27	26	1	49	32
105	255	7	25	24	1	48	31
108	252	7	22	23	1	47	31
111	249	7	17	21	1	45	31
114	246	7	10	20	1	43	30
117	243	7	2	18	1	40	30
120	240	6	52	16	1	38	29
123	237	6	42	15	1	35	28
126	234	6	32	14	1	32	27
129	231	6	17	12	1	29	25
132	228	6	5	11	1	25	24
135	225	5	45	10	1	21	23
138	222	5	30	9	1	17	22
141	219	5	13	7	1	12	21
144	216	4	54	6	1	7	20
147	213	4	32	5	1	3	18
150	210	4	12	4	0	58	17
153	207	3	48	3	0	53	14
156	204	3	25	3	0	47	13
159	201	3	2	2	0	42	12
162	198	2	39	1	0	36	10
165	195	2	13	1	0	30	9
168	192	1	48	1	0	24	7
171	189	1	21	0	0	18	5
174	186	0	53	0	0	12	4
177	183	0	27	0	0	6	2
180	180	0	0	0	0	0	0

COMPUTING THE APPARENT SUN *Chapter 25*

It is now quite clear, I believe, how the apparent position of the sun is computed from the foregoing [Table] for any given time. For that time, look for the true place of the vernal equinox or its precession together with the first, simple anomaly, as I explained above [III, 12]. Then through the Tables of the Uniform Motion [following III, 14, find] the mean simple motion of the center of the earth (or the motion of the sun, as you may wish to call it) and the annual anomaly. Add these [figures] to their established epochs [as given in III, 23]. Then, alongside the first, simple anomaly and its number, or an adjacent number, as recorded in the first or second column of the preceding Table, in the third column you will find the corresponding prosthaphaeresis of the annual anomaly. Set the accompanying proportional minutes aside. If the raw [annual anomaly] is less than a semicircle or its number occurs in the first column, add the prosthaphaeresis to the annual anomaly; otherwise, subtract [the prosthaphaeresis from the raw annual anomaly]. The remainder or sum will be the adjusted solar anomaly. Then with it obtain the prosthaphaeresis of the annual orbit, which occupies the fifth column, together with the accompanying increase. This increase, taken in conjunction with the proportional minutes, previously set aside, amounts to a quantity which is always added to this [orbital] prosthaphaeresis. This [sum] will become the adjusted prosthaphaeresis, which is subtracted from the sun's mean place if the number of the annual anomaly was found in the first column or was less than a semicircle. On the other hand, [the adjusted prosthaphaeresis] is added [to the sun's mean place] if [the annual anomaly was] greater [than a semicircle] or occupied [a line in] the second column of [common] numbers. The remainder or sum thus obtained will define the sun's true place, as measured from the beginning of the constellation of the Ram. Finally, the true precession of the vernal equinox, if added [to the sun's true place], will immediately also show the sun's position in relation to the equinox, in zodiacal signs and degrees of the zodiac.

If you wish to accomplish this result in another way, take the uniform composite [motion] instead of the simple motion. Perform all the aforementioned operations, except that you add or subtract, as the situation requires, only the prosthaphaeresis of the equinoctial precession instead of the precession itself. In this way the computation of the apparent sun is obtained through the motion of the earth in agreement with ancient and modern records, so that in addition the future motion has presumably already been foreseen.

Nevertheless I am also not unaware that if anybody believed the center of the annual revolution to be stationary as the center of the universe, while the sun moved with two motions similar and equal to those which I explained in connection with the center of the eccentric [III, 20], all the phenomena would appear as before – the same figures and the same proof. Nothing would be changed in them, especially the phenomena pertaining to the sun, except the position. For then the motion of the earth's center around the center of the universe would be regular and simple (the two remaining motions being ascribed to the sun). For this reason there will still remain a doubt about which of these two positions is occupied by the center of the universe, as I said ambiguously at the beginning that the center of the universe is in the sun [I, 9, 10] or near it [I, 10]. I shall discuss this question further, however, in my treatment of the five planets [V, 4].

There I shall also decide it to the best of my ability, with the thought that it is enough if I adopt reliable and nowise untrustworthy computations for the apparent sun.

THE NUCHTHEMERON, THAT IS, THE VARIABLE NATURAL DAY *Chapter 26*

With regard to the sun, something still remains to be said about the variation in the natural day, the time which is embraced in the period of 24 equal hours and which we have used up to the present as the general and precise measurement of the heavenly motions. Such a day, however, is defined differently by different people: as the interval between two sunrises, by the Babylonians and ancient Hebrews; between two sunsets, by the Athenians; from midnight to midnight, by the Romans; and from noon to noon by the Egyptians.

In this period, it is clear, the terrestrial globe completes its own rotation as well as what is added in the meantime by the annual revolution related to the apparent motion of the sun. But this addition is variable, as is shown in the first place by the sun's variable apparent motion, and secondly by the natural day's connection with the [rotation around the] poles of the equator, whereas the annual revolution [proceeds] along the ecliptic. For these reasons that apparent time cannot be the general and precise measurement of motion, since the days are not uniform with [the natural] day and with one another in every detail. It was therefore necessary to select from these [days] some mean and uniform day which would permit uniform motion to be measured without uncertainty.

Now around the poles of the earth in the course of an entire year 365 rotations take place. These are increased by approximately a whole additional rotation as the result of a daily prolongation due to the apparent advance of the sun. Therefore the natural day exceeds the uniform [day] by 1/365 of that [additional rotation]. Consequently we must define the uniform day and distinguish it from the nonuniform apparent [day]. Accordingly, I call that [day] which contains an entire rotation of the equator, plus as much as appears to be traversed by the sun in its uniform motion during that time, the "uniform day". By contrast, [I call that] day "nonuniform and apparent" which comprises the 360° of a rotation of the equator, plus that which rises on the horizon or meridian together with the apparent advance of the sun. Although the difference between these [uniform and nonuniform] days is quite small and imperceptible at the outset, nevertheless when several days are taken together, [the difference] adds up and becomes perceptible.

Of this [phenomenon] there are two causes: the nonuniformity of the apparent sun, and the nonuniform rising of the oblique ecliptic. The first cause, which is due to the nonuniform and apparent motion of the sun, has already been made clear [III, 16–17]. For in the semicircle whose midpoint is the higher apse, halfway between the two mean apsides, in comparison with the degrees of the ecliptic $4\,{}^3/_4$ time-degrees were lacking, according to Ptolemy [*Syntaxis*, III, 9]. The same number was in excess in the other semicircle, which contained the lower apse. Hence the entire surplus of one semicircle over the other was $9\,{}^1/_2$ time-degrees.

But in the [case of the] second cause (the one connected with the risings and settings) a very great difference occurs between the semicircles [containing] the two solstices. This is [the difference] between the shortest and longest day. It varies very much, being special for every single region. On the other hand, [the difference] related to noon or midnight is everywhere confined within four limits. For, the 88° from 16° within the Bull to 14° within the Lion cross the meridian in about 93 time-degrees. The 92° from 14° within the Lion to 16° within the Scorpion cross [the meridian] in 87 time-degrees. Hence in the latter case 5 time-degrees are lacking [92°– 87°], and in the former case the same number is in excess [93°– 88°]. Thus the sum of the days in the first interval exceeds those in the second [interval] by 10 time-degrees = $^2/_3$ of an hour. This happens similarly in the other semicircle, where the situation is reversed within the remaining, diametrically opposite, limits.

Now the astronomers decided to begin the natural day at noon or midnight, not at sunrise or sunset. For, the nonuniformity connected with the horizon is more complicated, since it extends over several hours. Moreover, it is not everywhere the same, but varies in a complex way depending on the obliquity of the sphere. On the other hand, [the nonuniformity] related to the meridian is the same everywhere, and simpler.

Consequently, the entire difference arising from the two aforementioned causes – the apparent nonuniform motion of the sun and the nonuniform crossing of the meridian – before Ptolemy, when the decrease started at the middle of the Water Bearer, and the increase at the beginning of the Scorpion, amounted to 8 $^1/_3$ time-degrees [*Syntaxis*, III, 9]. At present, when the decrease extends from 20° within the Water Bearer or thereabouts to 10° within the Scorpion, and the increase [extends] from 10° within the Scorpion to 20° within the Water Bearer, [the difference] has contracted to 7° 48′ time-degrees. For, these [phenomena] too change in time on account of the mutability of the perigee and the eccentricity.

Finally, if the maximum variation in the precession of the equinoxes is also added to the foregoing, the entire inequality in the natural days can rise above 10 time-degrees in several years. Herein a third cause of the nonuniformity in the days has hitherto remained hidden. For, the rotation of the equator has been found uniform with reference to the mean and uniform equinox, not to the apparent equinoxes, which (as was quite clear) are not entirely uniform. For, twice ten time-degrees = $1^1/_3$ hours, by which longer days can sometimes exceed shorter days. In connection with the sun's [apparent] annual motion and the relatively slow motion of the other planets, these [phenomena] could perhaps be neglected without any obvious error. But they should not be overlooked at all, on account of the moon's swift motion, which can cause a discrepancy of $^5/_6$°.

Now uniform time may be compared with apparent, nonuniform [time] by a method whereby all the variations are coordinated, as follows. Choose any time. For both limits of this time, I mean, the beginning and the end, look up the sun's mean displacement from the mean equinox resulting from what I have called the sun's composite uniform motion. Also [look up] the true apparent displacement from the true equinox. Determine how many time-degrees have crossed in right ascension at noon or midnight, or have intervened between [the right ascensions] from the first true place to the second true [place]. For if [the time-degrees] are equal to the degrees between both mean places, then the given apparent time will be equal to the mean [time]. But if the time-degrees are in excess, add the surplus to the given time. On the other hand, if [the time-degrees] are fewer, subtract the difference from the apparent time. By so doing, from the sum or remainder we shall obtain the time reduced to uniform [time], in taking four minutes of an hour or ten seconds of a sixtieth of a day [10^{ds}] for every time-degree. If the uniform time is given, however, and you want to know how much apparent time is equivalent to it, follow the opposite procedure.

Now for the first Olympiad we had the mean distance of the sun from the mean vernal equinox at noon on the first day of Hecatombaeon, the first Athenian month, as 90° 59′ [III, 19], and from the apparent equinox as 0° 36′ within the Crab. For the years since Christ, the sun's mean motion is 8° 2′ within the Goat [= 278° 2′; III, 19], while the true motion is 8° 48′ within the same sign. Therefore, in the right sphere from 0° 36′ within the Crab to 8° 48′ within the Goat 178 time-degrees 54′ rise, exceeding the distance between the mean places by one time-degree 51′ = 7 hour-minutes. The procedure is the same for the rest, by means of which a very precise examination can be made of the motion of the moon, with which the next Book deals.

Book Four

INTRODUCTION

In the preceding Book to the best of my limited ability I explained the phenomena caused by the motion of the earth around the sun, and through the same procedure I intend to analyze the motions of all the planets. Therefore the moon's motion confronts me now. This must be so, because it is principally through the moon, which takes part in the day and the night, that the positions of any asters whatever are found and verified. Secondly, of all [the heavenly bodies] only the moon relates its revolutions as a whole, even though they are very irregular, to the center of the earth, to which it is in the highest degree akin. Therefore the moon, taken by itself, gives no indication that the earth moves, except perhaps in its daily rotation. All the more for that reason it was believed that the earth was the center of the universe and the common center of all the revolutions. In expounding the moon's motion I do not disagree with the ancients' belief that it takes place around the earth. But I shall also present certain features at variance with what we have received from our predecessors and in closer agreement with one another. By means of those features I may determine the lunar motion too with greater certainty, as far as possible, in order that its secrets may be more clearly understood.

THE HYPOTHESES CONCERNING THE LUNAR CIRCLES, ACCORDING TO THE BELIEF OF THE ANCIENTS

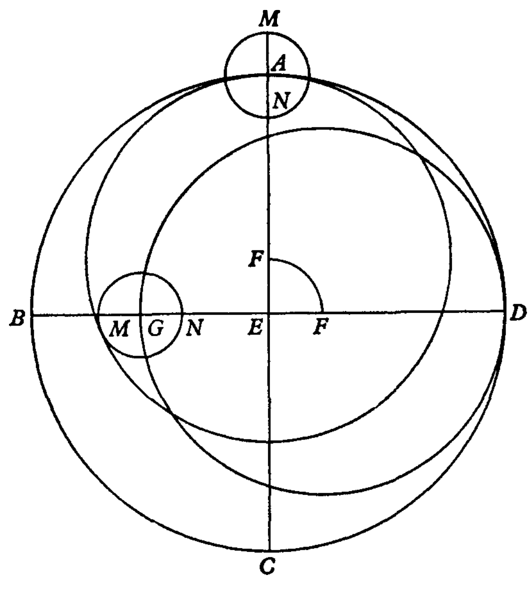
Chapter 1

A property of the moon's motion is that it follows, not the middle circle of the zodiac, but its own circle, which is inclined to the middle circle, bisects it, is in turn bisected by it, and crosses it into either latitude. These phenomena are very much like the tropics in the annual motion of the sun since, of course, what the year is to the sun, the month is to the moon. The mean places of the intersections are called "ecliptics" [by some astronomers] and "nodes" by others. The conjunctions and oppositions of the sun and moon which take place in these points are called "ecliptic". For apart from these points, where eclipses of the sun and moon can occur, the two circles have no other points in common. For when the moon is diverted to other places, the result is that [these two luminaries] do not block each other's light; on the contrary, as they pass on, they do not obstruct each other.

Furthermore, this tilted lunar circle, together with those four cardinal points belonging to it, moves uniformly around the center of the earth nearly 3′ a day, completing a revolution in the nineteenth year. In this circle and its plane the moon is seen moving always eastward. But sometimes its motion is very slight, and at other times very great. For it is slower, the higher it is; and faster, the nearer it is to the earth. This variation could be noticed more easily in the moon than in any other body on account of its proximity [to the earth].

This phenomenon was understood to occur through an epicycle. As the moon traveled along the upper [part of the epicycle's] circumference, its speed was less than the uniform speed; on the other hand, in traversing the lower [part of the epicycle's circumference], its speed exceeded the uniform speed. The results achieved by an epicycle, however, can be accomplished by an eccentric also, as has been proved [III, 15]. But an epicycle was chosen because the moon was seen to exhibit a twofold nonuniformity. For when it was in the epicycle's higher or lower apse, no departure from the uniform motion was apparent. On the other hand, when it was near the epicycle's intersections [with the deferent, the difference from the uniform motion occurred] not in a single way. On the contrary, it was far greater at the waxing and waning half moon than when the moon was full or new; and this variation occurred in a definite and regular pattern. For this reason it was believed that the deferent on which the epicycle moved was not concentric with the earth. On the contrary, an eccentrepicycle [was accepted]. The moon moved on the epicycle in accordance with the following rule: at every mean opposition and conjunction of the sun and moon, the epicycle was in the apogee of the eccentric, whereas the epicycle was in the perigee of the eccentric when the moon was halfway [between opposition and conjunction], at a quadrant's [distance from them]. The result was a conception of two uniform motions around the center of the earth in opposite directions, namely, an epicycle moving eastward, and the eccentric's center and apsides moving westward, with the line of the sun's mean place always halfway between both. In this way the epicycle traverses the eccentric twice a month.

To put these arrangements before the eyes, let the tilted lunar circle concentric with the earth be *ABCD*, quadrisected by the diameters *AEC* and *BED*. Let the center of the earth be *E*. Let the mean conjunction of the sun and moon lie on the line *AC*, and let the apogee of the eccentric, whose center is *F*, and the center of the epicycle *MN* be in the same place at the same time. Now let the eccentric's apogee move westward as much as the epicycle moves eastward, while they both uniformly execute around *E* equal monthly revolutions as measured by the mean

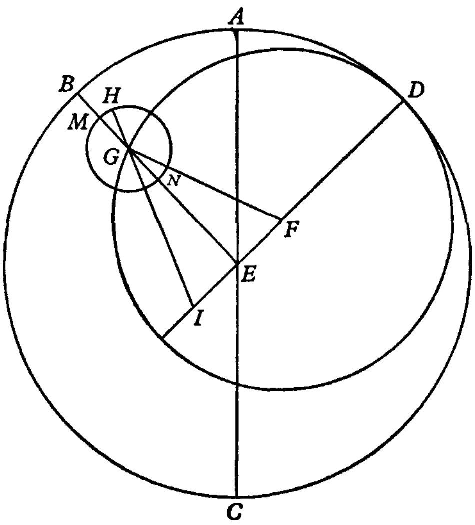


conjunctions with or oppositions to the sun. Let *AEC*, the line of the sun's mean place, always be midway between them, and let the moon also move westward from the epicycle's apogee. With matters so arranged, the phenomena are thought to be in order. For in half a month's time the epicycle moves half a circle away from the sun, but completes an entire revolution from the eccentric's apogee. As a result, in half of this time, which is about half moon, the epicycle and the eccentric's apogee are opposite each other along diameter *BD*, and the epicycle on the eccentric is at its perigee, as in point *G*. There, having come closer to the earth, it enlarges the nonuniformity's variations. For of equal magnitudes viewed at unequal distances, the one nearer to the eye looks bigger. The variations will therefore be smallest when the epicycle is in *A*, but greatest when the epicycle is in *G*. For *MN*, the diameter of the epicycle, will have the smallest ratio to line *AE*, but a larger ratio to *GE* than to all the other lines found in other places. For *GE* is the shortest of all the lines that can be drawn from the center of the earth to the eccentric circle, and the longest of them is *AE* or its equivalent *DE*.

THE DEFECT IN THOSE ASSUMPTIONS *Chapter 2*

This combination of circles was assumed by our predecessors to be in agreement with the lunar phenomena. But if we analyze the situation more carefully, we shall find this hypothesis neither suitable enough nor adequate, as we can prove by reason and by the senses. For while our predecessors declare that the motion of the epicycle's center is uniform around the center of the earth, they must also admit that it is nonuniform on its own eccentric (which it describes).

For example, take angle $AEB = 45°$, that is, half of a right angle, and equal to *AED*, so that the whole angle *BED* is a right angle. Put the epicycle's center in *G*, and join *GF*. *GFD*, being an exterior angle, obviously is greater than *GEF*, the opposite interior angle. Also unequal, therefore, are arcs *DAB* and *DG*, even though they are both described in the same time. Hence, since *DAB* is a quadrant, *DG*, which meanwhile is described by the epicycle's center, is greater than a quadrant. But at half moon both *DAB* and *DG* were shown to be semicircles [IV, 1,



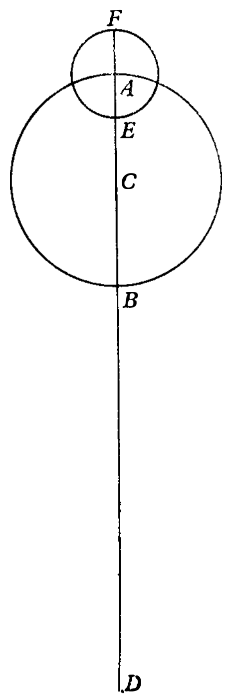
end]. Therefore, the epicycle's motion on the eccentric described by it is nonuniform. But if this is so, what shall we say about the axiom that the heavenly bodies' motion is uniform and only apparently seems nonuniform, if the epicycle's apparently uniform motion is really nonuniform and its occurrence absolutely contradicts an established principle and assumption? But suppose you say that the epicycle moves uniformly with respect to the earth's center, and that this is enough to safeguard uniformity. Then what sort of uniformity will that be on an extraneous circle on which the epicycle's motion does not occur, whereas it does occur on the epicycle's own eccentric?

I am likewise disturbed about the moon's uniform motion on the epicycle. My predecessors decided to interpret it as unrelated to the earth's center, to which uniform motion as measured by the epicycle's center should properly be related, to wit, through line *EGM*. But [they related the moon's uniform motion on the epicycle] to a certain other point. Halfway between it and the eccentric's center lay the earth, and line *IGH* served as the indicator of the moon's uniform motion on the epicycle. By itself, this also is enough to prove the nonuniformity of this motion, a conclusion required by the phenomena which follow in part from this hypothesis. Thus, the moon's motion on its epicycle is also nonuniform. If we now want to base the apparent nonuniformity on [really] nonuniform motions, it is evident what the nature of our reasoning will be. For will we do anything but furnish an opportunity to those who malign this science?

Secondly, experience and our senses themselves show us that the lunar parallaxes are different from those indicated by the ratio of the circles. The parallaxes, which are called "commutations", occur on account of the perceptible size of the earth in comparison with the proximity of the moon. For, straight lines drawn from the earth's surface and center to the moon do not appear parallel, but intersect each other at a detectable angle on the body of the moon. They must therefore produce a difference in the appearance of the moon. It seems to be in one place to those who view it at an angle from the earth's curvature and in a different place to those who inspect the moon [along a line] from the earth's center or point directly below [the moon]. Hence these parallaxes vary in accordance with the distance from the earth to the moon. By agreement of all astronomers, the greatest distance is $64\,^1/_6$ units, of which the earth's radius = 1. According to our predecessors' model, the smallest distance should be 33 units, 33′. As a result the moon would approach us nearly halfway. The resulting ratio would require the parallaxes at the smallest and greatest distances to differ from each other by almost 1 : 2. I observe, however, that the parallaxes occurring in the waxing and waning half moon, even when it is in the epicycle's perigee, differ very little or not at all from the parallaxes occurring in solar and lunar eclipses, as I shall prove satisfactorily at the proper place [IV, 22]. But the error is evinced most of all by the body of the moon, whose diameter would similarly look twice as large and half as large. Now, circles are to each other as the squares of their diameters. Thus, the moon would generally look four times larger, on the supposition that it shone with its full disk, in quadrature, when nearest to the earth, than when in opposition to the sun. But since [in quadrature] it glows with half its disk, it would nevertheless emit twice the light which the full moon would show if it were in that position. Although the contrary is self-evident, if anybody is dissatisfied with ordinary vision and wants to observe with a Hipparchan dioptra or any other instrument for measuring the moon's diameter, he will find that it varies only as much as required by the epicycle without that eccentric. Therefore, while investigating the fixed stars through the place of the moon, Menelaus and Timocharis did not hesitate to use at all times for the moon's diameter the same value of $^1/_2°$, which the moon was usually seen to occupy.

A DIFFERENT OPINION ABOUT THE MOON'S MOTION *Chapter 3*

It is accordingly quite clear that the epicycle looks bigger and smaller not on account of an eccentric but on account of some other system of circles. Let *AB* be an epicycle, which I shall call the first and larger epicycle. Let *C* be its center, and *D* the center of the earth, from which draw the straight line *DC* to the epicycle's higher apse [*A*]. With *A* as center, describe another epicyclet *EF*, of small dimensions. Let all these constructions lie in the same plane, that of the moon's tilted circle. Let *C* move eastward, but *A* westward. On the other hand, from *F* in the upper part of *EF* let the moon move eastward while maintaining the following pattern: when line *DC* is aligned with the sun's mean place, the moon is always nearest to center *C*, that is, in point *E*; at the quadratures, however, it is farthest [from center *C*] in *F*.



I say that the lunar phenomena agree with this model. For it follows [from this model] that the moon traverses the epicyclet *EF* twice a month, during which time *C* returns once to the sun. When new and full, the moon will be seen to trace its smallest circle, namely, the one with radius *CE*. On the other hand, in the quadratures it will describe its largest circle, with radius *CF*. Thus, it will also make the differences between uniform and apparent motion smaller in the former positions and larger in the latter positions, while it passes through similar but unequal arcs around center *C*. The [first] epicycle's center *C* will always be on a circle concentric with the earth. The moon will therefore display parallaxes which do not vary very much, but are connected only with the epicycle. This will at once provide the reason why the body of the moon is also seen virtually unchanged. All the other phenomena related to the moon's motion will emerge just as they are observed.

I shall demonstrate this agreement later on by means of my hypothesis, although the same phenomena can once more be produced by eccentrics, as I did with regard to the sun, if the required ratio is maintained [III, 15]. I shall begin, however, as I did above, [III, 13-14], with the uniform motions, without which the nonuniform motions cannot be ascertained. But here no mean problem arises because of the aforementioned parallaxes. On account of them the moon's place cannot be observed by astrolabes and any other instruments whatever. But in this area too nature's kindliness has been attentive to human desires, inasmuch as the moon's place is determined more reliably through its eclipses than through the use of instruments, and without any suspicion of error. For while the rest of the universe is bright and full of daylight, night is clearly nothing but the earth's shadow, which extends in the shape of a cone and ends in a point. When the moon encounters this shadow, it is darkened, and when it is immersed in the midst of the darkness it is indubitably known to have reached the place opposite the sun. On the other hand, solar eclipses, which are caused by the interposition of the moon [between the earth and the sun], do not provide precise evidence of the moon's place. For at that time we happen to see a conjunction of the sun and moon which, as regards the center of the earth, either has already passed beyond or has not yet occurred, on account of the aforementioned parallax. Therefore we do not see the same solar eclipse equal in extent and duration in all countries, nor similar in its details. In lunar eclipses, on the other hand, no such obstacle presents itself. They are everywhere identical, since the axis of that darkening shadow is cast by the earth from [the direction of] the sun through its own center. Lunar eclipses are therefore most suitable for ascertaining the moon's motion with the most highly certain computation.

THE MOON'S REVOLUTIONS, AND THE DETAILS OF ITS MOTIONS — *Chapter 4*

Among the earliest astronomers who strove to transmit numerical information about this subject to posterity there is found Meton the Athenian, who flourished in about the 87th Olympiad. He declared that 235 months were completed in 19 solar years. This great period is accordingly called the Metonic *enneadekaeteris,* that is, 19-year cycle. This number was so popular that it was displayed in the market place at Athens and other very famous cities. Even up to the present time it is widely accepted because it is believed to fix the beginning and end of the months in a precise order, and also to make the solar year of 365 $^1/_4$ days commensurable with the months. From it [came] the Callippic period of 76 years, in which 1 day is intercalated 19 times, and which is labelled the "Callippic cycle". But Hipparchus ingeniously discovered that in 304 years a whole day was in excess, which was corrected only by shortening the solar year by $^1/_{300}$ of a day. Hence some astronomers named that extensive period in which 3760 months were completed the "Hipparchan cycle".

These computations are stated too simply and too crudely when it is also a question of the cycles of the anomaly and latitude. These topics were therefore investigated further by Hipparchus [*Syntaxis,* IV, 2–3]. For he compared the records of his very careful observations of lunar eclipses with those which he received from the Babylonians. He determined the period in which the cycles of the months and of the anomaly were completed at the same time to be 345 Egyptian years, 82 days, 1 hour. In that interval 4267 months and 4573 cycles of the anomaly were completed. When the indicated number of days, to wit, 126007 days, 1 hour, is divided by the number of months, 1 month is found = 29 days 31′ 50″ 8‴ 9⁗ 20′′′′′. This result also made clear the motion in any time. For when the 360° of a monthly revolution are divided by the duration of a month, the daily motion of the moon away from the sun is 12° 11′ 26″ 41‴ 20⁗ 18′′′′′. This number, multiplied by 365, makes the annual motion 129° 37′ 21″ 28‴ 29⁗ in addition to 12 revolutions. Furthermore, 4267 months and 4573 revolutions of the anomaly are factorable numbers having 17 as a common factor. Reduced to their lowest terms, they stand in the ratio 251 : 269. This will give us, in accordance with Euclid, V, 15, the ratio of the moon's motion to the motion of the anomaly. When we multiply the moon's motion by 269 and divide the product by 251, we shall obtain the annual motion of the anomaly, after 13 complete revolutions, as 88° 43′ 8″ 40‴ 20⁗. Therefore, the daily motion will be 13° 3′ 53″ 56‴ 29⁗.

The cycle of latitude has a different rhythm, for it does not coincide with the precise interval in which the anomaly returns. We know that a lunar latitude has recurred only when a later lunar eclipse is in all respects similar and equal to an earlier eclipse so that, for instance, on the same side both darkened areas are equal, I mean in extent and duration. This happens when the moon's distances from the higher or lower apse are equal. For at that time the moon is known to have passed through equal shadows in equal times. Such a recurrence, according to Hipparchus, happens in 5458 months, corresponding to 5923 cycles of latitude. This ratio also made clear the detailed latitudinal motion in years and days, like the other motions. For when we multiply the moon's motion away from the sun by 5923 months, and divide the product by 5458, we shall have the moon's latitudinal motion in a year, after 13 revolutions, as $148^\circ 42' 46'' 49''' 3''''$, and in a day as $13^\circ 13' 45'' 39''' 40''''$. In this way Hipparchus computed the moon's uniform motions, which nobody before him had approached more closely. Nevertheless, later centuries showed that they were still not determined with complete accuracy. For Ptolemy found the same mean motion away from the sun as Hipparchus. Yet Ptolemy's value for the annual motion in anomaly was $1'' 11''' 39''''$ lower than Hipparchus', but for the annual motion in latitude $53''' 41''''$ higher. After the passage of more time I found that Hipparchus' [value for the] mean annual motion was $1'' 2''' 49''''$ too low, whereas for the anomaly he was only $24''' 49''''$ short. For the motion in latitude he is $1'' 1''' 42''''$ too high. Therefore the moon's annual uniform motion differs from the earth's by $129^\circ 37' 22'' 32''' 40''''$; its motion in anomaly by $88^\circ 43' 9'' 5''' 9''''$; and its motion in latitude by $148^\circ 42' 45'' 17''' 21''''$.

THE MOON'S MOTION IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 209° 58′

Egyptian Years	Motion					Egyptian Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	2	9	37	22	36	31	0	58	18	40	48
2	4	19	14	45	12	32	3	7	56	3	25
3	0	28	52	7	49	33	5	17	33	26	1
4	2	38	29	30	25	34	1	27	10	48	38
5	4	48	6	53	2	35	3	36	48	11	14
6	0	57	44	15	38	36	5	46	25	33	51
7	3	7	21	38	14	37	1	56	2	56	27
8	5	16	59	0	51	38	4	5	40	19	3
9	1	26	36	23	27	39	0	15	17	41	40
10	3	36	13	46	4	40	2	24	55	4	16
11	5	45	51	8	40	41	4	34	32	26	53
12	1	55	28	31	17	42	0	44	9	49	29
13	4	5	5	53	53	43	2	53	47	12	5
14	0	14	43	16	29	44	5	3	24	34	42
15	2	24	20	39	6	45	1	13	1	57	18
16	4	33	58	1	42	46	3	22	39	19	55
17	0	43	35	24	19	47	5	32	16	42	31
18	2	53	12	46	55	48	1	41	54	5	8
19	5	2	50	9	31	49	3	51	31	27	44
20	1	12	27	32	8	50	0	1	8	50	20
21	3	22	4	54	44	51	2	10	46	12	57
22	5	31	42	17	21	52	4	20	23	35	33
23	1	41	19	39	57	53	0	30	0	58	10
24	3	50	57	2	34	54	2	39	38	20	46
25	0	0	34	25	10	55	4	49	15	43	22
26	2	10	11	47	46	56	0	58	53	5	59
27	4	19	49	10	23	57	3	8	30	28	35
28	0	29	26	32	59	58	5	18	7	51	12
29	2	39	3	55	36	59	1	27	45	13	48
30	4	48	41	18	12	60	3	37	22	36	25

THE MOON'S MOTION IN DAYS, PERIODS OF SIXTY DAYS, AND DAY-MINUTES

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	12	11	26	41	31	6	17	54	47	26
2	0	24	22	53	23	32	6	30	6	14	8
3	0	36	34	20	4	33	6	42	17	40	49
4	0	48	45	46	46	34	6	54	29	7	31
5	1	0	57	13	27	35	7	6	40	34	12
6	1	13	8	40	9	36	7	18	52	0	54
7	1	25	20	6	50	37	7	31	3	27	35
8	1	37	31	33	32	38	7	43	14	54	17
9	1	49	43	0	13	39	7	55	26	20	58
10	2	1	54	26	55	40	8	7	37	47	40
11	2	14	5	53	36	41	8	19	49	14	21
12	2	26	17	20	18	42	8	32	0	41	3
13	2	38	28	47	0	43	8	44	12	7	44
14	2	50	40	13	41	44	8	56	23	34	26
15	3	2	51	40	22	45	9	8	35	1	7
16	3	15	3	7	4	46	9	20	46	27	49
17	3	27	14	33	45	47	9	32	57	54	30
18	3	39	26	0	27	48	9	45	9	21	12
19	3	51	37	27	8	49	9	57	20	47	53
20	4	3	48	53	50	50	10	9	32	14	35
21	4	16	0	20	31	51	10	21	43	41	16
22	4	28	11	47	13	52	10	33	55	7	58
23	4	40	23	13	54	53	10	46	6	34	40
24	4	52	34	40	36	54	10	58	18	1	21
25	5	4	46	7	17	55	11	10	29	28	2
26	5	16	57	33	59	56	11	22	40	54	43
27	5	29	9	0	40	57	11	34	52	21	25
28	5	41	20	27	22	58	11	47	3	48	7
29	5	53	31	54	3	59	11	59	15	14	48
30	6	5	43	20	45	60	12	11	26	41	31

THE MOON'S MOTION IN ANOMALY IN YEARS AND PERIODS OF SIXTY YEARS											
											Christian Era 207° 7′
Years	Motion					Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	1	28	43	9	7	31	3	50	17	42	44
2	2	57	26	18	14	32	5	19	0	51	52
3	4	26	9	27	21	33	0	47	44	0	59
4	5	54	52	36	29	34	2	16	27	10	6
5	1	23	35	45	36	35	3	45	10	19	13
6	2	52	18	54	43	36	5	13	53	28	21
7	4	21	2	3	50	37	0	42	36	37	28
8	5	49	45	12	58	38	2	11	19	46	35
9	1	18	28	22	5	39	3	40	2	55	42
10	2	47	11	31	12	40	5	8	46	4	50
11	4	15	54	40	19	41	0	37	29	13	57
12	5	44	37	49	27	42	2	6	12	23	4
13	1	13	20	58	34	43	3	34	55	32	11
14	2	42	4	7	41	44	5	3	38	41	19
15	4	10	47	16	48	45	0	32	21	50	26
16	5	39	30	25	56	46	2	1	4	59	33
17	1	8	13	35	3	47	3	29	48	8	40
18	2	36	56	44	10	48	4	58	31	17	48
19	4	5	39	53	17	49	0	27	14	26	55
20	5	34	23	2	25	50	1	55	57	36	2
21	1	3	6	11	32	51	3	24	40	45	9
22	2	31	49	20	39	52	4	53	23	54	17
23	4	0	32	29	46	53	0	22	7	3	24
24	5	29	15	38	54	54	1	50	50	12	31
25	0	57	58	48	1	55	3	19	33	21	38
26	2	26	41	57	8	56	4	48	16	30	46
27	3	55	25	6	15	57	0	16	59	39	53
28	5	24	8	15	23	58	1	45	42	49	0
29	0	52	51	24	30	59	3	14	25	58	7
30	2	21	34	33	37	60	4	43	9	7	15

THE MOON'S MOTION IN ANOMALY IN DAYS, PERIODS OF SIXTY DAYS, AND DAY-MINUTES

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	13	3	53	56	31	6	45	0	52	11
2	0	26	7	47	53	32	6	58	4	46	8
3	0	39	11	41	49	33	7	11	8	40	4
4	0	52	15	35	46	34	7	24	12	34	1
5	1	5	19	29	42	35	7	37	16	27	57
6	1	18	23	23	39	36	7	50	20	21	54
7	1	31	27	17	35	37	8	3	24	15	50
8	1	44	31	11	32	38	8	16	28	9	47
9	1	57	35	5	28	39	8	29	32	3	43
10	2	10	38	59	25	40	8	42	35	57	40
11	2	23	42	53	21	41	8	55	39	51	36
12	2	36	46	47	18	42	9	8	43	45	33
13	2	49	50	41	14	43	9	21	47	39	29
14	3	2	54	35	11	44	9	34	51	33	26
15	3	15	58	29	7	45	9	47	55	27	22
16	3	29	2	23	4	46	10	0	59	21	19
17	3	42	6	17	0	47	10	14	3	15	15
18	3	55	10	10	57	48	10	27	7	9	12
19	4	8	14	4	53	49	10	40	11	3	8
20	4	21	17	58	50	50	10	53	14	57	5
21	4	34	21	52	46	51	11	6	18	51	1
22	4	47	25	46	43	52	11	19	22	44	58
23	5	0	29	40	39	53	11	32	26	38	54
24	5	13	33	34	36	54	11	45	30	32	51
25	5	26	37	28	32	55	11	58	34	26	47
26	5	39	41	22	29	56	12	11	38	20	44
27	5	52	45	16	25	57	12	24	42	14	40
28	6	5	49	10	22	58	12	37	46	8	37
29	6	18	53	4	18	59	12	50	50	2	33
30	6	31	56	58	15	60	13	3	53	56	30

THE MOON'S MOTION IN LATITUDE IN YEARS AND PERIODS OF SIXTY YEARS

Christian Era 129° 45′

Years	Motion					Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	2	28	42	45	17	31	4	50	5	23	57
2	4	57	25	30	34	32	1	18	48	9	14
3	1	26	8	15	52	33	3	47	30	54	32
4	3	54	51	1	9	34	0	16	13	39	48
5	0	23	33	46	26	35	2	44	56	25	6
6	2	52	16	31	44	36	5	13	39	10	24
7	5	20	59	17	1	37	1	42	21	55	41
8	1	49	42	2	18	38	4	11	4	40	58
9	4	18	24	47	36	39	0	39	47	26	16
10	0	47	7	32	53	40	3	8	30	11	33
11	3	15	50	18	10	41	5	37	12	56	50
12	5	44	33	3	28	42	2	5	55	42	8
13	2	13	15	48	45	43	4	34	38	27	25
14	4	41	58	34	2	44	1	3	21	12	42
15	1	10	41	19	20	45	3	32	3	58	0
16	3	39	24	4	37	46	0	0	46	43	17
17	0	8	6	49	54	47	2	29	29	28	34
18	2	36	49	35	12	48	4	58	12	13	52
19	5	5	32	20	29	49	1	26	54	59	8
20	1	34	15	5	46	50	3	55	37	44	26
21	4	2	57	51	4	51	0	24	20	29	44
22	0	31	40	36	21	52	2	53	3	15	1
23	3	0	23	21	38	53	5	21	46	0	18
24	5	29	6	6	56	54	1	50	28	45	36
25	1	57	48	52	13	55	4	19	11	30	53
26	4	26	31	37	30	56	0	47	54	16	10
27	0	55	14	22	48	57	3	16	37	1	28
28	3	23	57	8	5	58	5	45	19	46	45
29	5	52	39	53	22	59	2	14	2	32	2
30	2	21	22	38	40	60	4	42	45	17	21

THE MOON'S MOTION IN LATITUDE IN DAYS, PERIODS OF SIXTY DAYS, AND DAY-MINUTES

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	13	13	45	39	31	6	50	6	35	20
2	0	26	27	31	18	32	7	3	20	20	59
3	0	39	41	16	58	33	7	16	34	6	39
4	0	52	55	2	37	34	7	29	47	52	18
5	1	6	8	48	16	35	7	43	1	37	58
6	1	19	22	33	56	36	7	56	15	23	37
7	1	32	36	19	35	37	8	9	29	9	16
8	1	45	50	5	14	38	8	22	42	54	56
9	1	59	3	50	54	39	8	35	56	40	35
10	2	12	17	36	33	40	8	49	10	26	14
11	2	25	31	22	13	41	9	2	24	11	54
12	2	38	45	7	52	42	9	15	37	57	33
13	2	51	58	53	31	43	9	28	51	43	13
14	3	5	12	39	11	44	9	42	5	28	52
15	3	18	26	24	50	45	9	55	19	14	31
16	3	31	40	10	29	46	10	8	33	0	11
17	3	44	53	56	9	47	10	21	46	45	50
18	3	58	7	41	48	48	10	35	0	31	29
19	4	11	21	27	28	49	10	48	14	17	9
20	4	24	35	13	7	50	11	1	28	2	48
21	4	37	48	58	46	51	11	14	41	48	28
22	4	51	2	44	26	52	11	27	55	34	7
23	5	4	16	30	5	53	11	41	9	19	46
24	5	17	30	15	44	54	11	54	23	5	26
25	5	30	44	1	24	55	12	7	36	51	5
26	5	43	57	47	3	56	12	20	50	36	44
27	5	57	11	32	43	57	12	34	4	22	24
28	6	10	25	18	22	58	12	47	18	8	3
29	6	23	39	4	1	59	13	0	31	53	43
30	6	36	52	49	41	60	13	13	45	39	22

EXPOSITION OF THE FIRST LUNAR INEQUALITY, WHICH OCCURS AT NEW AND FULL MOON *Chapter 5*

I have set forth the moon's uniform motions to the extent that I have been able to familiarize myself with them up to the present time. Now I must tackle the theory of the nonuniformity, which I shall expound by means of an epicycle. I shall begin with that inequality which occurs in conjunctions with and oppositions to the sun. With regard to this inequality the ancient astronomers used sets of three lunar eclipses with marvelous skill. I too shall follow this path which they have prepared for us. I shall take three eclipses carefully observed by Ptolemy. I shall compare them with three other no less carefully observed eclipses, in order to test whether the uniform motions set forth above are correct. In expounding them I shall imitate the ancients by treating the mean motions of the sun and moon from the place of the vernal equinox as uniform. For, the irregularity which occurs on account of the nonuniform precession of the equinoxes is not perceived in so short a time, even though it is ten years.

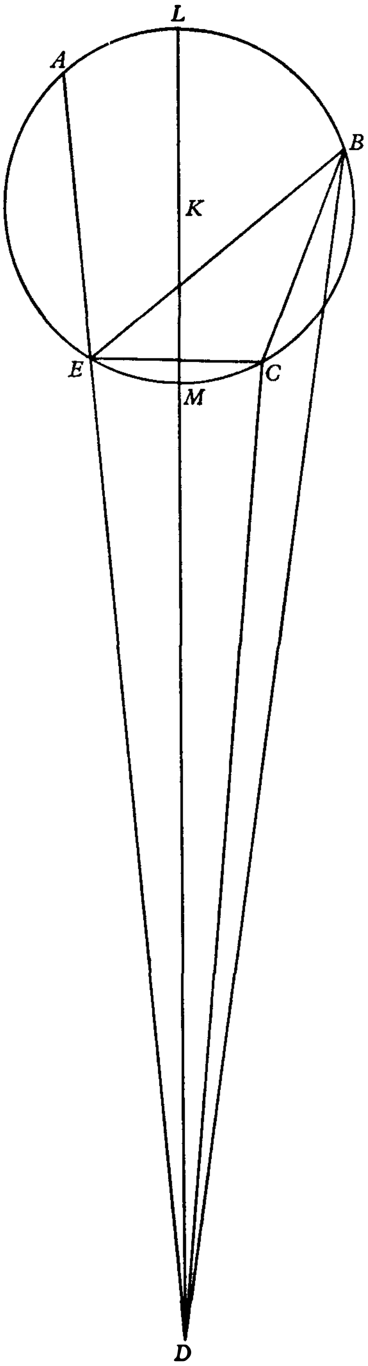
For his first eclipse Ptolemy [*Syntaxis*, IV, 6] takes the one which occurred in Emperor Hadrian's 17th year after the end of the 20th day of the month Pauni according to the Egyptian calendar. This was 133 C.E., 6 May = the day be-fore the Nones of May. The eclipse was total. Its midtime was $^{3}/_{4}$ of a uniform hour before midnight at Alexandria. But at Frombork or Cracow it would have been 1 $^{3}/_{4}$ hours before the midnight which was followed by 7 May. The sun was at 13 $^{1}/_{4}$° within the Bull, but at 12° 21′ within the Bull according to its mean motion.

Ptolemy says that the second eclipse occurred in Hadrian's 19th year after the end of the second day of Choiach, the fourth Egyptian month. This was 20 Octo-ber 134 C.E. The darkened area spread from the north over $^{5}/_{6}$ of the moon's diameter. The midtime preceded midnight by 1 uniform hour at Alexandria, but by 2 hours at Cracow. The sun was at 25 $^{1}/_{6}$° within the sign of the Balance, but at 26° 43′ within the same sign according to its mean motion.

The third eclipse occurred in Hadrian's year 20, after the end of the 19th day of Pharmuthi, the eighth Egyptian month. This was after the end of 6 March 136 C.E. The moon was again in shadow in the north up to half of its diameter. The midtime was 4 uniform hours at Alexandria, but at Cracow 3 hours, after the midnight followed by 7 March. The sun was then at 14° 5′ within the Fishes, but at 11° 44′ within the Fishes according to its mean motion.

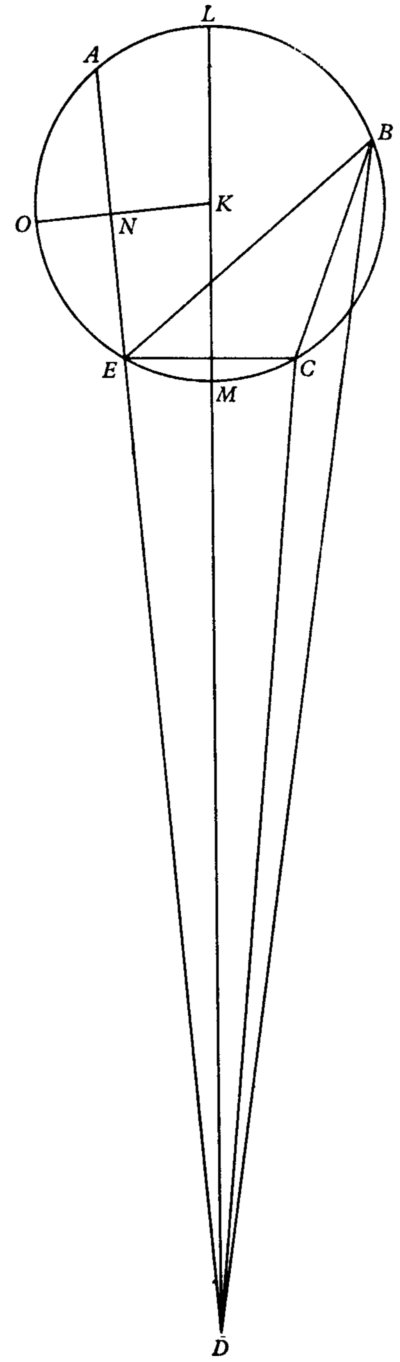
During the time between the first eclipse and the second the moon clearly traveled as far as the sun did in its apparent motion, that is (I mean, after whole circles are eliminated), 161° 55′; and 138° 55′ between the second eclipse and the third. In the first interval there were 1 year, 166 days, 23 $^{3}/_{4}$ uniform hours according to the appearances, but 23 $^{5}/_{8}$ hours after correction. In the second interval there were 1 year, 137 days, 5 hours simply, but 5 $^{1}/_{2}$ hours correctly. The combined uniform motion of the sun and moon in the first period, after the elimination of [complete] circles, was 169° 37′, and the [moon's] motion in anomaly was 110° 21′. In the second interval, similarly, the [combined] uniform motion of the sun and moon was 137° 34′, while the [moon's] motion in anomaly was $81°36'$. Clearly, then, in the first interval $110°21'$ of the epicycle subtract $7°42'$ from the moon's mean motion; and in the second interval $81°36'$ [of the epicycle] add $1°21'$ [to the moon's mean motion].

Now that this information has been set forth, draw the lunar epicycle *ABC*. On it let the first eclipse be at *A*, the second at *B*, and the last at *C*. Let the moon's motion be taken also in that direction, westward in the upper part [of the epicycle]. Let arc $AB = 110°21'$ which, as I said, subtracts $7°42'$ [from the moon's mean motion on the ecliptic]. Let $BC = 81°36'$, which adds $1°21'$ [to the moon's mean motion on the ecliptic]. *CA*, the rest of the circle [$360° - (110°21' + 81°36')$], will be $168°3'$, which adds the remaining $6°21'$ [of the prosthaphaeresis; $1°21' + 6°21' = 7°42'$]. The epicycle's higher apse is not in arcs *BC* and *CA*, since they are additive and less than a semicircle. Therefore it must be found in *AB*.



Now take *D* as the earth's center, around which the epicycle moves uniformly. From *D* draw lines *DA*, *DB*, and *DC* to the points of the eclipses. Join *BC*, *BE*, and *CE*. Since arc *AB* subtends $7°42'$ of the ecliptic, angle *ADB* will be $7°42'$ with $180° = 2$ right angles, but $15°24'$ [$= 2 \times 7°42'$] with $360° = 2$ right angles. In similar degrees, angle $AEB = 110°21'$ at the circumference, and it is an angle exterior to triangle *BDE*. Hence angle *EBD* is given as $94°57'$ [$= 110°21' - 15°24'$]. But when the angles of a triangle are given, the sides are given, and $DE = 147{,}396$ units, and $BE = 26{,}798$ units, of which the diameter of the circle circumscribing the triangle $= 200{,}000$. Furthermore, since arc *AEC* subtends $6°21'$ on the ecliptic, angle *EDC* will be $6°21'$ with $180° = 2$ right angles, but $12°42'$ with $360° = 2$ right angles. In those degrees angle $AEC = 191°57'$ [$= 110°21' + 81°36'$]. Being an angle exterior to triangle *CDE*, it leaves, after the subtraction of angle *D*, the third angle $ECD = 179°15'$ [$= 191°57' - 12°42'$] in the same degrees. Therefore sides *DE* and *CE* are given as 199,996 and 22,120 units, of which the diameter of the circumscribed circle $= 200{,}000$. But in the units of which $DE = 147{,}396$ and $BE = 26{,}798$, $CE = 16{,}302$. Once again, therefore, in triangle *BEC* two sides, *BE* and *EC*, are given, and angle $E = 81°36' =$ arc *BC*. Hence we shall also have the third side $BC = 17{,}960$ of those same units, in accordance with the theorems in Plane Triangles. When the epicycle's diameter $= 200{,}000$ units, chord *BC*, which subtends [an arc of] $81°36'$, will be 130,684 units. As for the other lines in the given ratio, in such units $ED = 1{,}072{,}684$ and $CE = 118{,}637$, while its arc $CE = 72°46'10''$. But by construction arc $CEA = 168°3'$. Therefore the remainder $EA = 95°16'50''$ [$= 168°3' - 72°46'10''$], and its subtending chord $= 147{,}786$ units. Hence the whole line $AED = 1{,}220{,}470$ of the same units [$= 147{,}786 + 1{,}072{,}684$]. But since segment *EA* is less than a semicircle, the epicycle's center will not be in it, but in the remainder *ABCE*.

Let the epicycle's center be *K*. Through both apsides draw *DMKL*. Let *L* be the higher apse, and *M* the lower apse. Clearly, in accordance with Euclid, III, 30, the rectangle formed by $AD \times DE =$ the rectangle formed by $LD \times DM$. But *K* is the midpoint of the circle's diameter *LM*, of which *DM* is an extension in a straight line. Therefore rectangle $LD \times DM + (KM)^2 = (DK)^2$. Consequently *DK* is given in length as 1,148,556 units, of which $LK = 100{,}000$. Hence, in units of which $DKL = 100{,}000$, *LK* will be 8706, and this is the epicycle's radius.



After completing these steps, draw *KNO* perpendicular to *AD*. The ratio of *KD, DE,* and *EA* to one another is given in units of which *LK* = 100,000. In those same units *NE* = $^1/_2$ (*AE* [= 147,786]) = 73,893. Therefore the whole line *DEN* = 1,146,577 [=*DE*+*EN*= 1,072,684+ 73,893]. But in triangle *DKN* two sides, *DK* and *ND,* are given, and *N* is a right angle. Therefore, central angle *NKD* = 86° 38 $^1/_2$′ = arc *MEO*. *LAO,* the rest of the semicircle = 93° 21 $^1/_2$′ [= 180°– 86° 38 $^1/_2$′]. From *LAO* subtract *AO* = $^1/_2$ (*AOE* [= 95° 16′ 50″]) = 47° 38 $^1/_2$′. The remainder *LA* = 45° 43′ [= 93° 21 $^1/_2$′– 47° 38 $^1/_2$′]. This is the moon's anomaly or its distance from the epicycle's higher apse in the first eclipse. But the whole of *AB* = 110° 21′. Therefore the remainder *LB* = the anomaly in the second eclipse = 64° 38′ [= 110° 21′–45° 43′]. The whole arc *LBC* = 146° 14′ [= 64° 38′ + 81° 36′], where the third eclipse occurred. Now, with 360° = 4 right angles, angle *DKN* = 86° 38′. When this is subtracted from a right angle, obviously the remaining angle *KDN* = 3° 22′ [= 90°– 86° 38′]. This is the prosthaphaeresis added by the anomaly in the first eclipse. But the whole angle *ADB* = 7° 42′. Therefore the remainder *LDB* = 4° 20′. This is what arc *LB* subtracts from the moon's uniform motion in the second eclipse. Angle *BDC* = 1° 21′. Therefore the remainder *CDM* = 2° 59′, the prosthaphaeresis subtracted by arc *LBC* in the third eclipse. Therefore the moon's mean place, that is, center *K,* in the first eclipse was 9° 53′ [= 13°15′–3°22′] within the Scorpion, because its apparent place was within the Scorpion at 13° 15′, I mean, exactly as much as the sun's place within the Bull, diametrically opposite. In the same way in the second eclipse the moon's mean motion was at 29 $^1/_2$° within the Ram [= Balance 25$^1/_6$°+180°+4°20′], and in the third eclipse at 17° 4′ within the Virgin [= Fishes 14°5′+180°+2°59′]. The moon's uniform distances from the sun were 177° 33′ in the first eclipse; 182° 47′ in the second; and 185° 20′ in the last eclipse. The foregoing was Ptolemy's procedure [*Syntaxis,* IV, 6].

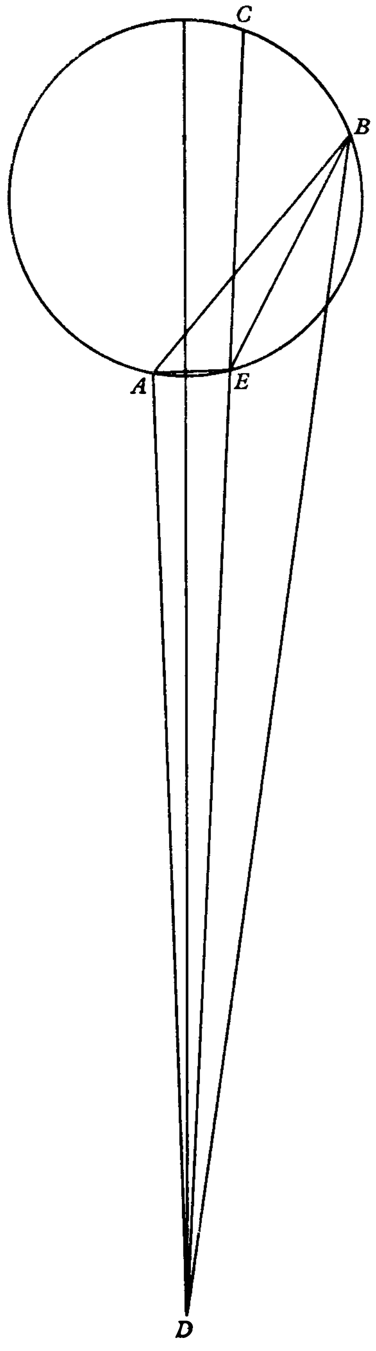
Following his example, let me now proceed to the second set of three lunar eclipses, which I observed very carefully, like him. The first one occurred at the end of 6 October 1511 C.E. The moon began to be eclipsed 1 $^1/_8$ uniform hours before midnight, and was fully illuminated again 2 $^1/_8$ hours after midnight. Thus, the middle of the eclipse was $^7/_{12}$ of an hour after the midnight followed by 7 October = the Nones of October. This was a total eclipse of the moon, when the sun was at 22° 25′ within the Balance, but at 24° 13′ within the Balance according to its uniform motion.

I observed the second eclipse at the end of 5 September 1522 C.E. This too was a total eclipse. It began at $^2/_5$ of a uniform hour before midnight, but its midtime was 1 $^1/_3$ hours after the midnight followed by 6 September = the eighth day before the Ides of September. The sun was at 22 $^1/_5$° within the Virgin, but at 23° 59′ within the Virgin according to its uniform motion.

The third eclipse occurred after the end of 25 August 1523 C.E. It began 2 $^4/_5$ hours after midnight. The midtime [of this eclipse], which also was total, was 4 $^5/_{12}$ hours after the midnight followed by 26 August. The sun was at 11° 21′ within the Virgin, but at 13° 2′ within the Virgin according to its mean motion.

Once again, the distance traversed by the true places of the sun and moon between the first eclipse and the second obviously was 329° 47′; and between the second eclipse and the third, 349° 9′. The time from the first eclipse to the second is 10 uniform years, 337 days, plus $^3/_4$ of an hour according to apparent time, but $^4/_5$ of an hour according to corrected uniform time. From the second eclipse to the third, there were 354 days, plus 3 hours and 5 minutes, but 3 hours, 9 minutes, according to uniform time. In the first interval the combined mean motion of the sun and moon, after the elimination of [complete] circles, amounts to 334° 47′; and the [moon's] motion in anomaly to 250° 36′, with about 5° to be subtracted from the uniform motion [334°47′−329°47′]. In the second interval the [combined] mean motion of the sun and moon is 346° 10′; and the [lunar] anomaly, 306° 43′, with 2° 59′ to be added to the mean motion [+346°10′ = 349°9′].

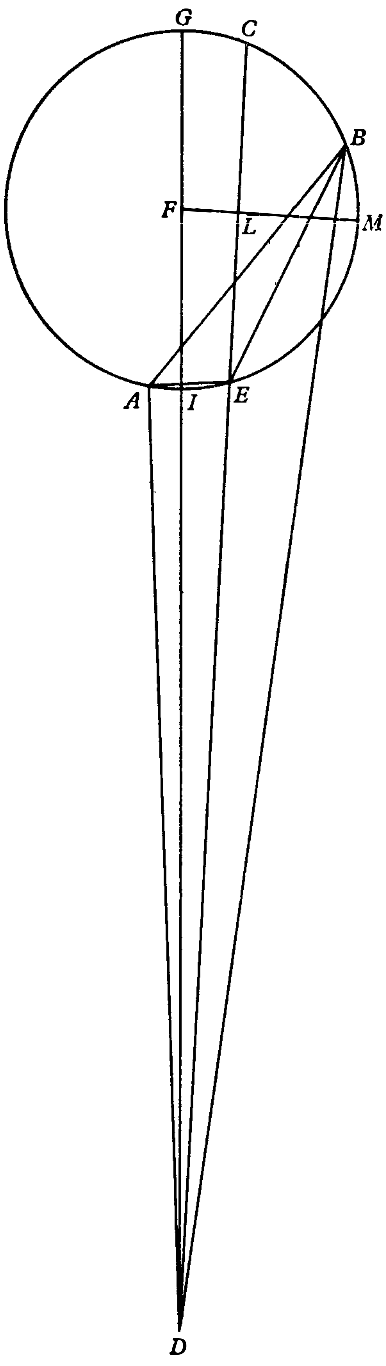
Now let *ABC* be the epicycle. Let *A* be the place of the moon at the middle of the first eclipse; *B* of the second; and *C*, of the third. Let the epicycle be regarded as moving from *C* to *B*, and from *B* to *A*; that is, westward in its upper circumference, and eastward in its lower circumference. Let arc *ACB* = 250° 36′, subtracting, as I said, 5° from the moon's mean motion in the first period of time. Let arc *BAC* = 306° 43′, adding 2° 59′ to the moon's mean motion. As a remainder, therefore, arc *AC* = 197° 19′, subtracting the remaining 2° 1′. Since *AC* is greater than a semicircle and is subtractive, it must contain the higher apse. For this cannot be in *BA* or *CBA*, each of which is less than a semicircle and additive, whereas the diminishing motion occurs near the apogee.



Opposite it take *D* as the center of the earth. Join *AD, DB, DEC, AB, AE,* and *EB*. As regards triangle *DBE*, exterior angle *CEB* is given = 53° 17′ = arc *CB*, the remainder when *BAC* is subtracted from the circle. Angle *BDE* at the center = 2° 59′, but at the circumference = 5° 58′. Therefore the remaining angle *EBD* = 47° 19′ [= 53°17′ − 5°58′]. Consequently side *BE* = 1042 units, and side *DE* = 8024 units, of which the radius of the circle circumscribing the triangle = 10,000. In like manner angle *AEC* = 197° 19′, since it intercepts arc *AC*. At the center angle *ADC* = 2° 1′, but = 4° 2′ at the circumference. Therefore in triangle [*ADE*] the remaining angle *DAE* = 193° 17′, with 360° = 2 right angles. Consequently the sides are also given. In units of which the radius of the circle circumscribing triangle *ADE* = 10,000, *AE* = 702, and *DE* = 19,865. But in units of which *DE* = 8024, and *EB* = 1042, *AE* = 283.

Once more, then, we have a triangle *ABE*, in which two sides, *AE* and *EB*, are given, and the whole angle *AEB* = 250° 36′, when 360° = 2 right angles. Hence, in accordance with the theorems on Plane Triangles, *AB* = 1227 units, of which *EB* = 1042. We have thus obtained the ratio of these three lines, *AB*, *EB*, and *ED*. In units of which the epicycle's radius = 10,000, and the given arc *AB* subtends 16,323, this ratio will also make known that *ED* = 106,751 and *EB* = 13,853. Hence arc *EB* is also given = 87° 41′. When this is added to *BC* [= 53° 17′], the total *EBC* = 140° 58′. Its subtending chord *CE* = 18,851 units, and *CED* as a whole = 125,602 units [= *ED*+*CE* = 106751 + 18,851].

Now emplace the epicycle's center, which must fall in segment *EAC*, since this is greater than a semicircle. Let the center be *F*. Prolong *DIFG* in a straight line through both apsides, *I* the lower, and *G* the higher. Once more, obviously, rectangle $CD \times DE$ = rectangle $GD \times DI$. But rectangle $GD \times DI + (FI)^2 = (DF)^2$. Therefore in length *DIF* is given = 116,226 units, of which *FG* = 10,000. Then, in units of which *DF* = 100,000, *FG* = 8,604 units, in agreement with what I find reported since Ptolemy by most other astronomers before me.



Now from center *F* drop *FL* perpendicular to *EC,* and extend it in straight line *FLM,* which will bisect *CE* in point *L.* Straight line *ED* = 106,751 units. Half of *CE* = *LE* = 9426 units. The sum, *DEL* = 116,177 units, of which *FG* = 10,000, and *DF* = 116,226. Therefore in triangle *DFL* two sides, *DF* and *DL,* are given. Angle *DFL* = 88° 21′ is also given, and the remaining angle *FDL* = 1° 39′. Arc *IEM* is likewise = 88° 21′. *MC* = $1/2$ *EBC* [= 140° 58′] = 70° 29′. *IMC* as a whole = 158° 50′ [= 88° 21′ + 70° 29′]. The rest of the semicircle = *GC* = 21° 10′ [= 180°–158° 50′].

This was the moon's distance from the epicycle's apogee, or the place of the anomaly in the third eclipse; in the second eclipse, *GCB* = 74° 27′ [= *GC*+ *CB* = 21°10′+53°17′]; and in the first eclipse, the whole arc *GBA* = 183° 51′ [= *GB*+*BA* = 74°27′+(109°24′ = 360° – 250°36′)]. Furthermore, in the third eclipse *IDE* as a central angle = 1° 39′, which is the subtractive prosthaphaeresis. In the second eclipse the whole angle *IDB,* also a subtractive prosthaphaeresis, = 4° 38′, for it consists of *GDC* = 1° 39′ and *CDB* = 2° 59′. Therefore, when *IDB* is subtracted from the whole angle *ADB* = 5°, the remainder is *ADI* = 22′, which are added to the uniform motion in the first eclipse. Therefore, the moon's uniform place in the first eclipse was 22° 3′ within the Ram, but its apparent place was 22° 25′, as many degrees as the sun had in the Balance, the opposite sign. So also in the second eclipse the moon's mean place was at 26° 50′ within the Fishes; and in the third eclipse at 13° within the Fishes. The moon's mean motion, by which it is separated from the earth's annual motion, in the first eclipse was 177° 51′; in the second, 182° 51′; and in the third, 179° 58′.

VERIFICATION OF THE STATEMENTS ABOUT THE MOON'S UNIFORM MOTIONS IN LONGITUDE AND ANOMALY

Chapter 6

What has been said about the lunar eclipses will also permit us to test whether the above statements about the moon's uniform motions are correct. In the first set of eclipses, the moon's distance from the sun in the second eclipse was shown to be 182° 47′, and the anomaly was 64° 38′. In the later set of eclipses in our time, in the second eclipse the moon's motion away from the sun was 182° 51′, and the anomaly was 74° 27′. Clearly, in the intervening period there are 17,166 complete months plus about 4 minutes, while the motion in anomaly was 9° 49′ [= 74° 27′– 64° 38′], after the elimination of complete circles. From Hadrian's year 19, in the Egyptian month Choiach, on the 2nd day, 2 hours before the midnight followed by the third day of the month, until 1 : 20 A. M., 5 Septem-ber 1522 C .E., there are 1388 Egyptian years, 302 days, plus 3 $1/3$ hours in apparent time = 3^h34^m in uniform time. In this interval, after the complete revo-lutions in 17,165 uniform months, there would have been 359° 38′ according to Hipparchus and Ptolemy. On the other hand, the anomaly was 9° 39′ according to Hipparchus but according to Ptolemy 9° 11′. For both of them the moon's motion is deficient by 26′ [= 360°4′ – 359°38′], while the anomaly lacks 38′ in Ptolemy's case [= 9°49′ – 9°1 1 ′], and in Hipparchus' case 10′ [9°49′ – 9°39′]. When these shortages are added, the results agree with the computations set out above.

THE EPOCHS OF THE LUNAR LONGITUDE AND ANOMALY

Chapter 7

Here too, as before [III, 23], I must determine the positions of the lunar longitude and anomaly at the established beginnings of eras: the Olympiads, Alexander's, Caesar's, Christ's, and any other which may be desired. Of the three ancient eclipses, let us consider the second one, which occurred in Hadrian's year 19, on the 2nd day of the Egyptian month Choiach, 1 uniform hour before midnight at Alexandria = 2 hours before midnight for us on the meridian of Cracow. From the beginning of the Christian era to this moment, we shall find 133 Egyptian years, 325 days, plus 22 hours simply, but 21 hours, 37 minutes exactly. During this time the moon's motion, according to my computation, is 332° 49′, and the motion in anomaly is 217° 32′. When each of these figures is subtracted from the corresponding figure found in the eclipse, the remainder for the moon's mean distance from the sun is 209° 58′, and 207° 7′ for the anomaly, at the beginning of the Christian era at midnight preceding 1 January.

Prior to this Christian epoch there are 193 Olympiads, 2 years, 194 $^1/_2$ days = 775 Egyptian years, 12 days, plus $^1/_2$ day, but 12 hours, 11 minutes in exact time. Similarly, from the death of Alexander to the birth of Christ there are reckoned 323 Egyptian years, 130 days, plus $^1/_2$ day in apparent time, but 12 hours, 16 minutes, in exact time. From Caesar to Christ there are 45 Egyptian years, 12 days, in which the computations for uniform and apparent time are in agreement.

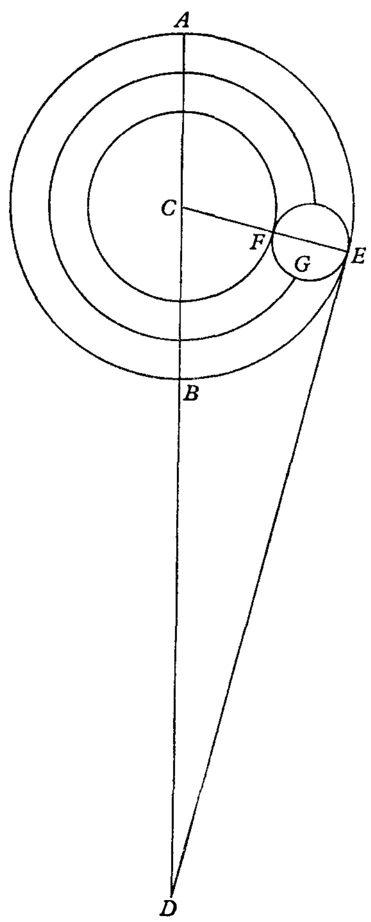
The motions corresponding to these differences of time are subtracted, each in its own category, from the places for Christ. For noon on the 1st day of the month Hecatombaeon of the 1st Olympiad, we shall have the moon's uniform distance from the sun as 39° 48′, and the anomaly as 46° 20′; for Alexander's era, at noon on the first day of the month Thoth, the moon's distance from the sun as 310° 44′, and the anomaly as 85° 41′; for Julius Caesar's era, at midnight before 1 January, the moon's distance from the sun as 350° 39′, and the anomaly as 17° 58′. All these values [are reduced] to the meridian of Cracow. For Gynopolis, which is commonly called Frombork, where I generally made my observations, is located at the mouths of the Vistula River and lies on the meridian of Cracow, as I learn from lunar and solar eclipses observed simultaneously in both places. Macedonia's Dyrrhachium, which was called Epidamnus in antiquity, is also located on this meridian.

THE MOON'S SECOND INEQUALITY, AND THE RATIO OF THE FIRST EPICYCLE TO THE SECOND

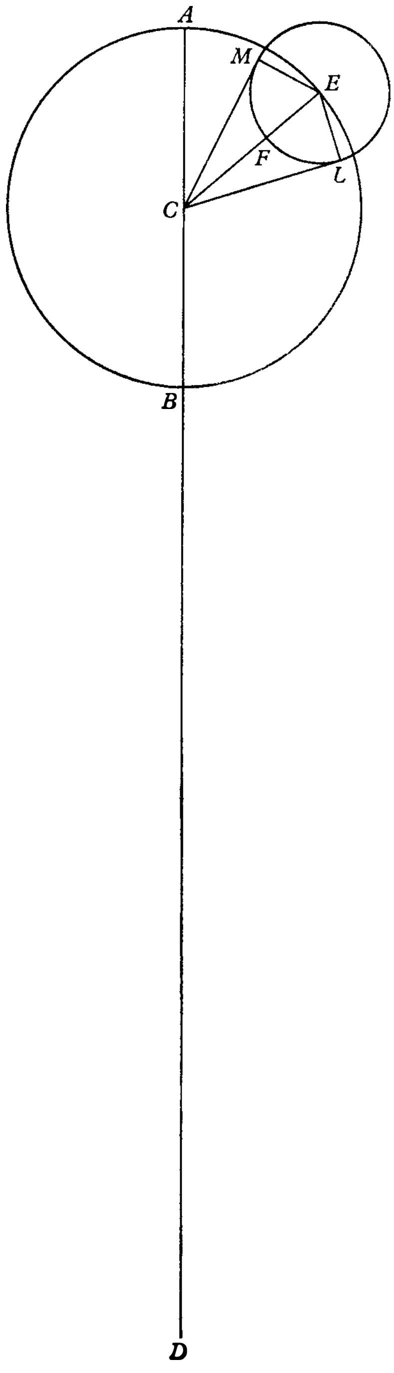
Chapter 8

Thus the moon's uniform motions together with its first inequality have been explained. Now I must investigate the ratio of the first epicycle to the second, and of both to their distance from the center of the earth. The greatest inequality [between the moon's mean and apparent motions] is found, as I said, halfway [between the higher apse and the lower] at the quadratures, when the waxing or waning moon is at the half. This inequality attains 7 $^2/_3$°, as reported also by the ancients [Ptolemy, *Syntaxis*, V, 3]. For they observed the time when the half moon approached most closely to the epicycle's mean distance. This [occurred] near the tangent drawn from the center of the earth, as could easily be perceived through the computation explained above. Since the moon was then about 90° of the ecliptic from its rising or setting, they avoided the error which could be produced in the longitudinal motion by parallax. For at that time the circle passing through the horizon's zenith intersects the ecliptic at right angles, and permits no variation in longitude, but the variation occurs entirely in latitude. Therefore they determined the moon's distance from the sun with the help of an instrument, the astrolabe. After the comparison was made, the moon was found to vary from its uniform motion by 7 $^2/_3$°, as I said, instead of 5°.

Now draw epicycle *AB*, with center *C*. From *D*, the center of the earth, draw the straight line *DBCA*. Let the epicycle's apogee be *A*, and its perigee *B*. Draw *DE* tangent to the epicycle, and join *CE*. At the tangent there is the greatest prosthaphaeresis. In this case let it be 7° 40′ = angle *BDE*. *CED* is a right angle, being at the point of tangency with the circle *AB*. Therefore *CE* will be 1334 units, of which radius *CD* = 10,000. But at full and new moon this distance was much smaller, since it was about 861 of the same units. Divide *CE*, letting *CF* = 860 units. Around the same center [*C*], *F* will [mark] the circumference which was traced by the new and full moon. Therefore the remainder *FE* = 474 units [= 1334 - 860] will be the diameter of the second epicycle. Bisect *FE* at its midpoint *G*. The whole line *CFG* = 1097 units [= *CF*+*FG*] is the radius of the circle described by the center of the second epicycle. Hence the ratio *CG* : *GE* = 1097 : 237 in units of which *CD* = 10,000.



THE REMAINING VARIATION, IN WHICH THE MOON IS SEEN MOVING NONUNIFORMLY AWAY FROM THE [FIRST] EPICYCLE'S HIGHER APSE *Chapter 9*

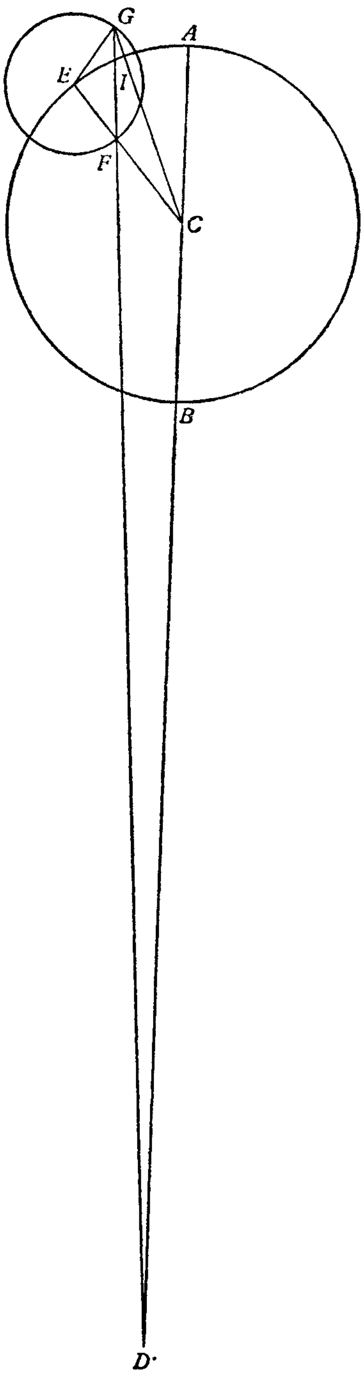


The foregoing demonstration also permits us to understand how the moon moves nonuniformly on its first epicycle, the greatest inequality occurring when it is crescent or gibbous as well as half full. Once more let *AB* be the first epicycle, described by the mean motion of the second epicycle's center. Let the first epicycle's center be *C*, its higher apse *A*, and its lower apse *B*. Take point *E* anywhere on the circumference, and join *CE*. Let $CE : EF = 1097 : 237$. With *E* as center, and radius *EF*, describe the second epicycle. Draw straight lines *CL* and *CM* tangent to it on both sides. Let the epicyclet move from *A* to *E*, that is, westward in the upper circumference [of the first epicycle]. Let the moon move from *F* to *L*, also westward. The motion *AE* being uniform, the second epicycle's motion through *FL* clearly adds arc *FL* to the uniform motion, and subtracts therefrom when it passes through *MF*. In triangle *CEL*, *L* is a right angle. $EL = 237$ units, of which $CE = 1097$. In units of which $CE = 10{,}000$, $EL = 2160$. It subtends angle *ECL* which, according to the Table, $= 12° 28' =$ angle *MCF*, since the triangles [*ECL* and *ECM*] are similar and equal. This is the greatest inequality of the moon's departure from the higher apse of the first epicycle. This happens when the moon in its mean motion deviates by 38° 46′ to either side of the line of the earth's mean motion. Thus these greatest prosthaphaereses quite clearly occur when the moon is at the mean distance of 38° 46′ from the sun and is at the same distance to either side of the mean opposition.

HOW THE MOON'S APPARENT MOTION IS DERIVED FROM THE GIVEN UNIFORM MOTIONS *Chapter 10*

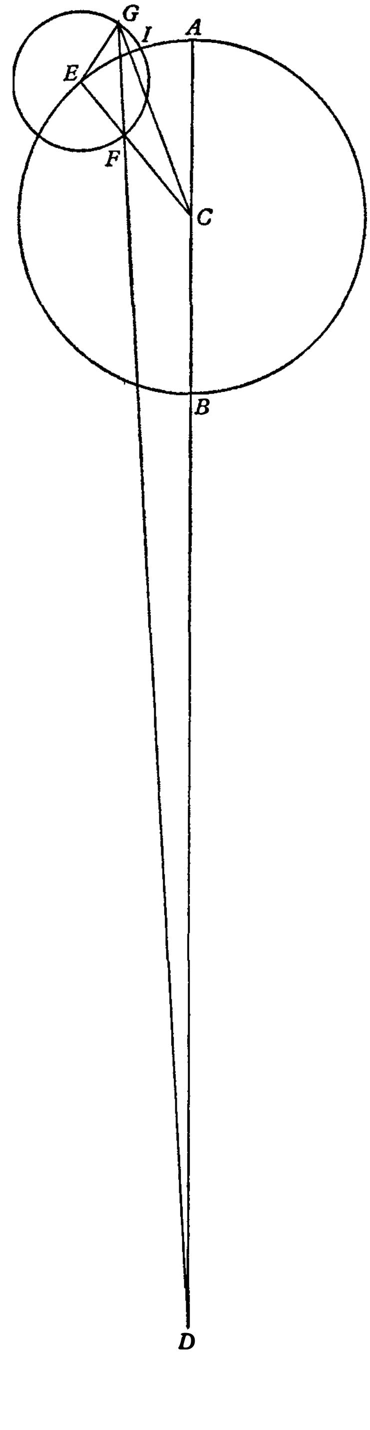
Having so disposed of all these topics, I now wish to show by way of a diagram how those uniform motions of the moon yield the apparent motion equal to the given uniform motions. I choose an example from Hipparchus' observations, by which at the same time the theory may be confirmed by experience [Ptolemy, *Syntaxis*, V, 5].

In the 197th year after the death of Alexander, on the 17th day of Pauni, which is the 10th Egyptian month, at $9\,^1/_3$ hours of the day, Hipparchus in Rhodes, observing the sun and moon with an astrolabe, found them $48\,^1/_{10}°$ apart, with the moon following the sun. He thought that the sun's place was $10\,^9/_{10}°$ within the Crab, and therefore the moon was at 29° within the Lion. At that time 29° within the Scorpion was rising, and 10° within the Virgin was culminating in Rhodes, where the elevation of the north pole is 36° [Ptolemy, *Syntaxis*, II, 2]. From this situation it was clear that the moon, located about 90° of the ecliptic from the horizon, at that time underwent no parallax in longitude or at any rate an imperceptible parallax. This observation was performed on that 17th day in the afternoon, at $3\,^1/_3$ hours = 4 uniform hours at Rhodes. This would have been $3\,^1/_6$ uniform hours at Cracow, since Rhodes is $^1/_6$ of an hour nearer to us than Alexandria is. From the death of Alexander there were 196 years, 286 days, plus $3\,^1/_6$ simple hours, but about $3\,^1/_3$ equal hours. At that time the sun in its mean motion reached 12° 3′ within the Crab, but in its apparent motion 10° 40′ within the Crab. Hence it is evident that the moon really was at 28° 37′ within the Lion. The moon's uniform motion in its monthly revolution was 45° 5′, and in anomaly away from the higher apse 333°, according to my calculation.



With this example before us, let us draw the first epicycle AB, with its center at C. Extend its diameter ACB in a straight line ABD to the center of the earth. On the epicycle take arc ABE = 333°. Join CE, and divide it at F, so that EF = 237 units, of which EC = 1097. With E as center, and radius EF, describe epicyclepicyclet FG. Let the moon be at point G, with arc FG = 90° 10′ = twice the uniform motion away from the sun = 45° 5′. Join CG, EG, and DG. In triangle CEG two sides are given, CE = 1097, and EG = EF = 237, with angle GEC = 90° 10′. Hence, in accordance with the theorems on Plane Triangles, the remaining side CG is given = 1123 of the same units, and so is angle ECG = 12° 11′. This makes clear also arc EI and the anomaly's additive prosthaphaeresis, with the whole of $ABEI$ = 345° 11′ [ABE+EI = 333°+12° 11′]. The remaining angle GCA = 14° 49′ [= 360°−345° 11′] = the moon's true distance from the higher apse of epicycle AB, and angle BCG = 165° 11′ [=180°−14°49′]. Consequently, also in triangle GDC two sides are given, GC = 1123 units, of which CD = 10,000, as well as angle GCD = 165° 11′. From them we obtain also angle CDG = 1° 29′ and the prosthaphaeresis, which was added to the moon's mean motion. As a result the moon's true distance from the sun's mean motion = 46° 34′ [= 45° 5′+1° 29′], and the moon's apparent place at 28° 37′ within the Lion differed from the sun's true place by 47° 57′ = 9′ less than in Hipparchus' observation [48° 6′−47° 57′ = 9′].

However, let nobody for this reason suspect that either his investigation or my computation was faulty. Although there is a slight discrepancy, I shall nevertheless show that neither he nor I committed an error, and that this is how things really were. For let us remember that the circle traversed by the moon is tilted. Then we shall also admit that in the ecliptic it produces some inequality in longitude, especially near the regions which lie midway between both limits, the northern and the southern, and both nodes. This situation is very much like the obliquity of the ecliptic and equator, as I explained in connection with the nonuniformity of the natural day [III, 26]. So also, if we transfer these ratios to the lunar circle, which Ptolemy asserted is inclined to the ecliptic [*Syntaxis*, V, 5], we shall find that in those places these ratios make a difference on the ecliptic of 7′ in longitude, which when doubled = 14′. This occurs as an addition and a subtraction in like manner. For, since the sun and moon are a quadrant apart, if the northern or southern limit of latitude is midway between them, then the arc intercepted on the ecliptic is 14′ larger than a quadrant of the moon's circle. On the contrary, in the other quadrant, in which the nodes are the midpoints, the circles through the poles of the ecliptic intercept the same quantity less than a quadrant. This is the situation in the present case. The moon was about halfway between the southern limit and its ascending intersection with the ecliptic (the intersection which the moderns call the "head of the Dragon"). The sun had already passed the other intersection, the descending one (which the moderns call the "tail [of the Dragon]"). There is no wonder, therefore, if that lunar distance of 47° 57′ on its tilted circle increased at least 7′ when related to the ecliptic, apart from the fact that the sun, in approaching its setting, also contributed some subtractive parallax. These topics will be discussed more fully in the explanation of the parallaxes [IV, 16]. Thus that distance of 48° 6′ between the luminaries, which Hipparchus had obtained instrumentally, accords with my computation with remarkable closeness and, as it were, by agreement.



TABULAR PRESENTATION OF THE LUNAR PROSTHAPHAERESES OR NORMALIZATIONS *Chapter 11*

The method of computing the lunar motions, I believe, is understood in general from the present example. In triangle *CEG* two sides, *GE* and *CE*, always remain the same. Through angle *GEC*, which constantly changes, but nevertheless is given, we obtain the remaining side *GC*, together with angle *ECG*, which is the prosthaphaeresis for normalizing the anomaly. Secondly, when two sides, *DC* and *CG*, in triangle *CDG*, as well as angle *DCE* are determined numerically, by the same procedure angle *D* at the center of the earth becomes known [as the difference] between the uniform and the true motions.

In order to make this information even handier, I shall construct a table of the prosthaphaereses in six columns. After two [columns containing the] common numbers of the deferent, the third column will show the prosthaphaereses which arise from the epicyclet's twice-monthly rotation and vary the uniformity of the first anomaly. Then, leaving the next column temporarily vacant to receive numbers later, I shall concern myself with the fifth column. In it I shall enter the first and larger epicycle's prosthaphaereses which occur at mean conjunctions and oppositions of the sun and moon. The biggest of these prosthaphaereses is 4° 56′. In the next to the last column are placed the numbers by which the prosthaphaereses occurring at half moon exceed the prosthaphaereses in column 4. Of these numbers, the largest is 2° 44′ [= 7° 40′– 4° 56′]. For the purpose of ascertaining the other numbers in excess, the proportional minutes have been worked out according to the following ratio. [The maximum number in excess] 2° 44′ was treated as 60′ in relation to any other excess occurring at the epicyclet's point of tangency [with the line drawn from the center of the earth]. Thus, in the same example [IV, 10], we had line $CG = 1123$ units of which $CD = 10{,}000$. This makes the largest prosthaphaeresis at the epicyclet's point of tangency 6° 29′, exceeding that first maximum by 1° 33′ [+4° 56′ = 6° 29′]. But 2° 44′ : 1° 33′ = 60′ : 34′. Therefore we have the ratio of the excess occurring in the epicyclet's semicircle to the excess caused by the given arc of 90° 10′. Accordingly, opposite 90° in the Table, I shall write 34′. In this way for every arc of the same circle entered in the Table we shall find the proportional minutes, which are to be recorded in the vacant fourth column. Finally, in the last column I added the northern and southern degrees of latitude, which I shall discuss below [IV, 13–14]. For, the convenience of the procedure and practice with it convinced me to preserve this arrangement.

TABLE OF THE MOON'S PROSTHAPHAERESES

Common Numbers		Second Epicycle's Prosthaphaeresis		Proportional Minutes	First Epicycle's Prosthaphaeresis		Increases		Northern Latitude	
°	′	°	′		°	′	°	′	°	′
3	357	0	51	0	0	14	0	7	4	59
6	354	1	40	0	0	28	0	14	4	58
9	351	2	28	1	0	43	0	21	4	56
12	348	3	15	1	0	57	0	28	4	53
15	345	4	1	2	1	11	0	35	4	50
18	342	4	47	3	1	24	0	43	4	45
21	339	5	31	3	1	38	0	50	4	40
24	336	6	13	4	1	51	0	56	4	34
27	333	6	54	5	2	5	1	4	4	27
30	330	7	34	5	2	17	1	12	4	20
33	327	8	10	6	2	30	1	18	4	12
36	324	8	44	7	2	42	1	25	4	3
39	321	9	16	8	2	54	1	30	3	53
42	318	9	47	10	3	6	1	37	3	43
45	315	10	14	11	3	17	1	42	3	32
48	312	10	30	12	3	27	1	48	3	20
51	309	11	0	13	3	38	1	52	3	8
54	306	11	21	15	3	47	1	57	2	56
57	303	11	38	16	3	56	2	2	2	44
60	300	11	50	18	4	5	2	6	2	30
63	297	12	2	19	4	13	2	10	2	16
66	294	12	12	21	4	20	2	15	2	2
69	291	12	18	22	4	27	2	18	1	47
72	288	12	23	24	4	33	2	21	1	33
75	285	12	27	25	4	39	2	25	1	18
78	282	12	28	27	4	43	2	28	1	2
81	279	12	26	28	4	47	2	30	0	47
84	276	12	23	30	4	51	2	34	0	31
87	273	12	17	32	4	53	2	37	0	16
90	270	12	12	34	4	55	2	40	0	0

TABLE OF THE MOON'S PROSTHAPHAERESES

Common Numbers		Second Epicycle's Prosthaphaeresis		Proportional Minutes	First Epicycle's Prosthaphaeresis		Increases		Southern Latitude	
°	′	°	′		°	′	°	′	°	′
93	267	12	3	35	4	56	2	42	0	16
96	264	11	53	37	4	56	2	42	0	31
99	261	11	41	38	4	55	2	43	0	47
102	258	11	27	39	4	54	2	43	1	2
105	255	11	10	41	4	51	2	44	1	18
108	252	10	52	42	4	48	2	44	1	33
111	249	10	35	43	4	44	2	43	1	47
114	246	10	17	45	4	39	2	41	2	2
117	243	9	57	46	4	34	2	38	2	16
120	240	9	35	47	4	27	2	35	2	30
123	237	9	13	48	4	20	2	31	2	44
126	234	8	50	49	4	11	2	27	2	56
129	231	8	25	50	4	2	2	22	3	9
132	228	7	59	51	3	53	2	18	3	21
135	225	7	33	52	3	42	2	13	3	32
138	222	7	7	53	3	31	2	8	3	43
141	219	6	38	54	3	19	2	1	3	53
144	216	6	9	55	3	7	1	53	4	3
147	213	5	40	56	2	53	1	46	4	12
150	210	5	11	57	2	40	1	37	4	20
153	207	4	42	57	2	25	1	28	4	27
156	204	4	11	58	2	10	1	20	4	34
159	201	3	41	58	1	55	1	12	4	40
162	198	3	10	59	1	39	1	4	4	45
165	195	2	39	59	1	23	0	53	4	50
168	192	2	7	59	1	7	0	43	4	53
171	189	1	36	60	0	51	0	33	4	56
174	186	1	4	60	0	34	0	22	4	58
177	183	0	32	60	0	17	0	11	4	59
180	180	0	0	60	0	0	0	0	5	0

COMPUTING THE MOON'S MOTION *Chapter 12*

The method of computing the apparent lunar [motion] is clear from the foregoing demonstrations, and is as follows. The proposed time for which we seek the moon's place will be reduced to uniform time. Through it, just as we did in the case of the sun [III, 25], we shall derive the [moon's] mean motions in longitude, anomaly, and also latitude, which I shall soon explain [IV, 13], from the given epoch of Christ or any other [given epoch]. We shall establish the place of each motion at the proposed time. Then, in the Table we shall look up the moon's uniform elongation or twice its distance from the sun. We shall note the appropriate prosthaphaeresis in column 3, and the accompanying proportional minutes. If the number with which we started is found in column 1 or is less than 180°, we shall add the prosthaphaeresis to the lunar anomaly. But if that number is greater than 180° or is in column 2, the prosthaphaeresis will be subtracted from the anomaly. Thus we shall obtain the normalized anomaly of the moon, and its true distance from the [first epicycle's] higher apse. With this we shall consult the Table again and take the prosthaphaeresis in column 5 corresponding to it, as well as the excess which follows in column 6. This excess is added by the second epicycle to the first. Its proportional part, computed from the ratio of the minutes found to 60 [minutes], is always added to this prosthaphaeresis. The sum thus obtained is subtracted from the mean motion in longitude and latitude, provided that the normalized anomaly is less than 180° or a semicircle, and it is added if the anomaly is greater [than 180°]. In this way we shall obtain the moon's true distance from the sun's mean place, and its normalized motion in latitude. There will therefore be no uncertainty about the moon's true distance either from the first star in the Ram through the sun's simple motion, or from the vernal equinox through its composite motion, affected by the precession of the equinox. Finally, through the normalized motion in latitude in the Table's seventh and last column we shall have the degrees of latitude by which the moon has deviated from the ecliptic. This latitude will be northern when the motion in longitude is found in the first part of the Table, that is, if it is less than 90° or greater than 270°. Otherwise its latitude will be southern. Up to 180°, therefore, the moon will descend from the north, and then ascend from its southern limit until it has completed the remaining degrees of the circle. To that extent the moon's apparent motion in a certain way has as many functions connected with the earth's center as the earth's center has with the sun.

HOW THE MOON'S MOTION IN LATITUDE IS ANALYZED AND DEMONSTRATED *Chapter 13*

Now I must give an account also of the moon's motion in latitude, which seems harder to find because more circumstances block the way. For, as I said before [IV, 4], suppose that two lunar eclipses are similar and equal in all respects; that is, the darkened areas occupy the same northern or southern position; the moon is near the same ascending or descending node; and its distance from the earth or from the higher apse is equal. If these two eclipses so agree, the moon is known to have completed whole circles of latitude in its true motion. For, the earth's shadow is conical. If a right cone is cut by a plane parallel to its base, the section is a circle. This is smaller at a greater distance from the base, and greater at a smaller distance from the base, and accordingly equal at an equal distance. Thus, at equal distances from the earth, the moon passes through equal circles of the shadow, and presents equal disks of itself to our sight. As a result, when it displays equal parts on the same side at equal distances from the center of the shadow, it informs us that the latitudes are equal. From this it necessarily follows that the moon has returned to an earlier place in latitude, and that its distances from the same node are also equal at those times, especially if the place of both bodies likewise agrees. For, an approach and withdrawal of the moon or of the earth change the whole size of the shadow. Yet the change is slight and barely ascertainable. Therefore, as was said with regard to the sun [III, 20], the longer the interval that has elapsed between the two eclipses, the more precisely will we be able to obtain the moon's motion in latitude. But two eclipses agreeing in these respects are rarely found (I for one have not encountered any thus far).

Nevertheless, I am aware that there is also another method by which this can be done. For suppose that while the other conditions remain, the moon is eclipsed on opposite sides and near opposite nodes. This will indicate that in the second eclipse the moon reached the place diametrically opposite the place of the first eclipse, and described a semicircle in addition to whole circles. This would seem to be satisfactory for the investigation of this topic. Accordingly, I have found two eclipses related to each other almost exactly in this way.

The first one occurred in the 7th year of Ptolemy Philometor = 150th year after Alexander, in Phamenoth, the 7th Egyptian month, after the 27th day, during the night followed by the 28th, as Claudius [Ptolemy] says [*Syntaxis*, VI, 5]. The moon was eclipsed from the beginning of the 8th hour until the end of the 10th hour, in seasonal hours of the night at Alexandria. The eclipse, near the descending node, at its greatest extent darkened 7/12ths of the moon's diameter from the north. The midtime of the eclipse, therefore, was 2 seasonal hours (according to Ptolemy) after midnight = $2\,{}^1/_3$ uniform hours, since the sun was at 6° within the Bull. At Cracow it would have been $1\,{}^1/_3$ hours [uniform time].

I observed the second eclipse on that same meridian of Cracow on 2 June 1509 C.E., when the sun was at 21° within the Twins. The eclipse's midtime was $11\,{}^3/_4$ uniform hours after noon of that day. About 8/12ths of the moon's diameter on its southern side were darkened. The eclipse occurred near the ascending node.

From the beginning of Alexander's era, therefore, [until the first eclipse] there are 149 Egyptian years, 206 days, plus $14\,{}^1/_3$ hours at Alexandria. At Cracow, however, there would have been $13\,{}^1/_3$ hours, local time, but $13\,{}^1/_2$ hours, uniform time. At that moment the uniform place of the anomaly, according to my computation in almost exact agreement with Ptolemy's [= 163°40′], was 163° 33′, and the prosthaphaeresis was 1° 23′, by which the moon's true place was less than its uniform [place]. From the same established epoch of Alexander to the second eclipse there are 1832 Egyptian years, 295 days, plus 11 hours, 45 minutes, apparent time = 11 hours, 55 minutes, uniform time. Hence the moon's uniform motion was 182° 18′; the place of the anomaly = 159° 55′ = 161° 13′, normalized; the prosthaphaeresis, by which the uniform motion was less than the apparent, was 1° 44′.

In both eclipses, therefore, the moon was clearly at an equal distance from the earth, and the sun was nearly at its apogee in both cases, but there was a difference of one digit between the darkened areas. The moon's diameter usually occupies about $^1/_2$°, as I shall show later on [IV, 18]. One digit = $^1/_{12}$th of the diameter = 2 $^1/_2$′, corresponding to about $^1/_2$° on the moon's tilted circle near the nodes. In the second eclipse the moon was $^1/_2$° farther away from the ascending node than from the descending node in the first eclipse. Hence, the moon's true motion in latitude after complete revolutions quite evidently was 179 $^1/_2$°. But between the first and second eclipse the lunar anomaly added to the uniform motion 21′, by which one prosthaphaeresis exceeds the other [1° 44′ –1° 23′]. We shall therefore have the moon's uniform motion in latitude as 179° 51′ [= 179°30′+21′] after complete circles. The interval between the two eclipses was 1683 years, 88 days, 22 hours, 25 minutes, apparent time, in agreement with uniform time. In this period, after 22,577 uniform revolutions were completed, there are 179° 51′, in agreement with the value which I just mentioned.

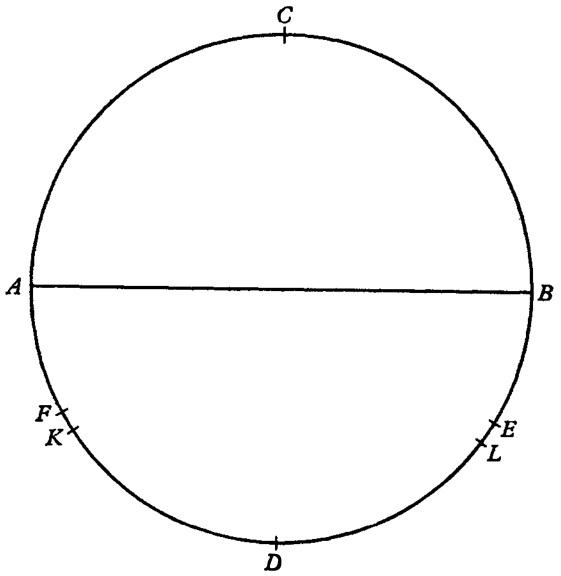
THE PLACES OF THE MOON'S ANOMALY IN LATITUDE *Chapter 14*

In order to determine the places of this motion too at the previously accepted epochs, here also I have taken two lunar eclipses. These occurred, not at the same node nor, as in the previous instances [IV, 13], in diametrically opposite regions, but in the same region, northern or southern (all the other conditions being met, as I said). By following Ptolemy's procedure [*Syntaxis,* IV, 9], with these eclipses we shall attain our goal without error.

The first eclipse, which I used for investigating other lunar motions also [IV, 5], was the one which I said was observed by Claudius Ptolemy in Hadrian's year 19, toward the end of the 2nd day of the month Choiach, one uniform hour before the midnight which was followed by the 3rd day at Alexandria. At Cracow it would have been 2 hours before midnight. At mid-eclipse $^5/_6$ of the diameter = 10 digits were darkened in the north. The sun was at 25° 10′ within the Balance. The place of the moon's anomaly was 64° 38′, and its subtractive prosthaphaeresis was 4° 20′. The eclipse occurred near the descending node.

I observed the second eclipse, also with great care, at Rome on 6 November 1500 C.E., two hours after the midnight which initiated 6 November. At Cracow, which lies 5° to the east, it was 2 $^1/_3$ hours after midnight. The sun was at 23° 16′ within the Scorpion. Once again, ten digits in the north were darkened. From the death of Alexander there is a total of 1824 Egyptian years, 84 days, plus 14 hours, 20 minutes, apparent time, but 14 hours, 16 minutes, uniform time. The moon's mean motion was at 174° 14′; the lunar anomaly was at 294° 44′, normalized at 291° 35′. The additive prosthaphaeresis was 4° 28′.

Also in these two eclipses, clearly, the moon's distances from the higher apse were almost equal. In both cases the sun was near its middle apse, and the size of the shadows was equal [10 digits]. These facts indicate that the moon's latitude was southern and equal, and therefore the moon's distances from the nodes were equal, in the latter case ascending, but in the former case descending. Between the two eclipses there are 1366 Egyptian years, 358 days, plus 4 hours, 20 minutes, apparent time, but 4 hours, 24 minutes, uniform time, during which the mean motion in latitude is 159° 55′.



Now in the moon's tilted circle let the diameter AB be the intersection with the ecliptic. Let C be the northern limit, and D the southern; A the descending node, and B the ascending node. In the southern region take two equal arcs, AF and BE, for the first eclipse at point F, and the second at point E. Furthermore, let FK be the subtractive prosthaphaeresis at the first eclipse, and EL the additive prosthaphaeresis at the second. Arc $KL = 159° 55'$. To it add $FK = 4° 20'$ and $EL = 4° 28'$. The whole arc $FKLE = 168° 43'$, and the rest of the semicircle $= 11° 17'$. Half of this $= 5° 39' = AF = BE$, the moon's true distances from the nodes A and B, and therefore $AFK = 9° 59'$ [$= 4° 20' + 5° 39'$]. Hence it is also clear that $CAFK =$ the distance of the latitude's mean place from the northern limit $= 99° 59'$ [$= 90° + 9° 59'$]. From the death of Alexander to the time of this observation by Ptolemy in this place there are 457 Egyptian years, 91 days, plus 10 hours by apparent time, but 9 hours, 54 minutes, by uniform time. In this interval the mean motion in latitude is $50° 59'$. When this figure is subtracted from $99° 59'$, the remainder is $49°$ for noon on the first day of Thoth, the first Egyptian month, at the epoch of Alexander, but on the meridian of Cracow.

Hence, the places of the moon's motion in latitude, starting from the northern limit, which I took as the origin of the motion, are given for all the other epochs according to the differences in the intervals. From the 1st Olympiad to the death of Alexander there are 451 Egyptian years, 247 days, from which 7 minutes are subtracted to normalize the time. In this period the motion in latitude $= 136° 57'$. Furthermore, from the 1st Olympiad to Caesar there are 730 Egyptian years, 12 hours, to which 10 minutes are added to normalize the time. In this period the uniform motion $= 206° 53'$. From then to Christ there are 45 years, 12 days. From $49°$ subtract $136° 57'$ by supplying the $360°$ of a circle; the remainder $= 272° 3'$ for noon on the first day of the month Hecatombaeon [in the first year] of the first Olympiad. Again, to this figure add $206° 53'$; the sum [$272° 3' + 206° 53' = 478° 56' - 360°$] $= 118° 56'$ for the midnight preceding 1 January of the Julian epoch. Finally, add $10° 49'$; the sum $= 129° 45'$, the place for the Christian epoch, likewise at midnight preceding 1 January.

THE CONSTRUCTION OF THE PARALLACTIC INSTRUMENT

Chapter 15

The moon's greatest latitude, corresponding to the angle of intersection between its circle and the ecliptic, = 5°, with the circle = 360°. An opportunity to make this observation was not vouchsafed by fate to me, hampered by lunar parallaxes, as it was to Claudius Ptolemy. For at Alexandria, where the north pole's elevation = 30° 58′, he focused on the moon's imminent closest approach to the zenith, that is, when it was at the beginning of the Crab and at its northern limit, which he was able to determine numerically in advance [*Syntaxis*, V, 12]. With the help of a certain device, which he calls the "parallactic instrument", constructed for the purpose of determining the moon's parallaxes, at that time he found its minimum distance from the zenith to be only 2 $^1/_8$°. Had this distance been affected by any parallax, this would necessarily have been quite small for so short a distance. Then, subtracting 2 $^1/_8$° from 30° 58′ leaves a remainder of 28° 50 $^1/_2$′. This figure exceeds the greatest obliquity of the ecliptic (which was then 23° 51′ 20″) by about 5 whole degrees. This lunar latitude, finally, is found to agree with the other details up to the present.

The parallactic instrument consists of three rulers. Two of them are of equal length, at least 4 cubits, while the third is somewhat larger. This [longer ruler] and one of the two shorter rulers are joined to either end of the third ruler by pins or pegs so fitted in careful perforations that while the rulers can move in the same plane, they do not wobble at all in those joints. From the center of the joint of the longer ruler produce a straight line down its entire length. On this straight line measure a segment as exactly as possible equal to the distance between the joints. Divide this segment into 1000 equal units, or more, if possible. With the same units continue this division on the rest of the ruler until you reach 1414 units. These constitute the side of the square inscribed in a circle whose radius = 1000 units. The rest of this ruler may be cut off as superfluous. From the center of the joint on the other ruler also draw a line equal to those 1000 units, or to the distance between the centers of the joints. To a side of this ruler attach eyepieces through which sight passes, as is customary with the dioptra. Arrange these eyepieces so that the lines of sight do not deviate at all from the line already drawn along the ruler, but are equally distant from it. Be sure also that when this line is moved toward the longer ruler, its end can touch the graduated line. In this way the rulers form an isosceles triangle, whose base will be in the units of the graduated line. Then a very well squared and polished pole is erected and made firm. To the pole fasten the ruler with the two joints by means of hinges, on which the instrument can rotate like a door. But the straight line passing through the centers of the ruler's joints is always vertical and, as though it were the axis of the horizon, points toward the zenith. Therefore, when you are looking for a star's distance from the zenith, keep the star in view along a straight line through the ruler's eyepieces. By placing the ruler with the graduated line underneath, you will find out how many units, of which the diameter of a circle = 20,000, subtend the angle between [the line of] sight and the axis of the horizon. From the Table [of Lines Subtended] you will obtain the desired arc of the great circle between the star and the zenith.

HOW THE LUNAR PARALLAXES ARE OBTAINED *Chapter 16*

With this instrument, as I said [IV, 15], Ptolemy learned that the moon's greatest latitude = 5°. Then, turning his attention to ascertaining its parallax, he says [*Syntaxis,* V, 13] that he found this at Alexandria to be 1° 7′; the sun was at 5° 28′ within the Balance; the moon's mean distance from the sun = 78° 13′; the uniform anomaly = 262° 20′; the motion in latitude = 354° 40′; the additive prosthaphaeresis = 7° 26′; therefore the moon's place was 3° 9′ within the Goat; the normalized motion in latitude = 2° 6′; the moon's northern latitude = 4° 59′; its declination from the equator = 23° 49′; and the latitude of Alexandria = 30° 58′. Near the meridian, he says, the moon was seen through the instrument at 50° 55′ from the zenith, that is, 1° 7′ more than required by computation. With this information, in accordance with the ancients' [lunar] theory of an eccentrepicycle, he shows that the moon's distance from the center of the earth at that time was 39 units, 45 minutes, with the radius of the earth = 1 unit. Then he demonstrates what follows from the ratio of the circles. For instance, the moon's greatest distance from the earth (which they say occurs at new and full moon in the apogee of the epicycle) is 64 units plus 10 minutes = 1/6th of a unit. But the moon's least distance [from the earth] (which occurs at the quadratures), when the half moon is in the perigee of the epicycle, is only 33 units, 33 minutes. Hence he also evaluated the parallaxes which occur about 90° from the zenith: the smallest = 53′ 34″, but the largest = 1° 43′ (as may be seen more fully from what he deduced therefrom).

But now to those who wish to consider the matter, it is clear that the situation is quite different, as I have frequently found. Nevertheless I shall review two observations which again establish that my lunar theory is more precise than theirs to the extent that it is found to agree better with the phenomena and to leave no residue of doubt.

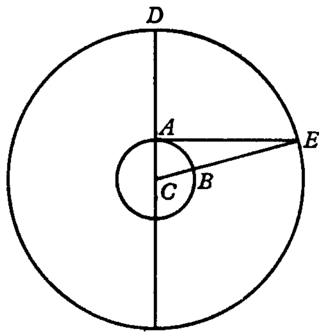
On 27 September 1522 C.E., 5 $^2/_3$ uniform hours after noon, about sunset at Frombork through the parallactic instrument I caught the center of the moon on the meridian, and found its distance from the zenith = 82° 50′. From the beginning of the Christian era to this moment there were 1522 Egyptian years, 284 days, plus 17 $^2/_3$ hours by apparent time, but 17 hours, 24 minutes by uniform time. Therefore the apparent place of the sun was computed to be 13° 29′ within the Balance; the moon's uniform distance from the sun = 87° 6′; the uniform anomaly = 357° 39′; the true anomaly = 358° 40′; and the additive [prosthaphaeresis] = 7′. Thus the moon's true place = 12° 33′ within the Goat. The mean motion in latitude from the northern limit = 197° 1′; the true motion in latitude = 197° 8′ [= 197°1′+7′]; the moon's southern latitude = 4° 47′; the declination from the equator = 27° 41′; and the latitude of my place of observation = 54° 19′. When this is added to the lunar declination, it makes the [moon's] true distance from the zenith = 82° [= 54° 19′+27° 41′]. Therefore the remaining 50′ [of 82° 50′, the apparent zenith distance] were the parallax, which should have been 1° 17′ according to Ptolemy's doctrine.

Moreover, I made another observation in the same place at 6 P. M. on 7 August 1524 C.E., and through the same instrument I saw the moon at 81° 55′ from the zenith. From the beginning of the Christian era until this hour there were 1524 Egyptian years, 234 days, 18 hours [by apparent time], and 18 hours by uniform time also. The sun's place was computed to be 24° 14′ within the Lion; the moon's mean distance from the sun = 97° 5′; the uniform anomaly = 242° 10′; the corrected anomaly = 239° 40′, adding about 7° to the mean motion. Therefore the moon's true place = 9° 39′ within the Archer; the mean motion in latitude = 193° 19′; the true [motion in latitude] = 200° 17′; the moon's southern latitude = 4° 41′; and its southern declination = 26° 36′. When this is added to the latitude of the place of the observation = 54° 19′, the sum = the moon's distance from the pole of the horizon = 80° 55′ [= 26° 36′+54° 19′]. But it appeared to be 81° 55′. Therefore the surplus of 1° was transferred to the lunar parallax which, according to Ptolemy and the ideas of my predecessors, should have been 1° 38′, a calculation required by consistency with the implications of their theory.

A DEMONSTRATION OF THE MOON'S DISTANCES FROM THE EARTH, AND OF THEIR RATIO IN UNITS OF WHICH THE EARTH'S RADIUS = 1

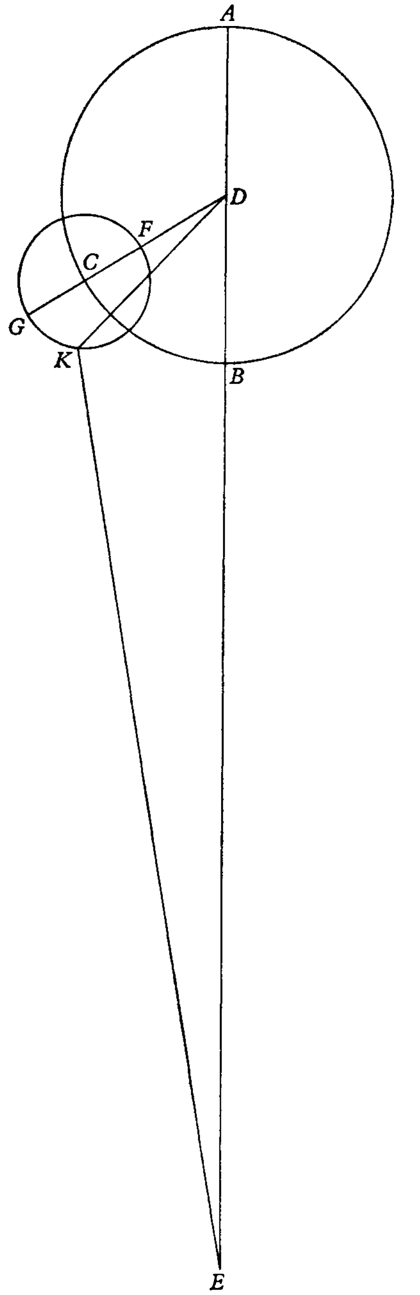
Chapter 17

From the foregoing information the size of the moon's distance from the earth will now be clear. Without this distance a definite value cannot be attached to the parallaxes, since these two quantities are related to each other. The distance will be determined as follows.



Let *AB* be a great circle of the earth, with its center at *C*. Around *C* describe another circle *DE*, in comparison with which the size of the earth's [circle] is significant. Let *D* be the pole of the horizon. Put the center of the moon at *E*, where *DE*, its distance from the zenith, is known. In the first observation [of IV, 16] angle *DAE* = 82° 50′; *ACE* was computed to be only 82°; and *AEC*, the difference between them = 50′ = the parallax. Accordingly we have triangle *ACE* with its angles given, and therefore its sides are given. For, since angle *CAE* is given [= 97°10′ = 180° − 82°50′], side *CE* = 99,219 units, of which the diameter of the circle circumscribed around triangle *AEC* = 100,000. In such units *AC* = 1454 ≅ 1/68 *CE* [= about 68 units], of which the earth's radius *AC* = 1. This was the moon's distance from the earth's center in the first observation.

But in the second [observation of IV, 16] the apparent angle *DAE* = 81° 55′; the computed angle *ACE* = 80° 55′; and the difference, angle *AEC* = 60′. Therefore side *EC* = 99,027 units, and *AC* = 1891 units, of which the diameter of the circle circumscribed around the triangle = 100,000. Thus *CE*, the moon's distance [from the earth's center] = 56 units, 42 minutes, of which the earth's radius $AC = 1$.



Now let the moon's greater epicycle be *ABC*, with center *D*. Take *E* as the earth's center, from which draw the straight line *EBDA* as far as the apogee *A*, while the perigee is at *B*. Measure arc $ABC = 242° 10'$ in accordance with the computed uniform lunar anomaly [in Copernicus' 2nd observation in IV, 16]. With center *C*, describe the second epicycle *FGK*. On it let arc $FGK = 194° 10'$ = twice the moon's distance from the sun [$= 2 \times 97° 5'$]. Join *DK*, which subtracts $2° 27'$ from the anomaly, leaving *KDB* = the angle of the normalized anomaly = $59° 43'$. The whole angle $CDB = 62° 10'$ [$= 59° 43' + 2° 27'$], being the excess over a semicircle [since $ABC = 242°10' = 62°10' + 180°$]. Angle $BEK = 7°$. In triangle *KDE*, therefore, the angles are given in degrees of which $180° = 2$ right angles. The ratio of the sides is also given: $DE = 91{,}856$ units, and $EK = 86{,}354$ units, of which the diameter of the circle circumscribing triangle $KDE = 100{,}000$. But in units of which $DE = 100{,}000$, $KE = 94{,}010$. It was shown above, however, that $DF = 8600$ units, and the whole line $DFG = 13{,}340$ units. In this given ratio, as was demonstrated [above in IV, 17], $EK = 56\ ^{42}/_{60}$ units, of which the earth's radius = 1 unit. It therefore follows that in the same units $DE = 60\ ^{18}/_{60}$, $DF = 5\ ^{11}/_{60}$, $DFG = 8\ ^{2}/_{60}$, and likewise the whole of *EDG*, if it were extended in a straight line = $68\ ^{1}/_{3}$ units [$60^p\ 18' + 8^p\ 2'$] = the greatest height of the half moon. Subtracting *DG* from *ED* [$60^p\ 18' - 8^p\ 2'$] leaves a remainder of $52\ ^{17}/_{60}$ as the half moon's smallest distance [from the earth]. So also the whole of *EDF*, the height occurring at full and new moon = $65\ ^{1}/_{2}$ units at its maximum [$60^p\ 18' + 5°\ 11' \cong 65°\ 30'$], and at its minimum = $55\ ^{8}/_{60}$ units, after *DF* has been subtracted [$60^p\ 18' - 5^p\ 11'$]. We should not be disturbed because the greatest distance of the full and new moon [from the earth] is thought to be $64\ ^{10}/_{60}$ units by others [IV, 16], especially by those who could become only partially familiar with the lunar parallaxes on account of the location of their residences. I have been permitted to understand them more completely by the moon's closer proximity to the horizon, near which its parallaxes reach their full value, as is clear. Yet I have found that the parallaxes vary by no more than 1′ on account of this difference.

THE DIAMETER OF THE MOON AND OF THE EARTH'S SHADOW AT THE PLACE WHERE THE MOON PASSES THROUGH IT — *Chapter 18*

Since the apparent diameters of the moon and of the shadow also vary with the moon's distance from the earth, a discussion of these topics too is important. To be sure, the diameters of the sun and moon are measured correctly by Hipparchus' dioptra. Nevertheless, this is done much more accurately in the case of the moon, it is believed, through some special lunar eclipses in which the moon is equally distant from its higher or lower apse. This is especially true if at those times the sun too is similarly situated, so that the circle of shadow through which the moon passes on both occasions is found equal, except that the darkened areas occupy unequal regions. Obviously, when the areas in shadow, and the lunar latitudes, are compared with each other, the difference shows how great an arc around the earth's center is subtended by the moon's diameter. When this is known, the radius of the shadow is also obtained quickly, as will be made clearer by an example.

Thus, suppose that at the middle of an earlier eclipse 3 digits or twelfths of the lunar diameter were darkened while the moon's latitude was 47′ 54″, whereas in a second eclipse 10 digits [were darkened] when the latitude was 29′ 37″. The difference between the darkened areas is 7 digits [= 10 − 3], and between the latitudes is 18′17″ [= 47′ 54″ − 29′ 37″], as compared with the proportion of 12 digits to 31′ 20″, subtending the diameter of the moon. In the middle of the first eclipse, therefore, the center of the moon clearly was outside the shadow by a quarter [the darkened area being 3 digits] of the diameter = 7′ 50″ of latitude [= 31′ 20″ ÷ 4]. If this figure is subtracted from the 47′ 54″ of the total latitude, the remainder = 40′ 4″ [= 47′ 54″ − 7′ 50″] = the radius of the shadow. Likewise, in the second eclipse the shadow occupied, in addition to the moon's latitude, $^1/_3$ of the lunar diameter [the darkened area being 10 digits = $^4/_{12}$ (= $^1/_3$) more than $^1/_2$] = 10′ 27″ [≅ 31′ 20″ ÷ 3]. To this add 29′ 37″, and the sum is again 40′ 4″ = the radius of the shadow. Ptolemy believes that when the sun is in conjunction or opposition with the moon at its greatest distance from the earth, the lunar diameter = 31 $^1/_3$′. He says that with Hipparchus' dioptra he found the sun's diameter to be the same, but the diameter of the shadow = 1° 21 $^1/_3$′. He thought that the ratio between these values = 13 : 5 = 2 $^3/_5$: 1 [*Syntaxis*, V, 14].

HOW TO DEMONSTRATE AT THE SAME TIME THE DISTANCES OF THE SUN AND MOON FROM THE EARTH, THEIR DIAMETERS, THE DIAMETER OF THE SHADOW WHERE THE MOON PASSES THROUGH IT, AND THE AXIS OF THE SHADOW *Chapter 19*

The sun too undergoes some parallax. Since this is slight, it is not easily perceived, except that the distances of the sun and moon from the earth, their diameters, the diameter of the shadow where the moon passes through it, and the axis of the shadow are mutually interrelated. Therefore these quantities disclose one another in analytical demonstrations. First, I shall review Ptolemy's conclusions about these quantities and his procedure for demonstrating them [*Syntaxis*, V, 15]. From this material I shall select what seems entirely correct.

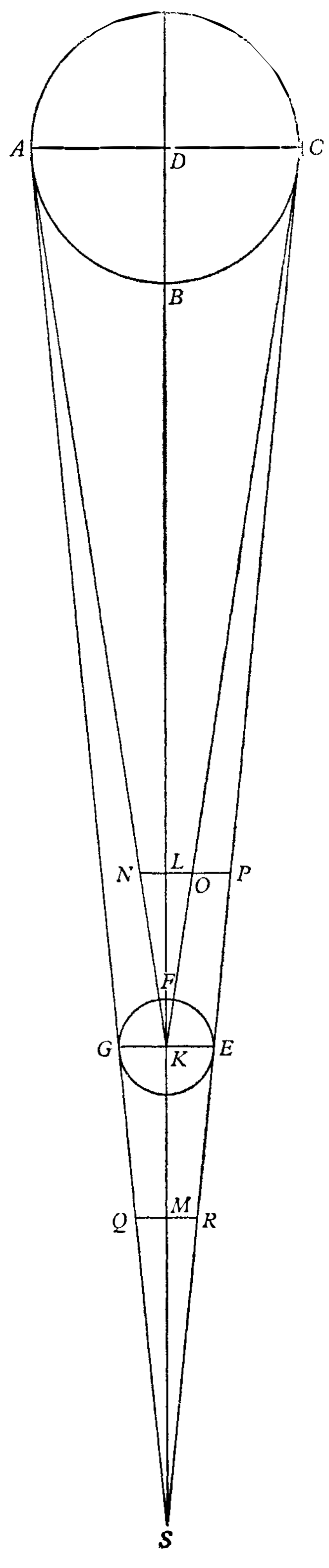
He assumes that the sun's apparent diameter = 31 $^1/_3$′, the value which he uses invariably. With it he equates the diameter of the full and new moon when it is at apogee. This, he says, is a distance of 64 10/60^p, with the earth's radius = 1^p. Hence he demonstrated the rest in the following way.

Let *ABC* be a circle of the solar globe through its center *D*. Let *EFG* be a circle of the terrestrial globe, at its greatest distance from the sun, through its own center *K*. Let *AG* and *CE* be straight lines tangent to both circles and, when they are extended, let them meet at *S*, the apex of the shadow. Draw the straight line *DKS* through the centers of the sun and earth. Also draw *AK* and *KC*. Join *AC* and *GE*, which should not differ at all from diameters on account of their enormous distance. On *DKS* take *LK* = *KM* at the distance of the full and new moon at apogee = 64 $^{10}/_{60}$^{p}, when *EK* = 1^p, in Ptolemy's opinion. Let *QMR* be the diameter of the shadow where the moon passes through it under these same conditions. Let NLO be the moon's diameter perpendicular to DK, and extend it as LOP.

The first problem is to find the ratio $DK : KE$. With 4 right angles $= 360°$, angle $NKO = 31\,{}^1/_3{}'$; half of it $= LKO = 15\,{}^2/_3{}'$. L is a right angle. Therefore, the angles of triangle LKO being given, the ratio of sides $KL : LO$ is given. As a length $LO = 17'\,33''$, when $LK = 64^p\,10'$ or $KE = 1^p$. Since $LO : MR = 5 : 13$, $MR = 45'\,38''$ in the same units. LOP and MR are parallel to KE at equal distances from it. Therefore $LOP + MR = 2KE$. Subtracting $MR + LO$ [$45'\,38'' + 17'\,33'' = 1^p\,3'\,11''$] from $2KE$ [$= 2^p$] leaves as a remainder $OP = 56'\,49''$. According to Euclid, VI, 2, $EC : PC = KC : OC = KD : LD = KE : OP = 60' : 56'\,49''$. Similarly LD is given $= 56'\,49''$, when the whole of $DLK = 1^p$. Therefore the remainder $KL = 3'\,11'$ [$= 1^p - 56'\,49''$]. But in units of which $KL = 64^p\,10'$ and $FK = 1^p$, the whole of $KD = 1210^p$. It has already been established that in such units $MR = 45'\,38''$. This makes clear the ratios $KE : MR$ [$60' : 45'38''$] and $KMS : MS$. Also in the whole of KMS, $KM = 14'\,22''$ [$= 60' - 45'38''$]. Alternately, in units of which $KM = 64^p\,10'$, the whole of $KMS = 268^p$ = the axis of the shadow. The foregoing is what Ptolemy did.

But after Ptolemy other astronomers found that the foregoing conclusions did not agree well enough with the phenomena, and reported other findings about these topics. Yet they admit that the greatest distance of the full and new moon from the earth $= 64^p\,10'$, and the apparent diameter of the sun at apogee $= 31\,{}^1/_3{}'$. They also concede that the shadow's diameter where the moon passes through it $= 13 : 5$ [in relation to the moon's diameter], as Ptolemy asserted. Nevertheless they deny that the moon's apparent diameter at that time is larger than $29\,{}^1/_2{}'$. Therefore they put the shadow's diameter at about $1°\,16\,{}^3/_4{}'$. Hence they believe it follows that the distance of the sun at apogee from the earth $= 1146^p$, and the shadow's axis $= 254^p$, where the earth's radius $= 1^p$. They designate [Al-Battani], the scientist from Raqqa, as the originator of these values, which nevertheless cannot be coordinated in any way.

With the intention of adjusting and correcting them, I put the apparent diameter of the sun at apogee $= 31'\,40''$, since it must now be somewhat bigger than before Ptolemy; [the apparent diameter] of the full or new moon when it is at its higher apse $= 30'$; the diameter of the shadow, where the moon passes through it, $= 80\,{}^3/_5{}'$, because the ratio between them is recognized to be slightly bigger than $5 : 13$, say, $150 : 403$ [$\cong 5 : 13\,{}^2/_5$]; the entire sun at apogee is not covered by the moon, unless the latter's distance from the earth is less than 62 earth-radii; and the greatest distance from the earth to the moon in conjunction with or opposition to the sun $= 65\,{}^1/_2$ earth-radii [IV, 17]. For when these values are assumed, they seem to be precisely in harmony not only with one another but also with other phenomena, and in agreement with the visible solar and lunar eclipses. Thus, in accordance with the foregoing demonstration, we shall have, in units and minutes whereof KE, the earth's radius, $= 1$ unit, $LO = 17'\,8''$; therefore $MR = 46'\,1''$ [$\cong 17'8'' \times 2.7$]; consequently $OP = 56'\,51''$ [$= 2^p - (17'8'' + 46'1'')$]; with $LK = 65\,{}^1/_2{}^p$, the whole of DLK = the distance from the earth to the sun at apogee $= 1179^p$; and KMS = the axis of the shadow $= 265^p$.



THE SIZE OF THESE THREE HEAVENLY BODIES, SUN, MOON, AND EARTH, AND A COMPARISON OF THEIR SIZES

Chapter 20

Consequently it is also clear that $KL = KD/18$, and $LO = DC/18$. But $18 \times LO \cong 5^p\,27'$, with $KE = 1^p$. Alternately, since $SK : KE = 265 : 1$, similarly the whole of $SKD : DC = 1444 : 5^p\,27'$, since these [sides] are [related to each other in the same] proportion. This will be the ratio of the diameters of the sun and earth. Spheres are to each other as the cubes of their diameters. Hence $(5^p\,27')^3 = 161\,{}^7/_8$, the factor by which the sun exceeds the terrestrial globe.

Furthermore, the moon's radius $= 17'\,9''$, whereof $KE = 1^p$. Therefore the ratio of the earth's diameter to the moon's diameter $= 7 : 2 = 3\,{}^1/_2 : 1$ [a convenient fraction for 3.498 : 1]. When this is raised to the third power, it shows that the earth is $42\,{}^7/_8$ times larger than the moon, and therefore the sun is 6937 times larger than the moon.

THE APPARENT DIAMETER AND PARALLAXES OF THE SUN

Chapter 21

The same magnitudes appear smaller when they are farther away than when they are closer. It therefore happens that the sun, moon, and earth's shadow vary with their different distances from the earth, no less than the parallaxes vary. All these variations are easily determined for any distance whatever on the basis of the foregoing results. This is clear, in the first place, in the case of the sun. For I have shown [III, 21] that the earth's greatest distance from it $= 10{,}322^p$, whereof the radius of the circle of the annual revolution $= 10{,}000^p$. The earth's closest approach [to the sun] $= 9678^p$ [$= 10{,}000 - 322$] in the other part of the diameter [of the circle of the annual revolution]. Therefore, with the higher apse $= 1179$ earth-radii [III, 19], the lower apse $= 1105$, and the mean apse $= 1142$. Dividing 1,000,000 by 1179, in the right triangle we shall have 848^p subtending the smallest angle $= 2'\,55''$ of the greatest parallax, which occurs near the horizon. Similarly, dividing 1,000,000 by $1105 =$ the least distance, we obtain 905^p, subtending an angle of $3'\,7'' =$ the largest parallax at the lower apse. But it has been shown [IV, 20] that the sun's diameter $= 5\,{}^{27}/_{60}$ earth-diameters, and at the higher apse appears $= 31'\,48''$. For, $1179 : 5\,{}^{27}/_{60} = 2{,}000{,}000 : 9245 =$ the diameter of the circle : the side subtending $31'\,48''$. Consequently at the least distance $=$ 1105 earth-radii, [the sun's apparent diameter] $= 33'\,54''$. The difference between these values [$33'\,54'' - 31'\,48''$] is therefore $2'\,6''$, but between the parallaxes only $12''$ [$3'\,7'' - 2'\,55''$]. Ptolemy [*Syntaxis*, V, 17] deemed both these differences negligible on account of their smallness, on the ground that the senses do not easily perceive $1'$ or $2'$, and such perception is even less feasible in the case of seconds. Therefore, if everywhere we put the sun's greatest parallax $= 3'$, we shall seem to have committed no error. But I shall take the sun's mean apparent diameters from its mean distances, or (as some astronomers do) from the sun's apparent hourly motion, which they think is to its diameter as $5 : 66 = 1 : 13\,{}^1/_5$. For, the hourly motion is nearly proportional to the sun's distance.

THE MOON'S VARYING APPARENT DIAMETER AND ITS PARALLAXES

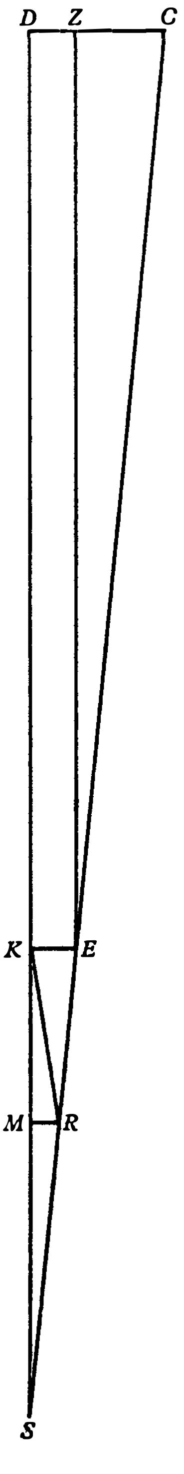
Chapter 22

A greater variation of both [apparent diameter and parallaxes] is evident in the moon as the nearest heavenly body. For, when it is new and full, its greatest distance from the earth = $65\,^1/_2$ earth-radii, and on the basis of the foregoing demonstrations [IV, 17], its least distance = $55\,^8/_{60}$. For the half moon, the greatest distance = $68\,^{21}/_{60}$, and the least distance = $52\,^{17}/_{60}$ earth-radii. Therefore, when we divide the radius of the [earth's] circumference by the distance earth-moon, at those four limits we shall obtain the parallaxes of the rising or setting moon: when it is most remote, 50′ 18″ for the half moon, and 52′ 24″ for the full and new moon; when these are at their nearest, 62′ 21″, and 65′ 45″ for the half moon at its nearest.

From these parallaxes the moon's apparent diameters also become clear. For, as has been shown [IV, 20], the ratio earth-diameter : moon-diameter = 7 : 2. Likewise, earth-radius : moon-diameter = 7 : 4, and this is also the ratio of the parallaxes to the moon's apparent diameters. For, the straight lines enclosing the angles of the greater parallaxes and of the apparent diameters at the same passage of the moon do not differ from one another at all. The angles are nearly proportional to the chords subtending them, nor is there any perceptible difference between them. This compact summary makes it clear that at the first limit of the parallaxes enumerated above, the moon's apparent diameter = $28\,^3/_4{}'$; at the second limit, about 30′; at the third limit, 35′ 38″; and at the last limit, 37′ 34″. This last value would have been nearly 1° according to the theory of Ptolemy and others, and the moon, with half [of its surface] shining at that time, would have to cast as much light on the earth as the full moon.

TO WHAT EXTENT DOES THE EARTH'S SHADOW VARY?

Chapter 23



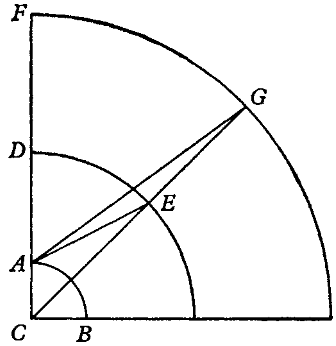
I also said above [IV, 19] that the ratio of the shadow's diameter to the moon's diameter = 403 : 150. Therefore, at full and new moon with the sun at apogee, the smallest shadow-diameter = 80′ 36″, the greatest = 95′ 44″, and the greatest difference = 15′ 8″ [=95′ 44″ − 80′ 36″]. Even when the moon passes through the same place, the earth's different distances from the sun also cause the earth's shadow to vary in the following way.

Again, as in the preceding diagram, draw straight line *DKS* through the centers of the sun and of the earth, as well as tangent *CES*. Join *DC* and *KE*. As has been shown, when distance *DK* = 1179 earth-radii, and *KM* = 62 earth-radii, *MR* = the radius of the shadow = $46\,^1/_{60}{}'$ of the earth-radius *KE; MKR*, made by joining *K* and *R* = the angle of the apparent [radius of the earth's shadow] = 42′ 32″; and *KMS* = the axis of the shadow = 265 earth-radii.

But when the earth is nearest to the sun, with *DK* = 1105 earth-radii, we shall compute the earth's shadow at the same [place of the] moon's passage as follows. Draw *EZ* parallel to *DK*. *CZ* : *ZE* = *EK* : *KS*. But *CZ* = $4\,^{27}/_{60}$ earth-radii, and *ZE* = 1105 earth-radii. For, *ZE* and the remainder *DZ* [= *CD* − *CZ* = $5^{27}/_{60} - 4^{27}/_{60}$ = 1] are equal to *DK* and *KE* [= 1], since *KZ* is a parallelogram.

Hence $KS = 248\ ^{19}/_{60}$ earth-radii. But $KM = 62$ earth-radii, and therefore the remainder $MS = 186\ ^{19}/_{60}$ earth-radii [$= 248^p\ 19' - 62^p$]. But since $SM : MR = SK : KE$, therefore $MR = 45\ ^1/_{60}{}'$ of an earth-radius, and MKR = the angle of the apparent [radius of the earth's shadow] $= 41'\ 35''$.

For this reason it happens that at the same place of the moon's crossing the approach and withdrawal of the sun and earth cause the shadow's diameter to vary at the most, with $KE = 1^p$, by $^1/_{60}{}'$, which is seen as $57''$, when $360° =$ 4 right angles. Furthermore, the ratio of the shadow's diameter to the moon's diameter in the first case [$46'\ 1''$] was greater, in the second case [$45'1''$] less than 13 : 5, which was a sort of mean value. Therefore we shall commit a negligible error if we use the same value everywhere, thereby saving work and following the opinion of the ancients.



TABULAR PRESENTATION OF THE INDIVIDUAL SOLAR AND LUNAR PARALLAXES IN THE CIRCLE WHICH PASSES THROUGH THE POLES OF THE HORIZON

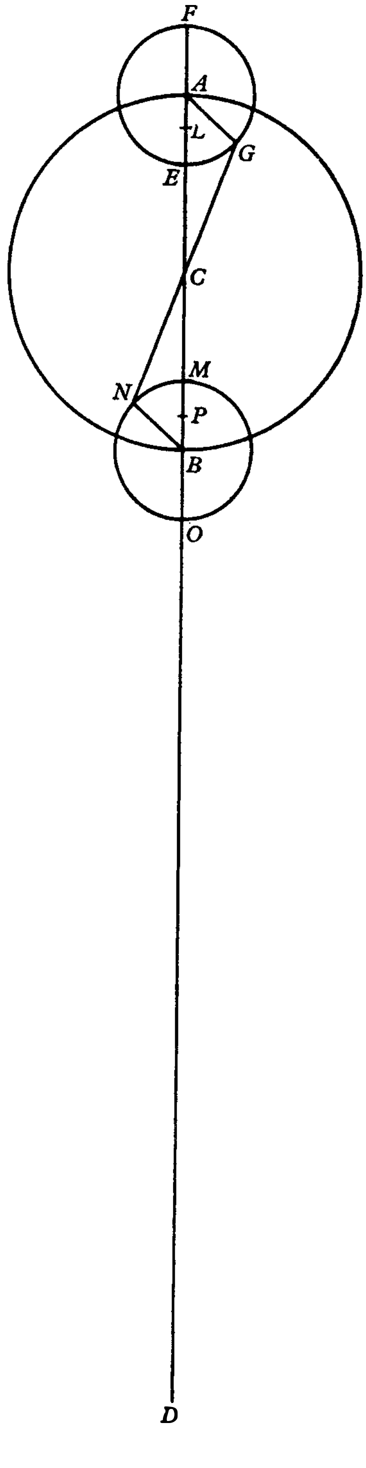
Chapter 24

Now there will be no uncertainty in ascertaining every single solar and lunar parallax too. Reproduce AB as [an arc of] the earth's circumference [passing] through [the earth's] center C and the point below the zenith. In the same plane let DE be the moon's circle; FG, the sun's circle; CDF, the line through the point below the zenith; and CEG, the line on which the true places of the sun and moon are taken. Draw AG and AE as the lines of sight to those places.

Then the solar parallax is indicated by angle AGC, and the lunar parallax by angle AEC. Moreover, the difference between the solar and lunar [parallaxes] is measured by angle GAE = the difference between angles AGC and AEC. Now let us take ACG as the angle to which we wish to compare those [other angles], and let ACG be, for example, 30°. According to the theorems on Plane Triangles, when we put line $CG = 1142^p$ whereof $AC = 1^p$, clearly angle AGC = the difference between the sun's true and apparent altitudes $= 1\ ^1/_2{}'$. But when angle $ACG = 60°$, $AGC = 2'\ 36''$. Similarly for the other [values of angle ACG, the solar parallaxes] will be obvious.

But in the case of the moon [we use] its four limits. For, suppose that we take angle DCE or arc $DE = 30°$, with $360° = 4$ right angles, when the moon is at its greatest distance from the earth, with $CE = 68^p\,21'$ whereof $CA = 1^p$, as I said [IV, 22]. Then we shall have triangle *ACE*, in which two sides *AC* and *CE* are given, as well as angle *ACE*. From this information we shall find *AEC* = the parallax angle $= 25'\,28''$. When $CE = 65\,{}^1/_2{}^p$, angle $AEC = 26'\,36''$. Similarly at the third limit, when $CE = 55^p\,8'$, the parallax angle $AEC = 31'\,42''$. Finally, at the [moon's] least distance [from the earth], when $CE = 52^p\,17'$, angle $AEC = 33'\,27''$. Moreover, when arc $DE = 60°$, the parallaxes will be, in the same order, first, $43'\,55''$; second, $45'\,51''$; third, $54\,{}^1/_2{}'$; and fourth, $57\,{}^1/_2{}'$.

I shall write down all these values in the order of the following Table. For greater convenience in use I shall extend it, like the others, to a series of 30 rows, but at intervals of 6°. These degrees are to be understood as twice the number of degrees reckoned from the zenith to a maximum of 90°. I have arranged the Table in 9 columns. The 1st and 2nd columns will contain the common numbers of the circle. In the 3rd column I shall put the solar parallaxes. After them come the lunar parallaxes [in columns 4–9]. The 4th column will show the differences by which the smallest parallaxes, occurring when the half moon is at apogee, are less than those in the following column, which occur at full and new moon. The 6th column will exhibit the parallaxes produced by the full or new moon at perigee. The minutes which follow [in the 7th column] are the differences by which the parallaxes of the half moon when it is nearest to us exceed those in their vicinity. Then the last two remaining columns are reserved for the proportional minutes, by which the parallaxes between those four limits can be computed. I shall explain these minutes also, first those near the apogee, and then those which fall between the first [two] limits [with the moon at apogee in the quadratures and in the syzygies, respectively]. The explanation proceeds as follows.



I say let circle *AB* be the moon's first epicycle, with center *C*. Taking *D* as the center of the earth, draw the straight line *DBCA*. With apogee *A* as center, describe the second epicycle *EFG*. Take arc $EG = 60°$. Join *AG* and *CG*. Straight line $CE = 5\,{}^{11}/_{60}$ earth-radii, as was shown above [IV, 17]. Moreover, $DC = 60\,{}^{18}/_{60}$ earth-radii, and $EF = 2\,{}^{51}/_{60}$ earth-radii. Therefore, in triangle *ACG*, side $GA = 1^p\,25'$, side $AC = 6^p\,36'$, and angle *CAG*, included between these sides, is also given. Hence, in accordance with the theorems on Plane Triangles, the third side $CG = 6^p\,7'$ in the same units. Consequently the whole of *DCG*, if formed into a straight line, or its equivalent $DCL = 66^p\,25'$ [$= 60^p\,18' + 6^p\,7'$]. But $DCE = 65\,{}^1/_2{}^p$ [$\cong 60^p 18' + 5^p 11'$]. Hence the remainder [$DCL - DCE$] $= EL \cong 55{}^1/_2{}'$ [$\cong 66^p\,25' - 65^p\,30'$]. Through this given ratio, when $DCE = 60^p$, $EF = 2^p\,37'$, and $EL = 46'$ in the same units. Accordingly, on the basis of $EF = 60'$, as the excess $EL \cong 18'$. I shall enter this value in the 8th column of the Table opposite 60° [in the 1st column].

I shall make a similar demonstration for the perigee *B*. With it as a center, reproduce the second epicycle *MNO*, with angle $MBN = 60°$. As before, triangle *BCN* will have its sides and angles given. Likewise the excess $MP \cong 55\,{}^1/_2{}'$, with an earth-radius $= 1^p$. In those units $DBM = 55^p\,8'$. However, if $DBM = 60^p$, in those units $MBO = 3^p\,7'$, and the excess $MP = 55'$. But $3^p\,7' : 55' \cong 60 : 18$, and [we have] the same results as before [in the case of the apogee], though they are a few seconds apart. I shall follow this procedure also in the other cases, with which I shall fill up the 8th column in the Table. But if, instead of these values, we use those enumerated in the [column of proportional minutes in the]

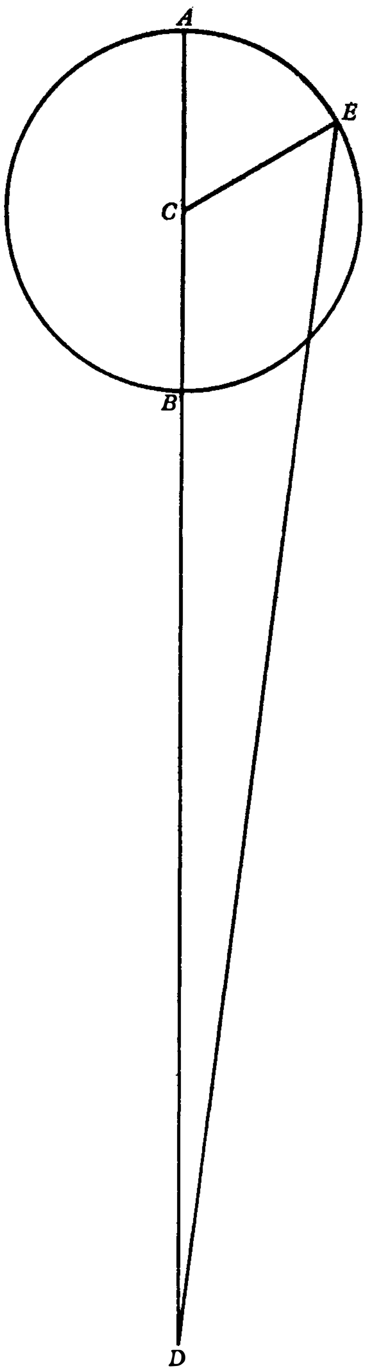


Table of Prosthaphaereses [after IV, 11], we shall not be committing any error, since they are almost identical and very small quantities are involved.

Remaining [to be considered] are the proportional minutes for the middle limits, that is, between the second and third. Now let the full and new moon describe the first epicycle *AB*, with center *C*. Take *D* as the center of the earth, and draw straight line *DBCA*. Starting from apogee *A*, take an arc, for example, $AE = 60°$. Join *DE* and *CE*. We shall have triangle *DCE*, of which two sides are given: $CD = 60^p\, 19'$, and $CE = 5^p\, 11'$. So is interior angle $DCE = 180° - ACE$. According to the theorems on Triangles, $DE = 63^p\, 4'$. But the whole of $DBA = 65\,{}^1/_2{}^p$, exceeding *ED* by $2^p\, 27'$ [$\cong 65^p\, 30' - 63^p\, 4'$]. But [$2 \times CE =$] $AB = 10^p\, 22' : 2^p\, 27' = 60 : 14$, which may be entered in the Table [in the 9th column] opposite 60°. With this as an example I have completed what was left, and I have finished the Table, which follows. I have added another Table of the Radii of the Sun, Moon, and Earth's Shadow in order that they may be available as far as possible.

TABLE OF SOLAR AND LUNAR PARALLAXES

Common Numbers		Solar Parallaxes		Difference to be Subtracted from the Lunar [Parallax at the] Second Limit [in Order to Obtain the Parallax at] the First Limit		Lunar Parallax at the Second Limit		Lunar Parallax at the Third Limit		Difference to be Added to the Lunar Parallax at the Third Limit in Order to Obtain the Parallax at the Fourth Limit		Proportional Minutes of the Smaller Epicycle	Proportional Minutes of the Larger Epicycle
°	°	′	″	′	″	′	″	′	″	′	″		
6	354	0	10	0	7	2	46	3	18	0	12	0	0
12	348	0	19	0	14	5	33	6	36	0	23	1	0
18	342	0	29	0	21	8	19	9	53	0	34	3	1
24	336	0	38	0	28	11	4	13	10	0	45	4	2
30	330	0	47	0	35	13	49	16	26	0	56	5	3
36	324	0	56	0	42	16	32	19	40	1	6	7	5
42	318	1	5	0	48	19	5	22	47	1	16	10	7
48	312	1	13	0	55	21	39	25	47	1	26	12	9
54	306	1	22	1	1	24	9	28	49	1	35	15	12
60	300	1	31	1	8	26	36	31	42	1	45	18	14
66	294	1	39	1	14	28	57	34	31	1	54	21	17
72	288	1	46	1	19	31	14	37	14	2	3	24	20
78	282	1	53	1	24	33	25	39	50	2	11	27	23
84	276	2	0	1	29	35	31	42	19	2	19	30	26
90	270	2	7	1	34	37	31	44	40	2	26	34	29
96	264	2	13	1	39	39	24	46	54	2	33	37	32
102	258	2	20	1	44	41	10	49	0	2	40	39	35
108	252	2	26	1	48	42	50	50	59	2	46	42	38
114	246	2	31	1	52	44	24	52	49	2	53	45	41
120	240	2	36	1	56	45	51	54	30	3	0	47	44
126	234	2	40	2	0	47	8	56	2	3	6	49	47
132	228	2	44	2	2	48	15	57	23	3	11	51	49
138	222	2	49	2	3	49	15	58	36	3	14	53	52
144	216	2	52	2	4	50	10	59	39	3	17	55	54
150	210	2	54	2	4	50	55	60	31	3	20	57	56
156	204	2	56	2	5	51	29	61	12	3	22	58	57
162	198	2	58	2	5	51	56	61	47	3	23	59	58
168	192	2	59	2	6	52	13	62	9	3	23	59	59
174	186	3	0	2	6	52	22	62	19	3	24	60	60
180	180	3	0	2	6	52	24	62	21	3	24	60	60

TABLE OF THE RADII OF THE SUN, MOON, AND [EARTH'S] SHADOW

Common Numbers		Sun's Radius		Moon's Radius		Shadow's Radius		Shadow's Variation
°	°	′	″	′	″	′	″	Minutes
6	354	15	50	15	0	40	18	0
12	348	15	50	15	1	40	21	0
18	342	15	51	15	3	40	26	1
24	336	15	52	15	6	40	34	2
30	330	15	53	15	9	40	42	3
36	324	15	55	15	14	40	56	4
42	318	15	57	15	19	41	10	6
48	312	16	0	15	25	41	26	9
54	306	16	3	15	32	41	44	11
60	300	16	6	15	39	42	2	14
66	294	16	9	15	47	42	24	16
72	288	16	12	15	56	42	40	19
78	282	16	15	16	5	43	13	22
84	276	16	19	16	13	43	34	25
90	270	16	22	16	22	43	58	27
96	264	16	26	16	30	44	20	31
102	258	16	29	16	39	44	44	33
108	252	16	32	16	47	45	6	36
114	246	16	36	16	55	45	20	39
120	240	16	39	17	4	45	52	42
126	234	16	42	17	12	46	13	45
132	228	16	45	17	19	46	32	47
138	222	16	48	17	26	46	51	49
144	216	16	50	17	32	47	7	51
150	210	16	53	17	38	47	23	53
156	204	16	54	17	41	47	31	54
162	198	16	55	17	44	47	39	55
168	192	16	56	17	46	47	44	56
174	186	16	57	17	48	47	49	56
180	180	16	57	17	49	47	52	57

COMPUTING THE SOLAR AND LUNAR PARALLAX

Chapter 25

I shall also briefly explain the method of computing the solar and lunar parallaxes by means of the Table. With the sun's distance from the zenith or twice the moon's distance therefrom, we take the corresponding parallaxes in the Table: the single entry in the case of the sun, but in the case of the moon the parallaxes at its four limits. Also, with twice the moon's motion or distance from the sun, we find the proportional minutes in the first [column of proportional minutes, that is, the 8th column]. With these proportional minutes we obtain, as proportional parts of 60, the excess for both the first and the last limits. We always subtract [the first of these proportional parts of 60] from the next parallax in the sequence [that is, the parallax at the second limit]; and we always add the second [of these proportional parts of 60] to the parallax at the next to the last limit. This [procedure] gives us a pair of lunar parallaxes, reduced to the apogee and perigee, and either increased or diminished by the smaller epicycle. Then with the lunar anomaly we take the proportional minutes in the last column. With these proportional minutes we next obtain the proportional part of the difference between the parallaxes just found. This proportional part [of 60] we always add to the first of the reduced parallaxes, that at the apogee. The result is the lunar parallax sought for a [given] place and time, as in the [following] example.

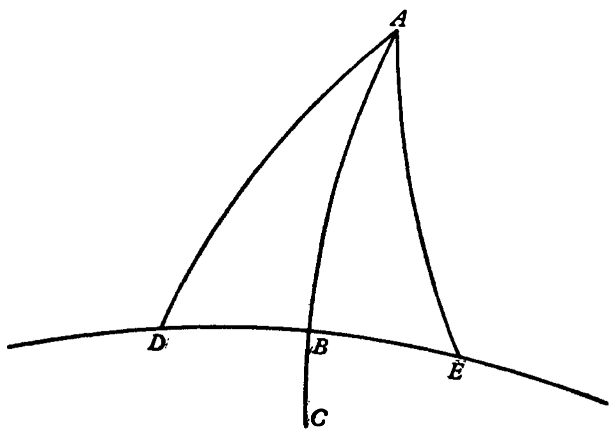
Let the moon's distance from the zenith = 54°; the moon's mean motion = 15°; and its normalized motion in anomaly = 100°. I wish to find the lunar parallax by means of the Table. I double the degrees of the [zenith] distance, making them 108°. Corresponding to 108° in the Table as the excess at the second limit over the first limit is 1′ 48″; the parallax at the second limit = 42′ 50″; the parallax at the third limit = 50′ 59″; the excess of the parallax at the fourth limit over the third = 2′ 46″. I note these values one by one. The moon's motion, when doubled, = 30°. For this figure I find 5′ in the first column of proportional minutes. With these 5′ I take the proportional part of 60 = 9″ of the excess [at the second limit] over the first [$1' 48'' \times {}^{5}/_{60} = 9''$]. I subtract these 9″ from 42′ 50″, the parallax [at the second limit]. The remainder is 42′ 41″. Similarly, of the second excess = 2′ 46″, the proportional part = 14″ [$2' 46'' \times {}^{1}/_{12} \cong 14''$]. These 14″ are added to 50′ 59″ = the parallax at the third limit, making the sum = 51′ 13″. The difference between these parallaxes = 8′ 32″ [= 51′ 13″– 42′ 41″]. After this, with the [100] degrees of the normalized anomaly, in the last column I take the proportional minutes = 34. With these I find the proportional part of the 8′ 32″ difference = 4′ 50″ [$= 8' 32'' \times {}^{34}/_{60}$]. When these 4′ 50″ are added to the first corrected parallax [42′ 41″], the sum is 47′ 31″. This is the required parallax of the moon in the vertical circle.

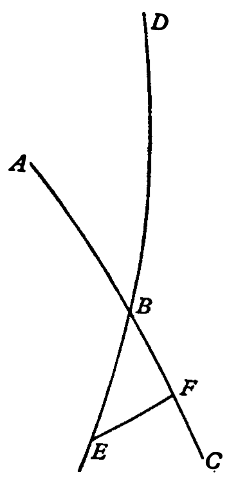
However, any lunar parallaxes differ so slightly from those which occur at full and new moon that it would seem sufficient if everywhere we kept between the middle limits. These we especially need for the prediction of eclipses. The others do not merit so extensive an investigation, which will perhaps be thought to serve curiosity rather than usefulness.

HOW THE PARALLAXES IN LONGITUDE AND LATITUDE ARE SEPARATED FROM EACH OTHER

Chapter 26

The parallax is readily separated into longitude and latitude; that is, [the distance] between the sun and moon [is measured] by arcs and angles of the ecliptic and of the vertical circle, which intersect each other. For when the vertical circle meets the ecliptic at right angles, obviously it produces no parallax in longitude. On the contrary, the entire parallax passes into the latitude, since the circles of latitude and altitude are the same. But, on the other hand, when the ecliptic happens to intersect the horizon at right angles and becomes identical with the circle of altitude, if the moon at that time lacks latitude, it undergoes only a parallax in longitude. But if it acquires any latitude, it does not escape having some parallax in longitude. Thus let *ABC* be the ecliptic, intersecting the horizon at right angles. Let *A* be the pole of the horizon. Then *ABC* will be identical with the vertical circle of the moon, which has no latitude. If its place is *B*, its entire parallax *BC* will be longitudinal.



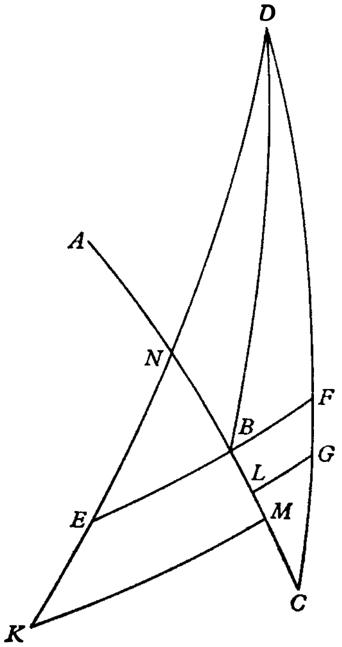


But suppose that the moon also has a latitude. Through the poles of the ecliptic draw circle *DBE*, and take *DB* or *BE* = the moon's latitude. Obviously, neither side *AD* nor side *AE* will be equal to *AB*. Nor will *D* or *E* be a right angle, since circles *DA* and *AE* do not pass through the poles of *DBE*. The parallax will participate somewhat in latitude, to a greater extent the nearer the moon is to the zenith. For while *DE*, the base of triangle *ADE*, remains constant, the shorter are sides *AD* and *AE*, the more acute are the angles made by them with the base. These angles become more like right angles, the farther removed the moon is from the zenith.

Now let *DBE*, the moon's vertical circle, intersect the ecliptic *ABC* obliquely. Let the moon have no latitude, as when it is at *B*, the intersection with the ecliptic. Let *BE* be the parallax in the vertical circle. Draw arc *EF* in the circle passing through the poles of *ABC*. Then in triangle *BEF*, angle *EBF* is given (as was shown above); *F* is a right angle; and side *BE* also is given. In accordance with the theorems on Spherical Triangles, *BF* and *FE*, the remaining sides, are given, corresponding to the parallax *BE*, the latitude being *FE*, and the longitude being

BF. However, on account of their small size *BE, EF*, and *FB* differ slightly and imperceptibly from straight lines. Therefore if we treat the right triangle as rectilinear, the computation will thereby become easy, and we shall commit no error.

The calculation is more difficult when the moon has some latitude. Reproduce the ecliptic *ABC*, intersected obliquely by *DB*, the circle passing through the poles of the horizon. Let *B* be the moon's place in longitude. Let its latitude be *BF* to the north or *BE* to the south. From the zenith *D* let fall on the moon *DEK* and *DFC* as vertical circles, on which are the parallaxes *EK* and *FG*. For, the moon's true places in longitude and latitude will be points *E* and *F*. But it will be seen at *K* and *G*, from which draw arcs *KM* and *LG* perpendicular to the ecliptic *ABC*. The moon's longitude and latitude are known, as well as the latitude of the region. Therefore, in triangle *DEB* two sides are known, *DB* and *BE*, as well as *ABD*, the angle of intersection [of the ecliptic and the vertical circle]. Adding *ABD* to the right angle [*ABE*] gives the whole angle *DBE*. Consequently the remaining side *DE* will be given, as well as angle *DEB*.



Similarly in triangle *DBF* two sides, *DB* and *BF*, are given, as well as angle *DBF*, which is the remainder when angle *ABD* is subtracted from the right angle [*ABF*]. Then *DF* also will be given, together with angle *DFB*. Therefore the parallaxes *EK* and *FG* of both arcs *DE* and *DF* are given through the Table. So is the moon's true distance *DE* or *DF* from the zenith, and likewise the apparent distance *DEK* or *DFG*.

But *DE* intersects the ecliptic at point *N*. In triangle *EBN, NBE* is a right angle; angle *NEB* is given; and so is the base *BE*. The remaining angle *BNE* will be known, as well as the remaining sides *BN* and *NE*. Similarly in the whole triangle *NKM*, from the given angles *M* and *N* and the whole side *KEN*, the base *KM* will be known. This is the moon's apparent southern latitude. Its excess over *EB* is the parallax in latitude. The remaining side *NBM* is given. When *NB* is subtracted from *NBM*, the remainder *BM* is the parallax in longitude.

Similarly in the northern triangle *BFC, B* is a right angle, while side *BF* and angle *BFC* are given. Therefore the remaining sides *BLC* and *FGC* are given, as well as the remaining angle *C*. Subtracting *FG* from *FGC* leaves *GC* as a side given in triangle *GLC*, in which *CLG* is a right angle, and angle *LCG* is given. Consequently the remaining sides *GL* and *LC* are given. So is the remainder when [*LC* is subtracted] from *BC*; it is *BL*, the parallax in longitude. Also given is the apparent latitude *GL*, whose parallax is the excess of the true latitude *BF* [over *GL*].

However (as you see) this computation, which is expended on very small magnitudes, costs more labor than it bears fruit. For it will be enough to use angle *ABD* for *DCB*, and *DBF* for *DEB*, and simply (as before) the mean arc *DB* always for arcs *DE* and *EF*, ignoring the lunar latitude. Nor will any error be apparent on this account, especially in the regions of the [earth's] northern side. On the other hand, in the extreme southern areas, when *B* touches the zenith at the maximum [lunar] latitude of 5° and the moon is nearest to the earth, the difference is about 6′. But during eclipses when the moon is in conjunction with the sun and its latitude cannot exceed $1\,{}^1/_2{}^\circ$, the difference can be only $1\,{}^3/_4{}'$. These considerations therefore make it clear that in the ecliptic's eastern quadrant the parallax in longitude is always added to the moon's true place, and in the other quadrant always subtracted from it, in order to obtain the moon's apparent longitude. Its apparent latitude is acquired through the parallax in latitude. For if they are on the same side [of the ecliptic], they are added together. But if they are on opposite [sides of the ecliptic], the smaller is subtracted from the larger, and the remainder is the apparent latitude on the same side as the larger.

CONFIRMATION OF THE ASSERTIONS ABOUT THE LUNAR PARALLAXES *Chapter 27*

The lunar parallaxes, as set forth above [IV, 22, 24–26], are in agreement with the phenomena, as I can assert on the basis of many other observations, such as the one I made in Bologna on 9 March 1497 C.E. after sunset. I watched the moon about to occult [Aldebaran,] the bright star in the Hyades which the Romans call Palilicium. After waiting, I saw the star touch the dark side of the lunar globe, with its light extinguished between the moon's horns at the end of the 5th hour of the night [= 11 P. M.]. It was closer to the southern horn by about $^{1}/_{3}$ of the moon's width or diameter. It was computed to be at 2° 52′ within the Twins and at 5 $^{1}/_{6}$° in southern latitude. Obviously, therefore, the center of the moon apparently was half of its diameter west of the star. Consequently, its apparent place was 2° 36′ [within the Twins = 2° 52′ $-^{1}/_{2}$ (32′)] in longitude, and about 5° 6′ in latitude. Accordingly, from the beginning of the Christian era there were 1497 Egyptian years, 76 days, plus 23 hours at Bologna. However, at Cracow, which lies nearly 9° farther east, the additional time would be 23 hours, 36 minutes, plus 4 minutes added for uniform time, since the sun was at 28 $^{1}/_{2}$° within the Fishes. The moon's uniform distance from the sun, then, was 74°; its normalized anomaly, 111° 10′; the moon's true place, 3° 24′ within the Twins; the southern latitude, 4° 35′; and the true motion in latitude, 203° 41′. At that time, moreover, at Bologna 26° within the Scorpion was rising at an angle of 59 $^{1}/_{2}$°; the moon was 84° from the zenith; the angle of intersection between the vertical circle and the ecliptic was about 29°; the lunar parallax in longitude, 51′, and in latitude, 30′. These values agree so thoroughly with the observation that nobody need doubt the correctness of my hypotheses and the statements based on them.

THE MEAN CONJUNCTIONS AND OPPOSITIONS OF THE SUN AND MOON *Chapter 28*

The statements made above about the motion of the moon and sun point to the method of investigating their conjunctions and oppositions. For any time close to when we think an opposition or conjunction will occur, we will look up the moon's uniform motion. If we find that it has just completed a circle, we know that there is a conjunction; if a semicircle, the moon is full [at opposition]. But since this [precision] is seldom encountered, we must examine the distance between the two bodies. When we divide this distance by the moon's daily motion, we will know the quantity of time since or until the occurrence of a syzygy, according as the motion was in excess or fell short. For this time, then, we will look up the motions and places, by which we will compute the true new and full moons, and distinguish the conjunctions at which eclipses occur from the others, in the manner indicated below [IV, 30]. Once we have established these phases, we may extend them to any other months and continue them for several years by means of a 12-month Table. This contains the [partial] times, the uniform motions of the sun and moon in anomaly, and of the moon in latitude, each value of which is linked with the individual uniform values previously found. But with regard to the solar anomaly, in order that we may have it at once, I shall appropriately record it in its normalized form. For, its nonuniformity will not be perceived in a single year, nor in several years, on account of the slowness of its origin, that is, of its higher apse.

TABLE OF THE CONJUNCTION AND OPPOSITION OF THE SUN AND MOON

Months	Partial Times				Moon's Motion in Anomaly				Moon's Motion in Latitude			
	Days	Day-Minutes	Day-Seconds	Sixtieths of Day-Seconds	60°	°	′	″	60°	°	′	″
1	29	31	50	9	0	25	49	0	0	30	40	14
2	59	3	40	18	0	51	38	0	1	1	20	28
3	88	35	30	27	1	17	27	1	1	32	0	42
4	118	7	20	36	1	43	16	1	2	2	40	56
5	147	39	10	45	2	9	5	2	2	33	21	10
6	177	11	0	54	2	34	54	2	3	4	1	24
7	206	42	51	3	3	0	43	2	3	34	41	38
8	236	14	41	12	3	26	32	3	4	5	21	52
9	265	46	31	21	3	52	21	3	4	36	2	6
10	295	18	21	30	4	18	10	3	5	6	42	20
11	324	50	11	39	4	43	59	4	5	37	22	34
12	354	22	1	48	5	9	48	4	0	8	2	48
FOR THE HALF-MONTH BETWEEN FULL AND NEW MOON												
	14	45	55	4½	3	12	54	30	3	15	20	7

SUN'S MOTION IN ANOMALY

Months	60°	°	′	″	Months	60°	°	′	″
1	0	29	6	18	7	3	23	44	7
2	0	58	12	36	8	3	52	50	25
3	1	27	18	54	9	4	21	56	43
4	1	56	25	12	10	4	51	3	1
5	2	25	31	31	11	5	20	9	20
6	2	54	37	49	12	5	49	15	38
FOR THE HALF-MONTH									
					½	0	14	33	9

INVESTIGATING THE TRUE CONJUNCTIONS AND OPPOSITIONS OF THE SUN AND MOON *Chapter 29*

After obtaining (by the aforesaid method) the time of the mean conjunction or opposition of these bodies as well as their motions, for the purpose of finding their true [syzygies] we must have the true distance by which they are west or east of each other. For if the moon is west of the sun in a [mean] conjunction or opposition, obviously a true [syzygy] will occur. If the sun [is west of the moon], the true [syzygy] for which we are looking has already happened. These [sequences] are made clear by the prosthaphaereses of both bodies. For if their prosthaphaereses are zero or equal and in the same sense, that is, both additive or both subtractive, the true conjunctions or oppositions obviously coincide with the mean [syzygies] at the same instant. But if the prosthaphaereses are unequal [in the same sense], the difference [between the prosthaphaereses] indicates the distance between the bodies. The body having the greater additive or subtractive [prosthaphaeresis] is west or east [of the other body]. But when the prosthaphaereses are in opposite senses, the body whose prosthaphaeresis is subtractive will be that much farther west, since the sum of the prosthaphaereses gives the distance between the bodies. With regard to this distance, we will consider in how many whole hours it can be traversed by the moon (taking 2 hours for each degree of distance).

Thus, if the distance between the bodies is about 6°, we will assume 12 hours for those degrees. Then for this time interval as thus determined, we will look for the moon's true distance from the sun. We will find this easily when we know that the moon's mean motion = $1° 1'$ in 2 hours, while its hourly true motion in anomaly around full and new moon $\cong 50'$. In 6 hours the uniform motion amounts to $3° 3'$ [$= 3 \times 1° 1'$], and the true motion in anomaly to $5°$ [$= 6 \times 50'$]. With these figures, in the Table of the Lunar Prosthaphaereses [after IV, 11] we will look up the difference between the prosthaphaereses. This difference is added to the mean motion if the anomaly is in the lower part of the circle; if it is in the upper part [of the circle], the difference will be subtracted. The sum or remainder is the moon's true motion in the assumed hours. This motion is sufficient if it is equal to the previously determined distance. Otherwise this distance, multiplied by the estimated number of hours, is divided by this motion; or we divide the distance, as it is, by what we have obtained as the true hourly motion. The quotient will be the true time difference in hours and minutes between the mean and true conjunction or opposition. We shall add this difference to the time of the mean conjunction or opposition, if the moon is west of the sun or of the place diametrically opposite the sun. If the moon is east [of these places], we will subtract this difference. Then we will have the time of the true conjunction or opposition. I admit, however, that the sun's nonuniformity also adds or subtracts something. But this quantity may rightly be ignored, since it cannot amount to $1'$ over the entire time, even [with the two bodies during syzygy] at their greatest distance, which surpasses 7°. This method of determining the lunations is more reliable. For, those who rely exclusively on the moon's hourly motion, which they call the "hourly surplus", are sometimes mistaken and are often compelled to repeat their computation, since [the motion of] the moon changes even from hour to hour and does not remain constant. Therefore, for the time of a true conjunction or opposition we shall establish the true motion in latitude in order to obtain the moon's latitude, and also the sun's true distance from the vernal equinox, that is, in the [zodiacal] signs, from which the moon's place is acquired, as being the same or the [diametrically] opposite.

In this way the mean and uniform time is known for the meridian of Cracow, and we reduce it to apparent time by the method explained above. But if we wish to determine these phenomena for some place other than Cracow, we consider its longitude. For each degree of that longitude we take 4 minutes of an hour, and 4 seconds of an hour for each minute of longitude. We add these intervals to Cracow time, if the other place is farther east; if it is farther west, we subtract the intervals. The remainder or sum will be the time of the [true] conjunction or opposition of the sun and moon.

HOW CONJUNCTIONS AND OPPOSITIONS OF THE SUN AND MOON AT WHICH ECLIPSES OCCUR MAY BE DISTINGUISHED FROM OTHERS *Chapter 30*

Whether or not eclipses occur [in syzygies] is easily decided in the case of the moon. For if its latitude is less than half [the sum] of the diameters of the moon and shadow, the moon undergoes an eclipse; but if its latitude is greater [than half the sum of those diameters], it will not be eclipsed.

The case of the sun, however, is more than enough troublesome, since it involves both [the solar and lunar] parallaxes, by which in general an apparent conjunction differs from the true conjunction. We therefore investigate the difference in longitude between the sun and the moon at the time of the true conjunction. Likewise, at 1 hour before the true conjunction in the eastern quadrant of the ecliptic, or in the western quadrant of the ecliptic at 1 hour after the true conjunction, we look for the moon's apparent longitudinal distance from the sun, in order to find out apparently how far the moon moves away from the sun in an hour. Dividing that difference in longitude by this hourly motion, we obtain the difference in time between the true and apparent conjunction. This difference in time is subtracted from the time of the true conjunction in the eastern part of the ecliptic, or it is added in the western part (since in the former case the apparent conjunction precedes, but in the latter case follows, the true conjunction). The result will be the desired time of the apparent conjunction. Then for this time we will compute the moon's apparent [distance in] latitude from the sun, or the distance between the centers of the sun and moon [at the time] of the apparent conjunction, after the solar parallax has been subtracted. If this latitude is greater than half [the sum] of the diameters of the sun and moon, the sun will not undergo an eclipse; but it will, if this latitude is less [than half the sum of those diameters]. These conclusions make it clear that if the moon at the time of a true conjunction has no parallax in longitude, the true and apparent conjunctions will coincide. This happens at about 90° of the ecliptic, measured from the east or from the west.

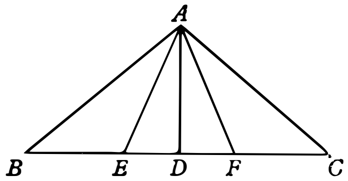
THE SIZE OF A SOLAR AND LUNAR ECLIPSE *Chapter 31*

After we learn that the sun or moon will be eclipsed, we will also easily know how great the eclipse will be. In the case of the sun [we use] the apparent [difference in] latitude between the sun and the moon at the time of the apparent conjunction. For if we subtract this latitude from half [the sum] of the diameters of the sun and moon, the remainder is the eclipsed portion of the sun, as measured along its diameter. When we multiply this remainder by 12, and divide the product by the sun's diameter, we will have the number of eclipsed digits in the sun. But if no latitude intervenes between the sun and moon, the entire sun will be eclipsed, or as much of it as the moon can cover.

In the case of a lunar eclipse [we proceed] in nearly the same way, except that instead of the apparent latitude we use the simple latitude. When this is subtracted from half [the sum] of the diameters of the moon and shadow, the remainder is the eclipsed portion of the moon, provided that the moon's latitude is not less than half [the sum] of [these] diameters by a lunar diameter. For then [if the moon's latitude is a lunar diameter less than half this sum] the entire moon will be eclipsed. Moreover, the smaller latitude will also somewhat prolong the time [spent by the moon] in the shadows. This time will be at its maximum when there is no latitude, as is entirely obvious, I believe, to those who consider the matter. In a partial lunar eclipse, then, when we multiply the eclipsed portion by twelve, and divide the product by the moon's diameter, we shall have the number of the eclipsed digits, exactly as was explained in the case of the sun.

PREDICTING HOW LONG AN ECLIPSE WILL LAST *Chapter 32*

It remains to see how long an eclipse will last. In this connection, it should be noted, we treat the arcs lying between the sun, moon, and shadow as straight lines on account of their small size, which makes them seem no different from straight lines.

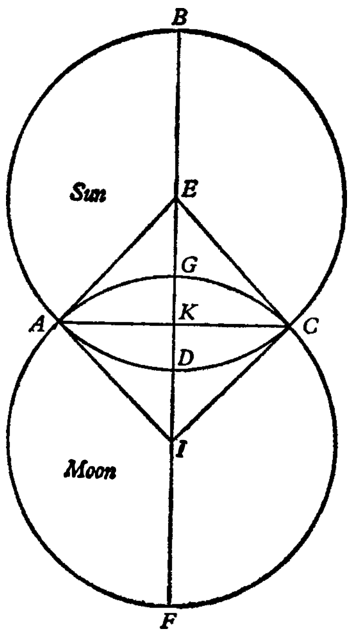


Thus, take the center of the sun or shadow in point *A*, and line *BC* as the passage of the moon's globe. Let *B* be its center as it touches the sun or shadow at the beginning of the contact, and *C* [its center] at the end of its emergence. Join *AB* and *AC*. Drop *AD* perpendicular to *BC*. When the moon's center is at *D*, obviously that will be the middle of the eclipse. For, *AD* is shorter than the other lines descending from *A* [to *BC*]. $BD = DC$, since $AB = AC$, each of which consists, in a solar eclipse, of half [the sum] of the diameters of the sun and moon, and of the moon and shadow in a lunar eclipse. *AD* is the true or apparent latitude of the moon at mid-eclipse. $(AB)^2 - (AD)^2 = (BD)^2$. Hence the length of *BD* will be given. When we divide this length by the true hourly motion of the moon in a lunar eclipse, or by the apparent [hourly motion of the moon] in a solar eclipse, we will have the time of half the duration [of the eclipse].

The moon, however, often tarries in the middle of the shadow. This happens when half the sum of the diameters of the moon and shadow exceeds the moon's latitude by more than its diameter, as I said [IV, 31]. Thus, assume *E* as the moon's center at the beginning of total immersion, when the moon contacts the shadow's circumference from within, and F [as the moon's center] at its second contact [with the shadow's circumference from within] where the moon first emerges [from the shadow]. Join AE and AF. Then, in the same way as before, ED and DF will clearly be half the time spent in the shadow. For, AD is known to be the latitude of the moon, and AE or AF the excess of half the shadow's diameter over half the moon's diameter. Therefore ED or DF will be determined. When either is once more divided by the true hourly motion of the moon, we will have half the time spent [in the shadow], which we were looking for.

Yet here it should be noticed that as the moon moves on its own circle, it does not tick off the degrees of longitude on the ecliptic exactly equal with the degrees on its own circle (as measured by the circles passing through the poles of the ecliptic). Nevertheless the difference is quite minute. At the full distance of 12° from the intersection with the ecliptic, close to the outermost limit of solar and lunar eclipses, the arcs of those circles do not differ from each other by $2' = {}^1/_{15}$ of an hour. For this reason I often use one of them instead of the other, as though they were identical. I likewise also use the same lunar latitude at the limits of an eclipse as at mid-eclipse, although the moon's latitude is always increasing or diminishing, and therefore the zones of immersion and emersion are not absolutely equal. On the other hand, the difference between them is so slight that it would seem to be a useless waste of time to investigate these details with greater precision. In the foregoing way the times, durations, and sizes of eclipses are explained by reference to the diameters.

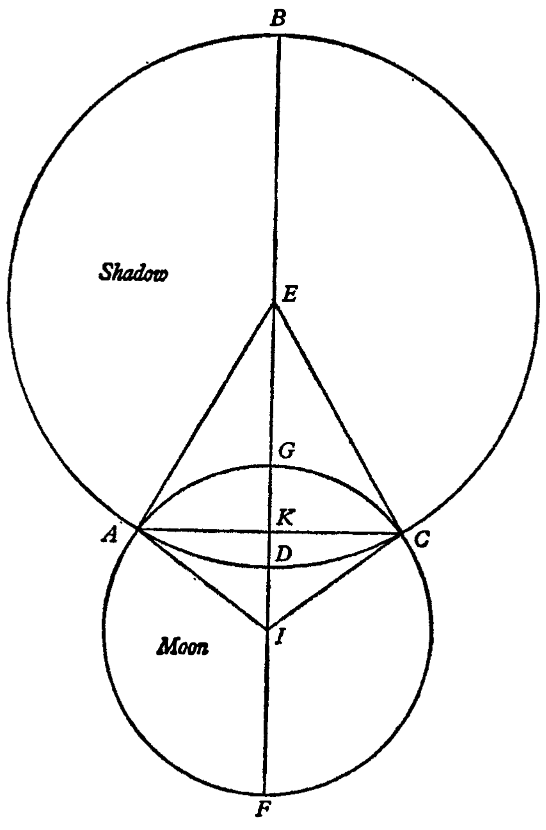
But in the opinion of many astronomers, the portions in eclipse should be determined by reference to surfaces, not diameters, since surfaces are eclipsed, not lines. Accordingly let the circle of the sun or shadow be $ABCD$, with its center at E. Let the moon's circle be $AFCG$, with its center at I. Let these two circles intersect each other at points A and C. Through both centers draw the straight line $BEIF$. Join AE, EC, AI, and IC. Draw AKC perpendicular to BF. From these



circles we wish to determine the size of the eclipsed surface *ADCG,* or the number of twelfths of the whole plane of the circle of the sun or moon when partially eclipsed.

Then, from what was said above, *AE* and *AI,* the radii of both circles are given. So is *EI,* the distance between their centers = the moon's latitude. Hence in triangle *AEI* we have the sides given, and therefore the angles are given, by what was proved above. *EIC* is similar and equal to *AEI.* Consequently arcs *ADC* and *AGC* will be given in degrees of which the circumference of a circle = 360°. According to the *Measurement of the Circle* by Archimedes of Syracuse, the ratio of the circumference to the diameter is less than $3\,{}^{1}/_{7}$ [: 1] but more than $3\,{}^{10}/_{71}$ [: 1]. Between these values Ptolemy assumes a ratio of $3^{p}\,8'\,30''$: 1^{p}. On the basis of this ratio, arcs *AGC* and *ADC* will be known also in the same units as their diameters, or as *AE* and *AI.* The areas contained under *EA* and *AD,* and under *IA* and *AG,* are equal to sectors *AEC* and *AIC,* respectively.

But in the isosceles triangles *AEC* and *AIC,* the common base *AKC* is given, and so are the perpendiculars *EK* and *KI.* Then the product of $AK \times KE$ is given as the area of triangle *AEC,* just as the product of $AK \times KI$ = the area of triangle *ACI.* Subtracting both triangles from their sectors [sector *EADC* − Δ*AEC*; sector *AGCI* − Δ*ACI*] leaves as remainders *AGC* and *ACD* as segments of the circles. These segments make known the whole of *ADCG,* which was sought. Also given is the entire circular area, which is defined by *BE* and *BAD* in a solar eclipse, or by *FI* and *FAG* in a lunar eclipse. Hence it will become clear how many twelfths



of the whole circle, either the sun's or the moon's, will be occupied by *ADCG*, the eclipsed area.

With regard to the moon, let the foregoing discussion, which has been treated more fully by other astronomers, suffice for the present. For I am hurrying on to the revolutions of the other five bodies, which will be the subject of the following Books.

End of the Fourth Book of the Revolutions

Book Five

INTRODUCTION

Thus far to the best of my ability I have discussed the earth's revolution around the sun [Book III], and the moon's revolution around the earth [Book IV]. Now I tackle the motions of the five planets. The order and size of their spheres are connected with remarkable agreement and precise symmetry by the earth's motion, as I indicated generally in Book I [ch. 9], when I showed that the centers of these spheres are not near the earth, but rather near the sun. It therefore remains for me to prove all these statements one at a time and more clearly, and to fulfill my promises as well as I can. In particular I shall utilize observations of phenomena, which I have taken not only from antiquity but also from our own times, and by which the theory of those motions is made more certain.

[Earlier version of the beginning of V, 1:

The planets move in various ways in longitude and latitude, their variations being nonuniform and observable on both sides [of their uniform motions]. Therefore it was worth-while to unravel their mean and uniform motions, from which the variation in their nonuniformity can be ascertained. In order to determine the uniform motion, however, it is important to know the periods of the revolutions, from which it is inferred that a nonuniformity has returned [to a state] similar to a previous [state], just as I did with respect to the sun and moon [III, 13; IV, 3].

[Printed version:

In Plato's *Timaeus* these five planets are each named according to its aspect. Saturn is called "Phaenon", as though you were to say "bright" or "visible", for it is invisible less than the others, and emerges sooner after being blotted out by the sun. Jupiter is called "Phaeton" from its brilliance. Mars is called "Pyrois" from its fiery splendor. Venus is sometimes called "Phosphorus", sometimes "Hesperus", that is, "Morning Star" and "Evening Star", according as it shines in the morning or evening. Finally, Mercury is called "Stilbon", on account of its twinkling and shimmering light.

These bodies move in longitude and latitude with greater irregularity than does the moon.

THE REVOLUTIONS AND MEAN MOTIONS [OF THE PLANETS] *Chapter 1*

Two entirely different motions in longitude appear in them. One is caused by the earth's aforementioned motion, and the other is each one's own proper motion. I have decided without any impropriety to call the first one a parallactic motion, since it is this which makes the stations, [resumptions of] direct motion, and retrogradations appear in all of them. These phenomena appear, not because the planet, which always moves forward with its own motion, is erratic in this way, but because a sort of parallax is produced by the earth's motion according as it differs in size from those spheres.

Clearly, then, the true places of Saturn, Jupiter, and Mars become visible to us only when they rise at sunset. This happens about the middle of their retrogradations. For at that time they coincide with the straight line through the mean place of the sun [and earth], and are unaffected by that parallax. For Venus and Mercury, however, a different relation prevails. For when they are in conjunction with the sun, they are completely blotted out, and are visible only while executing their elongations to either side of the sun, so that they are never found without this parallax. Consequently each planet has its own individual parallactic revolution, I mean, terrestrial motion in relation to the planet, which these two bodies perform mutually.

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Combined in this way, the motions of both bodies display themselves interconnected, and they incorporate the simple motion of the earth (or you may say, of the sun), since throughout this entire work, and now above all, it must be remembered that whatever is said in the ordinary way about the sun's motion is always to be understood as referring to the earth.]

I say that the motion in parallax is nothing but the difference by which the earth's uniform motion exceeds their motion, as in the cases of Saturn, Jupiter, and Mars, or is exceeded by it, as in the cases of Venus and Mercury. But these parallactic periods are found to be nonuniform with an obvious irregularity. The ancients accordingly recognized that the motions of these planets were likewise nonuniform, and their circles had apsides to which their nonuniformity returned. They believed that the apsides possessed permanent places in the sphere of the fixed stars. This consideration opened the way to mastering the planets' mean motions and uniform periods. For when they had a record of the place of a planet at a precise distance from the sun and a fixed star, and learned that after an interval of time the planet had arrived at the same place at a similar distance from the sun, the planet was seen to have passed through its entire irregularity and to have returned through all its aspects to its former relation with the earth. Thus by means of the intervening time they computed the number of whole uniform revolutions, and thereby the detailed motions of the planet.

These revolutions were reported by Ptolemy [*Syntaxis,* IX, 3] in terms of solar years, as he states that he received them from Hipparchus. But he wants solar years to be understood as measured from an equinox or solstice. Such years, however, it has now become quite clear, are not entirely uniform. Therefore I shall use those which are measured by the fixed stars. By means of these years I have also redetermined the motions of these five planets with greater accuracy, in accordance with my findings that in our time they lacked something or were in excess, as follows.

In what I have called the parallactic motion, the earth returns to Saturn 57 times in 59 of our solar years, plus 1 day, 6 day-minutes, and about 48 day-seconds; in that time the planet completes 2 revolutions plus $1° 6' 6''$ in its own motion. Jupiter is passed by the earth 65 times in 71 solar years, minus 5 days, 45 day-minutes, 27 day-seconds; in that time the planet revolves 6 times minus $5° 41' 2\,^1/_2''$ in its own motion. For Mars the parallactic revolutions are 37 in 79 solar years, 2 days, 27 day-minutes, 3 day-seconds; in that time the planet completes 42 periods plus $2° 24' 56''$ in its own motion. Venus passes the moving earth 5 times in 8 solar years minus 2 days, 26 day-minutes, 46 day-seconds; in this period it revolves around the sun 13 times minus $2° 24' 40''$. Finally in 46 solar years, plus 34 day-minutes, 23 day-seconds, Mercury completes 145 parallactic revolutions, in which it overtakes the moving earth, with which it revolves around the sun, 191 times plus 31′ and about 23″. For each planet, therefore, one parallactic revolution takes, for:

Saturn	378 days	5 day-minutes	32 day-seconds	11 day-thirds
Jupiter	398	23	2	56
Mars	779	56	19	7
Venus	583	55	17	24
Mercury	115	52	42	12

When we convert the foregoing figures to the [360] degrees of a circle multiplied by 365, and then divide this product by the [given] number of days and fractions of days, we will have the annual motion for

Saturn	347°	32′	2″	54‴	12⁗
Jupiter	329	25	8	15	6
Mars	168	28	29	13	12
Venus	225	1	48	54	30
Mercury	53	56	46	54	40, after 3 revolutions.

Of the above values, $1/_{365}$ is the daily motion for

Saturn		57′	7″	44‴	0⁗
Jupiter		54	9	3	49
Mars		27	41	40	8
Venus		36	59	28	35
Mercury	3°	6	24	7	43,

as set forth in the following Tables, on the model of the Tables of the Mean Motions of the Sun and Moon [following III, 14 and IV, 4]. However, I thought it unnecessary to tabulate in this manner the planets' proper motions. For these are obtained by subtracting the tabulated motions from the sun's mean motion, into which they enter as a component, as I said [earlier in V, 1]. Nevertheless, if anybody is dissatisfied with these arrangements, he may make the other table if he so wishes. For, the annual proper motion with respect to the sphere of the fixed stars is for

Saturn	12°	12′	46″	12‴	52⁗
Jupiter	30	19	40	51	58
Mars	191	16	19	53	52

But for Venus and Mercury, since [their annual proper motion] is not apparent to us, the sun's motion is used and furnishes a method of determining and demonstrating their appearances, as indicated below.

SATURN'S PARALLACTIC MOTION IN YEARS AND PERIODS OF 60 YEARS

Christian Era 205° 49′

Egyptian Years	Motion 60°	°	′	″	‴	Egyptian Years	Motion 60°	°	′	″	‴
1	5	47	32	3	9	31	5	33	33	37	59
2	5	35	4	6	19	32	5	21	5	41	9
3	5	22	36	9	29	33	5	8	37	44	19
4	5	10	8	12	38	34	4	56	9	47	28
5	4	57	40	15	48	35	4	43	41	50	38
6	4	45	12	18	58	36	4	31	13	53	48
7	4	32	44	22	7	37	4	18	45	56	57
8	4	20	16	25	17	38	4	6	18	0	7
9	4	7	48	28	27	39	3	53	50	3	17
10	3	55	20	31	36	40	3	41	22	6	26
11	3	42	52	34	46	41	3	28	54	9	36
12	3	30	24	37	56	42	3	16	26	12	46
13	3	17	56	41	5	43	3	3	58	15	55
14	3	5	28	44	15	44	2	51	30	19	5
15	2	53	0	47	25	45	2	39	2	22	15
16	2	40	32	50	34	46	2	26	34	25	24
17	2	28	4	53	44	47	2	14	6	28	34
18	2	15	36	56	54	48	2	1	38	31	44
19	2	3	9	0	3	49	1	49	10	34	53
20	1	50	41	3	13	50	1	36	42	38	3
21	1	38	13	6	23	51	1	24	14	41	13
22	1	25	45	9	32	52	1	11	46	44	22
23	1	13	17	12	42	53	0	59	18	47	32
24	1	0	49	15	52	54	0	46	50	50	42
25	0	48	21	19	1	55	0	34	22	53	51
26	0	35	53	22	11	56	0	21	54	57	1
27	0	23	25	25	21	57	0	9	27	0	11
28	0	10	57	28	30	58	5	56	59	3	20
29	5	58	29	31	40	59	5	44	31	6	30
30	5	46	1	34	50	60	5	32	3	9	40

SATURN'S PARALLACTIC MOTION IN DAYS, PERIODS OF 60 DAYS, AND FRACTIONS OF DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	57	7	44	31	0	29	30	59	46
2	0	1	54	15	28	32	0	30	28	7	30
3	0	2	51	23	12	33	0	31	25	15	14
4	0	3	48	30	56	34	0	32	22	22	58
5	0	4	45	38	40	35	0	33	19	30	42
6	0	5	42	46	24	36	0	34	16	38	26
7	0	6	39	54	8	37	0	35	13	46	1
8	0	7	37	1	52	38	0	36	10	53	55
9	0	8	34	9	36	39	0	37	8	1	39
10	0	9	31	17	20	40	0	38	5	9	23
11	0	10	28	25	4	41	0	39	2	17	7
12	0	11	25	32	49	42	0	39	59	24	51
13	0	12	22	40	33	43	0	40	56	32	35
14	0	13	19	48	17	44	0	41	53	40	19
15	0	14	16	56	1	45	0	42	50	48	3
16	0	15	14	3	45	46	0	43	47	55	47
17	0	16	11	11	29	47	0	44	45	3	31
18	0	17	8	19	13	48	0	45	42	11	16
19	0	18	5	26	57	49	0	46	39	19	0
20	0	19	2	34	41	50	0	47	36	26	44
21	0	19	59	42	25	51	0	48	33	34	28
22	0	20	56	50	9	52	0	49	30	42	12
23	0	21	53	57	53	53	0	50	27	49	56
24	0	22	51	5	38	54	0	51	24	57	40
25	0	23	48	13	22	55	0	52	22	5	24
26	0	24	45	21	6	56	0	53	19	13	8
27	0	25	42	28	50	57	0	54	16	20	52
28	0	26	39	36	34	58	0	55	13	28	36
29	0	27	36	44	18	59	0	56	10	36	20
30	0	28	33	52	2	60	0	57	7	44	5

JUPITER'S PARALLACTIC MOTION IN YEARS AND PERIODS OF 60 YEARS

Christian Era 98° 16′

Egyptian Years	Motion					Egyptian Years	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	5	29	25	8	15	31	2	11	59	15	48
2	4	58	50	16	30	32	1	41	24	24	3
3	4	28	15	24	45	33	1	10	49	32	18
4	3	57	40	33	0	34	0	40	14	40	33
5	3	27	5	41	15	35	0	9	39	48	48
6	2	56	30	49	30	36	5	39	4	57	3
7	2	25	55	57	45	37	5	8	30	5	18
8	1	55	21	6	0	38	4	37	55	13	33
9	1	24	46	14	15	39	4	7	20	21	48
10	0	54	11	22	31	40	3	36	45	30	4
11	0	23	36	30	46	41	3	6	10	38	19
12	5	53	1	39	1	42	2	35	35	46	34
13	5	22	26	47	16	43	2	5	0	54	49
14	4	51	51	55	31	44	1	34	26	3	4
15	4	21	17	3	46	45	1	3	51	11	19
16	3	50	42	12	1	46	0	33	16	19	34
17	3	20	7	20	16	47	0	2	41	27	49
18	2	49	32	28	31	48	5	32	5	36	4
19	2	18	57	36	46	49	5	1	31	44	19
20	1	48	22	45	2	50	4	30	56	52	34
21	1	17	47	53	17	51	4	0	22	0	50
22	0	47	13	1	32	52	3	29	47	9	5
23	0	16	38	9	47	53	2	59	12	17	20
24	5	46	3	18	2	54	2	28	37	25	35
25	5	15	28	26	17	55	1	58	2	33	50
26	4	44	53	34	32	56	1	27	27	42	5
27	4	14	18	42	47	57	0	56	52	50	20
28	3	43	43	51	2	58	0	26	17	58	35
29	3	13	8	59	17	59	5	55	43	6	50
30	2	42	34	7	33	60	5	25	8	15	6

JUPITER'S PARALLACTIC MOTION IN DAYS, PERIODS OF 60 DAYS, AND FRACTIONS OF DAYS

Days	Motion 60°	°	′	″	‴	Days	Motion 60°	°	′	″	‴
1	0	0	54	9	3	31	0	27	58	40	58
2	0	1	48	18	7	32	0	28	52	50	2
3	0	2	42	27	11	33	0	29	46	59	5
4	0	3	36	36	15	34	0	30	41	8	9
5	0	4	30	45	19	35	0	31	35	17	13
6	0	5	24	54	22	36	0	32	29	26	17
7	0	6	19	3	26	37	0	33	23	35	21
8	0	7	13	12	30	38	0	34	17	44	25
9	0	8	7	21	34	39	0	35	11	53	29
10	0	9	1	30	38	40	0	36	6	2	32
11	0	9	55	39	41	41	0	37	0	11	36
12	0	10	49	48	45	42	0	37	54	20	40
13	0	11	43	57	49	43	0	38	48	29	44
14	0	12	38	6	53	44	0	39	42	38	47
15	0	13	32	15	57	45	0	40	36	47	51
16	0	14	26	25	1	46	0	41	30	56	55
17	0	15	20	34	4	47	0	42	25	5	59
18	0	16	14	43	8	48	0	43	19	15	3
19	0	17	8	52	12	49	0	44	13	24	6
20	0	18	3	1	16	50	0	45	7	33	10
21	0	18	57	10	20	51	0	46	1	42	14
22	0	19	51	19	23	52	0	46	55	51	18
23	0	20	45	28	27	53	0	47	50	0	22
24	0	21	39	37	31	54	0	48	44	9	26
25	0	22	33	46	35	55	0	49	38	18	29
26	0	23	27	55	39	56	0	50	32	27	33
27	0	24	22	4	43	57	0	51	26	36	37
28	0	25	16	13	46	58	0	52	20	45	41
29	0	26	10	22	50	59	0	53	14	54	45
30	0	27	4	31	54	60	0	54	9	3	49

MARS' PARALLACTIC MOTION IN YEARS AND PERIODS OF 60 YEARS

Christian Era 238° 22′

Egyptian Years	Motion						Egyptian Years	Motion				
	60°	°	′	″	‴			60°	°	′	″	‴
1	2	48	28	30	36		31	3	2	43	48	38
2	5	36	57	1	12		32	5	51	12	19	14
3	2	25	25	31	48		33	2	39	40	49	50
4	5	13	54	2	24		34	5	28	9	20	26
5	2	2	22	33	0		35	2	16	37	51	2
6	4	50	51	3	36		36	5	5	6	21	38
7	1	39	19	34	12		37	1	53	34	52	14
8	4	27	48	4	48		38	4	42	3	22	50
9	1	16	16	35	24		39	1	30	31	53	26
10	4	4	45	6	0		40	4	19	0	24	2
11	0	53	13	36	36		41	1	7	28	54	38
12	3	41	42	7	12		42	3	55	57	25	14
13	0	30	10	37	48		43	0	44	25	55	50
14	3	18	39	8	24		44	3	32	54	26	26
15	0	7	7	39	1		45	0	21	22	57	3
16	2	55	36	9	37		46	3	9	51	27	39
17	5	44	4	40	13		47	5	58	19	58	15
18	2	32	33	10	49		48	2	46	48	28	51
19	5	21	1	41	25		49	5	35	16	59	27
20	2	9	30	12	1		50	2	23	45	30	3
21	4	57	58	42	37		51	5	12	14	0	39
22	1	46	27	13	13		52	2	0	42	31	15
23	4	34	55	43	49		53	4	49	11	1	51
24	1	23	24	14	25		54	1	37	39	32	27
25	4	11	52	45	1		55	4	26	8	3	3
26	1	0	21	15	37		56	1	14	36	33	39
27	3	48	49	46	13		57	4	3	5	4	15
28	0	37	18	16	49		58	0	51	33	34	51
29	3	25	46	47	25		59	3	40	2	5	27
30	0	14	15	18	2		60	0	28	30	36	4

MARS' PARALLACTIC MOTION IN DAYS, PERIODS OF 60 DAYS, AND FRACTIONS OF DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	27	41	40	31	0	14	18	31	51
2	0	0	55	23	20	32	0	14	46	13	31
3	0	1	23	5	1	33	0	15	14	55	12
4	0	1	50	46	41	34	0	15	41	36	52
5	0	2	18	28	21	35	0	16	9	18	32
6	0	2	46	10	2	36	0	16	37	0	13
7	0	3	13	51	42	37	0	17	4	41	53
8	0	3	41	33	22	38	0	17	32	23	33
9	0	4	9	15	3	39	0	18	0	5	14
10	0	4	36	56	43	40	0	18	27	46	54
11	0	5	4	38	24	41	0	18	55	28	35
12	0	5	32	20	4	42	0	19	23	10	15
13	0	6	0	1	44	43	0	19	50	51	55
14	0	6	27	43	25	44	0	20	18	33	36
15	0	6	55	25	5	45	0	20	46	15	16
16	0	7	23	6	45	46	0	21	13	56	56
17	0	7	50	48	26	47	0	21	41	38	37
18	0	8	18	30	6	48	0	22	9	20	17
19	0	8	46	11	47	49	0	22	37	1	57
20	0	9	13	53	27	50	0	23	4	43	38
21	0	9	41	35	7	51	0	23	32	25	18
22	0	10	9	16	48	52	0	24	0	6	59
23	0	10	36	58	28	53	0	24	27	48	39
24	0	11	4	40	8	54	0	24	55	30	19
25	0	11	32	21	49	55	0	25	23	12	0
26	0	12	0	3	29	56	0	25	50	53	40
27	0	12	27	45	9	57	0	26	18	35	20
28	0	12	55	26	49	58	0	26	46	17	1
29	0	13	23	8	30	59	0	27	13	58	41
30	0	13	50	50	11	60	0	27	41	40	22

VENUS' PARALLACTIC MOTION IN YEARS AND PERIODS OF 60 YEARS

Christian Era 126° 45′

Egyptian Years	Motion 60°	°	′	″	‴	Egyptian Years	Motion 60°	°	′	″	‴
1	3	45	1	45	3	31	2	15	54	16	53
2	1	30	3	30	7	32	0	0	56	1	57
3	5	15	5	15	11	33	3	45	57	47	1
4	3	0	7	0	14	34	1	30	59	32	4
5	0	45	8	45	18	35	5	16	1	17	8
6	4	30	10	30	22	36	3	1	3	2	12
7	2	15	12	15	25	37	0	46	4	47	15
8	0	0	14	0	29	38	4	31	6	32	19
9	3	45	15	45	33	39	2	16	8	17	23
10	1	30	17	30	36	40	0	1	10	2	26
11	5	15	19	15	40	41	3	46	11	47	30
12	3	0	21	0	44	42	1	31	13	32	34
13	0	45	22	45	47	43	5	16	15	17	37
14	4	30	24	30	51	44	3	1	17	2	41
15	2	15	26	15	55	45	0	46	18	47	45
16	0	0	28	0	58	46	4	31	20	32	48
17	3	45	29	46	2	47	2	16	22	17	52
18	1	30	31	31	6	48	0	1	24	2	56
19	5	15	33	16	9	49	3	46	25	47	59
20	3	0	35	1	13	50	1	31	27	33	3
21	0	45	36	46	17	51	5	16	29	18	7
22	4	30	38	31	20	52	3	1	31	3	10
23	2	15	40	16	24	53	0	46	32	48	14
24	0	0	42	1	28	54	4	31	34	33	18
25	3	45	43	46	31	55	2	16	36	18	21
26	1	30	45	31	35	56	0	1	38	3	25
27	5	15	47	16	39	57	3	46	39	48	29
28	3	0	49	1	42	58	1	31	41	33	32
29	0	45	50	46	46	59	5	16	43	18	36
30	4	30	52	31	50	60	3	1	45	3	40

VENUS' PARALLACTIC MOTION IN DAYS, PERIODS OF 60 DAYS, AND FRACTIONS OF DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	0	36	59	28	31	0	19	6	43	46
2	0	1	13	58	57	32	0	19	43	43	14
3	0	1	50	58	25	33	0	20	20	42	43
4	0	2	27	57	54	34	0	20	57	42	11
5	0	3	4	57	22	35	0	21	34	41	40
6	0	3	41	56	51	36	0	22	11	41	9
7	0	4	18	56	20	37	0	22	48	40	37
8	0	4	55	55	48	38	0	23	25	40	6
9	0	5	32	55	17	39	0	24	2	39	34
10	0	6	9	54	45	40	0	24	39	39	3
11	0	6	46	54	14	41	0	25	16	38	31
12	0	7	23	53	43	42	0	25	53	38	0
13	0	8	0	53	11	43	0	26	30	37	29
14	0	8	37	52	40	44	0	27	7	36	57
15	0	9	14	52	8	45	0	27	44	36	26
16	0	9	51	51	37	46	0	28	21	35	54
17	0	10	28	51	5	47	0	28	58	35	23
18	0	11	5	50	34	48	0	29	35	34	52
19	0	11	42	50	2	49	0	30	12	34	20
20	0	12	19	49	31	50	0	30	49	33	49
21	0	12	56	48	59	51	0	31	26	33	17
22	0	13	33	48	28	52	0	32	3	32	46
23	0	14	10	47	57	53	0	32	40	32	14
24	0	14	47	47	26	54	0	33	17	31	43
25	0	15	24	46	54	55	0	33	54	31	12
26	0	16	1	46	23	56	0	34	31	30	40
27	0	16	38	45	51	57	0	35	8	30	9
28	0	17	15	45	20	58	0	35	45	29	37
29	0	17	52	44	48	59	0	36	22	29	6
30	0	18	29	44	17	60	0	36	59	28	35

MERCURY'S PARALLACTIC MOTION IN YEARS AND PERIODS OF 60 YEARS

Christian Era 46° 24′

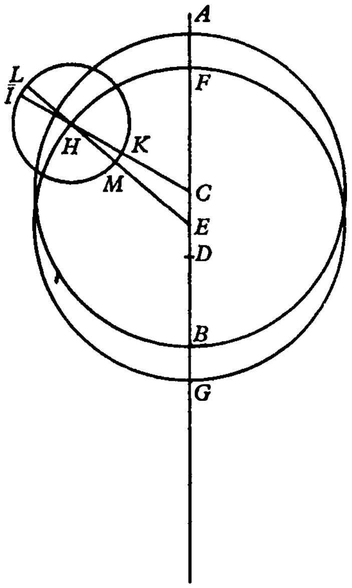
Egyptian Years	Motion 60°	°	′	″	‴	Egyptian Years	Motion 60°	°	′	″	‴
1	0	53	57	23	6	31	3	52	38	56	21
2	1	47	54	46	13	32	4	46	36	19	28
3	2	41	52	9	19	33	5	40	33	42	34
4	3	35	49	32	26	34	0	34	31	5	41
5	4	29	46	55	32	35	1	28	28	28	47
6	5	23	44	18	39	36	2	22	25	51	54
7	0	17	41	41	45	37	3	16	23	15	0
8	1	11	39	4	52	38	4	10	20	38	7
9	2	5	36	27	58	39	5	4	18	1	13
10	2	59	33	51	5	40	5	58	15	24	20
11	3	53	31	14	11	41	0	52	12	47	26
12	4	47	28	37	18	42	1	46	10	10	33
13	5	41	26	0	24	43	2	40	7	33	39
14	0	35	23	23	31	44	3	34	4	56	46
15	1	29	20	46	37	45	4	28	2	19	52
16	2	23	18	9	44	46	5	21	59	42	59
17	3	17	15	32	50	47	0	15	57	6	5
18	4	11	12	55	57	48	1	9	54	29	12
19	5	5	10	19	3	49	2	3	51	52	18
20	5	59	7	42	10	50	2	57	49	15	25
21	0	53	5	5	16	51	3	51	46	38	31
22	1	47	2	28	23	52	4	45	44	1	38
23	2	40	59	51	29	53	5	39	41	24	44
24	3	34	57	14	36	54	0	33	38	47	51
25	4	28	54	37	42	55	1	27	36	10	57
26	5	22	52	0	49	56	2	21	33	34	4
27	0	16	49	23	55	57	3	15	30	57	10
28	1	10	46	47	2	58	4	9	28	20	17
29	2	4	44	10	8	59	5	3	25	43	23
30	2	58	41	33	15	60	5	57	23	6	30

MERCURY'S PARALLACTIC MOTION IN DAYS, PERIODS OF 60 DAYS, AND FRACTIONS OF DAYS

Days	Motion					Days	Motion				
	60°	°	′	″	‴		60°	°	′	″	‴
1	0	3	6	24	13	31	1	36	18	31	3
2	0	6	12	48	27	32	1	39	24	55	17
3	0	9	19	12	41	33	1	42	31	19	31
4	0	12	25	36	54	34	1	45	37	43	44
5	0	15	32	1	8	35	1	48	44	7	58
6	0	18	38	25	22	36	1	51	50	32	12
7	0	21	44	49	35	37	1	54	56	56	25
8	0	24	51	13	49	38	1	58	3	20	39
9	0	27	57	38	3	39	2	1	9	44	53
10	0	31	4	2	16	40	2	4	16	9	6
11	0	34	10	26	30	41	2	7	22	33	20
12	0	37	16	50	44	42	2	10	28	57	34
13	0	40	23	14	57	43	2	13	35	21	47
14	0	43	29	39	11	44	2	16	41	46	1
15	0	46	36	3	25	45	2	19	48	10	15
16	0	49	42	27	38	46	2	22	54	34	28
17	0	52	48	51	52	47	2	26	0	58	42
18	0	55	55	16	6	48	2	29	7	22	56
19	0	59	1	40	19	49	2	32	13	47	9
20	1	2	8	4	33	50	2	35	20	11	23
21	1	5	14	28	47	51	2	38	26	35	37
22	1	8	20	53	0	52	2	41	32	59	50
23	1	11	27	17	14	53	2	44	39	24	4
24	1	14	33	41	28	54	2	47	45	48	18
25	1	17	40	5	41	55	2	50	52	12	31
26	1	20	46	29	55	56	2	53	58	36	45
27	1	23	52	54	9	57	2	57	5	0	59
28	1	26	59	18	22	58	3	0	11	25	12
29	1	30	5	42	36	59	3	3	17	49	26
30	1	33	12	6	50	60	3	6	24	13	40

THE PLANETS' UNIFORM AND APPARENT MOTION, AS EXPLAINED BY THE THEORY OF THE ANCIENTS — *Chapter 2*

Their mean motions occur as set forth above. Now let me turn to their nonuniform apparent motion. The ancient astronomers [for example, Ptolemy, *Syntaxis*, IX, 5], who regarded the earth as stationary, imagined an eccentrepicycle for Saturn, Jupiter, Mars, and Venus, as well as another eccentric, in relation to which the epicycle moved uniformly, and so did the planet on the epicycle.



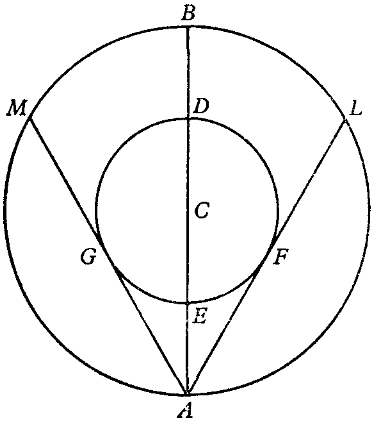
Thus, let *AB* be an eccentric circle, with its center at *C*. Let the diameter be *ACB*, on which the center of the earth is *D*, so that the apogee is in *A*, and the perigee in *B*. Bisect *DC* at *E*. With *E* as center, describe a second eccentric *FG*, equal to the first eccentric [*AB*]. Anywhere on *FG* take *H* as center, and describe epicycle *IK*. Through its center draw straight line *IHKC*, and likewise *LHME*. Let the eccentrics be understood to be inclined to the plane of the ecliptic, and the epicycle to the plane of the eccentric, on account of the latitudes displayed by the planet. Here, however, to simplify the explanation, [let all these circles] lie in one plane. This whole plane, according to the ancient astronomers, together with points *E* and *C*, moves around *D*, the center of the ecliptic, with the motion of the fixed stars. Through this [arrangement] they wish it to be understood that these points have unalterable places in the sphere of the fixed stars, while the epicycle also moves eastward on circle *FHG* but is regulated by line *IHC*, with reference to which the planet also revolves uniformly on epicycle *IK*.

The motion on the epicycle, however, clearly should be uniform with respect to *E*, the center of its deferent, and the planet's revolution should be uniform with respect to line *LME*. Here too, then, as they admit, a circular motion can be uniform with respect to an extraneous center not its own, a concept of which Scipio in Cicero would hardly have dreamed. And now in the case of Mercury the same thing is permitted, and even more. But (in my opinion) I have already adequately refuted this idea in connection with the moon [IV, 2]. These and similar situations gave me the occasion to consider the motion of the earth and other ways of preserving uniform motion and the principles of the science, as well as of making the computation of the apparent nonuniform motion more enduring.

GENERAL EXPLANATION OF THE APPARENT NONUNIFORMITY CAUSED BY THE EARTH'S MOTION — *Chapter 3*

There are two reasons why a planet's uniform motion appears nonuniform: the earth's motion, and the planet's own motion. I shall explain each of the nonuniformities in general and separately with a visual demonstration, in order that they may be better distinguished from each other. I shall begin with the nonuniformity which is intermingled with them all on account of the earth's motion, and I shall start with Venus and Mercury, which are enclosed within the earth's circle.

Let circle *AB*, eccentric to the sun, be described by the earth's center in the annual revolution as set forth above [III, 15]. Let *AB*'s center be *C*. Now let us

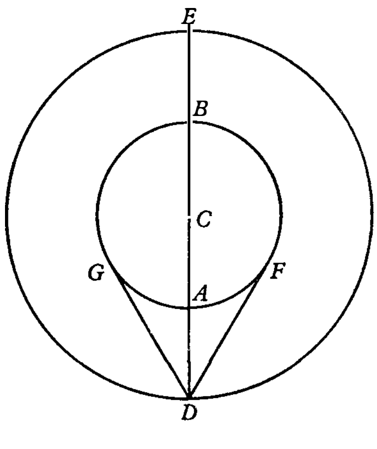


assume that the planet has no irregularity other than that which it would have if we made it concentric with *AB*. Let the concentric be *DE*, either Venus' or Mercury's. On account of their latitude *DE* must be inclined to *AB*. But for the sake of an easier explanation, let them be conceived in the same plane. Put the earth in point *A*, from which draw lines of sight *AFL* and *AGM*, tangent to the planet's circle at points *F* and *G*. Let *ACB* be a diameter common to both circles.

Let both bodies, I mean, the earth and the planet, move in the same direction, that is, eastward, but let the planet be faster than the earth. Hence *C* and line *ACB* will appear to an observer traveling with *A* to move with the sun's mean motion. On the other hand, on circle *DFG*, as though it were an epicycle, the planet will traverse arc *FDG* eastward in more time than the remaining arc *GEF* westward. In arc *FDG* it will add the entire angle *FAG* to the sun's mean motion, while in arc *GEF* it will subtract the same angle. Therefore, where the planet's subtractive motion, especially near perigee *E*, exceeds *C*'s additive motion, to the extent of that excess it seems to [the observer in] *A* to retrograde, as happens in these planets. In their cases, line *CE* : line *AE* > *A*'s motion : planet's motion, according to the theorems of Apollonius of Perga, as will be mentioned hereafter [V, 35]. But where the additive motion equals the subtractive (counteracting each other), the planet will seem stationary, all these aspects being in agreement with the phenomena.

Therefore, if there were no other irregularity in the planet's motion, as Apollonius thought, these constructions could be sufficient. But these planets' greatest elongations from the sun's mean place in the mornings and evenings, as indicated by angles *FAE* and *GAE*, are not found everywhere equal. Nor is either one of these greatest elongations equal to the other, nor are their sums equal to each other. The inference is obvious that they do not move on circles concentric with the earth's, but on certain other circles by which they produce a second inequality.

The same conclusion is proved also for the three outer planets, Saturn, Jupiter, and Mars, which completely encircle the earth. Reproduce the earth's circle from the preceding diagram. Assume *DE* outside it and concentric with it in the same plane. On *DE* put the planet at any point *D*, from which draw straight lines *DF* and *DG* tangent to the earth's circle at points *F* and *G*, and [from D also draw] *DACBE*, the diameter common [to both circles]. On *DE*, the line of the sun's motion, the true place of the planet, when it rises at sunset and is closest to the earth, will obviously be visible (only to an observer at *A*). For when the earth is at the opposite point *B*, although the planet is on the same line, it will not be visible, having become blotted out on account of the sun's closeness to *C*. But the earth's travel exceeds the planet's motion. Hence throughout the apogeal arc *GBF* it will appear to add the whole angle *GDF* to the planet's motion, and to subtract it in the remaining arc *FAG*, but for a shorter time, *FAG* being a smaller arc. Where the earth's subtractive motion exceeds the planet's additive motion (especially around *A*), the planet will seem to be left behind by the earth and to move westward, and to stand still where the observer sees the least difference between the opposing motions.



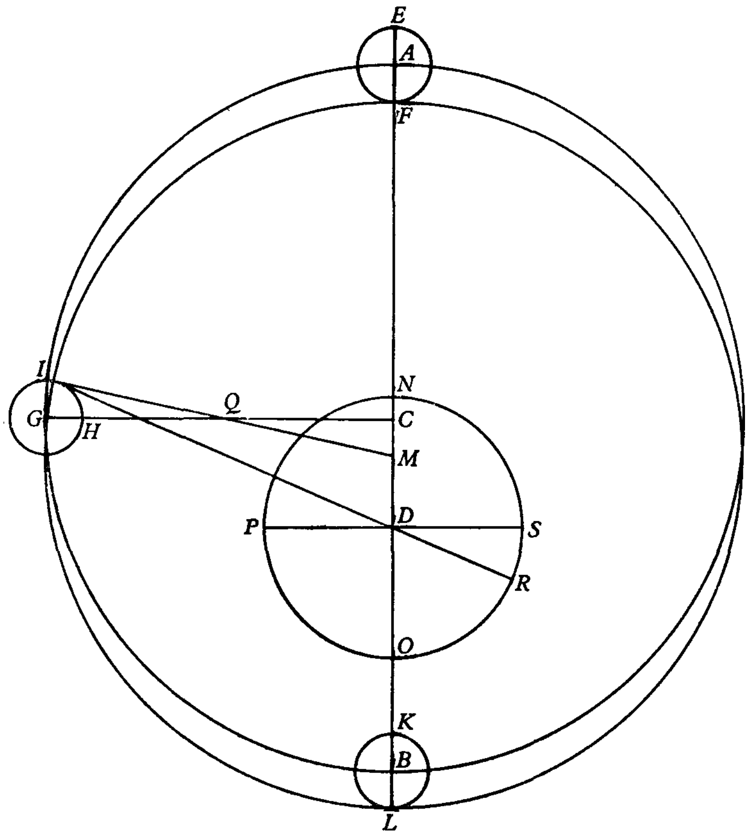
Thus all these phenomena, which the ancient astronomers sought [to explain] by means of an epicycle for each planet, happen on account of the single motion of the earth, as is again clear. Contrary to the view of Apollonius and the ancients, however, the planet's motion is not found uniform, as is proclaimed by the earth's irregular revolution with respect to the planet. Consequently the planets do not move on a concentric, but in another way, which I shall also explain next.

IN WHAT WAYS DO THE PLANETS' OWN MOTIONS APPEAR NONUNIFORM? *Chapter 4*

Their own motions in longitude have almost the same pattern, with the exception of Mercury, which seems to differ from them. Hence those four will be discussed together, and a separate place reserved for Mercury. Whereas the ancients put a single motion on two eccentrics, as has been recalled [V, 2], I think that there are two uniform motions of which the apparent nonuniformity is composed: either an eccentreccentric or an epicyclepicycle or also a mixed eccentr-epicycle, which can produce the same nonuniformity, as I proved above in connection with the sun and moon [III, 20; IV, 3].

Thus, let *AB* be an eccentric circle, with center *C*. Let the diameter *ACB*, drawn through the planet's higher and lower apse, be the line of the sun's mean place. On *ACB* let *D* be the center of the earth's circle. With the higher apse *A* as center, and radius $= {}^1/_3$ of distance *CD*, describe epicyclet *EF*. In *F*, its perigee, place the planet. Let the epicyclet move eastward along the eccentric *AB*. Let the planet likewise move eastward on the epicyclet's upper circumference, and westward on the rest of the circumference. Let the revolutions of both, I mean, the epicyclet and the planet, be equal to each other. It will therefore happen that with the epicyclet in the eccentric's higher apse, and the planet on the contrary in the epicyclet's perigee, when each of them has completed its semicircle, they change their relation to each other to the opposite. But at both quadratures midway between [the higher and lower apsides], each will be at its middle apse. Only in the former cases [higher and lower apsides], will the epicyclet's diameter lie on the line *AB*. Moreover, at the midpoints [between the higher and lower apsides, the epicyclet's diameter] will be perpendicular to *AB*. Elsewhere it always swings towards and away [from *AB*]. All these phenomena are easily understood from the sequence of the motions.

Hence it will also be demonstrated that by this composite motion the planet does not describe a perfect circle. [This departure from perfect circularity] is in conformity with the thinking of the ancient astronomers, yet the difference is imperceptible. Reproducing the same epicyclet, let it be *KL*, with center *B*. Taking *AG* as a quadrant of the [eccentric] circle, with *G* as center draw epicyclet *HI*. Trisecting *CD*, let ${}^1/_3\ CD = CM = GI$. Join *GC* and *IM*, which intersect each other in *Q*. Hence, arc *AG* is similar to arc *HI* by construction. *ACG* being a right angle, *HGI* is therefore also a right angle. Furthermore, the vertical angles at *Q* are likewise equal. Consequently triangles *GIQ* and *QCM* are equiangular. But their corresponding sides are also equal, since by hypothesis base $GI =$ base *CM*. Side $QI > GQ$, just as also $QM > QC$. Therefore, the whole of $IQM >$ the whole of *GQC*. But $FM = ML = AC = CG$. Then, the circle drawn around *M* as center through points *F* and $L =$ circle *AB*, and will intersect line *IM*. The demonstration will proceed in the same way in the other quadrant opposite [*AG*]. Therefore, the uniform motions of the epicyclet on the eccentric, and of the planet on the epicycle, cause the planet to describe not a perfect, but an almost perfect, circle. Q. E. D.



Now around D as center describe NO as the earth's annual circle. Draw IDR, and also PDS parallel to CG. Then IDR will be the straight line of the planet's true motion, and GC of its mean and uniform motion. In R the earth will be at its true greatest distance from the planet, and in S at its mean [greatest distance]. Therefore, angle RDS or IDP is the difference between these two, the uniform and apparent motions, that is, between angles ACG and CDI. But suppose that instead of eccentric AB, we took as its equal a concentric with D as center. This concentric would serve as deferent for an epicyclet, whose radius $= CD$. On this [first epicyclet] there would also be a second epicyclet, whose diameter $= {}^1/_2\, CD$. Let the first epicycle move eastward, and the second in the opposite direction with equal speed. Finally, on the second epicycle let the planet travel at twice this speed. The same results will follow as those described above, and they will not differ much from the lunar phenomena, or even [from those obtained] by any of the aforementioned arrangements.

But here I have chosen an eccentrepicycle. For though [the distance] between the sun and C always remains the same, D is meanwhile found to have shifted, as was shown in the solar phenomena [III, 20]. This shift is not accompanied equally by the others. Hence these must undergo an irregularity which, although slight, is nevertheless perceptible in Mars and Venus, as will be seen in the proper places [V, 16, 22].

Therefore, these hypotheses suffice for the phenomena, as I shall presently prove from observations. I shall do so first for Saturn, Jupiter, and Mars, for which the principal and hardest task is to find the place of the apogee and the distance *CD*, since from these data the rest is easily demonstrated. For these [three planets] I shall use practically the same procedure as I employed for the moon [IV, 5], namely, a comparison of three ancient oppositions to the sun with the same number of modern oppositions. These are called "acronycal risings" by the Greeks, and by us [risings and settings] "at the ends of the night". At those times the planet is in opposition to the sun and meets the straight line of the sun's mean motion, where it sloughs off the entire inequality imposed on it by the earth's motion. These positions are obtained instrumentally by observing with the astrolabe, as was explained above [II, 14], and also by applying the computations for the sun, until the planet has clearly arrived opposite it.

DERIVATIONS OF SATURN'S MOTION *Chapter 5*

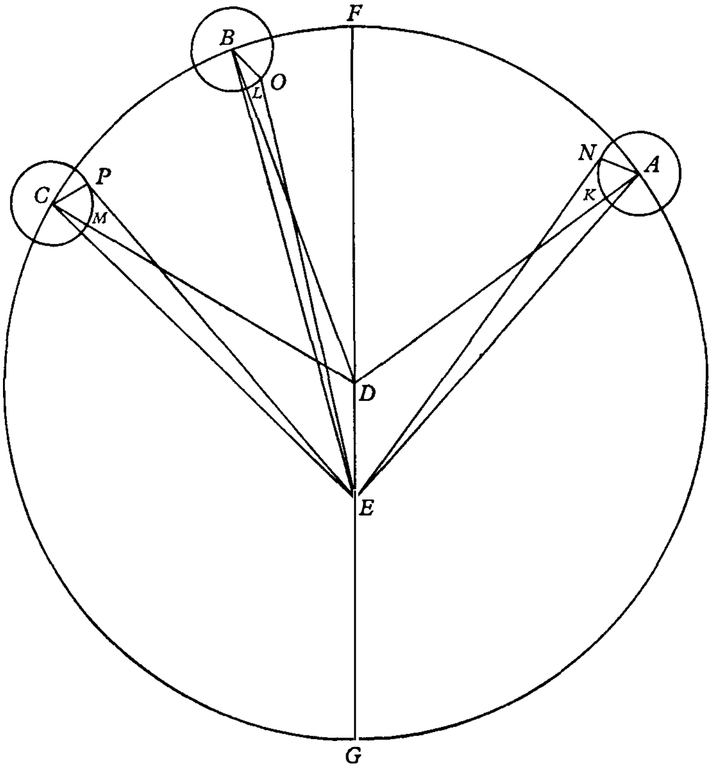
Let us begin with Saturn by taking three oppositions observed long ago by Ptolemy [*Syntaxis*, XI, 5]. The first of these occurred at the 1st hour of night on the 7th day of the month Pachon in Hadrian's 11th year. This was 26 March 127 C.E., 17 uniform hours after midnight, when the computation is reduced to the meridian of Cracow, which we have found to be 1 hour away from Alexan-dria. In the sphere of the fixed stars, to which we refer all these data for the origin of uniform motion, the planet's place was located at about 174° 40′. For at that time the sun in its simple motion was opposite [Saturn] at 354° 40′ [–180° = 174° 40′], the horn of the Ram being taken as the zero point.

The second opposition happened on the 18th day of the Egyptian month Epiphi in Hadrian's 17th year. This was 15 uniform hours after midnight on the 3rd day before the Nones of June in the Roman calendar [= 3 June] 133 C.E. Ptolemy finds the planet at 243° 3′, while the sun in its mean motion was at 63° 3′ [+180° = 243° 3′] at 15 hours after midnight.

Then he reported the third opposition as taking place on the 24th day of the Egyptian month Mesori in Hadrian's 20th year. This was 8 July 136 C.E., 11 hours after midnight, similarly reduced to the Cracow meridian. [The planet was] at 277° 37′, while the sun in its mean motion was at 97° 37′ [+180° = 277° 37′].

In the first interval, therefore, there are 6 years, 70 days, 55 day-minutes, during which the planet apparently moved 68° 23′ [= 243° 3′–174° 40′], while the earth's mean motion away from the planet – this is the motion in parallax – was 352° 44′. Hence the 7° 16′ missing from the circle [= 360°– 352° 44′] are added to make the planet's mean motion 75° 39′ [= 7° 16′+ 68° 23′]. In the second interval there are 3 Egyptian years, 35 days, 50 day-minutes; the planet's apparent motion is 34° 34′ [= 277° 37′–243° 3′], and the motion in parallax is 356° 43′. The remaining 3° 17′ of a circle [= 360°–356° 43′] are added to the plan-et's apparent motion, so that there are 37° 51′ in its mean motion [= 3° 17′+34° 34′].

Having reviewed these data, draw the planet's eccentric circle *ABC*, with center *D*, and diameter *FDG*, on which *E* is the center of the earth's grand circle. Let *A* be the epicyclet's center at the first opposition, *B* at the second, and *C* at the third. Around these [points as centers], describe this epicyclet, with radius = $^1/_3$ *DE*. Join centers *A, B,* and *C* with *D* and *E* by straight lines intersecting the



epicyclet's circumference in points *K, L,* and *M.* Take arc *KN* similar to *AF, LO* to *BF,* and *MP* to *FBC.* Join *EN, EO,* and *EP.* Then by [the preceding] computation arc $AB = 75° 39'$, $BC = 37° 51'$, $NEO =$ the angle of apparent motion $= 68° 23'$, and angle $OEP = 34° 34'$.

The first task is to investigate the places of the higher and lower apsides, that is, of *F* and *G,* as well as *DE,* the distance between the centers [of the planet's eccentric and the earth's grand circle]. Without this information there is no way of distinguishing between the uniform and apparent motions. But here too we encounter a difficulty no less than Ptolemy's in this discussion. For if the given angle *NEO* enclosed the given arc *AB,* and *OEP* included *BC,* the path would now be open to derive what we are seeking. However, the known arc *AB* subtends the unknown angle *AEB,* and similarly angle *BEC* lies unknown beneath the known arc *BC.* Yet both [*AEB* and *BEC*] must be known. But angles *AEN, BEO,* and *CEP,* which indicate the differences [between the apparent and mean motions], cannot be ascertained before the determination of arcs *AF, FB,* and *FBC,* which are similar to the arcs of the epicyclet. These values are so interconnected that they are unknown or known at the same time. Hence, lacking the means of deriving them, astronomers relied on *a posteriori* arguments and detours to what could not be reached directly and *a priori,* as happens in the squaring of the circle and many other problems. Thus in this investigation Ptolemy elaborated a verbose treatment and an enormous mass of calculations. To review these words and numbers is, in my judgement, burdensome and unnecessary, since in my discussion, which follows, I shall adopt practically the same procedure.

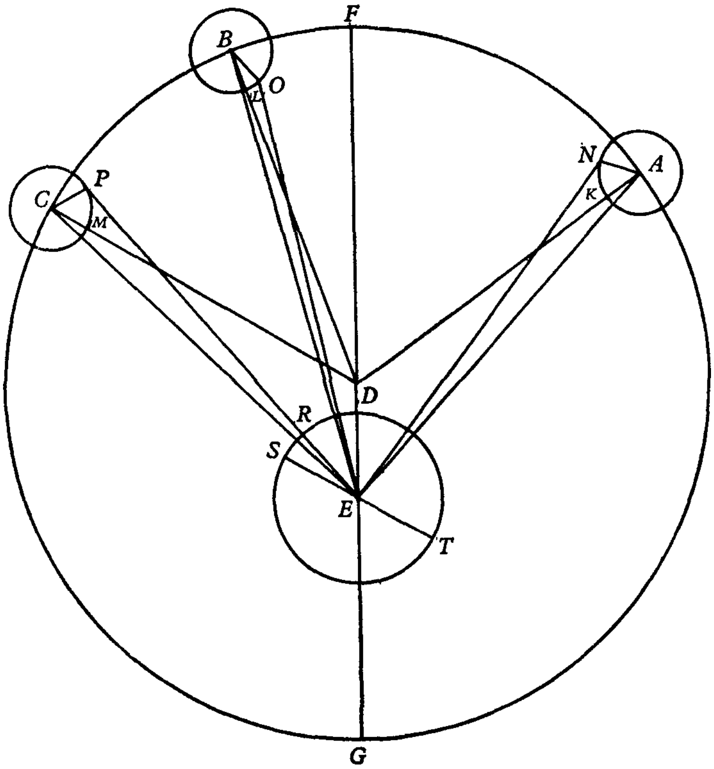
Reviewing his calculations, in the end [*Syntaxis*, XI, 5] he found arc *AF* = 57° 1′, *FB* = 18° 37′, *FBC* = 56 $^1/_2$°, and *DE* = the distance between the centers = 6^p 50′, whereof *DF* = 60^p. But with *DF* = 10,000 on our numerical scale, *DE* = 1139. Of this total, I have accepted $^3/_4$ for *DE* = 854, and I have assigned the remaining $^1/_4$ = 285 to the epicyclet. Assuming these values and borrowing them for my hypothesis, I shall show that they agree with the observed phenomena.

In the first opposition there are given in triangle *ADE* side *AD* = $10{,}000^p$, *DE* = 854^p, and angle *ADE* as the supplement of *ADF* [= 57° 1′]. From these values, in accordance with the theorems on Plane Triangles, *AE* = $10{,}489^p$ of the same units, while the remaining angles *DEA* = 53° 6′, and *DAE* = 3° 55′, when 4 right angles = 360°. But angle *KAN* = *ADF* = 57° 1′. Therefore the whole angle *NAE* = 60° 56′ [= 57° 1′+3° 55′]. Consequently in triangle *NAE* two sides are given: *AE* = $10{,}489^p$, and *NA* = 285^p, whereof *AD* = $10{,}000^p$, as well as angle *NAE*. Angle *AEN* will also be given = 1° 22′, and the remaining angle *NED* [=*AED*−*AEN*] = 51° 44′ [= 53° 6′- 1° 22′], whereof 4 right angles = 360°.

The situation is similar in the second opposition. For in triangle *BDE*, side *DE* is given = 854^p, whereof *BD* = $10{,}000^p$; and angle *BDE* = supplement of *BDF* = 161° 22′ [= 180°- 18° 38′]. The angles and sides of this triangle too will be given: side *BE* = $10{,}812^p$, whereof *BD* = $10{,}000^p$; angle *DBE* = 1° 27′; and the remaining angle *BED* = 17° 11′ [= 180°-(161° 22′+1° 27′)]. But angle *OBL* = *BDF* = 18° 38′. Therefore the whole angle *EBO* [= *DBE*+*OBL*] = 20° 5′ [= 18° 38′+1° 27′]. In triangle *EBO*, accordingly, besides angle *EBO* two sides are given: *BE* = $10{,}812^p$ and *BO* = 285^p. In accordance with the theorems on Plane Triangles, the remaining angle *BEO* is given = 32′. Hence *OED* = the remainder [when *BEO* is subtracted from *BED*] = 16° 39′ [= 17° 11′-32′].

Likewise in the third opposition, in triangle *CDE*, as before, two sides, *CD* and *DE*, are given, as well as angle *CDE* [the supplement of] 56° 29′ [= 123° 31′]. In accordance with Theorem IV on Plane Triangles, base *CE* is given = $10{,}512^p$, whereof *CE* = $10{,}000^p$; angle *DCE* = 3° 53′; and the remaining angle *CED* = 52° 36′ [= 180°- (3° 53′+123° 31′)]. Therefore the whole angle *ECP* = 60° 22′ [= 3° 53′+56° 29′], with 4 right angles = 360°. Then also in triangle *ECP* two sides are given, besides angle *ECP*. Angle *CEP* is also given = 1° 22′. Hence the remaining angle *PED* [= *CED*−*CEP*] = 51° 14′ [= 52° 36′- 1° 22′]. Accordingly the whole angle *OEN* [= *NED*+*BED*−*BEO*] of the apparent motion amounts to 68° 23′ [= 51° 44′+17° 11′- 32′], and *OEP* to 34° 35′ [= *PED*−*OED* = 51° 14′-16° 39′], in agreement with the observations. *F*, the place of the eccentric's higher apse, is 226° 20′ from the head of the Ram. To this figure add 6° 40′ for the precession of the vernal equinox as it was then, and the apse reaches 23° within the Scorpion, in conformity with Ptolemy's conclusion [*Syntaxis*, XI, 5]. For the planet's apparent place in this third opposition (as was mentioned) = 277° 37′. From this figure subtract 51° 14′ = *PEF*, the angle of the apparent motion, as has been shown, and the remainder is the place of the eccentric's higher apse in 226° 23′.

Now also describe the earth's annual circle *RST*, which will intersect line *PE* in point *R*. Draw diameter *SET* parallel to *CD*, the line of the planet's mean motion. Therefore angle *SED* = *CDF*. Hence angle *SER*, the difference and pros-

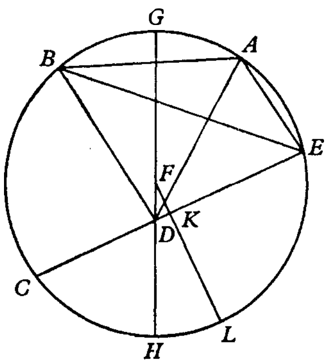


thaphaeresis between the apparent and mean motions, that is, between angles *CDF* and *PED*, = 5° 16′ [= 56° 30′ − 51° 14′]. Between the mean and true motions in parallax the difference is the same. When this is subtracted from a semicircle, it leaves arc *RT* = 174° 44′ [= 180° − 5° 16′] as the uniform motion in parallax from point *T*, the assumed origin, that is, from the mean conjunction of the sun and planet until this third "end of the night" or true opposition of the earth and planet.

Therefore we now have at the hour of this [third] observation, namely, in Hadrian's regnal year 20 = 136 C.E., 8 July, 11 hours after midnight, Saturn's anomalous motion from its eccentric's higher apse = 56 $^1/_2$°, and the mean motion in parallax = 174° 44′. The establishment of these values will be useful for what follows.

THREE OTHER MORE RECENTLY OBSERVED OPPOSITIONS OF SATURN *Chapter 6*

The computation of Saturn's motion as reported by Ptolemy differs, however, not a little from our times, nor could it be understood at once where the error lay hidden. Hence I was compelled to perform new observations, from which once more I took three oppositions of Saturn. The first occurred 1 $^1/_5$ hours before midnight on 5 May 1514 C.E., when Saturn was found at 205° 24′. The second happened at noon on 13 July 1520 C.E., [with Saturn] at 273° 25′. The third took place at 6 $^2/_5$ hours after midnight on 10 October 1527 C.E., when Saturn appeared at 7′ east of the Ram's horn. Then between the first and second oppositions there are 6 Egyptian years, 70 days, 33 day-minutes, during which Saturn's apparent motion is 68° 1′ [= 273° 25′ – 205° 24′]. From the second opposition to the third there are 7 Egyptian years, 89 days, 46 day-minutes, and the planet's apparent motion is 86° 42′ [= 360° 7′ – 273° 25′]. In the first interval its mean motion is 75° 39′; and in the second interval, 88° 29′. Therefore, in seeking the higher apse and eccentricity we must operate at first according to Ptolemy's procedure [*Syntaxis,* X, 7], as if the planet moved on a simple eccentric. Although this arrangement is not adequate, nevertheless by conforming to it we shall more easily reach the truth.



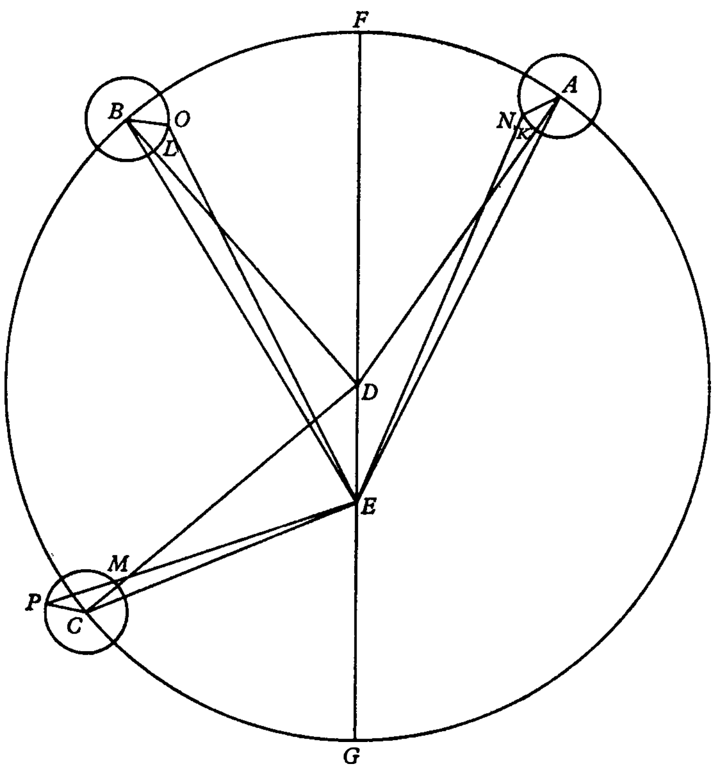
Hence take *ABC* as if it were the circle on which the planet moves uniformly. Let the first opposition be at point *A,* the second at *B,* and the third at *C.* Within *ABC* let the center of the earth's circle be *D.* Joining *AD, BD,* and *CD,* extend any one of them in a straight line to the opposite side of the circumference, for instance, *CDE.* Join *AE* and *BE.* Then angle *BDC* is given = 86° 42′. Hence with 2 central right angles = 180°, supplementary angle *BDE* = 93° 18′ [= 180° – 86° 42′], but 186° 36′ with 2 right angles = 360°. Angle *BED,* intercepting arc *BC,* = 88° 29′. Hence [in triangle *BDE*] the remaining angle *DBE* = 84° 55′ [= 360° – (186° 36′ + 88° 29′)]. Then in triangle *BDE,* the angles being given, the sides are obtained from the Table [of the Straight Lines Subtended in a Circle]: *BE* = $19,953^p$, and *DE* = $13,501^p$, whereof the diameter of the circle circumscribing the triangle = $20,000^p$. Similarly in triangle *ADE,* since *ADC* is given = 154° 43′ [= 68° 1′ + 86° 42′] with 2 right angles = 180°, supplementary angle *ADE* = 25° 17′ [= 180° – 154° 43′]. But with 2 right angles = 360°, *ADE* = 50° 34′. In those units angle *AED,* intercepting arc *ABC,* = 164° 8′ [= 75° 39′ + 88° 29′], and the remaining angle *DAE* = 145° 18′ [= 360° – (50° 34′ + 164° 8′)]. Therefore the sides too are known: *DE* = $19,090^p$, and *AE* = 8542^p, whereof the diameter of the circle circumscribed around triangle *ADE* = $20,000^p$. But in units whereof *DE* was given = $13,501^p$ and *BE* = $19,953^p$, *AE* will be 6041^p. Then in triangle *ABE* also, these two sides, *BE* and *EA,* are given, as well as angle *AEB,* inter-cepting arc *AB,* = 75° 39′. Hence, in accordance with the theorems on Plane Triangles, *AB* = $15,647^p$, whereof *BE* = $19,968^p$. But as *AB,* subtending a given arc, = $12,266^p$ whereof the eccentric's diameter = $20,000^p$, *EB* = $15,664^p$, and *DE* = $10,599^p$. Through chord *BE,* then, arc *BAE* is given = 103° 7′. Therefore the whole of *EABC* = 191° 36′ [= 103° 7′ + 88° 29′]. *CE,* the rest of the circle, = 168° 24′; hence its chord *CDE* = $19,898^p$; and *CD,* the remainder [when *DE* = 10,599 is subtracted from *CDE*], = 9299^p.

Now if *CDE* were the eccentric's diameter, obviously the places of the higher and lower apse would lie on it, and the distance between the centers [of the eccentric and the earth's grand circle] would be known. But because segment *EABC* is larger [than a semicircle], the center [of the eccentric] will fall within it. Let it be *F.* Through it and *D* draw diameter *GFDH,* and *FKL* perpendicular to *CDE.*

Clearly, rectangle $CD \times DE$ = rectangle $GD \times DH$. But rectangle $GD \times DH + (FD)^2 = (\frac{1}{2}GDH)^2 = (FDH)^2$. Therefore $(\frac{1}{2}\text{ diameter})^2$ – rectangle $GD \times DH$ or rectangle $CD \times DE = (FD)^2$. Then *FD* will be given as a length = 1200^p, whereof radius *GF* = $10,000^p$. But in units whereof *FG* = 60^p, *FD* = 7^p 12′, slightly different from Ptolemy [*Syntaxis,* XI, 6: 6^p 50′]. But *CDK* = 9949^p = ½ of the whole of *CDE* [= $19,898^p$]. *CD* has been shown = 9299^p. Therefore the remainder $DK = 650^p$ [$= 9949^p - 9299^p$], whereof GF is assumed $= 10{,}000^p$, and $FD = 1200^p$. But in units whereof $FD = 10{,}000^p$, $DK = 5411^p$ = half the chord subtending twice the angle DFK. The angle [DFK] $= 32°\,45'$, with 4 right angles $= 360°$. As an angle at the center of the circle, it subtends a similar quantity on arc HL. But the whole of $CHL = \frac{1}{2}\,CLE$ [$168°\,24'$] $\cong 84°\,13'$. Therefore CH, the remainder [when $HL = 32°\,45'$ is subtracted from $CHL = 84°\,13'$], extending from the third opposition to the perigee $= 51°\,28'$. Subtract this figure from the semicircle, and the remaining arc $CBG = 128°\,32'$, extending from the higher apse to the third opposition. Since arc $CB = 88°\,29'$, the remainder BG [when CB is subtracted from $CBG = 128°\,32'$] $= 40°\,3'$, extending from the higher apse to the second opposition. Then the following arc $BGA = 75°\,39'$ furnishes AG, which extends from the first opposition to apogee G, $= 35°\,36'$ [$= 75°\,39' - 40°\,3'$].

Now let ABC be a circle, with diameter $FDEG$, center D, apogee F, perigee G, arc $AF = 35°\,36'$, $FB = 40°\,3'$, and $FBC = 128°\,32'$. Of the previously demonstrated distance [1200^p] between the centers [of Saturn's eccentric and the earth's grand circle], take $\frac{3}{4}$ for $DE = 900^p$. With the remaining $\frac{1}{4} = 300^p$, whereof radius FD [of Saturn's eccentric] $= 10{,}000^p$, as radius, describe an epicyclet around A, B, and C as centers. Complete the diagram in accordance with the assumed conditions.

If we wish to derive Saturn's observed places from the foregoing arrangements by the method explained above and soon to be repeated, we shall find some discrepancies. To speak briefly, lest I overburden the reader and appear to have worked

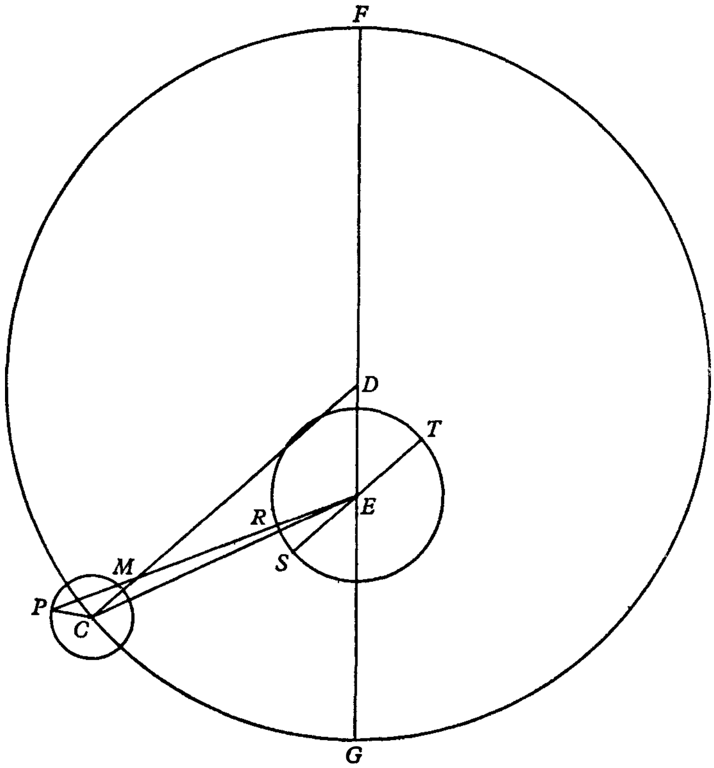


harder in showing bypaths than in indicating the right road forthwith, the foregoing data must lead through the solution of the triangles to angle $NEO = 67° 35'$ and the other angle $OEM = 87° 12'$. The latter is $^1/_2$° bigger than the apparent [angle $= 86° 42'$], and the former is $26'$ smaller [than $68° 1'$]. We find mutual agreement only by advancing the apogee a little [$3° 14'$] and setting $AF = 38° 50'$ [instead of $35° 36'$], and then arc $FB = 36° 49'$ [$= 40° 3' - 3° 14'$]; $FBC = 125° 18'$ [$= 128° 32' - 3° 14'$]; DE, the distance between the centers $= 854^p$ [instead of 900^p]; and the epicyclet's radius $= 285^p$ [instead of 300^p], whereof $FD = 10{,}000^p$. These figures nearly agree with Ptolemy, whose values were set forth above [V, 5].

The consistency of the above data with the phenomena and the three observed oppositions will become clear. For in the first opposition, in triangle ADE side DE is given $= 854^p$, whereof $AD = 10{,}000^p$. Angle $ADE = 141°10'$, and together with $ADF =$ [$38° 50'$] makes 2 right angles at the center. From the foregoing information the remaining side AE is shown $= 10{,}679^p$, whereof radius $FD = 10{,}000^p$. The remaining angles $DAE = 2° 52'$, and $DEA = 35° 58'$. Similarly in triangle AEN, since $KAN = ADF$ [$= 38° 50'$], the whole of $EAN = 41° 42'$ [$= DAE + KAN = 2° 52' + 38° 50'$], and side $AN = 285^p$, whereof $AE = 10{,}679^p$. Angle AEN will be shown $= 1° 3'$. But the whole of DEA consists of $35° 58'$. Hence DEN, the remainder [when AEN is subtracted from DEA], will be $34° 55'$ [$= 35° 58' - 1° 3'$].

Likewise in the second opposition, triangle BED has two sides given (for $DE = 854^p$, whereof $BD = 10{,}000^p$) as well as angle BDE [$= 180° - (BDF = 36°49') = 143°11'$]. Therefore $BE = 10{,}697^p$, angle $DBE = 2° 45'$, and the remaining angle $BED = 34° 4'$. But $LBO = BDF$ [$= 36° 49'$]. Therefore the whole of $EBO = 39° 34'$ at the center [$= DBO + DBE = 36° 49' + 2° 45'$]. Its enclosing sides are given: $BO = 285^p$, and $BE = 10{,}697^p$. From this information BEO is shown $= 59'$. When this value is subtracted from angle BED [$= 34° 4'$], the remainder $OED = 33° 5'$. But it has already been shown in the first opposition that angle $DEN = 34° 55'$. Therefore the whole angle OEN [$= DEN + OED$] $= 68°$ [$= 34°55' + 33°5'$]. It revealed the distance of the first opposition from the second, in agreement with the observations [$= 68°1'$].

A similar demonstration will apply to the third opposition. In triangle CDE angle CDE is given $= 54° 42'$ [$= 180° - (FDC = 125° 18')$], as well as sides CD and DE previously established [$= 10{,}000$; 854]. From this information the third side EC is shown $= 9532^p$, and the remaining angles $CED = 121° 5'$ and $DCE = 4° 13'$. Therefore the whole of $PCE = 129° 31'$ [$= 4° 13' + 125° 18'$]. Furthermore, in triangle EPC two sides, PC and CE, are given [$= 285$; 9532] as well as angle PCE [$= 129°31'$]. From this information angle PEC is shown $= 1° 18'$. When this figure is subtracted from CED [$= 121° 5'$], it will leave as a remainder angle $PED = 119° 47'$, the distance from the eccentric's higher apse to the planet's place in the third opposition. It has been shown, however, that in the second opposition there were $33° 5'$ [from the eccentric's higher apse to the planet's place]. Therefore, between Saturn's second and third oppositions there remain $86° 42'$ [$= 119° 47' - 33° 5'$]. This figure too is recognized to be in agreement with the observations. Saturn's place, however, was found by observation to be at that time $8'$ east of the Ram's first star, accepted as the zero point. The distance from Saturn's place to the eccentric's lower apse has been shown to be $60° 13'$



[= 180°– 119° 47′]. Therefore the lower apse was at about $60\,{}^1/_3{}^\circ$ [≅ 60° 13′+8′], and the place of the higher apse diametrically opposite at $240\,{}^1/_3{}^\circ$.

Now describe the earth's grand circle *RST*, with center *E*. Draw its diameter *SET* parallel to *CD*, the line of the [planet's] mean motion (by making angle *FDC* = *DES*). Then the earth and our place of observation will be on line *PE*, say, at point *R*. Angle *PES* [= *EMD*] or arc *RS* = the difference between angle *FDC* and *DEP* = the difference between the [planet's] uniform and apparent motions, has been shown = 5° 31′ [(*CES* = *DCE*)+*PEC* = 4°13′+1°18′]. When this figure is subtracted from the semicircle, the remainder, arc *RT*, = 174° 29′ = the planet's distance from the grand circle's apogee *T* = the sun's mean place. Thus we have the demonstration that at $6\,{}^2/_5$ hours after midnight on 10 October 1527 C.E., Saturn's motion in anomaly from the eccentric's higher apse = 125° 18′; the motion in parallax = 174° 29′; and the place of the higher apse = 240° 21′ from the first star of the Ram in the sphere of the fixed stars.

ANALYSIS OF SATURN'S MOTION *Chapter 7*

At the time of the last of Ptolemy's three observations, it has been shown [V, 5], Saturn's motion in parallax was at 174° 44′, and the place of its eccentric's higher apse was 226° 23′ from the beginning of the constellation Ram. Therefore, during the time intervening between the two observations [Ptolemy's last and Copernicus' last], it is clear, Saturn completed 1344 revolutions of its uniform motion in parallax minus ${}^1/_4{}^\circ$. From 1 hour before noon on the 24th day of the Egyptian month

Mesori in Hadrian's year 20, until the later observation at 6 o'clock on 10 October 1527 C.E., there are 1392 Egyptian years, 75 days, 48 day-minutes. For this time, furthermore, if we wish to obtain the motion from the Table [of Saturn's Parallactic Motion], we shall similarly find $5 \times 60°$ plus $59° 48'$ beyond 1343 revolutions of the parallax. Therefore, what was asserted [in V, 1] about Saturn's mean motions is correct.

In that [same] interval, moreover, the sun's simple motion is $82° 30'$. From this figure subtract $359° 45'$, and for Saturn's mean motion the remainder is $82° 45'$. This value has now accumulated in Saturn's 47th [sidereal] revolution, in agreement with the computation. Meanwhile the place of the eccentric's higher apse has also advanced $13° 58'$ [$= 240° 21' - 226° 23'$] in the sphere of the fixed stars. Ptolemy believed that the apse was fixed in the same way [as the stars], but now it is evident that the apse moves about 1° in 100 years.

DETERMINING SATURN'S PLACES *Chapter 8*

From the beginning of the Christian era to Ptolemy's observation at 1 hour before noon on the 24th day of the month Mesori in Hadrian's year 20, there are 135 Egyptian years, 222 days, 27 day-minutes. During that time Saturn's motion in parallax is $328° 55'$. When this figure is subtracted from $174° 44'$, the remainder $205° 49'$ gives the extent of the distance of the sun's mean place from Saturn's mean place, and this is the latter's motion in parallax at midnight preceding 1 January [1 C.E.] The motion in 775 Egyptian years, 12 ½ days, from the 1st Olympiad to this place [at the beginning of the Christian era] includes, in addition to complete revolutions, $70° 55'$. When this figure is subtracted from $205° 49'$, the remainder $134° 54'$ marks the beginning of the Olympiads at noon on the first day of the month Hecatombaeon. From that place, after 451 years, 247 days, there are, in addition to complete revolutions, $13° 7'$. When this figure is added to the previous value [$134° 54'$], the sum gives $148° 1'$ for the place of Alexander the Great at noon on the first day of the Egyptian month Thoth. For Caesar, in 278 years, 118 ½ days, the motion is $247° 20'$, making the place $35° 21'$ at midnight preceding 1 January [45 B. C.]

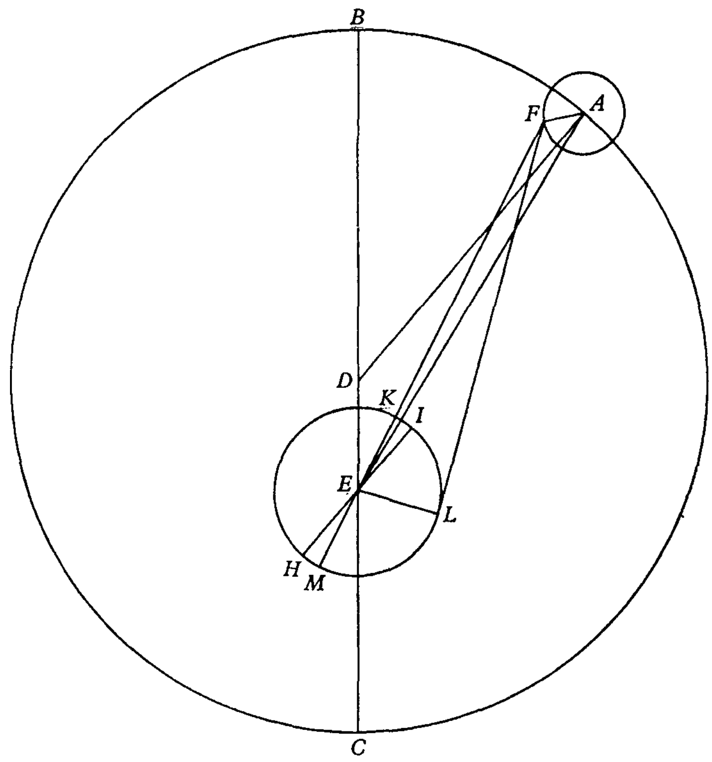
SATURN'S PARALLAXES ARISING FROM THE EARTH'S ANNUAL REVOLUTION, AND SATURN'S DISTANCE [FROM THE EARTH] *Chapter 9*

Saturn's uniform and apparent motions in longitude are set forth in the foregoing manner. For, the other phenomena to which it is subject are parallaxes, as I have called them [V, 1], arising from the earth's annual orbit. For just as the earth's size as compared with its distance from the moon creates parallaxes, so also the orbit in which it revolves annually has to produce parallaxes in the five planets. But because of the orbit's size the planetary parallaxes are far more conspicuous. These parallaxes cannot be ascertained, however, unless the planet's altitude is known previously. It is possible, nevertheless, to obtain the altitude from any observation of the parallax.

I made such an observation of Saturn at 5 uniform hours after midnight on

24 February 1514 C.E. Saturn was seen in a straight line with stars in the Scorpion's forehead, that is, the second and third [stars in that constellation], which have the same longitude, 209° in the sphere of the fixed stars. Through them, accordingly, Saturn's place was known. From the beginning of the Christian era until this hour there are 1514 Egyptian years, 67 days, 13 day-minutes. Hence the sun's mean place was computed to be 315° 41′; Saturn's parallactic anomaly, 116° 31′; and therefore Saturn's mean place was 199° 10′, and the place of the eccentric's higher apse was about 240 $^1/_3$°.

Now, in accordance with the previous model, let *ABC* be the eccentric, with its center at *D*. On its diameter *BDC* let *B* be the apogee, *C* the perigee, and *E* the center of the earth's orbit. Join *AD* and *AE*. With *A* as center, and radius = $^1/_3$ *DE,* describe the epicyclet. On it let *F* be the planet's place, making angle *DAF* = *ADB*. Draw *HI,* as though it were in the same plane as circle *ABC,* through *E,* the center of the earth's orbit. As the orbit's diameter, let *HI* be parallel to *AD,* so that *H* is understood to be the point on the earth's orbit farthest from the planet, and *I* is the nearest point. On the orbit take arc *HL* = 116° 31′ in agreement with the computation of the parallactic anomaly. Join *FL* and *EL*. Extend *FKEM* to intersect both sides of the orbit's circumference. Angle *ADB* = 41° 10′ = *DAF,* by hypothesis. Supplementary angle *ADE* = 138° 50′. *DE* = 854^p, whereof *AD* = 10,000^p. These data show that in triangle *ADE,* the third side *AE* = 10,667^p, angle *DEA* = 38° 9′, and the remaining angle *EAD* = 3° 1′. Therefore the whole of *EAF* [= *EAD*+*DAF*] = 44° 11′ [= 3° 1′+41° 10′]. Thus again in triangle *FAE,* side *FA* is given = 285^p, whereof *AE* also is given [= 10,667^p]. The remaining side *FKE* will be shown = 10,465^p, and angle



$AEF = 1° 5'$. Therefore the entire difference or prosthaphaeresis between the planet's mean and true places evidently $= 4° 6'$ = angle DAE + angle AEF [$= 3° 1' + 1° 5'$]. For this reason, had the earth's place been K or M, Saturn's place would have appeared to be 203° 16′ from the constellation of the Ram, as though it had been observed from center E. But with the earth at L, Saturn was seen at 209°. The difference of 5° 44′ [= 209°– 203° 16′] is the parallax, indicated by angle KFL. But arc HL in the [earth's] uniform motion = 116° 31′ [= Saturn's parallactic anomaly]. From this figure subtract the prosthaphaeresis HM. The remainder ML = 112° 25′ [= 116° 31′– 4° 6′], and LIK, the rest [of the semicircle] = 67° 35′ [= 180°–112° 25′]. From this information angle KEL is also obtained [= 67° 35′]. Therefore in triangle FEL, the angles being given [EFL = 5°44′; FEL = 67°35′; ELF = 106°41′], the ratio of the sides is also given, in units whereof EF = 10,465^p. In these units EL = 1090^p, whereof AD or BD = 10,000^p. But if BD = 60^p in accordance with the procedure of the ancients, EL = 6^p 32′, which also differs very slightly from Ptolemy's conclusion. Therefore the whole of BDE = 10,854^p, and CE = the rest of the diameter = 9146^p [= 20,000 – 10,854]. However, the epicyclet at B always subtracts 285^p from the planet's height, but at C adds the same quantity, that is, $^1/_2$ of its diameter. Therefore Saturn's greatest distance from center E = 10,569^p [= 10,854 – 285], and its least distance [from E] = 9431^p [9146+285], whereof BD = 10,000^p. According to this ratio, the height of Saturn's apogee = 9^p 42′, whereof the radius of the earth's orbit = 1^p, and the height of Saturn's perigee = 8^p 39′. From this information Saturn's larger parallaxes can be clearly obtained by the procedure explained in connection with the moon's small parallaxes [IV, 22, 24]. Saturn's greatest parallaxes = 5^p 55′ with the planet at apogee, and with the planet at perigee = 6^p 39′. The difference between these two values = 44′, which occurs when the lines coming from the planet are tangent to the [earth's] orbit. Through this example every individual variation in Saturn's motion is found. I shall set these variations forth hereafter at the same time for these five planets jointly [V, 33].

EXPOSITIONS OF JUPITER'S MOTION *Chapter 10*

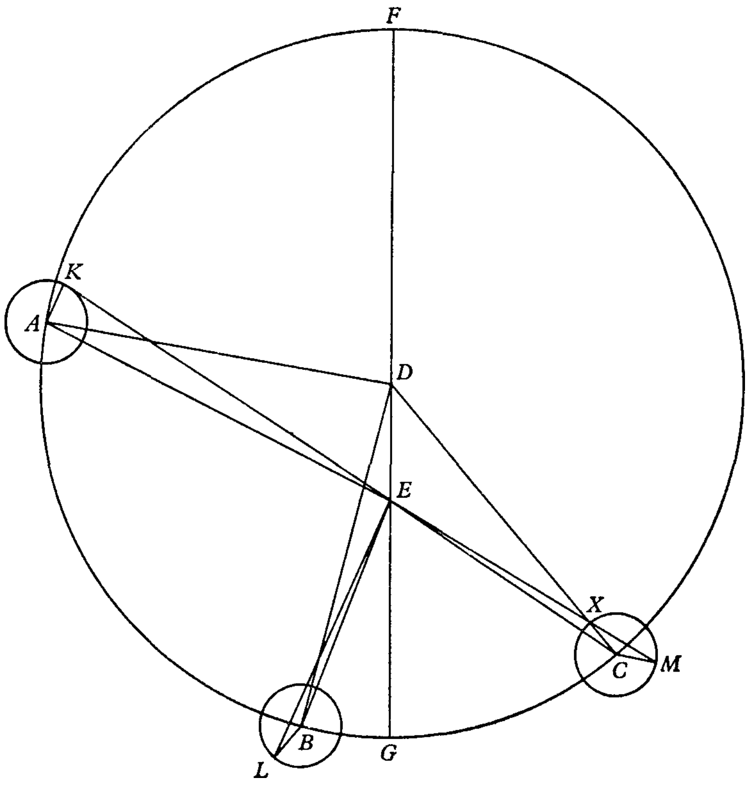
Having finished Saturn, I shall use the same procedure and order also for expounding Jupiter's motion. First, I shall repeat three places reported and analyzed by Ptolemy [*Syntaxis*, XI, 1]. I shall so reconstitute them by means of the previously exhibited transformation of the circles that they are the same as, or not much different from, his places.

The first of his oppositions occurred 1 hour before the midnight following the 1st day of the Egyptian month Epiphi in Hadrian's year 17 at 23° 11′ within the Scorpion [= 223°11′], according to Ptolemy, but at 226° 33′ after the precession of the equinoxes [= 6° 38′] is subtracted. He recorded the second opposition 2 hours before the midnight following the 13th day of the Egyptian month Phaophi in Hadrian's year 21 at 7° 54′ within the Fishes; in the sphere of the fixed stars, however, this was 331° 16′ [= 337° 54′– 6° 38′]. The third opposition happened 5 hours after the midnight following the 20th day of the month Athyr in Antoninus [Pius'] 1st year at 7° 45′ in the sphere of the fixed stars [= 14° 23′– 6° 38′].

Accordingly, from the first opposition to the second there are 3 Egyptian years 106 days 23 hours, and the planet's apparent motion = 104° 43′ [= 331° 16′–226° 33′]. From the second opposition to the third the interval is 1 year 37 days 7 hours, and the planet's apparent motion = 36° 29′ [= 360°+7° 45′–331° 16′]. In the first period of time the [planet's] mean motion = 99° 55′; and in the second, 33° 26′. Ptolemy found the eccentric's arc from the higher apse to the first opposition = 77° 15′; the following arc, from the second opposition to the lower apse = 2° 50′; and from there to the third opposition = 30° 36′; the entire eccentricity = $5\,{}^{1}/_{2}{}^{p}$, whereof the radius = 60^{p}; but if the radius = $10{,}000^{p}$, the eccentricity = 917^{p}. All these values agreed almost exactly with the observations.

Now let *ABC* be a circle, whose arc *AB* from the first opposition to the second contains the aforementioned 99° 55′, and *BC* = 33° 26′. Through center *D* draw diameter *FDG* so that, starting from the higher apse *F*, *FA* = 77° 15′, *FAB* = 177° 10′ [= 180°–2° 50′], and *GC* = 30° 36′. Take *E* as the center of the earth's circle, and let distance *DE* = 687^{p} = $^{3}/_{4}$ of [Ptolemy's eccentricity =] 917^{p}. With $^{1}/_{4}$ [of 917^{p}] = 229^{p} [as radius], describe an epicyclet around points *A*, *B*, and *C*. Join *AD*, *BD*, *CD*, *AE*, *BE*, and *CE*. In the epicyclets join *AK*, *BL*, and *CM*, so that angles *DAK*, *DBL*, *DCM* = *ADF*, *FDB*, *FDC*. Lastly, join *K*, *L*, and *M* by straight lines to *E* also.

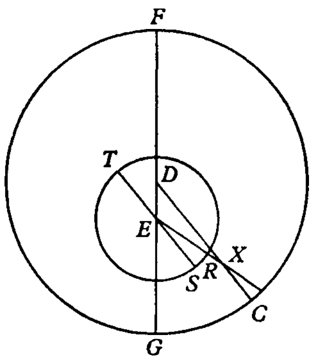
In triangle *ADE*, angle *ADE* is given = 102° 45′ because *ADF* is given [as its supplement = 77° 15′]; side *DE* = 687^{p}, whereof *AD* = $10{,}000^{p}$; the third side *AE* will be shown = $10{,}174^{p}$; angle *EAD* = 3° 48′; the remaining angle *DEA* = 73° 27′; and the whole of *EAK* = 81° 3′ [= *EAD*+(*DAK* = *ADF*) =



3° 48′ +77° 15′]. Therefore in triangle *AEK* likewise, two sides are given: EA = 10,174ᵖ, whereof AK = 229ᵖ, and since [the included] angle *EAK* is also given, angle *AEK* will be known = 1° 17′. Accordingly, the remaining angle *KED* = 72° 10′ [= *DEA*−*AEK* = 73° 27′− 1° 17′].

A similar demonstration will be made in triangle *BED*. For, sides *BD* and *DE* still remain equal to the previous [corresponding members], but angle *BDE* is given = 2° 50′ [=180°− (*FDB* = 177°10′)]. Therefore base *BE* will emerge = 9314ᵖ whereof *DB* = 10,000ᵖ, and angle *DBE* = 12′. Thus again in triangle *ELB* two sides are given [*BE*, *BL*] and the whole angle *EBL* [= (*DBL* = *FDB*) +*DBE*] = 177° 22′ [= 177° 10′+12′]. Angle *LEB* will also be given = 4′. When the sum of 16′ [= 12′ + 4′] is subtracted from angle *FDB* [= 177° 10′], the remainder 176° 54′ = angle *FEL*. From it subtract *KED* = 72° 10′, and the remainder = 104° 44′ = *KEL*, in almost exact agreement with the angle of the apparent motion between the first and second of the observed terminal points [= 104° 43′].

In like manner at the third position, in triangle *CDE* two sides, *CD* and *DE*, are given [= 10,000; 687] as well as angle *CDE* = 30° 36′. Base *EC* will be shown in the same way = 9410ᵖ, and angle *DCE* = 2° 8′. Hence the whole of *ECM* = 147° 44′ in triangle *ECM*. Thereby angle *CEM* is shown = 39′. Exterior angle *DXE* = interior angle *ECX* + opposite interior angle *CEX* = 2° 47′ [= 2° 8′+ 39′] = *FDC*−*DEM* [*FDC* = 180°− 30° 36′ = 149° 24′; *DEM* = 149° 24′− 2° 47′ =146° 37′]. Hence *GEM* = 180°−*DEM* = 33° 23′. The whole angle *LEM*, intervening between the second opposition and the third = 36° 29′, likewise in agreement with the observations. But this third opposition, 33° 23′ east of the lower apse (as was demonstrated), was found at 7° 45′. Hence the place of the higher apse is shown by the remainder of the semicircle to be 154° 22′ [= 180°− (33° 23′− 7° 45′)] in the sphere of the fixed stars.

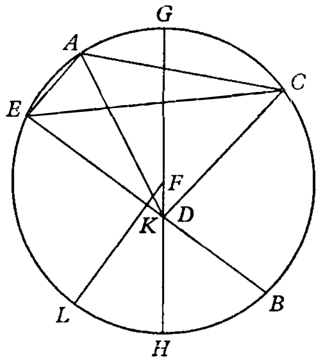


Now around *E* describe the earth's annual orbit *RST*, with diameter *SET* parallel to line *DC*. Angle *GDC* was shown = 30° 36′ = *GES*. Angle *DXE* = *RES* = arc *RS* = 2° 47′ = the planet's distance from the orbit's mean perigee. Thereby the whole of *TSR* = [the planet's distance] from the orbit's higher apse emerges = 182° 47′.

Thus it is confirmed that at this hour of Jupiter's third opposition, reported at 5 hours after the midnight following the 20th day of the Egyptian month Athyr in Antoninus [Pius'] year 1, the planet Jupiter in its anomaly of parallax was at 182° 47′; its uniform place in longitude = 4° 58′ [=7°45′− 2°47′]; and the place of the eccentric's higher apse = 154° 22′. All these results are in absolutely complete agreement also with my hypothesis of a moving earth and uniform motion.

THREE OTHER MORE RECENTLY OBSERVED OPPOSITIONS OF JUPITER — *Chapter 11*

To the three positions of the planet Jupiter as reported long ago and analyzed in the foregoing manner, I shall append three others, which I too observed with the greatest care in oppositions of Jupiter. The first occurred 11 hours after the midnight preceding 30 April 1520 C.E., at 200° 28′ in the sphere of the fixed stars. The second happened 3 hours after midnight on 28 November 1526 C.E., at 48° 34′. The third took place 19 hours after midnight on 1 February 1529 C.E., at 113° 44′. From the first opposition to the second, there are 6 years 212 days 40 day-minutes, during which Jupiter's motion appeared to be 208° 6′ [= 360° + 48° 34′ − 200° 28′]. From the second opposition to the third, there are 2 Egyptian years 66 days 39 day-minutes, and the planet's apparent motion = 65° 10′ [= 113° 44′ − 48° 34′]. In the first period of time, however, the uniform motion = 199° 40′, and in the second period, 66° 10′.



To illustrate this situation, describe an eccentric circle ABC, on which the planet is regarded as moving simply and uniformly. Designate the three observed places as A, B, and C in the order of the letters so that arc AB = 199° 40′, BC = 66° 10′, and therefore AC = the rest of the circle = 94° 10′. Also take D as the center of the earth's annual orbit. To D join AD, BD, and CD. Prolong any one of these, say DB, in a straight line BDE to both sides of the circle. Join AC, AE, and CE.

Angle BDC of the apparent motion = 65° 10′, with 4 right angles at the center = 360°. Supplementary angle CDE = 114° 50′ [= 180° − 65° 10′] in such degrees; but with 2 right angles (as at the circumference) = 360°, CDE = 229° 40′ [= 2 × 114° 50′]. Angle CED, intercepting arc BC, = 66° 10′. Therefore [in triangle CDE] the remaining angle DCE = 64° 10′ [= 360° − (229° 40′ + 66° 10′)]. Accordingly, in triangle CDE, the angles being given, the sides are given: CE = 18,150^p, and ED = 10,918^p, whereof the diameter of the circle circumscribed around the triangle = 20,000^p.

A similar demonstration holds for triangle ADE. Angle ADB is given = 151° 54′ = the remainder of the circle, from which is subtracted the given distance from the first opposition to the second [= 208° 6′]. Therefore, supplementary angle ADE = 28° 6′ [= 180° − 151° 54′] as a central angle, but at the circumference = 56° 12′ [= 2 × 28° 6′]. Angle AED, intercepting arc BCA [= BC+CA] = 160° 20′ [= 66° 10′ + 94° 10′]. The remaining [inscribed] angle EAD [in triangle ADE] = 143° 28′ [= 360° − (56° 12′ + 160° 20′)]. From this information, side AE emerges = 9420^p, and ED = 18,992^p, whereof the diameter of the circle circumscribed around triangle ADE = 20,000^p. But when ED = 10,918^p, AE = 5415^p in units whereof CE = 18,150^p also was known.

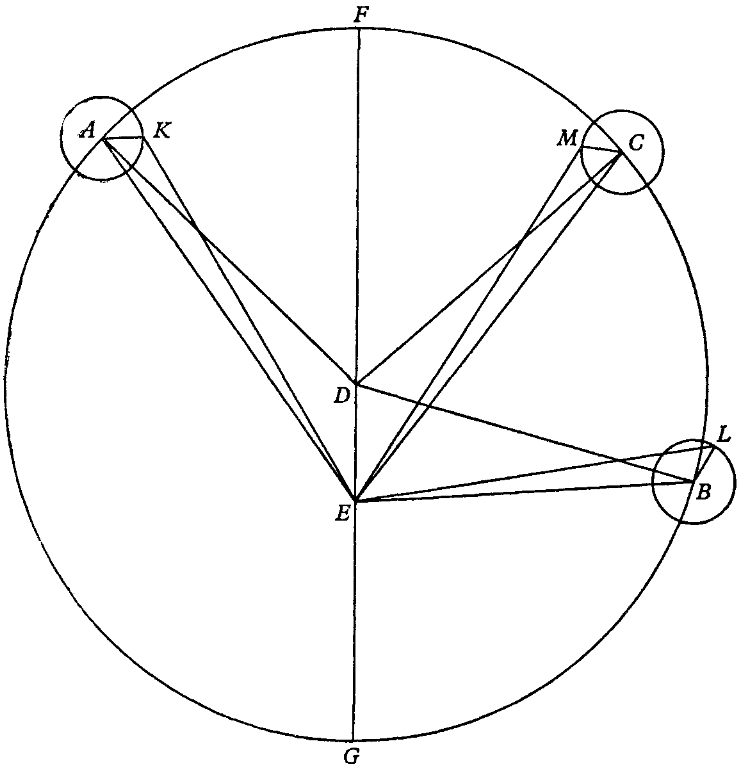
Hence we again have in triangle EAC two sides, EA and EC, given [5415; 18,150] as well as [the included] angle AEC, intercepting arc AC = 94° 10′. From this information, angle ACE, intercepting arc AE, will be shown = 30° 40′. When this figure is added to AC, the sum = 124° 50′ [= 94° 10′ + 30° 40′], subtended by CE = 17,727^p, whereof the eccentric's diameter = 20,000^p. In those same units, according to the previously established proportion, DE = 10,665^p. But the whole arc $BCAE$ = 191° [= BC+CA+AE = 66° 10′ + 94° 10′ + 30° 40′]. Consequently EB = the remainder of the circle [= 360° − ($BCAE$ = 191°)] = 169°, subtended by the whole of BDE = 19,908^p, whereof BD, the remainder [when DE = 10,665^p is subtracted from BDE = 19,908^p], = 9243^p. Therefore the larger segment is $BCAE$, within which will lie the [eccentric] circle's center. Let this be F.

Now draw diameter $GFDH$. Obviously rectangle $ED \times DB$ = rectangle $GD \times DH$, which therefore is also given. But rectangle $GD \times DH + (FD)^2 = (FDH)^2$, and when rectangle $GD \times DH$ is subtracted from $(FDH)^2$, the remainder = $(FD)^2$. Therefore as a length *FD* is given = 1193^p whereof *FG* = $10{,}000^p$. But when *FG* = 60^p, *FD* = $7^p\ 9'$. Now bisect *BE* at *K*, and draw *FKL*, which will therefore be perpendicular to *BE*. Since *BDK* = $^1/_2$ [*BDE* = $19{,}908^p$] = 9954^p, and *DB* = 9243^p, the remainder *DK* [when *DB* = 9243^p is subtracted from *BDK* = 9954^p] = 711^p. Hence, in [right] triangle *DFK*, the sides being given, [*FD* = 1193; *DK* = 711; $(KF)^2 = (FD)^2 - (DK)^2$], angle *DFK* also is given = 36° 35′, and arc *LH* likewise = 36° 35′. But the whole of *LHB* = 84 $^1/_2$° [= $^1/_2$ (*EB* = 169°)]. The remainder *BH* [when *LH* = 36° 35′ is subtracted from *LHB* = 84 $^1/_2$°] = 47° 55′ = the distance of the place of the second [opposition] from the perigee. The remainder *BCG* [when *BH* = 47° 55′ is subtracted from the semicircle] = the distance from the second opposition to the apogee = 132° 5′. From *BCG* [= 132° 5′] subtract *BC* = 66° 10′, and the remainder = 65° 55′ [= *CG*, the distance] from the place of the third [opposition] to the apogee. When this figure [65° 55′] is subtracted from 94° 10′ [= *CA*], the remainder [*GA*] = 28° 15′ = the distance from the apogee to the epicycle's first place.

The foregoing results unquestionably agree only slightly with the phenomena, since the planet does not run along the aforementioned eccentric. Consequently this method of exposition, based on an erroneous foundation, cannot produce any sound result. Among the many proofs of its fallibility is the fact that in Ptolemy it yielded an eccentricity greater than was proper for Saturn, and for Jupiter smaller, whereas in my case the eccentricity for Jupiter was quite excessive. Thus it appears obvious that when different arcs of a circle are assumed for a planet, the desired result does not come out in the same way. A comparison of Jupiter's uniform and apparent motion at the three aforementioned terminal points, and thereafter at all places, would have been impossible had I not accepted the entire eccentricity declared by Ptolemy = $5^p\ 30'$ whereof the eccentric's radius = 60^p, but with the radius = $10{,}000^p$, the eccentricity = 917^p [V, 10], and put the arc from the higher apse to the first opposition = 45° 2′ [instead of 28°15′]; from the lower apse to the second opposition = 64° 42′ [instead of 47°55′]; and from the third opposition to the higher apse = 49° 8′ [instead of 65°55′].

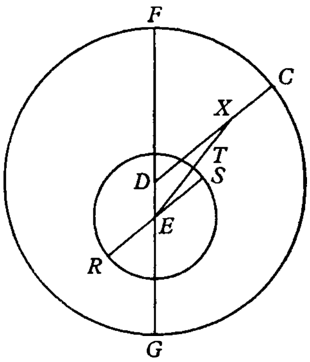
Reproduce the previous diagram of an eccentrepicycle, insofar as it fits this situation. In accordance with my hypothesis, $^3/_4$ of the entire distance [916 instead of 1193] between the centers = 687^p = *DE*, while the epicyclet receives the remaining $^1/_4$ = 229^p whereof *FD* = $10{,}000^p$. Angle *ADF* = 45° 2′. Hence, in triangle *ADE*, two sides, *AD* and *DE*, are given [$10{,}000^p$, 687^p], as well as [the included] angle *ADE* [= 134° 58′ = 180°–(*ADF* = 45° 2′)]. Thereby the third side *AE* will be shown = $10{,}496^p$ whereof *AD* = $10{,}000^p$, and angle *DAE* = 2° 39′. Angle *DAK* being assumed = *ADF* [= 45° 2′], the whole of *EAK* = 47° 41′ [=*DAK*+ *DAE* = 45° 2′ +2° 39′]. Moreover, in triangle *AEK* two sides, *AK* and *AE*, are also given [229^p, $10{,}496^p$]. This makes angle *AEK* = 57′. When this angle + *DAE* [= 2° 39′] are subtracted from *ADF* [= 45° 2′], the remainder *KED* = 41° 26′ at the first opposition.

A similar result will be shown in triangle *BDE*. Two sides, *BD* and *DE*, are given [$10{,}000^p$, 687^p], and [the included] angle *BDE* = 64° 42′. Hence here too the third side *BE* will be known = 9725^p whereof *BD* = $10{,}000^p$, as well as angle *DBE* = 3° 40′. Consequently, also in triangle *BEL* two sides, *BE* and *BL*,



are given [9725ᵖ, 229ᵖ], together with the [included] whole angle EBL = 118° 58′ [= DBE = 3° 40′+DBL = FDB = 180°- (BDG = 64° 42′) = 115° 18′]. BEL will also be given = 1° 10′, and thereby DEL = 110° 28′. But KED was previously known = 41° 26′. Therefore the whole of KEL [= KED+DEL] = 151° 54′ [= 110° 28′+ 41° 26′]. Then, as the remainder from 4 right angles = 360° [- 151°54′], 208° 6′ = the apparent motion between the first and second oppositions, in agreement with the [revised] observations [(180°- 45° 2′ = 134° 58′)+64° 42′ = 199° 40′].

Lastly, at the third place, sides DC and DE of triangle CDE are given in the same way [10,000ᵖ, 687ᵖ]. Moreover, [the included] angle CDE = 130° 52′ because FDC is given [= 49° 8′ = the distance from the third opposition to the higher apse]. The third side CE will emerge = 10,463ᵖ whereof CD = 10,000ᵖ, and angle DCE = 2° 51′. Therefore the whole of ECM = 51° 59′ [= 2° 51′+ 49° 8′ = DCE+(DCM = FDC)]. Consequently in triangle CEM likewise two sides, CM and CE, are given [229ᵖ, 10,463ᵖ], as well as [the included] angle MCE [= 51° 59′]. Angle MEC will also be known = 1°. MEC+DCE, previously found [= 2° 51′], = the difference between FDC and DEM, the angles of the uniform and apparent motions. Therefore DEM at the third opposition = 45° 17′. But DEL has already been shown = 110° 28′. Therefore LEM = the difference [between DEL and DEM = 110° 28′- 45° 17′] = 65° 10′ = the angle from the second observed opposition to the third, likewise in agreement with the observations [= 180°- (64°42′+49° 8′ = 113° 50′) = 66° 10′]. But since Jupiter's third place was seen at 113° 44′ in the sphere of the fixed stars, the place of Jupiter's higher apse is shown ≅ 159° [113° 44′+45° 17′ = 159° 1′].



Now around E describe the earth's orbit RST, with diameter RES parallel to DC. Evidently, at Jupiter's third opposition, angle $FDC = 49° \ 8' = DES$, and R = the apogee of the uniform motion in parallax. But after the earth has traversed a semicircle plus arc ST, it enters into conjunction with Jupiter in opposition [to the sun]. Arc $ST = 3° \ 51'$ = angle SET, as was shown numerically [in the previous diagram: $DCE = 2° \ 51' + MEC = 1°$]. These figures therefore show that 19 hours after midnight on 1 February 1529 C.E. Jupiter's uni-form anomaly in parallax = $183° \ 51'$ [$= RS + ST = 180° + 3° \ 51'$]; its true motion = $109° \ 52'$; and the eccentric's apogee now ≅ 159° from the horn of the constellation Ram. This is the information we were seeking.

CONFIRMATION OF JUPITER'S UNIFORM MOTION — *Chapter 12*

As we saw above [V, 10], in the last of the three oppositions observed by Ptole-my the planet Jupiter in its mean motion was at $4° \ 58'$, while the parallactic anom-aly was $182° \ 47'$. Hence, in the period intervening between both observations [Ptolemy's last and Copernicus' last], Jupiter's motion in parallax evidently traversed $1° \ 5'$ [≅ $183° \ 51' - 182° \ 47'$] in addition to complete revolutions; and its own motion, about $104° \ 54'$ [$= 109° \ 52' - 4° \ 58'$]. The time elapsed between 5 hours after the midnight following the 20th day of the Egyptian month Athyr in Antoninus [Pius'] year 1, and 19 hours after the midnight preceding 1 February 1529 C.E., amounts to 1392 Egyptian years 99 days 37 day-minutes. To this time, according to the computation set forth above, the corresponding [motion in parallax] sim-ilarly $= 1° \ 5'$ after complete revolutions, in which the earth in its uniform motion overtook Jupiter 1274 times. Thus the calculation is considered to be certain and confirmed because it agrees with the results obtained visually. In this time also, the eccentric's higher and lower apsides clearly shifted eastward $4 \frac{1}{2}°$ [≅ $159° - 154° \ 22'$]. An average distribution assigns approximately 1° to 300 years.

DETERMINING THE PLACES OF JUPITER'S MOTION — *Chapter 13*

The last of [Ptolemy's] three observations occurred at 5 hours after the midnight following the 20th day of the month Athyr in Antoninus [Pius'] year 1. The time reckoned backwards from then to the beginning of the Christian era = 136 Egyptian years 314 days 10 day-minutes. In that period the mean motion in parallax = $84° \ 31'$. When this figure is subtracted from $182° \ 47'$ [at Ptolemy's third observation], the remainder = $98° \ 16'$ for the midnight preceding 1 January at the beginning of the Christian era. From that time to the 1st Olympiad in 775 Egyptian years $12 \frac{1}{2}$ days, the motion is computed = $70° \ 58'$ in addition to complete circles. When this figure is subtracted from $98° \ 16'$ [for the Christian era], the remainder = $27° \ 18'$ for the place of the Olympiads. Thereafter in 451 years 247 days the motion amounts to $110° \ 52'$. When this figure is added to the place of the Olympiads, the sum = $138° \ 10'$ for the place of Alexander at noon on the 1st day of the Egyptian month Thoth. This method will serve for any other epochs.

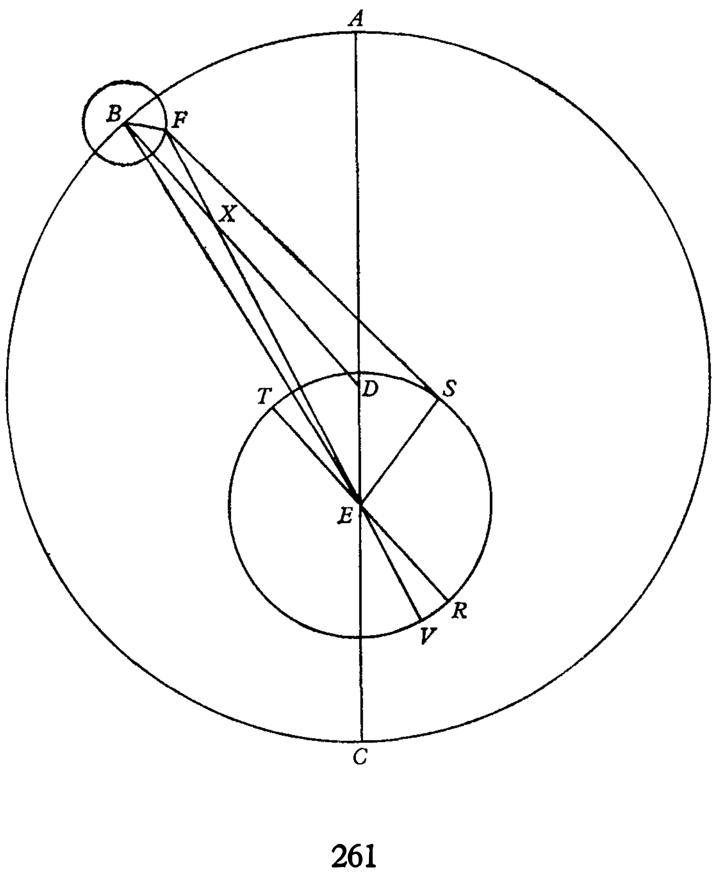
DETERMINING JUPITER'S PARALLAXES, AND ITS HEIGHT IN RELATION TO THE EARTH'S ORBITAL REVOLUTION

Chapter 14

For the purpose of determining the other phenomena connected with Jupiter, namely, its parallax, I very carefully observed its position at 6 hours before noon on 19 February 1520 C.E. Through the instrument I saw Jupiter 4° 31′ west of the first, brighter star in the forehead of the Scorpion. Since the fixed star's place = 209° 40′, Jupiter's position obviously = 205° 9′ in the sphere of the fixed stars. From the beginning of the Christian era to the hour of this observation there are 1520 uniform years 62 days 15 day-minutes. Thereby the sun's mean motion is derived = 309° 16′, and the [mean] parallactic anomaly = 111° 15′. Hence the planet Jupiter's mean place is determined = 198° 1′ [= 309°16′− 111° 15′]. In our time the place of the eccentric's higher apse has been found = 159° [V, 11]. Therefore, the anomaly of Jupiter's eccentric = 39° 1′ [= 198° 1′− 159°].

To illustrate this situation, describe the eccentric circle *ABC*, with center *D* and diameter *ADC*. Let the apogee be at *A*, the perigee at *C*, and therefore let *E*, the center of the earth's annual orbit, be on *DC*. Take arc *AB* = 39° 1′. With *B* as center, describe the epicyclet, with [radius] $BF = \frac{1}{3}\,DE$ = the distance [between the centers]. Let angle *DBF* = *ADB*. Draw straight lines *BD*, *BE*, and *FE*.

In triangle *BDE* two sides are given: $DE = 687^p$ whereof $BD = 10{,}000^p$. They enclose the given angle *BDE* = 140° 59′ [= 180°−(*ADB* = 39° 1′)].



From this information base BE will therefore be shown $= 10{,}543^p$, and angle $DBE = 2°\,21' = ADB - BED$. Consequently the whole of angle $EBF = 41°\,22'$ [$= (DBE = 2°\,21') + (DBF = ADB = 39°\,1')$]. Hence in triangle EBF, angle EBF is given, together with the two sides enclosing it: $EB = 10{,}543^p$ whereof $BD = 10{,}000^p$, and $BF = 229^p = {}^1/_3\,(DE =$ the distance) [between the centers]. From this information the remaining side FE is deduced $= 10{,}373^p$, and angle $BEF = 50'$. Lines BD and FE intersect each other in point X. Hence angle DXE at the intersection $= BDA - FED =$ the mean motion minus the true. $DXE = DBE + BEF$ [$= 2°\,21' + 50'$] $= 3°\,11'$. When this figure is subtracted from $39°\,1'$ [$= ADB$], the remainder = angle $FED = 35°\,50'$ = the angle between the eccentric's higher apse and the planet. But the place of the higher apse $= 159°$ [V, 11]. Together they amount to $194°\,50'$. This was Jupiter's true place with respect to center E, but the planet was seen at $205°\,9'$ [V, 14, above]. Therefore, the difference $= 10°\,19'$ belongs to the parallax.

Now around E as center, describe the earth's orbit RST, with diameter RET parallel to BD, so that R is the parallactic apogee. Also take arc $RS = 111°\,15'$ in accordance with the determination [at the beginning of V, 14] of the mean parallactic anomaly. Prolong FEV in a straight line through both sides of the earth's orbit. V will be the planet's true apogee. REV = the angular difference [between the mean and true apogees], $= DXE$, makes the whole arc $VRS = 114°\,26'$ [$= RS + RV = 111°\,15' + 3°\,11'$], and FES, the remainder [when $SEV = 114°\,26'$ is subtracted from $180°$] $= 65°\,34'$. But [just above, the parallax] EFS was found $= 10°\,19'$, and FSE, the remaining angle [in triangle EFS] $= 104°\,7'$. Therefore in triangle EFS, the angles being given, the ratio of the sides is given: $FE : ES = 9698 : 1791$. Then, with $FE = 10{,}373^p$, $ES = 1916$, with $BD = 10{,}000$. Ptolemy, however, found $ES = 11^p\,30'$, with the eccentric's radius $= 60^p$ [*Syntaxis*, XI, 2]. This is nearly the same ratio as $1916 : 10{,}000$. In this respect, therefore, I seem not to differ from him at all.

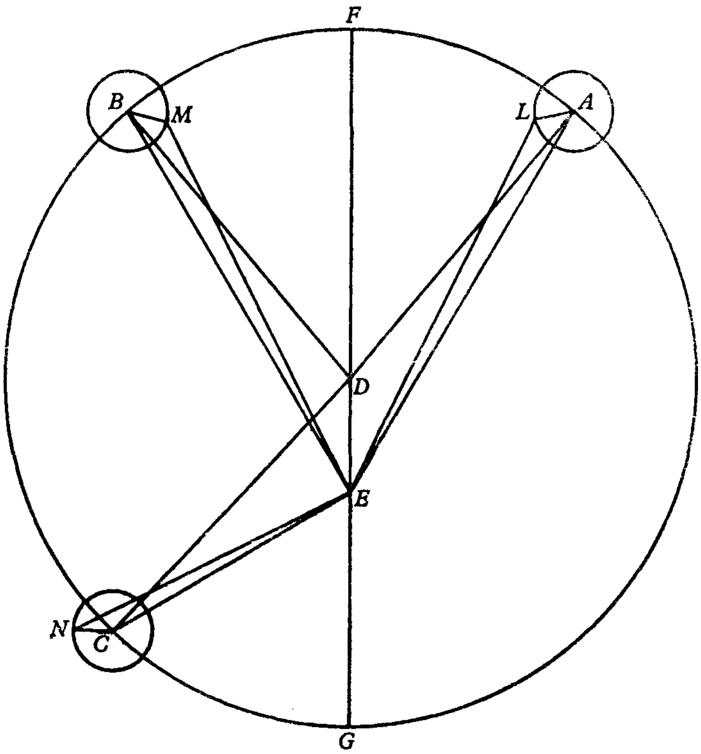
Then diameter ADC : diameter $RET = 5^p\,13' : 1^p$. Similarly, $AD : ES$ or $RE = 5^p\,13'\,9'' : 1^p$. In like manner $DE = 21'\,29''$, and $BF = 7'\,10''$. Therefore, with the radius of the earth's orbit $= 1^p$, the whole of $ADE - BF = 5^p\,27'\,29''$ [$= 5^p\,13'9'' + 21'29'' - 7'9''$], with Jupiter at apogee; [with the planet] at perigee, the remainder $EC + BF$ [$= 5^p\,13'\,9'' - 21'\,29'' + 7'\,9''$] $= 4^p\,58'\,49''$; and with the planet at places between [apogee and perigee], there is a corresponding value. These figures lead to the conclusion that at apogee Jupiter makes its greatest parallax $= 10°\,35'$; at perigee, $11°\,35'$; and between these [extremes] the difference $= 1°$. Accordingly, Jupiter's uniform motions as well as its apparent motions have been determined.

THE PLANET MARS — *Chapter 15*

Now I must analyze Mars' revolutions by taking three of its ancient oppositions, with which I shall once again combine the earth's motion in antiquity. Of the oppositions reported by Ptolemy [*Syntaxis*, X, 7], the first occurred 1 uniform hour after the midnight following the 26th day of Tybi, the 5th Egyptian month, in Hadrian's year 15; according to Ptolemy, the planet was at $21°$ within the Twins, but at $74°\,20'$ in relation to the sphere of the fixed stars [$21°$ Twins $= 81°\,0' - 6°\,40' = 74°\,20'$]. He recorded the second opposition at 3 hours before midnight following the 6th day of Pharmuthi, the 8th Egyptian month, in Hadrian's year 19, with the planet at 28° 50′ within the Lion, but at 142° 10′ in the sphere of the fixed stars [28° 50′ Lion = 148° 50′(−6° 40′) = 142° 10′]. The third opposition happened at 2 uniform hours before the midnight following the 12th day of Epiphi, the 11th Egyptian month, in Antoninus [Pius'] year 2, with the planet at 2° 34′ within the Archer, but at 235° 54′ in the sphere of the fixed stars [2° 34′ Archer = 242° 34′(−6° 40′) = 235° 54′].

Between the first opposition and the second, then, there are 4 Egyptian years 69 days, plus 20 hours = 50 day-minutes, with the planet's apparent motion, after complete revolutions, = 67° 50′ [= 142° 10′ − 74° 20′]. From the second opposition to the third, there are 4 years 96 days 1 hour, with the planet's apparent motion = 93° 44′ [= 235° 54′ − 142° 10′]. But in the first interval the mean motion = 81° 44′ in addition to complete revolutions; and in the second interval, 95° 28′. Then Ptolemy found [*Syntaxis,* X, 7] the entire distance between the centers = 12^p whereof the eccentric's radius = 60^p; but with the radius = $10{,}000^p$, the proportionate distance = $2{,}000^p$. From the first opposition to the higher apse, the mean motion = 41° 33′; then, next in order, from the higher apse to the second opposition, = 40° 11′; and from the third opposition to the lower apse = 44° 21′.

In accordance with my hypothesis of uniform motion, however, the distance between the centers of the eccentric and of the earth's orbit = 1500^p = $^3/_4$ [of Ptolemy's eccentricity = 2000^p], while the remaining $^1/_4$ = 500^p makes up the epicyclet's radius. In this way now describe the eccentric circle *ABC,* with center *D.* Through both apsides draw diameter *FDG,* on which let *E* be the center of the circle of the annual revolution. Let *A, B, C* in that order be the places of the

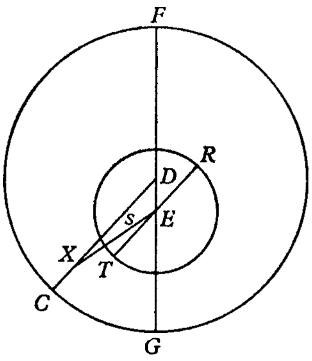


observed oppositions, with arcs $AF = 41° 33'$, $FB = 40° 11'$, and $CG = 44° 21'$. At each of the points A, B, and C describe the epicyclet, with radius $= {}^1/_3$ of the distance DE. Join AD, BD, CD, AE, BE, and CE. In the epicyclets draw AL, BM, and CN so that angles DAL, DBM, and $DCN = ADF$, BDF, and CDF.

In triangle ADE, angle ADE is given $= 138°$ [27′], because angle FDA is given [$= 41° 33'$]. Furthermore, two sides are given: $DE = 1500^p$ whereof $AD = 10{,}000^p$. From this information it follows that the remaining side $AE = 11{,}172^p$ in the same units, and angle $DAE = 5° 7'$. Hence, the whole of EAL [$= DAE+DAL = 5° 7'+41° 33'$] $= 46° 40'$. So also in triangle EAL, angle EAL is given [$= 46° 40'$] as well as two sides : $AE = 11{,}172^p$, and $AL = 500^p$ whereof $AD = 10{,}000^p$. Angle AEL will also be given $= 1° 56'$. When added to angle DAE, AEL makes the entire difference between ADF and $LED = 7° 3'$, and $DEL = 34\,{}^1/_2°$.

Similarly, at the second opposition, in triangle BDE angle BDE is given $= 139° 49'$ [$= 180°-(FDB = 40° 11')$], and side $DE = 1500^p$ whereof $BD = 10{,}000^p$. This makes side $BE = 11{,}188^p$, angle $BED = 35° 13'$, and the remaining angle DBE [$= 180°-(139° 49'+35° 13')$] $= 4° 58'$. Therefore the whole of EBM [$= DBE+(DBM = BDF) = 4° 58'+40° 11'$] $= 45° 9'$, enclosed by the given sides BE and BM [= 11,188, 500]. Hence it follows that angle $BEM = 1° 53'$, and the remaining angle DEM [$= BED-BEM = 35° 13'-1° 53'$] $= 33° 20'$. Therefore the whole of MEL [$= DEM+DEL = 33° 20'+34\,{}^1/_2°$] $= 67° 50'$ = the angle through which the planet was seen to move from the first opposition to the second, a numerical result in agreement with experience [$= 67° 50'$].

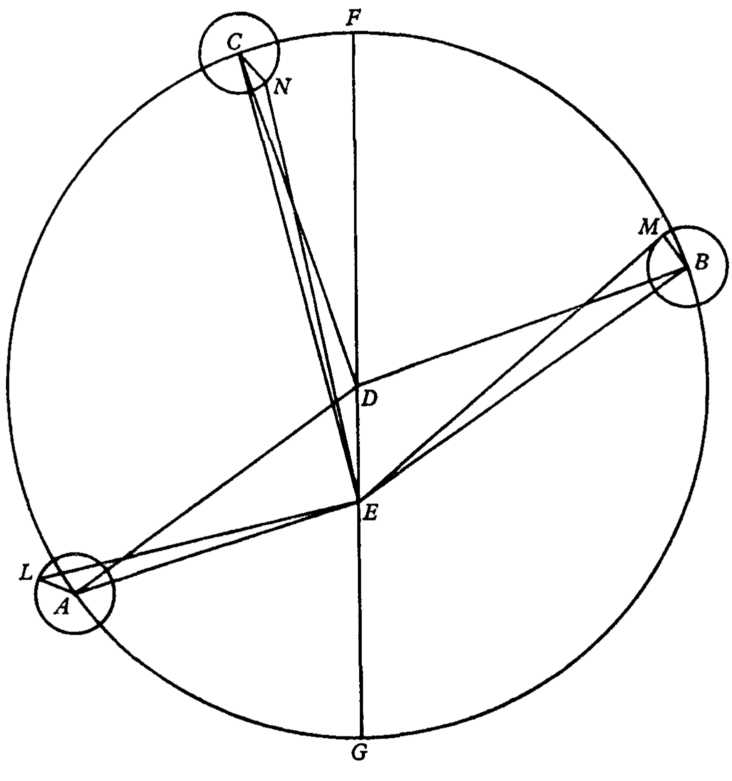
Again, at the third opposition, triangle CDE has two sides, CD and DE, given [$= 10{,}000^p$, 1500^p]. They enclose angle CDE [= arc CG] $= 44° 21'$. Hence, base CE comes out $= 8988^p$ whereof $CD = 10{,}000^p$ or $DE = 1500^p$, angle $CED = 128° 57'$, and the remaining angle $DCE = 6° 42'$ [$= 180°-(44° 21' +128° 57')$]. Thus once more in triangle CEN, the whole angle ECN [$= (DCN = CDF = 180°-44° 21' = 135° 39')+(DCE = 6° 42')$] $= 142° 21'$, and is enclosed by known sides EC and CN [8988^p, 500^p]. Hence angle CEN will also be given $= 1° 52'$. Therefore the remaining angle NED [$= CED-CEN = 128° 57'-1° 52'$] $= 127° 5'$ at the third opposition. But DEM has already been shown $= 33° 20'$. The remainder MEN [$NED-DEM = 127° 5'-33° 20'$] $= 93° 45'$ = the angle of the apparent motion between the second and third oppositions. Here also the numerical result agrees quite well with the observations [93° 45′ as compared with 93° 44′]. In this last observed opposition of Mars, the planet was seen at 235° 54′, at a distance of 127° 5′ from the eccentric's apogee [$= \sphericalangle NEF$], as was shown. Hence, the place of the apogee of Mars' eccentric was 108° 49′ [$= 235° 54'-127° 5'$] in the sphere of the fixed stars.



Now around E as center, describe the earth's annual orbit RST, with diameter RET parallel to DC, in order that R may be the parallactic apogee, and T the perigee. The planet was sighted along EX at 235° 54′ in longitude. Angle DXE has been shown $= 8° 34'$ = the difference between the uniform and apparent motions [$= DCE+CEN = 6° 42'+1° 52'$, in the preceding diagram]. Therefore, the mean motion $= 244\,{}^1/_2°$ [$\cong 235° 54'+8° 34' = 244° 28'$]. But angle DXE = central angle SET, which similarly $= 8° 34'$. Hence, if arc $ST = 8° 34'$ is subtracted from a semicircle, we shall have the planet's mean motion in parallax = arc $RS = 171° 26'$. Consequently, in addition to other results, I have also shown by means of this hypothesis of the moving earth that at 10 uniform hours after noon on the 12th day of the Egyptian month Epiphi in Antoninus [Pius'] year 2, the planet Mars' mean motion in longitude = 244 $^1/_2$°, and its parallactic anomaly = 171° 26′.

THREE OTHER RECENTLY OBSERVED OPPOSITIONS OF THE PLANET MARS *Chapter 16*

Once more, with these observations of Mars by Ptolemy, I compared three others, which I performed not without some care. The first occurred at 1 hour after midnight on 5 June 1512 C.E., when Mars' place was found to be 235° 33′, just as the sun was directly opposite at 55° 33′ from the first star in the Ram, taken as the beginning of the sphere of the fixed stars. The second observation happened 8 hours after noon on 12 December 1518 C.E., when the planet appeared at 63° 2′. The third observation took place at 7 hours before noon on 22 February 1523 C.E., with the planet at 133° 20′. From the first observation to the second, there are 6 Egyptian years 191 days 45 day-minutes; and from the second observation to the third, 4 years 72 days 23 day-minutes. In the first period of time, the apparent motion = 187° 29′ [= 63° 2′ + 360° – 235° 33′], but the uniform motion = 168° 7′; and in the second interval, the apparent motion = 70° 18′ [= 133° 20′ – 63° 2′], but the uniform motion = 83°. Now reproduce Mars' eccentric circle, except that this time AB = 168° 7′, and BC = 83°. Then by a method like that which I used for Saturn and Jupiter (to pass silently over the multitude, complexity, and boredom of those computa-



tions), I finally found Mars' apogee on arc BC. Obviously it could not be on *AB*, because [there] the apparent motion exceeded the mean motion, namely, by 19° 22′ [= 187° 29′ − 168° 7′]. Nor [could the apogee be] on *CA*. For even though [there the apparent motion 102° 13′ = 360° − (187° 29′ + 70° 18′)] is smaller [than the mean motion, 360° − (168° 7′ + 83° = 251° 7′) = 108° 53′], nevertheless on *BC*, preceding *CA*, [the mean motion = 83°] exceeds the apparent motion [= 70° 18′] by a wider margin [12° 42′] than on *CA* [where 108° 53′ mean − 102° 13′ apparent = 6° 40′]. But, as was shown above [V, 4], on the eccentric the smaller and diminished [apparent] motion occurs near the apogee. Therefore, the apogee will rightly be regarded as located on *BC*.

Let it be *F*, and let the circle's diameter be *FDG*, on which [*E*], the center of the earth's orbit, is located as well as [*D*, the center of the eccentric]. I then found *FCA* = 125° 29′ and, in order, *BF* = 66° 25′, *FC* = 16° 36′, *DE* = the distance between the centers = 1460^p whereof the radius = $10{,}000^p$, and the epicyclet's radius = 500^p in the same units. These figures show that the apparent and uniform motions are mutually consistent and entirely in agreement with the observations.

Accordingly, complete the diagram, as before. In triangle *ADE* two sides, *AD* and *DE*, are known [$10{,}000^p$, 1460^p], as well as angle *ADE*, from Mars' first opposition to the perigee, = 54° 31′ [= arc *AG* = 180° − (*FCA* = 125° 29′)]. Therefore, angle *DAE* will be shown to emerge = 7° 24′, the remaining angle *AED* = 118° 5′ [= 180° − (*ADE* + *DAE* = 54° 31′ + 7° 24′)], and the third side *AE* = 9229^p. But angle *DAL* = *FDA* by hypothesis. Therefore the whole of *EAL* [= *DAE* + *DAL* = 7° 24′ + 125° 29′] = 132° 53′. Thus also in triangle *EAL* two sides, *EA* and *AL*, are given [9229^p, 500^p], enclosing the given angle *A* [= 132° 53′]. Therefore the remaining angle *AEL* = 2° 12′, and the residual angle *LED* = 115° 53′ [= *AED* − *AEL* = 118° 5′ − 2° 12′].

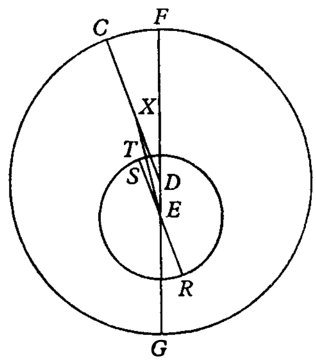
Similarly, at the second opposition, in triangle *BDE* two sides, *DB* and *DE*, are given [$10{,}000^p$, 1460^p]. They enclose angle *BDE* [= arc *BG* = 180° − (*BF* = 66° 25′)] = 113° 35′. Therefore, in accordance with the theorems on Plane Triangles, angle *DBE* will be shown = 7° 11′, the remaining angle *DEB* = 59° 14′ [= 180° − (113° 35′ + 7° 11′)], base *BE* = $10{,}668^p$ whereof *DB* = $10{,}000^p$ and *BM* = 500^p, and the whole of *EBM* [= *DBE* + (*DBM* = *BF*) = 7° 11′ + 66° 25′] = 73° 36′.

Thus, also in triangle *EBM*, whose given sides [*BE* = 10,668; *BM* = 500] enclose the given angle [*EBM* = 73° 36′], angle *BEM* will be shown = 2° 36′. *DEM*, the remainder when *BEM* is subtracted [from *DEB* = 59° 14′] = 56° 38′. Then exterior angle *MEG*, from the perigee [to the second opposition], = the supplement [of *DEM* = 56° 38′] = 123° 22′. But angle *LED* has already been shown = 115° 53′. Its supplement *LEG* = 64° 7′. When this is added to *GEM*, which has already been found [= 123° 22′], the sum = 187° 29′, with 4 right angles = 360°. This figure [187° 29′] agrees with the apparent distance [= 187° 29′] from the first opposition to the second.

The third opposition may likewise be analyzed by the same method. For, angle *DCE* is shown = 2° 6′, and side *EC* = $11{,}407^p$ whereof *CD* = $10{,}000^p$. Therefore the whole of angle *ECN* [= *DCE* + (*DCN* = *FDC*) = 2° 6′ + 16° 36′] = 18° 42′. In triangle *ECN*, sides *CE* and *CN* are already given [$11{,}407^p$, 500^p]. Hence, angle *CEN* will come out = 50′. When this figure is added to *DCE*

[= 2° 6′], the sum = 2° 56′ = the amount by which *DEN* = the angle of the apparent motion, is smaller than *FDC* = the angle of the uniform motion [= arc *FC* = 16° 36′]. Therefore *DEN* is given = 13° 40′. These figures [*DEN*+*DEM* = 13° 40′+56° 38′ = 70° 18′] are once more in close agreement with the apparent motion [= 70° 18′] observed between the second and third oppositions.

On this later occasion, as I said [near the beginning of V, 16], the planet Mars appeared at 133° 20′ from the head of the constellation Ram. Angle *FEN* has been shown ≅ 13° 40′. Therefore, computed backwards, the place of the eccentric's apogee in this last observation obviously = 119° 40′ [= 133° 20′– 13° 40′] in the sphere of the fixed stars. In Antoninus [Pius'] time Ptolemy found the apogee at 108° 50′ [*Syntaxis*, X, 7 : 25° 30′ within the Crab = 115° 30′–6° 40′]. It has therefore shifted eastward 10° 50′ [= 119° 40′– 108° 50′ from that time] to ours. I have also found the distance between the centers smaller by 40^p [1460^p as compared with 1500^p] whereof the eccentric's radius = $10{,}000^p$. The reason is not that Ptolemy or I made an error, but that, as is clearly proved, the center of the earth's grand circle has approached the center of Mars' orbit, with the sun meanwhile remaining stationary. For, these conclusions are mutually consistent to a high degree, as will become plainer than daylight hereafter [V, 19].



Now around *E* as center describe the earth's annual orbit [*RST*], with its diameter *SER* parallel to *CD*, on account of the equality of their revolutions. Let *R* = the uniform apogee with respect to the planet, and *S* = the perigee. Put the earth at *T*. When *ET*, along which the planet is sighted, is extended, it will intersect *CD* at point *X*. But in this last position the planet was seen along *ETX* at 133° 20′ of longitude, as was mentioned [near the beginning of V, 16]. Moreover, angle *DXE* [= *CEN*+*DCE* in the preceding diagram = 50′+2° 6′] has been shown = 2° 56′. Now *DXE* is the difference by which *XDF* = the angle of the uniform motion, exceeds *XED* = the angle of the apparent motion. But *SET* = alternate interior angle *DXE* = the parallactic prosthaphaeresis. When this is subtracted from the semicircle [*STR*], it leaves as the remainder 177° 4′ [= 180°–2° 56′] = the uniform parallactic anomaly, computed from *R* = the apogee of the uniform motion. Consequently we have established here again that at 7 uniform hours before noon on 22 February 1523 C.*E*., the planet Mars' mean motion in longitude = 136° 16′ [= 2° 56′+(133° 20′ = apparent place)]; its uniform parallactic anomaly = 177° 4′ [= 180°–2° 56′]; and the eccentric's higher apse = 119° 40′. Q. E. D.

CONFIRMATION OF MARS' MOTION — *Chapter 17*

In the last of Ptolemy's three observations, as was made clear above [V, 15], Mars' mean motion [in longitude] = $244\frac{1}{2}$°, and its parallactic anomaly = 171° 26′. Therefore, in the time intervening [between Ptolemy's last observation and Copernicus' last observation] there was an accumulation, in addition to complete revolutions, of 5° 38′ [+171° 26′ = 177° 4′]. From 9 hours after noon = 3 uniform hours, with respect to the meridian of Cracow, before the midnight following the 12th day of Epiphi = the 11th Egyptian month in Antoninus [Pius'] year 2, until 7 hours before noon on 22 February 1523 C.*E*., there are 1384 Egyptian years 251 days 19 day-minutes. During this interval, according to the computation set forth above, there is an accumulation of 5° 38′ in the parallactic anomaly after 648 complete revolutions. The anticipated uniform motion of the sun = 257 $^1/_2$°. From this figure subtract 5° 38′ of the parallactic motion, and the remainder = 251° 52′ = Mars' mean motion in longitude. All these results agree fairly well with what was just set forth.

DETERMINING MARS' PLACES — *Chapter 18*

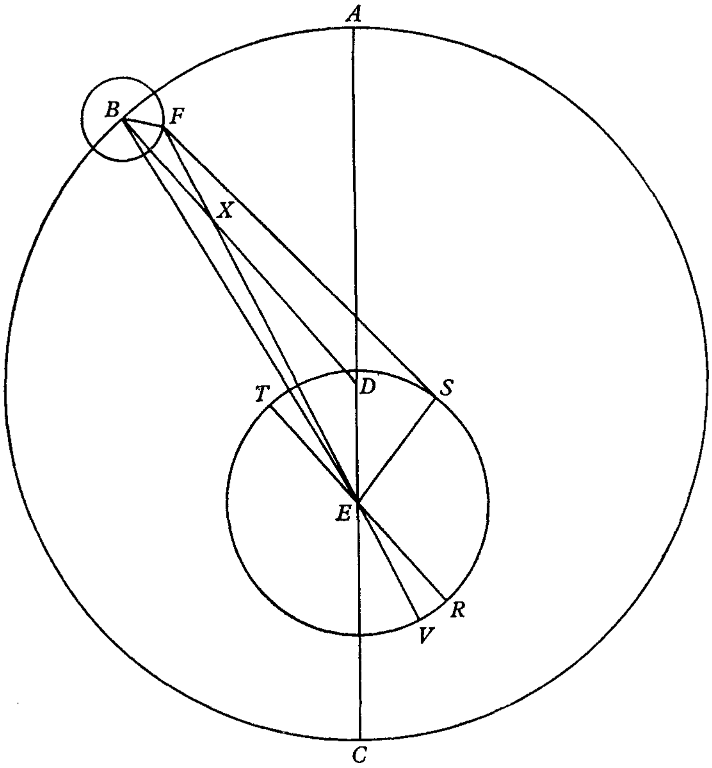
From the beginning of the Christian era to 3 hours before midnight on the 12th day of the Egyptian month Epiphi in Antoninus [Pius'] year 2, there are counted 138 Egyptian years 180 days 52 day-minutes. During that time the parallactic motion = 293° 4′. When this figure is subtracted from the 171° 26′ of Ptolemy's last observation [V, 15, end], an entire revolution being borrowed, [171° 26′+360° = 531° 26′], the remainder = 238° 22′ [= 531° 26′− 293° 4′] for midnight, 1 January 1 C.E. To this place, from the 1st Olympiad there are 775 Egyptian years 12 $^1/_2$ days. During that time the parallactic motion = 254° 1′. When this figure is similarly subtracted from 238° 22′, a revolution being borrowed [238° 22′+360° = 598° 22′], the remainder for the place of the 1st Olympiad = 344° 21′. By similarly separating out the motions for the periods of other eras, we shall have the place of Alexander's era = 120° 39′, and Caesar's = 111° 25′.

THE SIZE OF MARS' ORBIT IN UNITS WHEREOF THE EARTH'S ANNUAL ORBIT IS ONE UNIT — *Chapter 19*

In addition, I also observed Mars occulting the star called the "southern claw", the first bright star in the Claws. I made this observation on 1 January 1512 C.E. Early in the morning, 6 uniform hours before noon on that day, I saw Mars $^1/_4$° away from the fixed star, but in the direction of the sun's solstitial rising [in the winter, that is, northeast]. This indicated that Mars was $^1/_8$° east of the star in longitude, but in latitude $^1/_5$° to the north. The star's place being known = 191° 20′ from the first star in the Ram, with a northern latitude = 40′, Mars' place was clearly = 191° 28′ [≅191° 20′+$^1/_8$°], with a northern latitude = 51′ [≅ 40′+$^1/_5$°]. At that time the parallactic anomaly by computation = 98° 28′; the sun's mean place = 262°; Mars' mean place = 163° 32′; and the eccentric's anomaly = 43° 52′.

With this information available, describe the eccentric *ABC*, with center *D*, diameter *ADC*, apogee *A*, perigee *C*, and eccentricity *DE* = 1460^p whereof *AD* = 10,000^p. Arc *AB* is given = 43° 52′. With *B* as center, and radius *BF* = 500^p whereof *AD* = 10,000^p, describe the epicyclet so that angle *DBF* = *ADB*. Join *BD, BE,* and *FE.* Furthermore, around *E* as center, construct the earth's grand circle *RST*. On its diameter *RET*, parallel to *BD*, let *R* = the [uniform] apogee of the planet's parallax, and *T* = the perigee of its uniform motion. Put the earth at *S*, with arc *RS* = the uniform parallactic anomaly, computed = 98° 28′. Extend *FE* as straight line *FEV*, intersecting *BD* at point *X*, and the convex circumference of the earth's orbit at *V* = the true apogee of the parallax.

In triangle *BDE* two sides are given: *DE* = 1460^p whereof *BD* = 10,000^p. They enclose angle *BDE*, given = 136° 8′ = the supplement of *ADB*, given =



43° 52′. From this information the third side BE will be shown $= 11{,}097^p$, and angle $DBE = 5°\ 13'$. But angle $DBF = ADB$ by hypothesis. The whole angle $EBF = 49°\ 5'$ [$= DBE + DBF = 5°\ 13' + 43°\ 52'$], enclosed by the given sides EB and BF [$11{,}097^p$, 500^p]. [In triangle BEF] we shall therefore have angle $BEF = 2°$, and the remaining side $FE = 10{,}776^p$ whereof $DB = 10{,}000^p$. Hence $DXE = 7°\ 13' = XBE + XEB$ = the opposite interior angles [$= 5°\ 13' + 2°$]. DXE is the subtractive prosthaphaeresis, by which angle ADB exceeded XED [$= 36°\ 39' = 43°\ 52' - 7°\ 13'$], and Mars' mean place exceeded its true place. But its mean place was computed $= 163°\ 32'$. Therefore its true place was to the west, at $156°\ 19'$ [$+7°\ 13' = 163°\ 32'$]. But to those who were observing it from a place near S it appeared at $191°\ 28'$. Therefore its parallax or commutation became $35°\ 9'$ eastward [$= 191°\ 28' - 156°\ 19'$]. Clearly, then, angle $EFS = 35°\ 9'$. Since RT is parallel to BD, angle $DXE = REV$, and arc RV likewise $= 7°\ 13'$. Thus the whole of VRS [$= RV + RS = 7°\ 13' + 98°\ 28'$] $= 105°\ 41'$ = the normalized parallactic anomaly. Thus is obtained angle VES, exterior to triangle FES, [$= 105°\ 41'$]. Hence, the opposite interior angle FSE is also given $= 70°\ 32'$ [$= VES - EFS = 105°\ 41' - 35°\ 9'$]. All these angles are given in degrees whereof $180° = 2$ right angles.

But in a triangle whose angles are given, the ratio of the sides is given. Therefore as a length $FE = 9428^p$, and $ES = 5757^p$, whereof the diameter of the circle circumscribed around the triangle $= 10{,}000^p$. Then, with $EF = 10{,}776^p$, $ES \cong 6580^p$ whereof $BD = 10{,}000^p$. This too differs only slightly from what Ptolemy found [*Syntaxis*, X, 8;39 $^1/_2$:60], and is almost identical therewith [39 $^1/_2$:60 $= 6583^1/_3$:$10{,}000^p$]. But in the same units all of $ADE = 11{,}460^p$ [$= AD + DE =$

10,000+1460], and the remainder EC = 8540^p [$ADEC$ = 20,000^p]. At A = the eccentric's higher apse, the epicyclet subtracts 500^p, and adds the same quantity at the lower apse, so that at the higher apse 10,960^p remain [= 11,460 − 500], and at the lower apse 9040^p [= 8540+500]. Therefore, with the radius of the earth's orbit = 1^p, Mars' apogee and greatest distance = $1^p\,39'\,57''$; its least distance = $1^p\,22'\,26''$; and its mean distance = $1^p\,31'\,11''$ [$1^p\,39'\,57'' - 1^p\,22'\,26'' = 17'\,31''$; $17'\,31'' \div 2 \cong 8'\,45''$; $8'\,45'' + 1^p\,22'\,26'' \cong 1^p\,39'\,57'' - 8'\,45''$]. Thus also in the case of Mars the sizes and distances of its motion have been explained through sound computation by means of the earth's motion.

THE PLANET VENUS *Chapter 20*

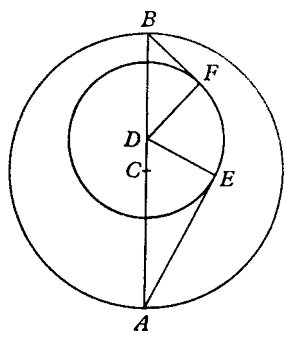
After the explanation of the motions of the three outer planets, Saturn, Jupiter, and Mars, which encircle the earth, it is now time to discuss those which are enclosed by the earth. I shall deal first with Venus, which permits an easier and clearer demonstration of its motion than the outer planets do, provided that the necessary observations of certain places are not lacking. For if its greatest elongations, morning and evening, to either side of the sun's mean place are found equal to each other, then we know for certain that halfway between those two places of the sun is the higher or lower apse of Venus' eccentric. These apsides are distinguished from each other by the fact that when the matched [greatest] elongations are smaller, they occur around the apogee, with the bigger pairs around the opposite apse. In all the other places [between the apsides], finally, the relative size of the elongations reveals without any uncertainty the distance of Venus' globe from the higher or lower apse, and also its eccentricity, as these topics are treated very perspicuously by Ptolemy [*Syntaxis,* X, 1–4]. Hence there is no need to repeat these matters one after the other, except insofar as they are adapted to my hypothesis of a moving earth from Ptolemy's observations.

He took the first of these from the astronomer Theon (of Smyrna?). It was performed at the first hour of the night following the 21st day of the month Pharmuthi in Hadrian's year 16, as Ptolemy says [*Syntaxis,* X, 1] = twilight, 8 March 132 C.E. Venus was seen at its greatest evening elongation = 47 $^1/_4$° from the mean place of the sun, when that mean place of the sun was computed = 337° 41′ in the sphere of the fixed stars. To this observation Ptolemy compared another, which he says he made at dawn on the 12th day of the month Thoth in Antoninus [Pius'] year 4 = daybreak, 30 July 140 C.E. Here again he states that Venus' greatest morning elongation = 47° 15′ = the previous distance from the sun's mean place, which was ≅ 119° in the sphere of the fixed stars, and previously had been = 337° 41′. Halfway between these places, clearly, are the apsides opposite each other at 48 $^1/_3$° and 228 $^1/_3$° [337° 41′–119° = 218° 41′; 218° 41′ ÷ 2 ≅ 109° 20′; 109° 20′+119° = 228° 20′; 228° 20′– 180° = 48° 20′]. To both these figures add 6 $^2/_3$° for the precession of the equinoxes, and the apsides come out, as Ptolemy says [*Syntaxis,* X, 1], at 25° within the Bull [= 55° = 48 $^1/_3$° +6 $^2/_3$°] and Scorpion [= 235° = 228 $^1/_3$°+6 $^2/_3$°], where Venus' higher and lower apsides had to be diametrically opposite each other.

Moreover, for stronger support of this result, he takes another observation by Theon at dawn on the 20th day of the month Athyr in Hadrian's year 12 = the morning of 12 October 127 C.E. At that time Venus was again found at its greatest elongation $= 47°\,32'$ from the sun's mean place $= 191°\,13'$. To this observation Ptolemy adds his own in Hadrian's year 21 = 136 C.E., on the 9th day of the Egyptian month Mechir = 25 December in the Roman calendar, at the first hour of the following night, when the evening elongation was again found = $47°\,32'$ from the mean sun $= 265°$. But in the previous observation by Theon the sun's mean place $= 191°\,13'$. The midpoints between these places [$265° - 191°\,13' = 73°\,47'$; $73°\,47' \div 2 \cong 36°\,53'$; $36°\,53' + 191°\,13' = 228°\,6'$; $228°\,6' - 180° = 48°\,6'$] again come out $\cong 48°\,20'$, $228°\,20'$, where the apogee and perigee must lie. As measured from the equinoxes, these points $= 25°$ within the Bull and Scorpion, which Ptolemy then distinguished by two other observations, as follows [*Syntaxis*, X, 2].

One of them was Theon's on the 3rd day of the month Epiphi in Hadrian's year 13 = 21 May 129 C.E., at dawn, when he found Venus' morning greatest elongation $= 44°\,48'$, with the sun's mean motion $= 48\,{}^5/_6°$, and Venus appearing at $4°$ [$\cong 48°\,50' - 44°\,48'$] in the sphere of the fixed stars. Ptolemy himself made the other observation on the 2nd day of the Egyptian month Tybi in Hadrian's year 21, which I equate with 18 November 136 C.E. in the Roman calendar. At the 1st hour of the following night the sun's mean motion $= 228°\,54'$, from which Venus' evening greatest elongation $= 47°\,16'$, with the planet itself appearing at $276\,{}^1/_6°$ [$= 228°\,54' + 47°\,16'$]. By means of these observations the apsides are distinguished from each other; namely, the higher apse $= 48\,{}^1/_3°$, where Venus' [greatest] elongations are narrower, and the lower apse $= 228\,{}^1/_3°$, where they are wider. Q. E. D.

THE RATIO OF THE EARTH'S AND VENUS' ORBITAL DIAMETERS *Chapter 21*



This information will accordingly also make clear the ratio of the earth's and Venus' orbital diameters. Describe the earth's orbit *AB* around *C* as center. Through both apsides draw diameter *ACB*, on which take *D* as the center of Venus' orbit, eccentric to circle *AB*. Let A = the place of the apogee. When the earth is in the apogee, the center of Venus' orbit is at its greatest distance [from the earth]. *AB*, the line of the sun's mean motion, is at $48\,{}^1/_3°$ [at *A*], with B = Venus' perigee, at $228\,{}^1/_3°$. Also draw straight lines *AE* and *BF*, tangent to Venus' orbit at points *E* and *F*. Join *DE* and *DF*.

DAE, as an angle at the center of a circle, subtends an arc $= 44\,{}^4/_5°$ [= greatest elongation in Theon's 3rd observation, V, 20], and *AED* is a right angle. Therefore, triangle *DAE* will have its angles given, and consequently its sides, namely, DE = half the chord subtending twice $DAE = 7046^p$ whereof $AD = 10{,}000^p$. In the same way, in right triangle *BDF*, angle *DBF* is given $= 47°\,16'$, and chord $DF = 7346^p$ whereof $BD = 10{,}000^p$. Then, with $DF = DE = 7046^p$, in those units $BD = 9582^p$. Hence, the whole of $ACB = 19{,}582^p$ [$= BD + AD = 9582^p + 10{,}000^p$]; $AC = {}^1/_2$ [ACB] $= 9791^p$, and *CD*, the remainder [when *BD* is subtracted from *BC* $(= AC) = 9791 - 9582$] $= 209^p$. Then, with $AC = 1^p$, $DE = 43\,{}^1/_6{}'$, and $CD \cong 1\,{}^1/_4{}'$. With $AC = 10{,}000^p$, $DE = DF = 7193^p$, and $CD \cong 208^p$. Q. E. D.

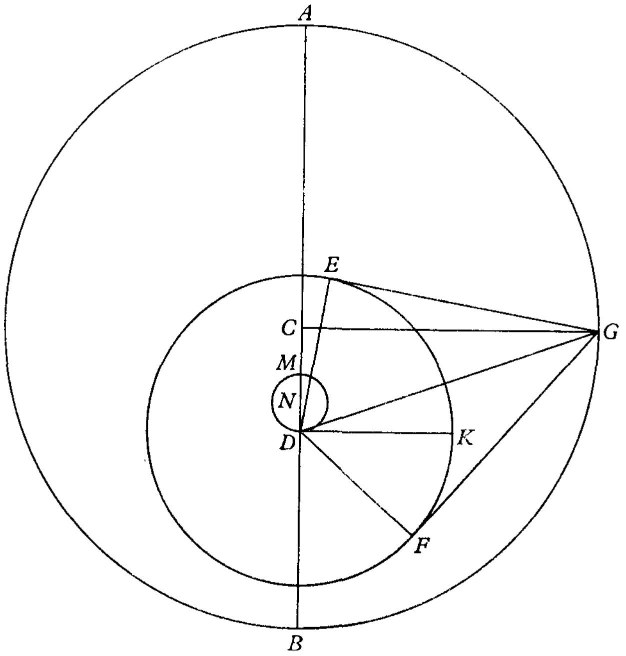
VENUS' TWOFOLD MOTION *Chapter 22*

Nevertheless, there is no simple uniform motion of Venus around *D*, as is proved particularly by two of Ptolemy's observations [*Syntaxis*, X, 3]. He made one of them on the 2nd day of the Egyptian month Pharmuthi in Hadrian's year 18 = dawn, 18 February 134 C.E. in the Roman calendar. At that time, with the sun's mean motion = $318\,{}^{5}/_{6}°$, Venus, appearing in the morning at $275\,{}^{1}/_{4}°$ in the ecliptic, had reached the outermost limit of its elongation = 43° 35′ [+275° 15′ = 318° 50′]. Ptolemy performed the second observation on the 4th day of the same Egyptian month Pharmuthi in Antoninus [Pius'] year 3 = twilight, 19 February 140 C.E. in the Roman calendar. At that time too the sun's mean place = $318\,{}^{5}/_{6}°$; Venus, at its evening greatest elongation therefrom = $48\,{}^{1}/_{3}°$, was seen at $7\,{}^{1}/_{6}°$ in longitude [= 48° 20′+318° 50′− 360°].

With this information available, on the same terrestrial orbit take point *G*, where the earth is located, such that *AG* = the quadrant of a circle, the distance at which in both observations the sun in its mean motion was seen on the opposite side [of the circle] west of the apogee of Venus' eccentric [$48{}^{1}/_{3}°$+360° −90° = 318° 20′ ≅ $318\,{}^{5}/_{6}°$]. Join *GC*, and construct *DK* parallel to it. Draw *GE* and *GF* tangent to Venus' orbit. Join *DE*, *DF*, and *DG*.

In the first observation angle *EGC* = the morning elongation = 43° 35′. In the second observation, *CGF* = the evening elongation = $48\,{}^{1}/_{3}°$. The sum of both = the whole of *EGF* = $91\,{}^{11}/_{12}°$. Therefore *DGF* = $^{1}/_{2}$ [*EGF*] = 45° 57 $^{1}/_{2}$′. *CGD*, the remainder [when *DGF* is subtracted from *CGF* = $48\,{}^{1}/_{3}°$− 45° 57 $^{1}/_{2}$′ = 2° 22$^{1}/_{2}$′] ≅ 2° 23′. But *DCG* is a right angle. Therefore in [right] triangle *CGD*, the angles being given, the ratio of the sides is given, and as a length *CD* = 416^{p} whereof *CG* = $10{,}000^{p}$. However, the distance between the centers was shown above = 208^{p} in the same units [V, 21]. Now it has become just twice as large. Hence, when *CD* is bisected at point *M*, *DM* will similarly = 208^{p} = the entire variation of this approach and withdrawal. If this variation is bisected again at *N*, this will appear to be the midpoint and normalizer of this motion. Consequently, as in the three outer planets, Venus' motion too happens to be compounded out of two uniform motions, whether that occurs through an eccentrepicycle, as in those cases [V, 4], or in any other of the aforementioned ways.

Nevertheless, this planet differs somewhat from the others in the pattern and measurement of its motions, as will be demonstrated more easily and more conveniently by an eccentreccentric (in my opinion). Thus, suppose that around *N* as center and with *DN* as radius, we describe a circlet on which [the center of] Venus' circle revolves and shifts in accordance with the following rule. Whenever the earth touches diameter *ACB*, which contains the eccentric's higher and lower apsides, the center of the planet's circle is always at its least distance [from the center of the earth's orbit in *C*], that is, at point *M*. But when the earth is at a middle apse (such as *G*), the center of the [planet's] circle reaches point *D*, with *CD* the greatest distance [from the center of the earth's orbit in *C*]. Hence, as may be inferred, while the earth traverses its own orbit once, the center of the planet's circle revolves twice around center *N*, and in the same direction as the earth, that is, eastward. Through this hypothesis for Venus, its uniform and apparent motions agree with every kind of situation, as will soon be clear. Everything proved thus



far with regard to Venus is found to fit our times too, except that the eccentricity has decreased by about $^1/_6$. Formerly it was all of 416^p [Ptolemy, *Syntaxis,* X, 3; $2\ ^1/_2{}^p : 60^p = 416\ ^2/_3$], but now it is 350^p, as many observations show us [$416 \times {}^5/_6 = 347$].

ANALYZING VENUS' MOTION — *Chapter 23*

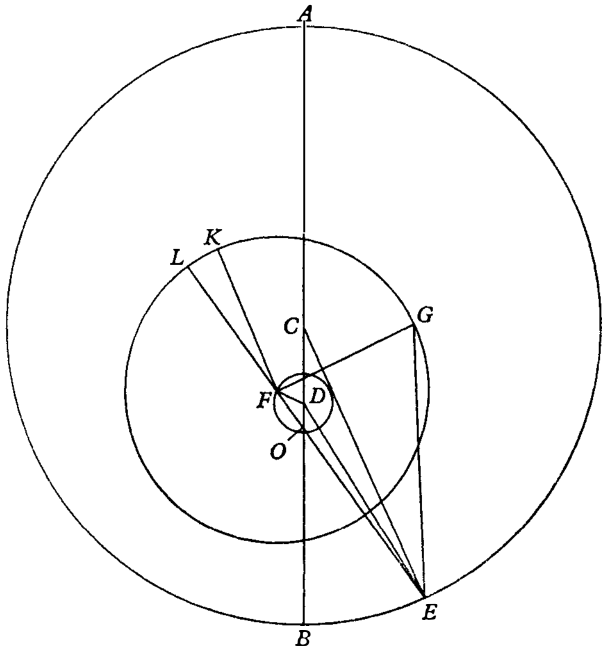
From these observations I took two places, observed with the greatest accuracy [*Syntaxis,* X, 4].

[Earlier version:

One was the work of Ptolemy in Antoninus [Pius'] year 2 on the 29th day of the month Tybi before dawn. On a straight line between the moon and the first bright star, the farthest north of the [three], in the forehead of the Scorpion, Ptolemy saw Venus $1\ ^1/_2$ times farther away from the moon than from the fixed star. The position of the fixed star being known, namely, 209° 40′ [longitude] and $1\ ^1/_3$° north latitude, it was worth-while ascertaining the observed place of the moon for the purpose of determining the position of Venus.

From the birth of Christ to the time of this observation there were 138 Egyptian years, 18 days, $4\ ^3/_4$ hours after midnight at Alexandria, but at Cracow $3\ ^3/_4{}^h$ local time, uniform time $3^h\ 41^m = 9^{dm}\ 23^{ds}$. In its mean uniform motion the sun was at $255\ ^1/_2$°; in its apparent motion, at 23° within the Archer [= 263°]. Hence, the moon's uniform distance from the sun = 319° 18′; its mean anomaly = 87° 37′; and its mean latitudinal anomaly from its northern limit = 12° 19′. From this information the moon's true place was computed = 209° 4′, north latitude 4° 58′. But the addition of the precession of the equinoxes, then = 6° 41′, put the moon at 5° 45′ within the Scorpion [= 215° 45′ = 209° 4′+6° 41′]. By instrumental means 2° within the Virgin were seen to culminate at Alexandria, and 25° within the Scorpion were rising. Consequently according to my calculation the moon's parallax was 51′ in longitude, 16′ in latitude. Hence the place of the moon, as observed at Alexandria and corrected, emerged as 209° 55′ [= 209° 4′+51′], north latitude 4° 42′ [= 4° 58′–16′]. Accordingly, Venus' position was determined = 209° 46′, north latitude 2° 40′.

Now let the earth's orbit be *AB*, with its center at *C* and diameter *ACB* passing through both



apsides. Let A be the point from which the body of Venus is seen at its apogee $= 48\frac{1}{3}^\circ$, and B the opposite point $= 228\frac{1}{3}^\circ$. On the diameter take distance $CD = 312^p$, whereof $AC = 10{,}000$. With D as center, and radius $DF = 1/3\ CD$, that is, 104, describe a circlet.

Now since the mean place of the sun $= 255\frac{1}{2}^\circ$, therefore the earth's distance from [Venus'] lower apse $= 27^\circ\ 10'$ [$+228\frac{1}{3}^\circ = 255\frac{1}{2}^\circ$]. Hence let arc $BE = 27^\circ\ 10'$. Join EC, ED, and DF, so that angle $CDF = 2 \times BCE$. Then with F as center, describe Venus' orbit. Let its concave circumference be intersected at L by the prolongation of straight line EF, intersecting diameter AB at O. To the same circumference also draw FK parallel to CE. Put the planet at point G. Join GE and GF.

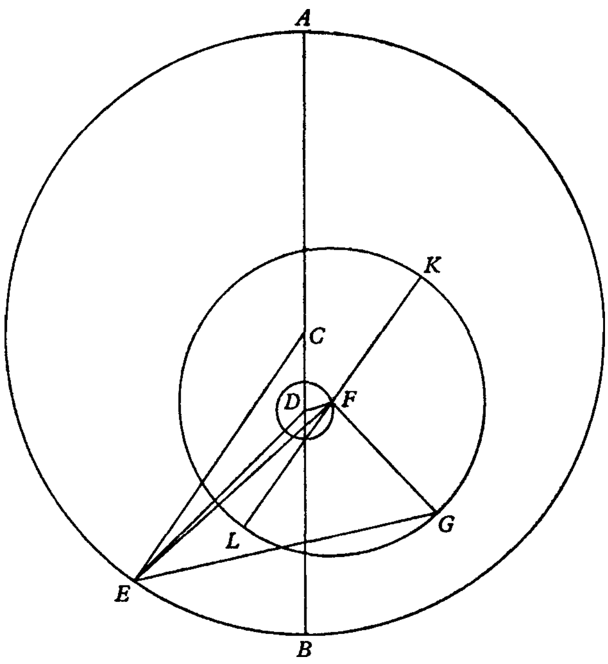
Now that these preparations have been made, our tasks are to find arc KG = the planet's distance from its orbit's mean apogee $= K$, and angle CEO. In triangle CDE, angle $DCE = 27^\circ\ 10'$ and side $CD = 312^p$, whereof $CE = 10{,}000$. Hence the remaining side $DE = 9724$, and angle $CED = 50'$. Similarly in triangle DEF, two sides are given, $DE = 9724^p$, whereof $DF = 104$, and $CE = 10{,}000$. The angle [EDF] enclosed by sides ED and DF is given. For, CDF is given $= 54^\circ\ 20'$ [$= 2 \times (BCE = 27^\circ\ 10')$] and FDB = the remainder of the semicircle [from which $CDF = 54^\circ\ 20'$ is subtracted] $= 125^\circ\ 40'$. Therefore, the whole of $FDE = 153^\circ\ 40'$. Hence, the remaining side EF is obtained $= 9817$ in those units, and angle $DEF = 16'$.

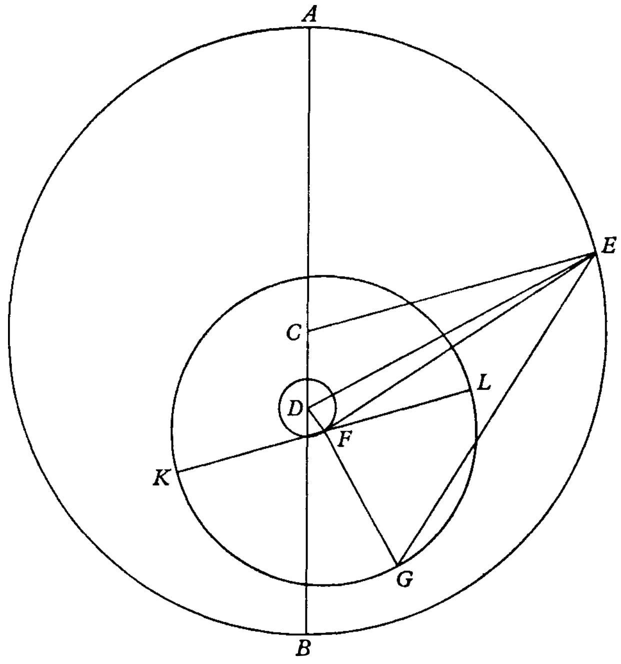
The whole of CEF [$= DEF + CED = 16' + 50'$] $= 1^\circ\ 6'$. This is the difference between the mean motion and the apparent motion around center F, that is, between angles BCE and EOB. Therefore BOE is obtained $= 28^\circ\ 16'$ [$= 27^\circ\ 10' + 1^\circ\ 6'$], which was our first task.

Secondly, angle $CEG = 45^\circ\ 44'$ = the planet's distance from the mean place of the sun [$= 255\frac{1}{2}^\circ - 209^\circ\ 46'$]. Hence, the whole of FEG [$= CEG + FEC = 45^\circ\ 44' + 1^\circ\ 6'$] $= 46^\circ\ 50'$. But EF is given $= 9817^p$, whereof $AC = 10{,}000$, and FG is ascertained $= 7193$ also in those aforementioned units. Hence in triangle EFG, the ratio of the sides EF and FG is given [$9817 : 7193$], together with angle FEG [$= 46^\circ\ 50'$]. Angle EFG will also be given $= 84^\circ\ 19'$. Thereby exterior angle LFG is given $= 131^\circ\ 6'$ = arc LKG = the planet's distance from the apparent apogee of its orbit. But angle $KFL = CEF$ = the difference between the mean and true apsides $= 1^\circ\ 6'$, as has been shown. When this is subtracted from $131^\circ\ 6'$, the remainder $= 130^\circ$ = arc KG, from the planet to the mean apse. The remainder of the circle $= 230^\circ$ = the uniform anomaly, measured from point K. Hence we have for Antoninus [Pius'] year 2 = 138 C.E., at Cracow, on 16 December, 3 hours, 45 minutes after midnight, Venus' uniform anomaly $= 230^\circ$, the quantity for which we were looking].

One was the work of Timocharis at dawn on the 18th day of the Egyptian month Mesori in Ptolemy Philadelphus' year 13 = year 52 after Alexander's death. In this observation Venus was reported as having been seen occulting the westernmost of the four fixed stars in the Virgin's left wing. In the description of this constellation, this is the sixth star, with longitude = $151\,{}^1/_2°$, latitude = $1\,{}^1/_6°$ north, and magnitude = 3. Thus Venus' place was evident [= $151\,{}^1/_2°$]. The sun's mean place was computed = 194° 23′.

This being the situation, in the illustrative diagram, with point *A* remaining at 48° 20′, arc *AE* = 146° 3′ [= 194° 23′ − 48° 20′]. *BE* = the remainder [when *AE* is subtracted from the semicircle = 180° − 146° 3′] = 33° 57′. Furthermore, angle *CEG* = the planet's distance from the sun's mean place = 42° 53′ [= 194° 23′ − 151 $^1/_2$°]. Line *CD* = 312^p [= $208^p + 104^p$] whereof *CE* = $10{,}000^p$. Angle *BCE* [= arc *BE*] = 33° 57′. Hence in triangle *CDE*, the remaining angles *CED* = 1° 1′ [and *CDE* = 145° 2′], while the third side *DE* = 9743^p. But angle *CDF* = 2 × *BCE* [= 33° 57′] = 67° 54′. When *CDF* is subtracted from the semicircle, the remainder = *BDF* = 112° 6′. *BDE*, being an angle exterior to triangle *CDE*, [= *CED* + (*DCE* = *BCE*) = 1° 1′ + 33° 57′] = 34° 58′. Hence, all of *EDF* [= *BDE* + *BDF* = 34° 58′ + 112° 6′] = 147° 4′. *DF* is given = 104^p whereof *DE* = 9743^p. Moreover, in triangle *DEF*, angle *DEF* = 20′. The whole of *CEF* [= *CED* + *DEF* = 1° 1′ + 20′] = 1° 21′, and side *EF* = 9831^p. But the whole of *CEG* is already known = 42° 53′. Therefore *FEG*, the remainder [when *CEF* (= 1° 21′) is subtracted from *CEG* (= 42° 53′)] = 41° 32′. *FG* = the radius of [Venus'] orbit = 7193^p whereof *EF* = 9831^p. In triangle *EFG*, therefore, through the given ratio of the sides and through angle *FEG*, the remaining angles are given, and *EFG* = 72° 5′. When this is added to a semicircle the sum = 252° 5′ = arc *KLG*, from the higher apse of [Venus'] orbit. Thus again we have established that at dawn on the 18th day of the month Mesori in Ptolemy Philadelphus' year 13, Venus' parallactic anomaly = 252° 5′.





I myself observed Venus' other place at 1 hour after sunset = the start of the 8th hour after noon on 12 March 1529 C.E. I saw Venus beginning to be occulted by the moon's dark side midway between both horns. This occultation lasted until the end of that hour or a little longer, when the planet was observed emerging westward on the [moon's] other side in the middle of the curvature between the horns. Therefore, at or about the middle of this hour, clearly there was a central conjunction of the moon and Venus, a spectacle which I witnessed at Frombork. Venus was still increasing its evening elongation, and had not yet reached the tangent to its orbit. From the beginning of the Christian era there are 1529 Egyptian years 87 days plus $7\,{}^1/_2$ hours by apparent time, but 7 hours 34 minutes by uniform time. The sun's mean place in its simple motion = 332° 11′; the precession of the equinoxes = 27° 24′; the moon's uniform motion away from the sun = 33° 57′; its uniform anomaly = 205° 1′; and its [motion in] latitude = 71° 59′. From this information the moon's true place was computed = 10°, but with respect to the equinox = 7° 24′ within the Bull [= 37° 24′ = 10° + 27° 24′], with latitude = 1° 13′ north. Since 15° within the Balance were rising, the moon's parallax in longitude = 48′, and in latitude = 32′. Hence, its apparent place = 6° 36′ within the Bull [= 7° 24′ − 48′]. But its longitude in the sphere of the fixed stars = 9° 12′ [= 10° − 48′], with north latitude = 41′ [= 1° 13′ − 32′]. The same was Venus' apparent place in the evening when it was 37° 1′ away from the sun's mean place [332° 11′ + 37° 1′ = 369° 12′ = 9° 12′], with the earth's distance to Venus' higher apse = 76° 9′ to the west [+332° 11′ = 408° 20′ − 360° = 48° 20′].

Now reproduce the diagram, following the model of the preceding construction, except that arc EA or angle ECA = 76° 9′. $CDF = 2 \times ECA$ = 152° 18′. The eccentricity CD, as it is found nowadays, = 246ᵖ, and DF = 104ᵖ whereof CE = 10,000ᵖ. Therefore, in triangle CDE, we have angle DCE = the remainder [when $ECA = 76° 9'$ is subtracted from 180°] given $= 103° 51'$, and enclosed by given sides [$CD = 246^p$, $CE = 10{,}000^p$]. From this information angle CED will be shown $= 1° 15'$, the third side $DE = 10{,}056^p$, and the remaining angle $CDE = 74° 54'$ [$= 180° - (DCE + CED = 103° 51' + 1° 15')$]. But $CDF = 2 \times ACE$ [$= 76° 9'$] $= 152° 18'$. From CDF, subtract angle CDE [$= 74° 54'$], and the remainder $EDF = 77° 24'$ [$= 152° 18' - 74° 54'$]. Thus again in triangle DEF, two sides, $DF = 104^p$ whereof $DE = 10{,}056^p$, enclose the given angle EDF [$= 77° 24'$]. Angle DEF is also given $= 35'$, as well as the remaining side $EF = 10{,}034^p$. Hence, the whole angle CEF [$= CED + DEF = 1° 15' + 35'$] $= 1° 50'$. Furthermore, the whole angle $CEG = 37° 1'$ = the planet's apparent distance from the sun's mean place. When CEF is subtracted from CEG, the remainder FEG [$= 37° 1' - 1° 50'$] $= 35° 11'$. Accordingly, in triangle EFG also, with angle E given [$= 35° 11'$], two sides are likewise given: $EF = 10{,}034^p$ whereof $FG = 7193^p$. Hence, the remaining angles will also be determined: $EGF = 53\,^1/_2°$, and $EFG = 91° 19'$ = the planet's distance from its orbit's true perigee.

But diameter KFL was drawn parallel to CE, so that K = the apogee of [the planet's] uniform motion, and L = the perigee. [From $EFG = 91° 19'$], subtract angle $EFL = CEF$ [$= 1° 50'$]. The remainder = angle LFG = arc $LG = 89° 29'$. KG = the remainder [when LG is subtracted] from the semicircle $= 90° 31'$ = the planet's parallactic anomaly as measured from the uniform higher apse of its orbit. This is what we wanted for this hour of my observation.

In Timocharis' observation, however, the corresponding figure $= 252° 5'$. In the intervening period, then, besides 1115 complete revolutions, there are $198° 26'$ [$= (90° 31' + 360° = 450° 31') - 252° 5'$]. From dawn on the 18th day of the month Mesori in Ptolemy Philadelphus' year 13 to 7 $^1/_2$ hours after noon on 12 March 1529 *C.E.*, there are 1800 Egyptian years 236 days plus about 40 day-minutes. Multiply the motion in 1115 revolutions plus $198° 26'$ by 365 days. Divide the product by 1800 years 236 days 40 day-minutes. The result will be the annual motion $= 3 \times 60°$ plus $45° 1' 45'' 3''' 40''''$. When this figure is distributed over 365 days, the outcome = the daily motion $= 36' 59'' 28'''$. This was the basis on which was constructed the Table exhibited above [after V, 1].

[Earlier version of the concluding paragraph of V, 23:
In the preceding observation by Ptolemy, however, the value was 230°. In the interval, therefore, in addition to complete revolutions there are $220° 31'$ [$= (90° 31' + 360° = 450° 31') - 230°$]. From Antoninus [Pius'] year 2, 8 $^1/_4$ hours before noon, Cracow time, on the 20th day of the month Tybi until 1529 C.E., 12 March, 7 $^1/_2$ hours after noon, there are 1391 Egyptian years, 69 days, 39 day-minutes, 23 day-seconds. In this time there are likewise counted $220° 31'$ in addition to complete revolutions, which are 859 according to the Table of Mean Motions [after V, 1], which therefore is correct. Meanwhile, the positions of the eccentric's apsides have remained unchanged at $48\,^1/_3°$ and $228° 20'$].

THE PLACES OF VENUS' ANOMALY *Chapter 24*

[Earlier version:
THE PLACES OF VENUS' MEAN ANOMALY
Hence the places of Venus' parallactic anomaly are easily established. For from the birth of Christ to Ptolemy's observation there are 138 Egyptian years, 18 days, 9 $^1/_2$ day-minutes. The motion

corresponding to this interval is 105° 25′. When this value is subtracted from the 230° of Ptolemy's observation, the remainder is Venus' anomaly of 124° 35′ [= 230° −105° 25′] at midnight preceding 1 January [1 C.E.]. Then, in accordance with the reckoning of the motion and the time, which has often been repeated, the remaining places are, for the 1st Olympiad, 318° 9′; for Alexander, 79° 14′; for Caesar, 70° 48′].

[Printed version:

From the 1st Olympiad to dawn on the 18th day of the month Mesori in Ptolemy Philadelphus' year 13 there are 503 Egyptian years 228 days 40 day-minutes, during which the motion is computed = 290° 39′. Subtract this figure from 252° 5′ plus 1 revolution [612° 5′−290° 39′], and the remainder = 321° 26′ = the place of the 1st Olympiad. From this place, the remaining places are obtained by computing the motion and the time, which has often been mentioned: Alexander's = 81° 52′, Caesar's = 70° 26′, and Christ's = 126° 45′.

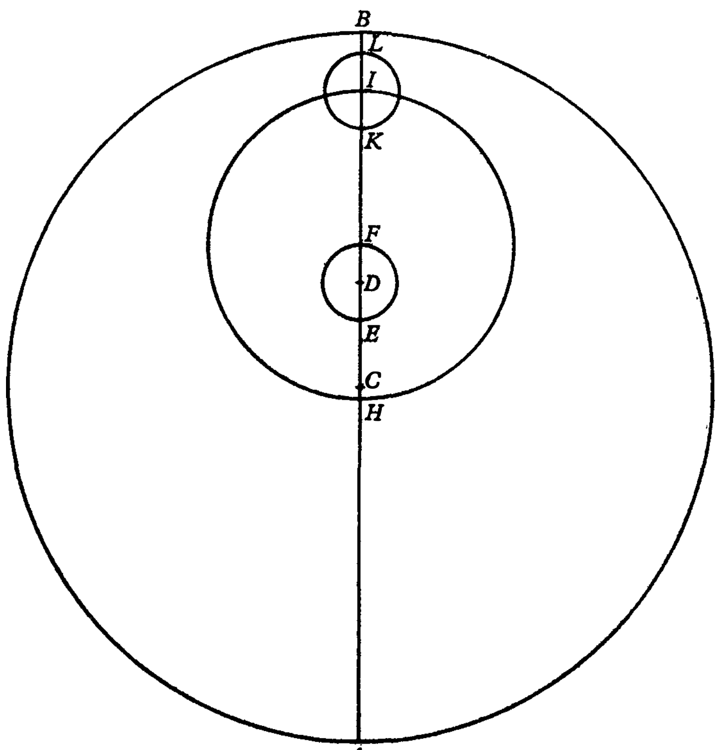
MERCURY *Chapter 25*

Now that I have shown how Venus is linked with the earth's motion, and beneath what ratio of its circles its uniform motion lies concealed, Mercury remains. It too will doubtless conform to the same basic assumption, even though it wanders in more convolutions than does Venus or any [other] of the [planets] discussed above. As is clear from the experience of the ancient observers, the narrowest of Mercury's [greatest] elongations from the sun occur in the sign of the Balance, and wider [greatest] elongations (as is proper) in the opposite sign [the Ram]. Yet its widest [greatest] elongations do not occur in this place, but in certain others to either side [of the Ram], namely, in the Twins and Water Bearer, especially in Antoninus [Pius'] time, according to Ptolemy's conclusion [*Syntaxis,* IX, 8]. This displacement occurs in no other planet.

The explanation of this phenomenon was believed by the ancient astronomers to be the earth's motionlessness and Mercury's motion on its large epicycle, [carried] by an eccentric. They realized that a single, simple eccentric could not account for these phenomena (even when they permitted the eccentric to move not around its own center, but around a different center). They were further obliged to grant that the same eccentric which carried the epicycle moved on another circlet, such as they accepted in connection with the moon's eccentric [IV, 1]. Thus there were three centers: namely, that belonging to the eccentric which carried the epicycle; secondly, to the circlet; and thirdly, to that circle which more recent astronomers call the "equant". Passing over the first two centers, the ancients allowed the epicycle to move uniformly only around the equant's center. This procedure was in gross conflict with the true center [of the epicycle's motion], its relative [distances], and the prior centers of both [other circles]. The ancients were convinced that the phenomena of this planet could be explained in no other way than that expounded at considerable length in Ptolemy's *Syntaxis* [IX, 6].

However, in order that this last planet too may be rescued from the affronts and pretenses of its detractors, and that its uniform motion, no less than that of the other aforementioned planets, may be revealed in relation to the earth's motion, I shall attribute to it too, [as the circle mounted] on its eccentric, an eccentric instead of the epicycle accepted in antiquity. The pattern, however, is different from Venus' [V, 22], and yet on the [outer] eccentric there moves an epicyclet. The planet is carried not around the epicyclet's circumference, but up and down along its diameter. This [motion along a straight line] can be the result also of uniform circular motions, as was shown above in connection with the precession of the equinoxes [III, 4]. There is nothing surprising in this, since Proclus too in his *Commentary on Euclid's Elements* declares that a straight line can also be produced by multiple motions. Mercury's appearances will be demonstrated by all these [devices]. But to make the hypothesis clearer, let the earth's grand circle be *AB*, with its center at *C*. On diameter *ACB*, between points *B* and *C*, take *D* as center and with radius $= {}^1/_3$ *CD* describe circlet *EF*, so that the greatest distance from *C* is at *F*, and at *E* the least distance. Around *F* as center describe *HI* as Mercury's [outer eccentric] circle. Then, around its higher apse *I*, taken as center, add the epicyclet [*KL*] traversed by the planet. Let *HI*, an eccentreccentric, function as an epicycle on an eccentric.

After the diagram has been drawn in this way, let all these [points] occur in order on straight line *AHCEDFKILB*. But meanwhile put the planet at *K*, that is, at the least distance $= KF$ from $F =$ the center of the circle carrying the epicyclet. Make this [*K*] the beginning of Mercury's revolutions. Conceive center *F* performing two revolutions to one of the earth's and in the same direction, that is, eastward. The same [speed applies] also to the planet on *KL*, but up and down along the diameter with respect to the center of circle *HI*.



From these arrangements it follows that whenever the earth is in *A* or *B*, the center of Mercury's [outer eccentric] circle is at *F* = its greatest distance from point *C*. But when the earth is midway [between *A* and *B*] at a quadrant's distance from them, [the center of Mercury's outer eccentric] is at *E* = its closest [approach to *C*]. In accordance with this sequence the pattern is the opposite of Venus' [V, 22]. Furthermore, as a result of this rule, while Mercury traverses the diameter of epicyclet *KL*, it is closest to the center of the circle carrying the epicyclet, that is, it is at *K*, when the earth crosses diameter *AB*. When the earth on either side is at the places midway [between *A* and *B*], the planet arrives at *L* = its greatest distance [from the center of the circle carrying the epicyclet]. In this way, commensurate with the earth's annual period, two double revolutions equal to each other occur, that of the center of the [outer eccentric] circle on the circumference of the circlet *EF*, and that of the planet along diameter *LK*.

But in the meantime the epicyclet or line *FI* moves uniformly with its own motion around circle *HI* and its center in about 88 days, completing one revolution independently with respect to the sphere of the fixed stars. However, in what I call the "motion in parallax", which exceeds the earth's motion, the epicyclet overtakes the earth in 116 days, as can be inferred more precisely from the Table of Mean Motions [after V, 1]. It therefore follows that in its own motion Mercury does not always describe the same circular circumference. On the contrary, in proportion to its distance from the center of its deferent, it traces an exceedingly varying circuit, smallest in point *K*, greatest in *L*, and mean in *I*. Almost the same variation may be noticed in the lunar epicyclepicyclet [IV, 3]. But what the moon does along the circumference, Mercury accomplishes along the diameter in a reciprocating motion. Yet this is compounded out of uniform motions. How this is done, I explained above in connection with the precession of the equinoxes [III, 4]. However, I shall add some other remarks about this subject later on in connection with the latitudes [VI, 2]. The foregoing hypothesis suffices for all the observed phenomena of Mercury, as will become clear from a review of the observations made by Ptolemy and others.

THE PLACE OF MERCURY'S HIGHER AND LOWER APSIDES — *Chapter 26*

Ptolemy observed Mercury in Antoninus [Pius'] year 1 on the 20th day of the month Epiphi after sunset, when the planet was at its greatest evening elongation from the sun's mean place [*Syntaxis*, IX, 7]. This = 138 C.E., 188 days, 42 $^1/_2$ day-minutes, Cracow time. According to my computation, therefore, the sun's mean place = 63° 50′, and the planet [was observed] through the instrument (as Ptolemy says) at 7° within the Crab [= 97°]. But after the subtraction of the equinoctial precession, then = 6° 40′, Mercury's place clearly = 90° 20′ [=97°–6° 40′] from the beginning of the Ram in the sphere of the fixed stars, and its greatest elongation from the mean sun = 26 $^1/_2$° [= 90° 20′– 63° 50′].

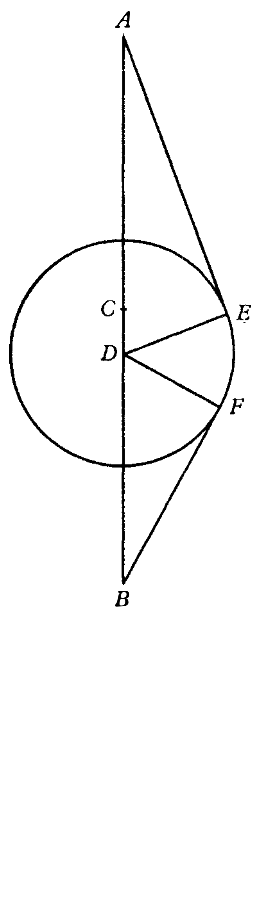
Ptolemy made a second observation at dawn on the 19th day of the month Phamenoth in Antoninus [Pius'] year 4 = 140 years 67 days from the beginning of the Christian era, plus about 12 day-minutes, with the mean sun at 303° 19′. Through the instrument Mercury appeared at 13 $^1/_2$° within the Goat [= 283 $^1/_2$°], but at about 276° 49′ [≅ 283 $^1/_2$°–6° 40′] from the beginning of the Ram among the fixed stars. Therefore, its greatest morning elongation likewise = $26\,{}^1\!/_2°$ [= 303° 19′ − 276° 49′]. The limits of its elongations from the sun's mean place being equal on both sides, Mercury's apsides must be halfway between both places, that is, between 276° 49′ and 90° 20′: 3° 34′ and, diametrically opposite, 183° 34′ [276° 49′ − 90° 20′ = 186° 29′; 186° 29′ ÷ 2 ≅ 93° 15′; 276° 49′ − 93° 15′ = 183° 34′; 183° 34′ − 180° = 3° 34′]. These must be the places of both of Mercury's apsides, the higher and the lower.

These are distinguished, as in the case of Venus [V, 20] by two observations. The first of these was made [by Ptolemy, *Syntaxis,* IX, 8] at dawn on the 15th day of the month Athyr in Hadrian's year 19, with the sun's mean place = 182° 38′. Mercury's greatest morning elongation from it = 19° 3′, since Mercury's apparent place = 163° 35′ [+19° 3′ = 182° 38′]. In the same year 19 of Hadrian = 135 C.E., on the 19th day of the Egyptian month Pachon at twilight Mercury was found with the aid of the instrument at 27° 43′ in the sphere of the fixed stars, with the sun in its mean motion = 4° 28′. Once more [as in the case of Venus, V, 20] the planet's greatest evening elongation = 23° 15′ was larger than the previous [morning elongation = 19° 3′]. Hence Mercury's apogee quite clearly was nowhere else but at about $183\,{}^1\!/_8°$ [≅ 183° 34′] at that time. Q. E. D.

THE SIZE OF MERCURY'S ECCENTRICITY, AND THE RATIO OF ITS CIRCLES *Chapter 27*

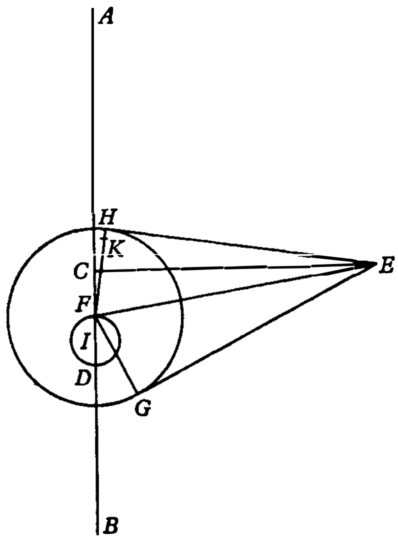
By means of these observations the distance between the centers and the sizes of the circles are likewise demonstrated at the same time. For let straight line *AB* pass through Mercury's apsides, *A* the higher, and *B* the lower, and let *AB* also be the diameter of the [earth's] grand circle with center *C*. With center *D*, describe the planet's orbit. Then draw lines *AE* and *BF* tangent to the orbit. Join *DE* and *DF*.

In the former of the two observations above, the greatest morning elongation was seen = 19° 3′; therefore, angle *CAE* = 19° 3′. But in the other observation the greatest evening elongation was seen = $23\,{}^1\!/_4°$. Consequently, in both right triangles *AED* and *BFD*, the angles being given, the ratios of the sides will also be given. Thus, with *AD* = 100,000^p, *ED* = the radius of the orbit = 32,639^p. However, with *BD* = 100,000^p, in those units *FD* = 39,474^p. Yet *FD* (being a radius of the orbit) = *ED* = 32,639^p whereof *AD* = 100,000^p. In those units *DB*, the remainder [of *AB* − *AD*] = 82,685^p. Hence *AC* = $^1\!/_2$ [*AD* + *DB* = 100,000^p + 82,685^p = 182,685^p] = 91,342^p, and *CD* = the remainder [= *AD* − *AC* = 100,000^p − 91,342^p] = 8658^p = the distance between the centers [of the earth's orbit and Mercury's orbit]. With *AC* = 1^p or 60′, however, the radius of Mercury's orbit = 21′ 26″, and *CD* = 5′ 41″. With *AC* = 100,000^p, *DF* = 35,733^p, and *CD* = 9479^p. Q. E. D.



But these sizes too do not remain everywhere the same, and they are quite different from those occurring near the mean apsides, as is shown by the apparent morning and evening elongations observed in those positions, as reported by Theon and Ptolemy [*Syntaxis,* IX, 9]. Theon observed Mercury's greatest evening elongation after sunset on the 18th day of the month Mesori in Hadrian's year 14 = 129 years 216 days 45 day-minutes after the birth of Christ, with the sun's mean place = $93\,{}^1\!/_2°$, that is, near Mercury's mean apse [≅ $^1\!/_2$ (183° 34′ − 3° 34′); = 90°

$+3^\circ\,34'$]. Through the instrument the planet was seen at $3\frac{5}{6}^\circ$ east of the Little King in the Lion. Therefore its place $= 119\frac{3}{4}^\circ$ [$\cong 3^\circ\,50'+115^\circ\,50'$], and its greatest evening elongation $= 26\frac{1}{4}^\circ$ [$= 119\frac{3}{4}^\circ - 93\frac{1}{2}^\circ$]. The other greatest elongation was reported by Ptolemy as observed by himself at dawn on the 21st day of the month Mesori in Antoninus [Pius'] year 2 = 138 years 219 days 12 day-minutes in the Christian calendar. In like manner the sun's mean place $= 93^\circ\,39'$, from which he found Mercury's greatest morning elongation $= 20\frac{1}{4}^\circ$, since it was seen at $73\frac{2}{5}^\circ$ in the sphere of the fixed stars [$73^\circ\,24'+20^\circ\,15' = 93^\circ\,39'$].



Now reproduce *ACDB* as the diameter of the [earth's] grand circle. As before, let it pass through Mercury's apsides. At point *C* erect the perpendicular *CE* as the line of the sun's mean motion. Between *C* and *D* take point *F*. Around it describe Mercury's orbit, to which straight lines *EH* and *EG* are tangent. Join *FG*, *FH*, and *EF*.

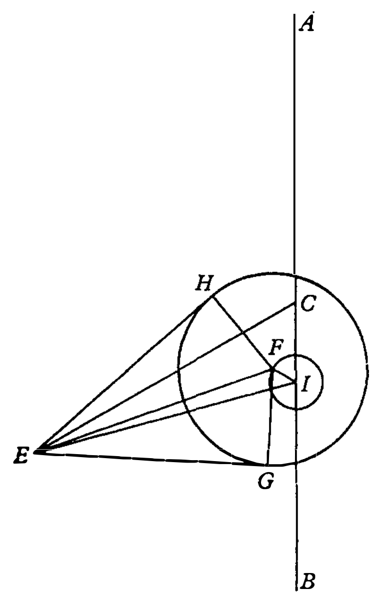
It is proposed once more to find point *F*, and the ratio of radius *FG* to *AC*. Angle *CEG* is given $= 26\frac{1}{4}^\circ$, and $CEH = 20\frac{1}{4}^\circ$. Therefore, the whole of HEG [$= CEH+CEG = 20^\circ\,15'+26^\circ\,15'$] $= 46\frac{1}{2}^\circ$. $HEF = \frac{1}{2}$ [$HEG = 46\frac{1}{2}^\circ$] $= 23\frac{1}{4}^\circ$. CEF = the remainder [$= HEF - CEH = 23\frac{1}{4}^\circ - 20\frac{1}{4}^\circ$] $= 3^\circ$. Therefore, in right triangle *CEF*, side *CF* is given $= 524^p$, and hypotenuse $FE = 10{,}014^p$ whereof $CE = AC = 10{,}000^p$. The whole of *CD* has been shown above [earlier in V, 27] $= 948^p$ when the earth is in the planet's higher or lower apse. *DF* = the diameter of the circlet traversed by the center of Mercury's orbit = the excess [of $CD = 948^p$ over $CF = 524^p$] $= 424^p$, and radius $IF = 212^p$ [$= \frac{1}{2}$ diameter *DF*]. Hence, the whole of *CFI* [$= CF+FI = 524^p+212^p$] $\cong 736\frac{1}{2}^p$.

Similarly, in triangle *HEF* (in which *H* is a right angle) *HEF* is also given $= 23\frac{1}{4}^\circ$. Hence, *FH* clearly $= 3947^p$ whereof $EF = 10{,}000^p$. But with $EF = 10{,}014^p$ whereof $CE = 10{,}000^p$, $FH = 3953^p$. However, *FH* was shown above [at the beginning of V, 27, where it was lettered *DF*] $= 3573^p$. Let $FK = 3573^p$. Then *HK* = the remainder [= this $FH - FK = 3953^p - 3573^p$] $= 380^p$ = the greatest variation in the planet's distance from *F* = the center of its orbit, which occurs [as the planet moves] from the higher and lower apsides to the mean apsides. On account of this distance and its variation, the planet describes around *F*, the center of its orbit, unequal circles depending on the various distances, the smallest $= 3573^p$ [$= FK$], and the greatest $= 3953^p$ [$= FH$]. The mean between them must be $= 3763^p$ [$380^p \div 2 = 190^p$; 190^p+3573^p; 3953^p-190^p]. Q. E. D.

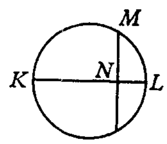
WHY MERCURY'S ELONGATIONS AT ABOUT THE SIDE OF A HEXAGON [= 60°, FROM THE PERIGEE] LOOK BIGGER THAN THE ELONGATIONS OCCURRING AT PERIGEE *Chapter 28*

Furthermore, it will therefore not seem surprising that at about [the points where] the sides of a hexagon [touch a circumscribed] circle, Mercury's elongations are greater than at perigee. [These elongations at 60° from perigee] exceed even those which I have already demonstrated [at the end of V, 27]. Consequently, the ancients believed that Mercury's orbit comes closest to the earth twice in one revolution of the earth.

Construct angle $BCE = 60^\circ$. Hence angle $BIF = 120^\circ$, since *F* is assumed to make two revolutions for one of E = the earth. Join EF and EI. [In V, 27] CI was shown $= 736\frac{1}{2}^p$ whereof $EC = 10{,}000^p$, and angle ECI is given $= 60°$. Therefore in triangle ECI, the remaining side $EI = 9655^p$, and angle $CEI \cong 3°\ 47'$. $CEI = ACE - CIE$. But ACE is given $= 120°$ [= supplement of ($BCE = 60°$) by construction]. Therefore $CIE = 116°\ 13'$ [$= ACE - CEI = 120° - 3°\ 47'$]. But angle FIB likewise $= 120° = 2 \times ECI$ [$= 60°$] by construction. CIF, which [together with $FIB = 120°$] completes the semicircle, $= 60°$. EIF = the remainder [$= CIE - CIF = 116°\ 13' - 60°$] $= 56°\ 13'$. But IF was shown [in V, 27] $= 212^p$ whereof $EI = 9655^p$ [V, 28, above]. These sides enclose angle EIF given [$= 56°\ 13'$]. This information yields angle $FEI = 1°\ 4'$. CEF = the remainder [$= CEI - FEI = 3°\ 47' - 1°\ 4'$] $= 2°\ 43'$ = the difference between the center of the planet's orbit and the sun's mean place. The remaining side EF [in triangle EFI] $= 9540^p$.



Now describe Mercury's orbit GH around center F. From E draw EG and EH tangent to the orbit. Join FG and FH. We must first ascertain the size of radius FG or FH in this situation. We shall do so in the following way. Take a circlet whose diameter $KL = 380^p$ [= greatest variation; V, 27] whereof $AC = 10{,}000^p$. Along this diameter, or its equivalent, conceive the planet approaching toward, or receding from, center F on straight line FG or FH in the manner explained above in connection with the precession of the equinoxes [III, 4]. According to the hypothesis that BCE intercepts an arc $= 60°$, take $KM = 120°$ in the same degrees. Draw MN perpendicular to KL. MN = half the chord subtending $2 \times KM$ or $2 \times ML$, will intercept $LN = \frac{1}{4}$ of the diameter $= 95^p$ [$= \frac{1}{4} \times 380^p$], as is proved in Euclid's *Elements*, XIII, 12, combined with V, 15. Then KN = the remaining $\frac{3}{4}$ of the diameter $= 285^p$ [$= 380 - 95$]. This, added to the planet's least distance [$= 3573^p$; V, 27] = the desired line FG or FH in this instance $= 3858^p$ [$= 3573^p + 285^p$], with AC similarly $= 10{,}000^p$ and EF also shown $= 9540^p$ [V, 28 above]. Therefore, in right triangle FEG or FEH, two sides are given [EF with FG or FH]. Hence angle FEG or FEH will also be given. With $EF = 10{,}000^p$, FG or $FH = 4044^p$, subtending an angle $= 23°52\frac{1}{2}'$. Thus, the whole of GEH [$= FEG + FEH = 2 \times 23°\ 52\frac{1}{2}'$] $= 47°\ 45'$. But at the lower apse only $46\frac{1}{2}°$ were seen; and at the mean apse, similarly $46\frac{1}{2}°$ [V, 27]. Consequently, here the angle has become greater than in both those situations by $1°\ 14'$ [$\cong 47°\ 45' - 46°\ 30'$]. The reason is not that the planet's orbit is nearer to the earth than it is at perigee, but that here the planet describes a larger circle than it does there. All these results are in agreement with both past and present observations, and are produced by uniform motions.

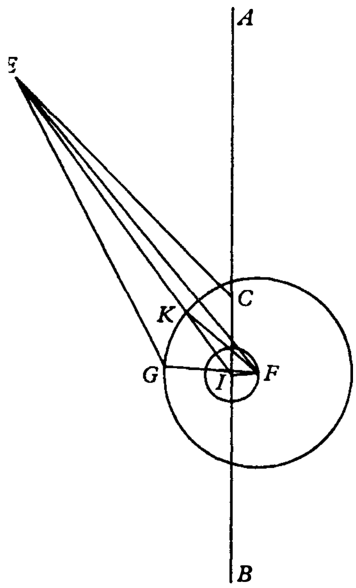


ANALYSIS OF MERCURY'S MEAN MOTION *Chapter 29*

Among the more ancient observations [*Syntaxis*, IX, 10] there is found an appearance of Mercury, at dawn on the 19th day of the Egyptian month Thoth in Ptolemy Philadelphus' year 21, 2 lunar diameters east of the straight line passing through the first and second of the stars in the Scorpion's forehead, and 1 lunar diameter north of the first star. The place of the first star is known $= 209°\ 40'$ longitude, $1\frac{1}{3}°$ north latitude; of the second star $= 209°$ longitude, $1°\ \frac{1}{2}°\ \frac{1}{3}° = 1\frac{5}{6}°$ south latitude. From this information Mercury's place was inferred $= 210°\ 40'$ longitude [$= 209°\ 40' + (2 \times \frac{1}{2}°)$], $\cong 1\frac{5}{6}°$ north latitude [$= 1\frac{1}{3}° + \frac{1}{2}°$].

From Alexander's death there were 59 years 17 days 45 day-minutes; the sun's mean place = 228° 8′, according to my computation; and the planet's morning elongation = 17° 28′. This was still increasing, as was noticed during the next 4 days. Hence the planet had certainly not yet reached its greatest morning elongation nor the point of tangency on its orbit, but was still traveling in the lower arc, closer to the earth. Since the higher apse = 183° 20′ [V, 26], its distance from the sun's mean place = 44° 48′ [= 228° 8′ − 183° 20′].

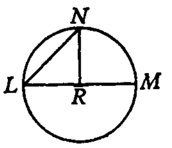
Then once more let ACB = the grand circle's diameter, as above [V, 27]. From C = the [grand circle's] center, draw CE as the line of the sun's mean motion so that angle ACE = 44° 48′. With I as center, describe the circlet which carries the eccentric's center = F. Take angle BIF by hypothesis = $2 \times ACE$ [= 2 × 44° 48′] = 89° 36′. Join EF and EI.



In triangle ECI two sides are given: $CI = 736\,{}^1/_2{}^p$ [V, 27] whereof $CE = 10{,}000^p$. These sides enclose angle ECI given = 135° 12′ = supplement of ACE [= 44° 48′]. The remaining side $EI = 10{,}534^p$, and angle CEI = 2° 49′ = $ACE - EIC$. Therefore CIE is given = 41° 59′ [= 44° 48′ − 2° 49′]. But CIF = supplement of BIF [= 89° 36′] = 90° 24′. Hence, the whole of EIF [= $CIF + EIC$ = 90° 24′ + 41° 59′] = 132° 23′.

In triangle EFI, EIF is likewise enclosed by given sides, namely, $EI = 10{,}534^p$, and $IF = 211\,{}^1/_2{}^p$, whereof AC is assumed = $10{,}000^p$. This information discloses angle FEI = 50′, with the remaining side $EF = 10{,}678^p$. CEF = the remainder [= $CEI - FEI$ = 2° 49′ − 50′] = 1° 59′.

Now take circlet LM, with diameter $LM = 380^p$ whereof $AC = 10{,}000^p$. Let arc LN = 89° 36′ by hypothesis. Draw its chord LN, and NR perpendicular to LM. Then $(LN)^2 = LM \times LR$. According to this given ratio, LR in particular is given as a length $\cong 189^p$ whereof diameter $LM = 380^p$. Along this straight line [LR], or its equivalent, the planet is known to have diverged from F, the center of its orbit, while line EC has traversed angle ACE. Hence, when these [189] units are added to 3573^p = the minimum distance [V, 27], in this situation the sum = 3762^p.



Therefore, with center F, and radius = 3762^p, describe a circle. Draw EG, cutting the convex circumference [of Mercury's orbit] at point G so that angle CEG = 17° 28′ = the planet's apparent elongation from the sun's mean place [= 228° 8′ − 210° 40′]. Join FG, and FK parallel to CE. When angle CEF is subtracted from the whole of CEG, the remainder FEG = 15° 29′ [= 17° 28′ − 1° 59′]. Hence, in triangle EFG, two sides are given: $EF = 10{,}678^p$, and $FG = 3762^p$, as well as angle FEG = 15° 29′. This information yields angle EFG = 33° 46′. EFG − (EFK = CEF [its alternate interior angle]) = KFG = arc KG = 31° 47′ [= 33° 46′ − 1° 59′]. This is the planet's distance from its orbit's mean perigee = K. If a semicircle is added to KG, the sum = 211° 47′ [= 180° + 31° 47′] = the mean motion in parallactic anomaly in this observation. Q. E. D.

MORE RECENT OBSERVATIONS OF MERCURY'S MOTIONS *Chapter 30*

The foregoing method of analyzing this planet's motion was shown to us by the ancients. But they were helped by clearer skies where the Nile (it is said) does not give off such mists as does the Vistula for us. We inhabitants of a more severe region have been denied that advantage by nature. The less frequent calmness of our air, in addition to the great obliquity of the sphere, allows us to see Mercury more rarely, even when it is at its greatest elongation from the sun. For, Mercury's rising in the Ram and Fishes is not visible to us nor, on the other hand, is its setting in the Virgin and Balance. Indeed, in the Crab or Twins it does not show itself in any position whatsoever when there is only twilight or dawn, whereas it never appears at night unless the sun has moved well into the Lion. This planet has accordingly inflicted many perplexities and labors on us in our investigation of its wanderings.

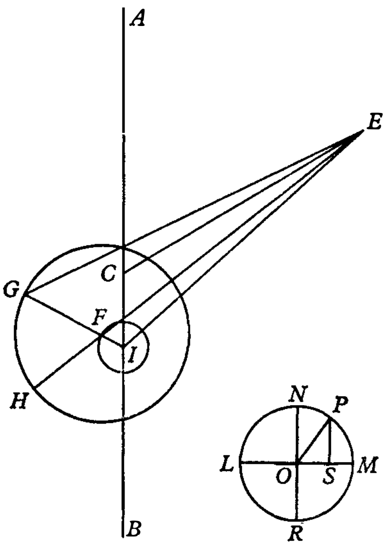
I have therefore borrowed three positions from those which were carefully observed at Nuremberg. The first was determined by Bernhard Walther, Regiomontanus' pupil, 5 uniform hours after midnight on 9 September = 5 days before the Ides, 1491 C.E., by means of an armillary astrolabe directed toward Palili-cium [= Aldebaran]. He saw Mercury at 13 $^1/_2$° within the Virgin [= 163 $^1/_2$°], 1° 50′ north latitude. At that time the planet was beginning to set in the morning, while it had steadily diminished its morning appearances during the preceding days. From the beginning of the Christian era there were 1491 Egyptian years 258 days 12 $^1/_2$ day-minutes. The sun's mean place in itself = 149° 48′, but in relation to the vernal equinox = 26° 47′ within the Virgin [= 176° 47′]. Hence Mercury's elongation ≅ 13 $^1/_4$° [176° 47′ − 163° 30′ = 13° 17′].

The second position was observed by Johann Schöner 6 $^1/_2$ hours after mid-night on 9 January 1504 C.E., when 10° within the Scorpion were culminating over Nuremberg. He saw the planet at 3 $^1/_3$° within the Goat, 0° 45′ north latitude. The sun's mean place in relation to the vernal equinox was computed = 27° 7′ within the Goat [= 297° 7′], with Mercury 23° 47′ to the west in the morning.

The third observation was made by the same Johann [Schöner] on 18 March in the same year 1504. He found Mercury at 26° 55′ within the Ram, about 3° north latitude, when 25° within the Crab were culminating over Nuremberg. His armillary sphere was directed toward the same star Palilicium [Aldebaran] at 7 $^1/_2$ hours after noon, with the sun's mean place in respect to the vernal equinox = 5° 39′ within the Ram, and Mercury's elongation from the sun in the evening = 21° 17′ [≅ 26° 55′ − 5° 39′].

From the first position to the second, there are 12 Egyptian years 125 days 3 day-minutes 45 day-seconds. During this time the sun's simple motion = 120° 14′ and Mercury's anomaly in parallax = 316° 1′. In the second interval there are 69 days 31 day-minutes 45 day-seconds; the sun's mean simple motion = 68° 32′, and Mercury's mean anomaly in parallax = 216°.

I wish to analyze Mercury's motion at the present time on the basis of these three observations. In them, I believe it must be granted, the measurements of the circles have remained valid from Ptolemy until now, since also in the other planets the earlier sound writers are not found to have gone astray in this respect. If in addition to these observations we had the place of the eccentric's apse, nothing further would be missing in the apparent motion of this planet too. I have assumed that the place of the higher apse = 211 $^1/_2$°, that is, 18 $^1/_2$° within the sign of the Scorpion. For I might not make it smaller without injuring the observers. Thus we shall have the eccentric's anomaly, I mean, the distance of the sun's mean motion from the apogee, at the first determination, = 298° 15′; at the second, = 58° 29′; and at the third, = 127° 1′.

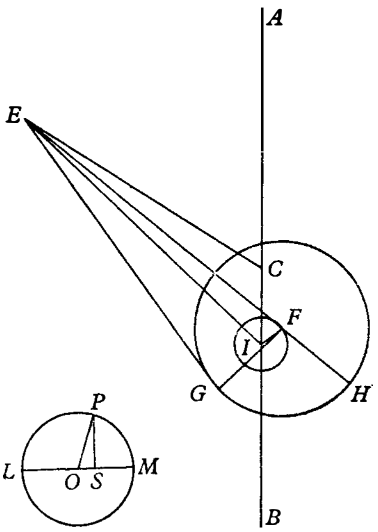


Now draw the diagram according to the preceding model, except that angle ACE is taken $= 61°\,45'$ [$= 360° - 298°\,15'$] = the distance by which the line of the mean sun was west of the apogee in the first observation. Let everything which follows therefrom be in agreement with the hypothesis. IC is given [V, 29] $= 736\frac{1}{2}^{p}$ whereof $AC = 10{,}000^{p}$. In triangle ECI, angle ECI also is given [$= 180° - (ACE = 61°\,45') = 118°\,15'$]. Angle CEI will be given too $= 3°\,35'$, and side $IE = 10{,}369^{p}$ whereof $EC = 10{,}000^{p}$, and $IF = 211\frac{1}{2}^{p}$ [V, 29].

Then also in triangle EFI, there are two sides having a given ratio [$IE : IF = 10{,}369^{p} : 211\frac{1}{2}^{p}$]. By construction angle $BIF = 123\frac{1}{2}° = 2 \times ACE$ [$= 61°\,45'$]. CIF = the supplement [of $BIF = 123\frac{1}{2}°$] $= 56\frac{1}{2}°$. Therefore the whole of EIF [$= CIF + EIC = 56°\,30' + (EIC = ACE - CEI = 61°\,45' - 3°\,35' = 58°\,10')$] $= 114°\,40'$. Therefore $IEF = 1°\,5'$, and side $EF = 10{,}371^{p}$. Hence angle $CEF = 2\frac{1}{2}°$ [$= CEI - IEF = 3°\,35' - 1°\,5'$].

However, in order to determine how much the motion of approach and withdrawal has increased [the distance of] the circle centered at F from the apogee or perigee, describe a circlet quadrisected by diameters LM and NR at center O. Take angle $POL = 2 \times ACE$ [$= 61°\,45'$] $= 123\frac{1}{2}°$. From point P drop PS perpendicular to LM. Then, according to the given ratio, OP (or its equivalent LO) : $OS = 10{,}000^{p} : 5519^{p} = 190 : 105$. These numbers, added together, constitute $LS = 295^{p}$ whereof $AC = 10{,}000^{p}$, and the extent to which the planet has become more remote from center F. When 295^{p} is added to 3573^{p} = the least distance [V, 27], the sum $= 3868^{p}$ = the present value.

With this as radius, describe circle HG around center F. Join EG, and extend EF as straight line EFH. Angle CEF has been shown $= 2\frac{1}{2}°$. GEC was observed $= 13\frac{1}{4}°$ = the distance between the planet in the morning and the mean sun [in the observation attributed to Walther]. Then the whole of FEG [$= GEC + CEF = 13°\,15' + 2°\,30'$] $= 15\frac{3}{4}°$. But in triangle EFG, $EF : FG = 10{,}371^{p} : 3868^{p}$, and angle E is given [$= 15°\,45'$]. This information will give us also angle $EGF = 49°\,8'$. Hence the remaining exterior angle [$GFH = EGF + GEF = 49°\,8' + 15°\,45'$] $= 64°\,53'$. When this quantity is subtracted from the whole circle, the remainder $= 295°\,7'$ = the true anomaly in parallax. To this, add angle CEF [$= 2°\,30'$], and the sum = the mean and uniform [anomaly in parallax] $= 297°\,37'$, which we were looking for. To this, add $316°\,1'$ [= the parallactic anomaly between the first observation and the second], and for the second observation we shall have the uniform parallactic anomaly $= 253°\,38'$ [$= 297°\,37' + 316°\,1' = 613°\,38' - 360°$], which I shall also show to be correct and in agreement with the observation.

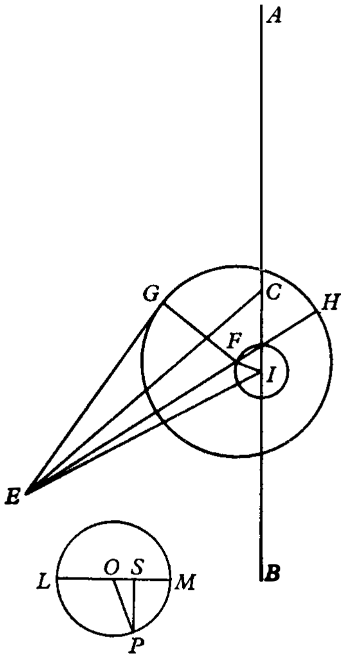


As the measure of the anomaly of the eccentric in the second observation, let us take angle $ACE = 58°\,29'$. Then, once more, in triangle CEI two sides are given: $IC = 736^{p}$ [previously and hereafter $736\frac{1}{2}^{p}$] whereof $EC = 10{,}000^{p}$, and also angle ECI, the supplement [of $ACE = 58°\,29'$] $= 121°\,31'$. Therefore, the third side $EI = 10{,}404^{p}$ in the same units, and angle $CEI = 3°\,28'$. Similarly, in triangle EIF, angle $EIF = 118°\,3'$, and side $IF = 211\frac{1}{2}^{p}$ whereof $IE = 10{,}404^{p}$. Therefore, the third side $EF = 10{,}505^{p}$ in the same units, and angle $IEF = 61'$. Hence, the remainder FEC [$= CEI - IEF = 3°\,28' - 1°\,1'$] $= 2°\,27'$ = the eccentric's prosthaphaeresis. When this quantity is added to the mean motion in parallax, the sum = the true [motion in parallax] $= 256°\,5'$ [$= 2°\,27' + 253°\,38'$].

Now on the epicyclet [which produces the] approach and withdrawal let us take arc LP or angle $LOP = 2 \times ACE$ $[= 58^\circ 29'] = 116^\circ 58'$. Once more, then, in right triangle OPS, because the ratio of the sides $OP : OS$ is given $=$ $10{,}000^p : 4535^p$, $OS = 86^p$ whereof OP or $LO = 190^p$. As a length the whole of LOS $[= LO+OS = 190^p+86^p] = 276^p$. When this quantity is added to the smallest distance $= 3573^p$ [V, 27], the sum $= 3849^p$.

With this as radius, around F as center describe circle HG so that the apogee of the parallax is at point H. Let the planet's distance from point H be arc HG, extending westward $103^\circ 55'$. This is the amount by which an entire revolution differs from the corrected motion in parallax [= mean motion + additive prosthaphaeresis = true motion] $= 256^\circ 5'$ $[+103^\circ 55' = 360^\circ]$. Therefore EFG, the supplement [of $HFG = 103^\circ 55'$] $= 76^\circ 5'$. Thus again in triangle EFG two sides are given: $FG = 3849^p$ whereof $EF = 10{,}505^p$. Hence angle $FEG =$ $21^\circ 19'$. When this quantity is added to CEF $[= 2^\circ 27']$, the whole of $CEG =$ $23^\circ 46'$ = the apparent distance between C = the center of the grand circle, and the planet G. This distance too takes only a little away from the observed elongation $[= 23^\circ 47']$.

This agreement will be further confirmed a third time when we take angle $ACE = 127^\circ 1'$ or its supplement $BCE = 52^\circ 59'$. Again we shall have a triangle [ECI], two of whose sides are known: $CI = 736\,{}^1/_2{}^p$ whereof $EC = 10{,}000^p$. These sides enclose angle $ECI = 52^\circ 59'$. From this information angle CEI is shown $= 3^\circ 31'$, and side $IE = 9575^p$ whereof $EC = 10{,}000^p$. By construction angle EIF is given $= 49^\circ 28'$, and also is enclosed by given sides $FI = 211\,{}^1/_2{}^p$ whereof $EI = 9575^p$. Hence [in triangle EIF] the remaining side [EF] $= 9440^p$ in those units, and angle $IEF = 59'$. When this quantity is subtracted from the whole of IEC $[= 3^\circ 31']$, the remainder $= FEC = 2^\circ 32'$. This is the subtractive prosthaphaeresis of the eccentric's anomaly. When this quantity $[2^\circ 32']$ is added to the mean parallactic anomaly, which I determined by adding 216° [= the mean parallactic anomaly] of the second interval [to $253^\circ 38'$ = the uniform parallactic anomaly in the second observation; $216^\circ+253^\circ 38' = 469^\circ 38' - 360^\circ] = 109^\circ 38'$, the true [parallactic anomaly] comes out $= 112^\circ 10'$ $[= 2^\circ 32' + 109^\circ 38']$.



Now on the epicyclet take angle $LOP = 2 \times ECI$ $[= 52^\circ 59'] = 105^\circ 58'$. Here too, on the basis of the ratio $PO : OS$, we shall have $OS = 52^p$, so that the whole of $LOS = 242^p$ $[= LO+OS = 190^p+52^p]$. When this quantity $[242^p]$ is added to the smallest distance $= 3573^p$, we shall have the corrected [distance] $=$ 3815^p. With this as radius, around F as center describe a circle in which the parallax's higher apse $= H$ on EFH, prolonged as a straight line. As a measure of the true parallactic anomaly, take arc $HG = 112^\circ 10'$, and join GF. Then the supplementary angle $GFE = 67^\circ 50'$. This is enclosed by the given sides $GF = 3815^p$ whereof $EF = 9440^p$. From this information angle FEG will be determined $= 23^\circ 50'$. From this quantity subtract the prosthaphaeresis CEF $[= 2^\circ 32']$, and the remainder $CEG = 21^\circ 18'$ = the apparent distance between the evening planet [G] and [C,] the center of the grand circle. This is practically the same distance as was found by observation $[= 21^\circ 17']$.

This agreement of these three positions with the observations, therefore, unquestionably guarantees that the eccentric's higher apse is located, as I assumed, at $211\,{}^1/_2{}^\circ$ in the sphere of the fixed stars in our time, and also that the entailed consequences are correct; namely, the uniform parallactic anomaly in the first position = 297° 37′, in the second = 253° 38′, and in the third = 109° 38′. These are the results we were seeking.

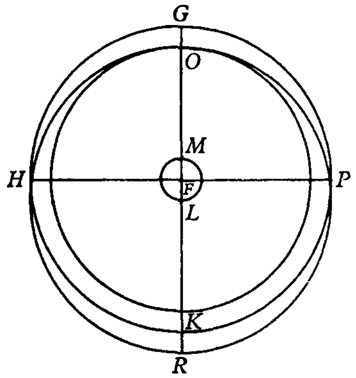
In that ancient observation at dawn on the 19th day of Thoth, the 1st Egyptian month, in Ptolemy Philadelphus' year 21, the place of the eccentric's higher apse (in Ptolemy's opinion) = 183° 20′ in the sphere of the fixed stars, while the mean parallactic anomaly = 211° 47′ [V, 29]. The interval between this most recent and that ancient observation = 1768 Egyptian years 200 days 33 day-minutes. In that time the eccentric's higher apse moved 28° 10′ [= 211° 30′ − 183° 20′] in the sphere of the fixed stars, and the parallactic motion, in addition to 5570 whole revolutions = 257° 51′ [+211° 47′ = 469° 38′; 469° 38′ − 360° = 109° 38′ in the 3rd observation]. For in 20 years about 63 periods are completed, amounting in [20 × 88 =] 1760 years to [88 × 63 =] 5544 periods. In the remaining 8 years 200 days there are 26 revolutions [$20 : 8\,{}^1/_2 \cong 63 : 26$]. Accordingly in 1768 years 200 days 33 day-minutes there is an excess, after 5570 revolutions [= 5544+26], of 257° 51′. This is the difference between the observed places in that first ancient observation and ours. This difference also agrees with the numbers set down in my Tables [after V, 1]. When we compare to this interval the 28° 10′ through which the eccentric's apogee moved, its motion will be recognized = 1° in 63 years, provided it was uniform [$1768\,{}^1/_2{}^y \div 28\,{}^1/_6 = 63^y$].

DETERMINING MERCURY'S PLACES *Chapter 31*

From the beginning of the Christian era to the most recent observation there are 1504 Egyptian years 87 days 48 day-minutes. During that time Mercury's parallactic motion in anomaly = 63° 14′, disregarding whole revolutions. When this quantity is subtracted from 109° 38′ [the anomaly in the third modern observation], the remainder = 46° 24′ = the place of Mercury's parallactic anomaly at the beginning of the Christian era. From that time backward to the beginning of the 1st Olympiad there are 775 Egyptian years 12 ½ days. For this interval the computation is 95° 3′ after complete revolutions. When this quantity is subtracted from the place of Christ (one revolution being borrowed), the remainder = the place of the 1st Olympiad = 311° 21′ [= 46° 24′+360° = 406° 24′ − 95° 3′]. Moreover, the computation being made for the 451 years 247 days from this time to Alexander's death, his place comes out = 213° 3′.

AN ALTERNATIVE ACCOUNT OF APPROACH AND WITHDRAWAL *Chapter 32*

Before leaving Mercury, I have decided to consider another method, no less plausible than the foregoing, by which that approach and withdrawal can be produced and explained. Let circle *GHKP* be quadrisected at center *F*. Around *F* describe a concentric circlet *LM*. In addition describe another circle *OR*, with center *L* and radius *LFO* = *FG* or *FH*. Suppose that this whole combination of circles, together with their intersections *GFR* and *HFP*, moves eastward away from the apogee of the planet's eccentric around center *F* about 2° 7′ every day, that is, as much as the planet's parallactic motion exceeds the earth's motion in



the ecliptic. Let the planet meanwhile supply the rest of the parallactic motion, nearly equal to the earth's motion, away from point *G* on its own circle *OR*. Also assume that in this same revolution, which is annual, the center of *OR*, the circle which carries the planet, moves back and forth, as was stated above [V, 25], in a motion of libration along diameter *LFM*, which is twice as great as the one posited previously.

Now that these arrangements have been made, put the earth in its mean motion opposite the apogee of the planet's eccentric. At that time place the center of the planet-carrying circle at *L*, but the planet itself at point *O*. Since it is then at its least distance from *F*, as the entire [configuration] moves, the planet will describe the smallest circle, whose radius is *FO*. What follows thereafter is that when the earth is near the middle apse, the planet arrives at point *H*, corresponding to its greatest distance from *F*, and describes the largest arcs, that is, along the circle centered at *F*. For then the deferent *OR* will coincide with the circle *GH* because their centers merge in *F*. As the earth proceeds from this position in the direction of the perigee [of the planet's eccentric], and the center of the circle *OR* [oscillates] to the other limit *M*, the circle itself rises above *GK*, and the planet at *R* will again attain its least distance from *F*, and traverse the paths assigned to it at the start. For here the three equal revolutions coincide, namely, the earth's return to the apogee of Mercury's eccentric, the libration of the center along diameter *LM*, and the planet's circuit from line *FG* to the same line. The only deviation from these revolutions is the motion [$\cong 2° 7'$ daily] of the intersections *G*, *H*, *K*, and *P* away from the eccentric's apse, as I said [earlier in V, 32].

Thus nature has played a game with this planet and its remarkable variability, which has nevertheless been confirmed by its perpetual, precise, and unchangeable orderliness. But here it should be noted that the planet does not pass through the middle regions of quadrants *GH* and *KP* without a deviation in longitude. For, as the variation in the centers intervenes, it must produce a prosthaphaeresis. Yet the center's impermanence interposes an obstacle. For example, suppose that while the center remained at *L*, the planet started out from *O*. Near *H* it would undergo its greatest deviation, as measured by eccentricity *FL*. But it follows from the assumptions that as the planet moves away from *O*, it initiates and increases the deviation which must be produced by the distance *FL* of the centers. However, as the movable center approaches its mean position at *F*, more and more of the anticipated deviation diminishes and is overcome so that it entirely disappears near the middle intersections *H* and *P*, where the greatest deviation should have been expected. Nevertheless (as I admit) even when the deviation becomes small, it is hidden in the sun's rays, and is not perceived at all along the circumference of the circle when the planet rises or sets in the morning or evening. I did not wish to omit this model, which is no less reasonable than the foregoing model, and which will be highly suitable for use in connection with the variations in latitude [VI, 2].

TABLES OF THE PROSTHAPHAERESES OF THE FIVE PLANETS

Chapter 33

The uniform and apparent motions of Mercury and the other planets have been demonstrated above and expounded by computations, which will serve as examples to open the way to calculating the differences in these motions at any other places. However, for the purpose of facilitating the procedure, for each planet I have prepared its own Tables, consisting of 6 columns and 30 rows in steps of 3°, in the usual manner. The first 2 columns will contain the common numbers not only of the eccentric's anomaly but also of the parallaxes. The 3rd column shows the eccentric's collected, I mean, total differences occurring between the uniform and nonuniform motions of those circles. In the 4th column there are the proportional minutes, computed as sixtieths, by which the parallaxes increase or decrease on account of the earth's greater or smaller distance. In the 5th column there are the prosthaphaereses themselves, which are the parallaxes occurring at the higher apse of the planet's eccentric with reference to the grand circle. In the 6th and last column are found the surpluses by which the parallaxes occurring at the higher apse are exceeded by those happening at the eccentric's lower apse. The Tables are as follows.

TABLE OF SATURN'S PROSTHAPHAERESES								
Common Numbers		Correction of the Eccentric		Propor- tional Minutes	Parallaxes of the Grand Circle [at the Higher Apse]		Surplus of the Parallax [at the Lower Apse]	
°	°	°	′		°	′	°	′
3	357	0	20	0	0	17	0	2
6	354	0	40	0	0	34	0	4
9	351	0	58	0	0	51	0	6
12	348	1	17	0	1	7	0	8
15	345	1	36	1	1	23	0	10
18	342	1	55	1	1	40	0	12
21	339	2	13	1	1	56	0	14
24	336	2	31	2	2	11	0	16
27	333	2	49	2	2	26	0	18
30	330	3	6	3	2	42	0	19
33	327	3	23	3	2	56	0	21
36	324	3	39	4	3	10	0	23
39	321	3	55	4	3	25	0	24
42	318	4	10	5	3	38	0	26
45	315	4	25	6	3	52	0	27
48	312	4	39	7	4	5	0	29
51	309	4	52	8	4	17	0	31
54	306	5	5	9	4	28	0	33
57	303	5	17	10	4	38	0	34
60	300	5	29	11	4	49	0	35
63	297	5	41	12	4	59	0	36
66	294	5	50	13	5	8	0	37
69	291	5	59	14	5	17	0	38
72	288	6	7	16	5	24	0	38
75	285	6	14	17	5	31	0	39
78	282	6	19	18	5	37	0	39
81	279	6	23	19	5	42	0	40
84	276	6	27	21	5	46	0	41
87	273	6	29	22	5	50	0	42
90	270	6	31	23	5	52	0	42

TABLE OF SATURN'S PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Proportional Minutes	Parallaxes of the Grand Circle at the Higher Apse		Surplus [of the Parallax] at the Lower Apse	
°	°	°	′		°	′	°	′
93	267	6	31	25	5	52	0	43
96	264	6	30	27	5	53	0	44
99	261	6	28	29	5	53	0	45
102	258	6	26	31	5	51	0	46
105	255	6	22	32	5	48	0	46
108	252	6	17	34	5	45	0	45
111	249	6	12	35	5	40	0	45
114	246	6	6	36	5	36	0	44
117	243	5	58	38	5	29	0	43
120	240	5	49	39	5	22	0	42
123	237	5	40	41	5	13	0	41
126	234	5	28	42	5	3	0	40
129	231	5	16	44	4	52	0	39
132	228	5	3	46	4	41	0	37
135	225	4	48	47	4	29	0	35
138	222	4	33	48	4	15	0	34
141	219	4	17	50	4	1	0	32
144	216	4	0	51	3	46	0	30
147	213	3	42	52	3	30	0	28
150	210	3	24	53	3	13	0	26
153	207	3	6	54	2	56	0	24
156	204	2	46	55	2	38	0	22
159	201	2	27	56	2	21	0	19
162	198	2	7	57	2	2	0	17
165	195	1	46	58	1	42	0	14
168	192	1	25	59	1	22	0	12
171	189	1	4	59	1	2	0	9
174	186	0	43	60	0	42	0	7
177	183	0	22	60	0	21	0	4
180	180	0	0	60	0	0	0	0

TABLE OF JUPITER'S PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Proportional		Parallaxes of [the Grand] Circle [at the Higher Apse]		Surplus [of the Parallax at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
3	357	0	16	0	3	0	28	0	2
6	354	0	31	0	12	0	56	0	4
9	351	0	47	0	18	1	25	0	6
12	348	1	2	0	30	1	53	0	8
15	345	1	18	0	45	2	19	0	10
18	342	1	33	1	3	2	46	0	13
21	339	1	48	1	23	3	13	0	15
24	336	2	2	1	48	3	40	0	17
27	333	2	17	2	18	4	6	0	19
30	330	2	31	2	50	4	32	0	21
33	327	2	44	3	26	4	57	0	23
36	324	2	58	4	10	5	22	0	25
39	321	3	11	5	40	5	47	0	27
42	318	3	23	6	43	6	11	0	29
45	315	3	35	7	48	6	34	0	31
48	312	3	47	8	50	6	56	0	34
51	309	3	58	9	53	7	18	0	36
54	306	4	8	10	57	7	39	0	38
57	303	4	17	12	0	7	58	0	40
60	300	4	26	13	10	8	17	0	42
63	297	4	35	14	20	8	35	0	44
66	294	4	42	15	30	8	52	0	46
69	291	4	50	16	50	9	8	0	48
72	288	4	56	18	10	9	22	0	50
75	285	5	1	19	17	9	35	0	52
78	282	5	5	20	40	9	47	0	54
81	279	5	9	22	20	9	59	0	55
84	276	5	12	23	50	10	8	0	56
87	273	5	14	25	23	10	17	0	57
90	270	5	15	26	57	10	24	0	58

TABLE OF JUPITER'S PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Proportional		Parallaxes of [the Grand] Circle [at the Higher Apse]		Surplus [of the Parallax] at [the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
93	267	5	15	28	33	10	25	0	59
96	264	5	15	30	12	10	33	1	0
99	261	5	14	31	43	10	34	1	1
102	258	5	12	33	17	10	34	1	1
105	255	5	10	34	50	10	33	1	2
108	252	5	6	36	21	10	29	1	3
111	249	5	1	37	47	10	23	1	3
114	246	4	55	39	0	10	15	1	3
117	243	4	49	40	25	10	5	1	3
120	240	4	41	41	50	9	54	1	2
123	237	4	32	43	18	9	41	1	1
126	234	4	23	44	46	9	25	1	0
129	231	4	13	46	11	9	8	0	59
132	228	4	2	47	37	8	56	0	58
135	225	3	50	49	2	8	27	0	57
138	222	3	38	50	22	8	5	0	55
141	219	3	25	51	46	7	39	0	53
144	216	3	13	53	6	7	12	0	50
147	213	2	59	54	10	6	43	0	47
150	210	2	45	55	15	6	13	0	43
153	207	2	30	56	12	5	41	0	39
156	204	2	15	57	0	5	7	0	35
159	201	1	59	57	37	4	32	0	31
162	198	1	43	58	6	3	56	0	27
165	195	1	27	58	34	3	18	0	23
168	192	1	11	59	3	2	40	0	19
171	189	0	53	59	36	2	0	0	15
174	186	0	35	59	58	1	20	0	11
177	183	0	17	60	0	0	40	0	6
180	180	0	0	60	0	0	0	0	0

TABLE OF MARS' PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Proportional		Parallaxes of [the Grand] Circle [at the Higher Apse]		Surplus of the Parallax [at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
3	357	0	32	0	0	1	8	0	8
6	354	1	5	0	2	2	16	0	17
9	351	1	37	0	7	3	24	0	25
12	348	2	8	0	15	4	31	0	33
15	345	2	39	0	28	5	38	0	41
18	342	3	10	0	42	6	45	0	50
21	339	3	41	0	57	7	52	0	59
24	336	4	11	1	13	8	58	1	8
27	333	4	41	1	34	10	5	1	16
30	330	5	10	2	1	11	11	1	25
33	327	5	38	2	31	12	16	1	34
36	324	6	6	3	2	13	22	1	43
39	321	6	32	3	32	14	26	1	52
42	318	6	58	4	3	15	31	2	2
45	315	7	23	4	37	16	35	2	11
48	312	7	47	5	16	17	39	2	20
51	309	8	10	6	2	18	42	2	30
54	306	8	32	6	50	19	45	2	40
57	303	8	53	7	39	20	47	2	50
60	300	9	12	8	30	21	49	3	0
63	297	9	30	9	27	22	50	3	11
66	294	9	47	10	25	23	48	3	22
69	291	10	3	11	28	24	47	3	34
72	288	10	19	12	33	25	44	3	46
75	285	10	32	13	38	26	40	3	59
78	282	10	42	14	46	27	35	4	11
81	279	10	50	16	4	28	29	4	24
84	276	10	56	17	24	29	21	4	36
87	273	11	1	18	45	30	12	4	50
90	270	11	5	20	8	31	0	5	5

TABLE OF MARS' PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Proportional		Parallaxes of [the Grand] Circle [at the Higher Apse]		Surplus [of the Parallax at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
93	267	11	7	21	32	31	45	5	20
96	264	11	8	22	58	32	30	5	35
99	261	11	7	24	32	33	13	5	51
102	258	11	5	26	7	33	53	6	7
105	255	11	1	27	43	34	30	6	25
108	252	10	56	29	21	35	3	6	45
111	249	10	45	31	2	35	34	7	4
114	246	10	33	32	46	35	59	7	25
117	243	10	11	34	31	36	21	7	46
120	240	10	7	36	16	36	37	8	11
123	237	9	51	38	1	36	49	8	34
126	234	9	33	39	46	36	54	8	59
129	231	9	13	41	30	36	53	9	24
132	228	8	50	43	12	36	45	9	49
135	225	8	27	44	50	36	25	10	17
138	222	8	2	46	26	35	59	10	47
141	219	7	36	48	1	35	25	11	15
144	216	7	7	49	35	34	30	11	45
147	213	6	37	51	2	33	24	12	12
150	210	6	7	52	22	32	3	12	35
153	207	5	34	53	38	30	26	12	54
156	204	5	0	54	50	28	5	13	28
159	201	4	25	56	0	26	8	13	7
162	198	3	49	57	6	23	28	12	47
165	195	3	12	57	54	20	21	12	12
168	192	2	35	58	22	16	51	10	59
171	189	1	57	58	50	13	1	9	1
174	186	1	18	59	11	8	51	6	40
177	183	0	39	59	44	4	32	3	28
180	180	0	0	60	0	0	0	0	0

TABLE OF VENUS' PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Propor- tional		Parallaxes of [the Grand] Circle [at the Higher Apse]		Surplus [of the Parallax at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
3	357	0	6	0	0	1	15	0	1
6	354	0	13	0	0	2	30	0	2
9	351	0	19	0	10	3	45	0	3
12	348	0	25	0	39	4	59	0	5
15	345	0	31	0	58	6	13	0	6
18	342	0	36	1	20	7	28	0	7
21	339	0	42	1	39	8	42	0	9
24	336	0	48	2	23	9	56	0	11
27	333	0	53	2	59	11	10	0	12
30	330	0	59	3	38	12	24	0	13
33	327	1	4	4	18	13	37	0	14
36	324	1	10	5	3	14	50	0	16
39	321	1	15	5	45	16	3	0	17
42	318	1	20	6	32	17	16	0	18
45	315	1	25	7	22	18	28	0	20
48	312	1	29	8	18	19	40	0	21
51	309	1	33	9	31	20	52	0	22
54	306	1	36	10	48	22	3	0	24
57	303	1	40	12	8	23	14	0	26
60	300	1	43	13	32	24	24	0	27
63	297	1	46	15	8	25	34	0	28
66	294	1	49	16	35	26	43	0	30
69	291	1	52	18	0	27	52	0	32
72	288	1	54	19	33	28	57	0	34
75	285	1	56	21	8	30	4	0	36
78	282	1	58	22	32	31	9	0	38
81	279	1	59	24	7	32	13	0	41
84	276	2	0	25	30	33	17	0	43
87	273	2	0	27	5	34	20	0	45
90	270	2	0	28	28	35	21	0	47

TABLE OF VENUS' PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Propor-tional		Parallaxes [of the Grand Circle at the Higher Apse]		Surplus [of the Parallax at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
93	267	2	0	29	58	36	20	0	50
96	264	2	0	31	28	37	17	0	53
99	261	1	59	32	57	38	13	0	55
102	258	1	58	34	26	39	7	0	58
105	255	1	57	35	55	40	0	1	0
108	252	1	55	37	23	40	49	1	4
111	249	1	53	38	52	41	36	1	8
114	246	1	51	40	19	42	18	1	11
117	243	1	48	41	45	42	59	1	14
120	240	1	45	43	10	43	35	1	18
123	237	1	42	44	37	44	7	1	22
126	234	1	39	46	6	44	32	1	26
129	231	1	35	47	36	44	49	1	30
132	228	1	31	49	6	45	4	1	36
135	225	1	27	50	12	45	10	1	41
138	222	1	22	51	17	45	5	1	47
141	219	1	17	52	33	44	51	1	53
144	216	1	12	53	48	44	22	2	0
147	213	1	7	54	28	43	36	2	6
150	210	1	1	55	0	42	34	2	13
153	207	0	55	55	57	41	12	2	19
156	204	0	49	56	47	39	20	2	34
159	201	0	43	57	33	36	58	2	27
162	198	0	37	58	16	33	58	2	27
165	195	0	31	58	59	30	14	2	27
168	192	0	25	59	39	25	42	2	16
171	189	0	19	59	48	20	20	1	56
174	186	0	13	59	54	14	7	1	26
177	183	0	7	59	58	7	16	0	46
180	180	0	0	60	0	0	16	0	0

TABLE OF MERCURY'S PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Propor-tional		Parallaxes [of the Grand Circle at the Higher Apse]		Surplus of the Parallax [at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
3	357	0	8	0	3	0	44	0	8
6	354	0	17	0	12	1	28	0	15
9	351	0	26	0	24	2	12	0	23
12	348	0	34	0	50	2	56	0	31
15	345	0	43	1	43	3	41	0	38
18	342	0	51	2	42	4	25	0	45
21	339	0	59	3	51	5	8	0	53
24	336	1	8	5	10	5	51	1	1
27	333	1	16	6	41	6	34	1	8
30	330	1	24	8	29	7	15	1	16
33	327	1	32	10	35	7	57	1	24
36	324	1	39	12	50	8	38	1	32
39	321	1	46	15	7	9	18	1	40
42	318	1	53	17	26	9	59	1	47
45	315	2	0	19	47	10	38	1	55
48	312	2	6	22	8	11	17	2	2
51	309	2	12	24	31	11	54	2	10
54	306	2	18	26	17	12	31	2	18
57	303	2	24	29	17	13	7	2	26
60	300	2	29	31	39	13	41	2	34
63	297	2	34	33	59	14	14	2	42
66	294	2	38	36	12	14	46	2	51
69	291	2	43	38	29	15	17	2	59
72	288	2	47	40	45	15	46	3	8
75	285	2	50	42	58	16	14	3	16
78	282	2	53	45	6	16	40	3	24
81	279	2	56	46	59	17	4	3	32
84	276	2	58	48	50	17	27	3	40
87	273	2	59	50	36	17	48	3	48
90	270	3	0	52	2	18	6	3	56

TABLE OF MERCURY'S PROSTHAPHAERESES

Common Numbers		Correction of the Eccentric		Propor-tional		Parallaxes [of the Grand Circle at the Higher Apse]		Surplus of the Parallax [at the Lower Apse]	
°	°	°	′	Minutes	Seconds	°	′	°	′
93	267	3	0	53	43	18	23	4	3
96	264	3	1	55	4	18	37	4	11
99	261	3	0	56	14	18	48	4	19
102	258	2	59	57	14	18	56	4	27
105	255	2	58	58	1	19	2	4	34
108	252	2	56	58	40	19	3	4	42
111	249	2	55	59	14	19	3	4	49
114	246	2	53	59	40	18	59	4	54
117	243	2	49	59	57	18	53	4	58
120	240	2	44	60	0	18	42	5	2
123	237	2	39	59	49	18	27	5	4
126	234	2	34	59	35	18	8	5	6
129	231	2	28	59	19	17	44	5	9
132	228	2	22	58	59	17	17	5	9
135	225	2	16	58	32	16	44	5	6
138	222	2	10	57	56	16	7	5	3
141	219	2	3	56	41	15	25	4	59
144	216	1	55	55	27	14	38	4	52
147	213	1	47	54	55	13	47	4	41
150	210	1	38	54	25	12	52	4	26
153	207	1	29	53	54	11	51	4	10
156	204	1	19	53	23	10	44	3	53
159	201	1	10	52	54	9	34	3	33
162	198	1	0	52	33	8	20	3	10
165	195	0	51	52	18	7	4	2	43
168	192	0	41	52	8	5	43	2	14
171	189	0	31	52	3	4	19	1	43
174	186	0	21	52	2	2	54	1	9
177	183	0	10	52	2	1	27	0	35
180	180	0	0	52	2	0	0	0	0

HOW TO COMPUTE THE LONGITUDINAL PLACES OF THESE FIVE PLANETS *Chapter 34*

By means of these Tables so drawn up by me, we shall compute the longitudinal places of these five planets without any difficulty. For nearly the same computational procedure applies to them all. Yet in this respect the three outer planets differ somewhat from Venus and Mercury.

Hence let me speak first about Saturn, Jupiter, and Mars, for which the computation proceeds as follows. For any given time seek the mean motions, I mean, the sun's simple motion and the planet's parallactic motion, by the method explained above [III, 14; V, 1]. Then subtract the place of the higher apse of the planet's eccentric from the sun's simple place. From the remainder subtract the parallactic motion. The resulting remainder is the anomaly of the planet's eccentric. We look up its number among the common numbers in either of the first two columns of the Table. Opposite this number we take the normalization of the eccentric from the 3rd column, and the proportional minutes from the following column. We add this correction to the parallactic motion, and subtract it from the eccentric's anomaly, if the number with which we entered the Table is found in the 1st column. Conversely, we subtract it from the parallactic anomaly and add it to the eccentric's anomaly, if the [initial] number occupied the 2nd column. The sum or remainder will be the normalized anomalies of the parallax and the eccentric, while the proportional minutes are reserved for a purpose soon to be explained.

Then we look up also this normalized parallactic anomaly among the common numbers in the first [two columns], and opposite it in the 5th column we take the parallactic prosthaphaeresis, together with its surplus, placed in the last column. In accordance with the number of proportional minutes we take the proportional part of this surplus. We always add this proportional part to the prosthaphaeresis. The sum is the planet's true parallax. This must be subtracted from the normalized parallactic anomaly if that is less than a semicircle, or added if the anomaly is greater than a semicircle. For in this way we shall have the planet's true and apparent distance westward from the sun's mean place. When this distance is subtracted from the [place of the] sun, the remainder will be the required place of the planet in the sphere of the fixed stars. Finally, if the precession of the equinoxes is added to the place of the planet, its distance from the vernal equinox will be ascertained.

In the cases of Venus and Mercury, instead of the eccentric's anomaly we use the higher apse's distance from the sun's mean place. With the aid of this anomaly we normalize the parallactic motion and the eccentric's anomaly, as has already been explained. But the eccentric's prosthaphaeresis, together with the normalized parallax, if they are of one direction or kind, are simultaneously added to or subtracted from the sun's mean place. However, if they are of different kind, the smaller is subtracted from the larger. Operate with the remainder as I just explained about the additive or subtractive property of the larger number, and the result will be the apparent place of the required planet.

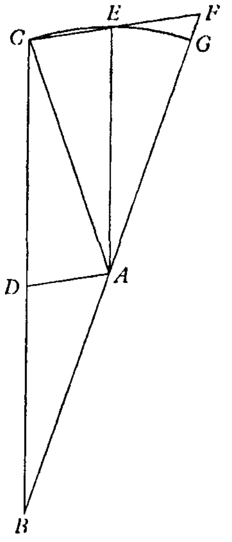
THE STATIONS AND RETROGRADATIONS OF THE FIVE PLANETS *Chapter 35*

Evidently there is a connection between the explanation of the [planets'] motion in longitude and the understanding of their stations, regressions, and retrogradations, and of the place, time, and extent of these phenomena. These topics too were discussed not a little by astronomers, especially Apollonius of Perga [Ptolemy, *Syntaxis,* XII, 1]. But their discussion proceeded as though the planets moved with only one nonuniformity, that which occurs with respect to the sun, and which I have called the parallax due to the motion of the earth's grand circle.

Suppose the earth's grand circle to be concentric with the planets' circles, by which all the planets are carried at unequal speeds in the same direction, that is, eastward. Also assume that a planet, like Venus and Mercury, inside the grand circle is faster on its own orbit than the earth's motion. From the earth draw a straight line intersecting the planet's orbit. Bisect the segment within the orbit. This half-segment has the same ratio to the line extending from our observatory, which is the earth, to the lower and convex arc of the intersected orbit as the earth's motion has to the planet's velocity. The point then made by the line so drawn to the perigean arc of the planet's circle separates the retrogradation from the direct motion, so that the planet gives the appearance of being stationary when it is located in that place.

The situation is similar in the remaining three outer planets, whose motion is slower than the earth's speed. A straight line drawn through our eye will intersect the grand circle so that the half-segment within that circle has the same ratio to the line extending from the planet to our eye located on the nearer and convex arc of the grand circle as the planet's motion has to the earth's speed. To our eye the planet at that time and place will give the impression of standing still.

But if the ratio of the half-segment within the aforesaid [inner] circle to the remaining outer segment exceeds the ratio of the earth's speed to the velocity of Venus or Mercury, or the ratio of the motion of any of the three outer planets to the earth's speed, the planet will advance eastward. On the other hand, if the [first] ratio is smaller [than the second], the planet will retrograde westward.

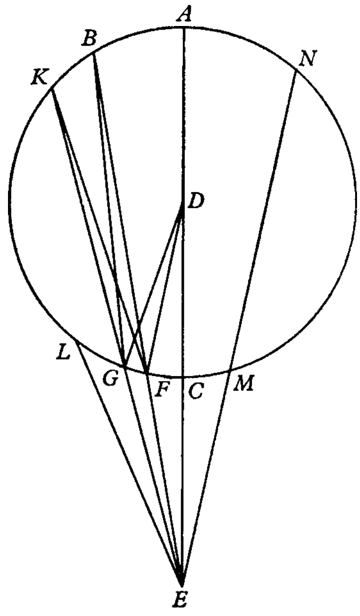


For the purpose of proving the foregoing statements Apollonius adduces a certain auxiliary theorem. Although it conforms to the hypothesis of a stationary earth, nevertheless it is compatible also with my principles based on the mobility of the earth, and therefore I too shall use it. I can enunciate it in the following form. Suppose that in a triangle a longer side is divided so that one of the segments is not less than the adjoining side. The ratio of that segment to the remaining segment will exceed the inverse ratio of the angles at the divided side [the angle at the remaining segment: the angle at the adjoining side]. In triangle *ABC* let the longer side be *BC*. On it take *CD* not less than *AC*. I say that *CD* : *BD* > angle *ABC* : angle *BCA*.

The proof proceeds as follows. Complete parallelogram *ADCE*. Extend *BA* and *CE* to meet at point *F*. With center *A* and radius *AE* describe a circle. This will pass through *C* or beyond it, since *AE* [= *CD*] is not smaller than *AC*. For the present let the circle pass through *C*, and let it be *GEC*. Triangle *AEF* is greater than sector *AEG*. But triangle *AEC* is smaller than sector *AEC*. Therefore, triangle *AEF* : [triangle] *AEC* > sector *AEG* : sector *AEC*. But triangle *AEF* :

triangle AEC = base FE : base EC. Therefore $FE : EC >$ angle FAE : angle EAC. But $FE : EC = CD : DB$, since angle FAE = angle ABC, and angle EAC = angle BCA. Therefore $CD : DB >$ angle ABC : angle ACB. The [first] ratio will clearly be much greater, moreover, if CD, that is, AE, is assumed not equal to AC, but AE is taken greater than AC.

Now around D as center let ABC be Venus' or Mercury's circle. Outside the circle let the earth E move around the same center D. From our observatory at E draw straight line $ECDA$ through the center of the circle. Let A be the place most distant from the earth, and C the place nearest to the earth. Assume that the ratio $DC : CE$ is greater than the ratio of the observer's motion to the planet's speed. Therefore a line EFB can be found such that $\frac{1}{2} BF : FE$ = observer's motion : planet's speed. For as line EFB recedes from center D, along FB it shrinks and along EF it lengthens until the required condition occurs. I say that when the planet is located at point F, it will give us the appearance of being stationary. However little arc we choose on either side of F, in the direction of the apogee we shall find it progressive, but retrogressive if toward the perigee.



First, take arc FG extending toward the apogee. Prolong EGK. Join BG, DG, and DF. In triangle BGE, segment BF of the longer side BE exceeds BG. Hence $BF : EF >$ angle FEG : angle GBF. Therefore $\frac{1}{2} BF : FE >$ angle $FEG : 2 \times$ angle GBF = angle GDF. But $\frac{1}{2} BF : FE$ = earth's motion : planet's motion. Therefore angle FEG : angle GDF < earth's speed : planet's speed. Consequently, the angle which has the same ratio to angle FDG as the ratio of the earth's motion to the planet's motion exceeds angle FEG. Let this greater angle = FEL. Hence, during the time in which the planet traverses arc GF of the circle, our line of sight will be thought to have passed through an opposite distance, that lying between line EF and line EL. Clearly, in the same interval in which arc GF has transported the planet, as seen by us, westward through the smaller angle FEG, the earth's passage has drawn the planet back eastward through the greater angle FEL. As a result the planet is still retrojected through angle GEL, and seems to have progressed, not to have, remained stationary.

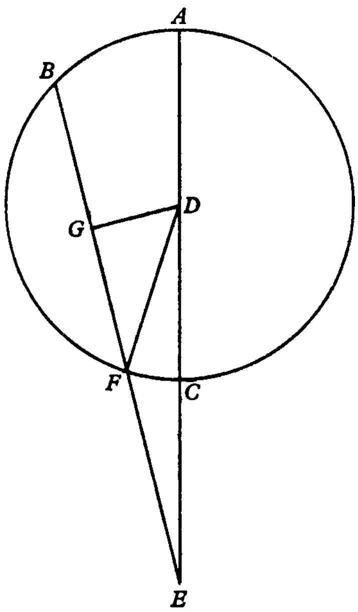
The reverse of this proposition will clearly be demonstrated by the same means. In the same diagram suppose that we take $\frac{1}{2} GK : GE$ = earth's motion : planet's speed. Assume that arc GF extends toward the perigee from straight line EK. Join KF, making triangle KEF. In it GE is drawn longer than EF. $KG : GE <$ angle FEG : angle FKG. So also $\frac{1}{2} KG : GE <$ angle $FEG : 2 \times$ angle FKG = angle GDF. This relation is the reverse of that demonstrated above. By the same means it will be established that angle GDF : angle FEG < planet's speed : speed of the [line of] sight. Accordingly, when these ratios become equal as angle GDF becomes greater, the planet will likewise execute a greater movement westward than the forward motion demands.

These considerations make it clear also that if we assume arcs FC and CM to be equal, the second station will be at point M. Draw line EMN. Just like $\frac{1}{2} BF : FE$, so too $\frac{1}{2} MN : ME$ = earth's speed : planet's speed. Therefore, points F and M will occupy both stations, delimit the whole of arc FCM as retrogressive, and the rest of the circle as progressive. It also follows that at whatever distances $DC : CE$ does not exceed the ratio earth's speed : planet's speed, another straight line cannot be drawn having a ratio equal to the ratio earth's speed : planet's speed, and the planet will seem to be neither stationary nor retrogressive. For in triangle *DGE*, when straight line *DC* is assumed to be not smaller than *EG*, angle *CEG* : angle *CDG* $<$ *DC* : *CE*. But *DC* : *CE* does not exceed the ratio earth's speed : planet's speed. Therefore, angle *CEG* : *CDG* $<$ earth's speed : planet's speed. When this occurs, the planet will move eastward, and we will not find anywhere on the planet's orbit an arc through which it would seem to retrograde. The foregoing discussion applies to Venus and Mercury, which are inside the grand circle.

For the three other outer planets the proof proceeds in the same way and with the same diagram (only the designations being changed). We make *ABC* the earth's grand circle and the orbit of our observatory. In *E* we put the planet, whose motion on its own orbit is slower than the speed of our observatory on the grand circle. As for the rest, the proof will proceed in all respects as before.

HOW THE TIMES, PLACES, AND ARCS OF RETROGRESSION ARE DETERMINED — *Chapter 36*

Now if the circles which carry the planets were concentric with the grand circle, what the preceding demonstrations promise would readily be confirmed (since the ratio planet's speed : observatory's speed would always remain the same). However, these circles are eccentric, and this is the reason why the apparent motions are nonuniform. Consequently we must everywhere assume disparate and normalized motions with variations in their velocities, and use them in our proofs, and not simple and uniform motions, unless the planet happens to be near its middle longitudes, the only places on its orbit where it seems to be carried with a mean motion.



I shall demonstrate these propositions by the example of Mars, which will clarify the retrogradations of the other planets too. Let the grand circle be *ABC*, on which our observatory is situated. Put the planet at point *E*, from which draw straight line *ECDA* through the center of the grand circle. Also draw *EFB*, and *DG* perpendicular to *EFB*; $\frac{1}{2}$ *BF* = *GF*. *GF* : *EF* = planet's momentary speed : observatory's speed, which exceeds the planet's speed.

Our task is to find *FC* = $\frac{1}{2}$ of the arc of retrogression, or *ABF* [= 180°−*FC*], in order to know the planet's greatest [angular] distance from *A* when the planet is stationary, and the amount of angle *FEC*. For, from this information, we shall predict the time and place of this planetary phenomenon. Put the planet near the eccentric's middle apse, where its observed motions in longitude and anomaly differ little from the uniform motions.

In the case of the planet Mars, when its mean motion = $1^p\,8'\,7''$ = line *GF*, its parallactic motion, that is, the motion of our [line of] sight : planet's mean motion = 1^p = straight line *EF*. Hence the whole of *EB* = $3^p\,16'\,14''$ [= 2 × $1^p\,8'\,7''$ (= $2^p\,16'\,14''$)+1^p], and rectangle *BE* × *EF* likewise = $3^p\,16'\,14''$. But I have shown [V, 19] that radius *DA* = 6580^p whereof *DE* = $10{,}000^p$.

[Earlier version:
The whole of *EA* = 16580 [= 6580+10000], and the remainder *BC* [when 2×*DA* = 13160 is subtracted from *EA* = 16580] = 3420. The rectangle formed by *AE*×*EC* = 56,703,600 = the rectangle formed by *BE*×*EF*. But *BE* : *EF* is the given ratio from which we obtain the ratio of the rectangle *EB*×*EF* (to which the rectangle *AE*×*EC* is equal, namely, 56, 703, 600) to $(EF)^2$. We shall therefore also have *EF* as a length = 4164^p, whereof *DE* = 10000^p and *DF* = 6580^p

[as well as the other whole line $EB = 13618$ and the remainder GF [$= \frac{1}{2}(BF = 13618 - 4164 = 9454)$] $= 4727^p$. In triangle DFG, sides DF and FG are given [$= 6580, 4727$] while G is a right angle. Hence we shall have angle $FDG = 39°\ 15'$]. In triangle DEF, the sides being given [$DE = 10000$; $DF = 6580$; $EF = 4164$], angles $FED = 17°\ 3'$ and $FDE = 17°\ 2'$ are given. Hence ABF, the arc of the anomaly to the first stationary point, $= 162°\ 58'$ [$+17°\ 2' = 180°$]. When we add to this value $2 \times FC$ [$= 17°\ 2'$], we shall have $197°\ 2'$ [$= 162°\ 58' + (2 \times 17°\ 2' = 34°\ 4')$] for the second arc measured from A. By means of arc FC we shall know how much time elapsed from the first stationary point to the opposition, which is C. When this time is doubled, it shows us the entire period of the retrogradation.

The foregoing is what happens at the middle longitudes of the eccentric. But according to the computations performed for the greatest distance, the prosthaphaeresis, which approximates 1°, makes the ratio of the planet's disparate motion to the disparate motion of [the line of] sight or of the parallactic anomaly, that is, line GF : line $EF = 10000 : 8917$, and [$(2 \times GF) + EF =$] the whole of $BE : EF = 28917 : 8917$. DE has been shown $= 10960^p$ whereof $AD = 6580^p$ [V, 19]. Therefore, with $DE = 10000^p$, $AD = 6004^p$ [6003.6]. The whole of AE [$= AD + DE = 6004 + 10000$] $= 16004$. The remainder EC [$= DE - (DC = AD) = 10000 - 6004$] $= 3996$. The included rectangle [$AE \times EC = 16004 \times 3996$] $= 63,951,984$ is less than $(EF)^2$, in proportion to the ratio $BE : EF$. Hence we shall have EF as a length $= 4441^p$ whereof $DE = 10000^p$ or $DF = 6004^p$. Therefore we again have triangle DEF with its sides given, and angles...]

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However, with $DE = 60^p$, in such units $AD = 39^p\ 29'$. [$DE + AD = 60^p + 39^p\ 29' =$] the whole of $AE : EC = 99^p\ 29' : 20^p\ 31'$ [$= 60^p - 39^p\ 29' = DE - DC$]. A rectangle formed from these [segments $AE \times EC$] $= 2041^p\ 4'$, known $= BE \times EF$. The result of the comparison, I mean, the division of $2041^p\ 4'$ by $3^p\ 16'\ 14''$ [$=$ the previous value of $BE \times EF$] $= 624^p\ 4'$, and a side [$=$ square root] of it $= 24^p\ 58'\ 52'' = EF$ in units whereof DE was assumed $= 60^p$. However, with $DE = 10{,}000^p$, $EF = 4163^p\ 5'$ whereof $DF = 6580^p$.

Since the sides of triangle DEF are given, we shall have angle $DEF = 27°\ 15' =$ angle of planet's retrogradation, and $CDF =$ angle of parallactic anomaly $= 16°\ 50'$. At its first station the planet appears along line EF, and along EC at opposition. If the planet did not move eastward at all, arc $CF = 16°\ 50'$ would comprise the $27°\ 15'$ of retrogradation found in angle AEF. However, in accordance with the established ratio planet's speed : observatory's speed, to the parallactic anomaly of $16°\ 50'$ corresponds a planetary longitude of approximately $19°\ 6'\ 39''$. When this quantity is subtracted from $27°\ 15'$, the remainder from the second station to opposition $= 8°\ 8'$, and about $36\frac{1}{2}$ days. In that time that longitude of $19°\ 6'\ 39''$ is traversed, and hence the entire retrogression of $16°\ 16'$ [$= 2 \times 8°\ 8'$] is completed in 73 days [$= 2 \times 36\frac{1}{2}$ days].

The foregoing analysis is made for the eccentric's middle longitudes.

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But according to the computations executed for the greatest distance, the prosthaphaeresis which retards the uniform motions makes the ratio of the planet's disparate motion to the disparate motion of [the line of] sight or the parallactic anomaly, that is, line GF : line $EF = 46'\ 20''\ 6''' : 1^p$. [$2 \times (GF = 46'\ 20'') = 1^p\ 32'\ 40''$, $+(1^p = EF)$] $=$ the whole of $BE : EF = 2^p\ 32'\ 40'' : 1^p$, and the rectangle formed by $BE \times EF$ likewise $= 2^p\ 32'\ 40''$. At the higher apse DE has been shown $= 10960^p$ whereof $DA = 6580^p$ [V, 19]. Hence, with $DE = 60^p$, $DA = 36^p\ 1'\ 20''$. Thus, the whole of AE [$= DE + DA = 60^p + 36^p\ 1'\ 20''$] $= 96^p\ 1'\ 20''$. The remainder [$= EC$, when $2 \times DA$ is subtracted from AE] $= 23^p\ 58'\ 40''$, and $AE \times EC = 2302^p\ 23'\ 58''$. When this [product] is divided by $2^p\ 32'\ 40''$ [$= BE$], the quotient is $904^p\ 51'\ 12''$ [should be $52'\ 23''$]. A side [square root] of this $= 30^p\ 4'\ 51''$, and this is the line EF in units whereof $DE = 60$. But with $DE = 100{,}000$, $EF = 50135$ and $DF = 60037$ also in those units. In triangle DEF, therefore, all the sides being given, the angles are given: $DEF = 27°\ 18'\ 40''$ for the motion of the retrograding planet, and

$EDF = 22° 9' 50''$ for the parallactic anomaly of [the line of] sight. Connected with them, according to the proportion of the apogee, is the disparate longitude $= 17° 19' 3''$, whereas the uniform motion $= 20° 59' 3''$. Half of the retrogradation is estimated $= 9° 59' 37''$ in approximately 40 days, while the entire retrogradation $= 19° 59' 14''$ [$= 2 \times 9° 59' 37''$] in 80 days.

With regard to the perigee we shall also reason in like manner. There we find the ratio of the planet's disparate motion: disparate motion of [the line of] sight $= 1^p 50' 40'' : 1^p = GF : FE$. Hence, the rectangle formed by $BE \times EF = 4^p 41' 21''$ [$2 \times (GF = 1^p 50' 40'') = 3^p 41' 20''$; $3^p 41' 20'' + 1^p = 4^p 41' 20'' \times 1^p$]. But line DE has been shown $= 9040^p$ whereof $AD = 6580^p$ [V, 19]. Hence, with $DE = 60^p$, in such units $AD = 43^p 40' 21''$; the whole of AE [$= AD + DE = 43^p 40' 21'' + 60^p$] $= 103^p 40' 21''$; and the remainder CE [$= AE - 2 \times AD = 103^p 40' 21'' - 87^p 20' 42''$] $= 16^p 19' 39''$. Hence the rectangle formed by $AE \times EC$ [$= 103^p 40' 21'' \times 16^p 19' 39''$] $= 1672^p 42' 52''$ [should be 1692^p]. When this value is divided by $4^p 41' 21''$ [$= BE \times EF$], the quotient is $360^p 59' 1''$, of which a side [square root] $= EF = 18^p 59' 58''$ whereof $DE = 60^p$. But with $DE = 100{,}000^p$, in such units $EF = 31665$ and also $DF = 72787^p$. Hence in triangle DEF all the sides being given, the angles are given: $DEF = 25° 45' 16''$ = the planet's retrogressive parallax, and $EDF = 10° 53' 13''$, the angular separation between [the line of] sight and the mid-point of the retrogression at opposition. However, in the interval in which [the line of] sight passes through arc $FC = 10° 53' 13''$, in its disparate motion the planet traverses $19° 44' 58''$, but $16° 17' 21''$ in its uniform motion, crossing half of the retrogression $\cong 6°$ in $31\,{}^1/_{12}$ days, while the entire retrogression amounts to $12° 1'$ in about $62\,{}^1/_6$ days].

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For other places the procedure is similar, but the planet's momentary velocity as determined by the place is always applied, as I pointed out [near the beginning of V, 36].

Hence, the same method of analysis is available for Saturn, Jupiter, and Mars, no less than for Venus and Mercury, provided that we put the observatory in the planet's place and the planet in the observatory's place. Naturally, in these orbits enclosed by the earth, what occurs is the opposite of what happens in the orbits surrounding the earth. Therefore, let the foregoing remarks suffice, lest I repeat the same song over and over again.

Nevertheless, as the planet's motion varies with the line of sight, it produces no small difficulty and uncertainty concerning the stations. That assumption on the part of Apollonius [V, 35] provides us no relief from these perplexities. Hence I do not know whether it would not be better to investigate the stations simply and in relation to the nearest place. In like manner we seek the opposition of a planet by its impingement on the line of the sun's mean motion, or the conjunction of any planets from the known quantities of their motions. I leave this problem for everybody to pursue to his own satisfaction.

End of the Fifth Book of the Revolutions

Book Six

INTRODUCTION

I have to the best of my ability indicated how the assumed revolution of the earth influences and affects the planets' apparent motion in longitude, and how it forces all these phenomena into a precise and necessary regularity. It remains for me to consider the movements which impart to the planets a deviation in latitude, and to show how the earth's motion exercises control over these phenomena too, and prescribes rules for them also in this division. This division of the science is indispensable because the planets' deviations [in latitude] produce no small modification in the risings and settings, first visibilities, occultations, and other phenomena which were explained in general above. Indeed, the planets' true places are said to be known when their longitude is determined together with their latitudinal deviation from the ecliptic. What the ancient astronomers believed they had demonstrated here too by means of a stationary earth, I shall accomplish perhaps more compactly and more appropriately by assuming that the earth moves.

GENERAL EXPLANATION OF THE FIVE PLANETS' DEVIATION IN LATITUDE

Chapter 1

In all these planets the ancients found a twofold deviation in latitude, corresponding to the twofold longitudinal nonuniformity of each of these planets. One [of these latitudinal deviations, in their opinion,] was produced by the eccentrics, and the other by the epicycles. Instead of these epicycles I have accepted the earth's one grand circle (which has already been mentioned often). [I did] not [accept the grand circle] because it deviates in any way from the plane of the ecliptic, with which it is conjoined once and for all, since they are identical. On the other hand, [I did accept the grand circle] because the planets' orbits are inclined to this plane [the ecliptic] at an angle which is not fixed, and this variation is geared to the motion and revolutions of the earth's grand circle.

The three outer planets, Saturn, Jupiter, and Mars, however, move in longitude according to certain principles different from [those governing the longitudinal motion of] the other two. In their latitudinal motion, too, the outer planets differ not a little. Hence the ancients first investigated the location and quantity of the extreme limits of their northern latitudes. For Saturn and Jupiter, Ptolemy found those limits near the beginning of the Balance, but for Mars near the end of the Crab close to the eccentric's apogee [*Syntaxis*, XIII, 1].

In our time, however, I have found these northern limits for Saturn at 7° within the Scorpion, for Jupiter at 27° within the Balance, and for Mars at 27° within the Lion, just as the apogees have likewise shifted in the period extending to us [V, 7, 12, 16], since the motion of those circles is followed by the inclinations and cardinal points of the latitudes. At a normalized or apparent quadrant's distance from these limits [these planets] seem to make absolutely no deviation in latitude, wherever the earth may happen to be at that time. At these middle longitudes, then, these planets are understood to be at the intersection of their orbits with the ecliptic, like the moon at its intersections with the ecliptic. In the present instance Ptolemy [*Syntaxis*, XIII, 1] calls these intersections the "nodes"; from the ascending node the planet enters the northern regions, and from the descending node it crosses over into the southern regions. [These deviations do] not [occur] because the earth's grand circle, which always remains invariably in the plane of the ecliptic, produces any latitude in these planets. On the contrary, the entire deviation in latitude comes from them, and reaches its peak at the places midway between the nodes. When the planets are seen there in opposition to the sun and culminating at midnight, as the earth approaches they always execute a greater deviation than in any other position of the earth, moving northward in the northern hemisphere and southward in the southern hemisphere. This deviation is greater than is required by the earth's approach and withdrawal. This circumstance led to the recognition that the inclination of the planets' orbits is not fixed, but shifts in a certain motion of libration commensurable with the revolutions of the earth's grand circle, as will be explained a little later on [VI, 2].

Venus and Mercury, however, seem to deviate in certain other ways, although they conform to a precise rule linked with their middle, higher, and lower apsides. For at their middle longitudes, that is, when the line of the sun's mean motion is at a quadrant's distance from their higher or lower apse, and the planets themselves in the evening or morning are at the distance of a quadrant of their orbits away from the same line of [the sun's] mean motion, the ancients found in them no deviation from the ecliptic. Through this circumstance the ancients recognized that these planets were then at the intersection of their orbits and the ecliptic. Since this intersection passes through their apogees and perigees, when they are farther from or closer to the earth, at those times they execute conspicuous deviations. But these are at their maximum when the planets are at their greatest distance from the earth, that is, around first visibility in the evening or setting in the morning, when Venus is seen farthest north, and Mercury farthest south.

On the other hand, at a place nearer to the earth, when they set in the evening or rise in the morning, Venus is to the south, and Mercury to the north. Conversely, when the earth is opposite this place and in the other middle apse, that is, when the anomaly of the eccentric = 270°, Venus appears to the south at a greater distance from the earth, and Mercury to the north. At a place nearer to the earth, Venus appears to the north, and Mercury to the south.

But when the earth approached the apogees of these planets, Ptolemy found Venus' latitude northern in the morning and southern in the evening. The opposite was true for Mercury, whose latitude was southern in the morning and northern in the evening. At the opposite place, [with the earth near these planets'] perigee, these directions are similarly reversed, so that Venus as the morning star is seen in the south, and as evening star in the north, whereas in the morning Mercury is to the north, and to the south in the evening. However, [with the earth] in both these places [the apogee and perigee of these planets], the ancients found Venus' deviation always greater to the north than to the south, and Mercury's greater to the south than to the north.

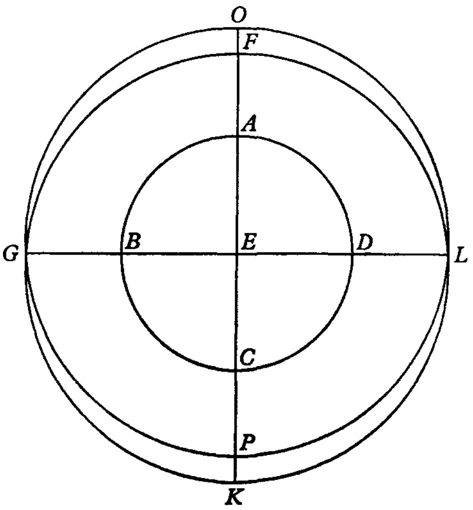
On account of this fact, for this situation [with the earth at the planetary apogees and perigees], the ancients devised a twofold latitude, and in general a threefold latitude. The first, which occurs at the middle longitudes, they called the "declination". The second, which takes place at the higher and lower apsides, they named the "obliquation". The last, linked with the second, they labeled the "deviation", always to the north for Venus, and to the south for Mercury. Between these four limits [the higher, lower, and two middle apsides] the latitudes mingle with one another, alternately increasing and decreasing, and give way to one another. To all these phenomena I shall assign the appropriate circumstances.

THE THEORY OF THE CIRCLES BY WHICH THESE PLANETS ARE MOVED IN LATITUDE *Chapter 2*

The orbits of these five planets, then, must be assumed to be tilted at a variable but regular inclination to the plane of the ecliptic, the intersection being a diameter of the ecliptic. Around that intersection as an axis, in the cases of Saturn, Jupiter, and Mars, the angle of intersection undergoes a certain oscillation, such as I demonstrated in connection with the precession of the equinoxes [III, 3]. In these three planets, however, it is simple and commensurable with the motion in parallax, with which it increases and decreases in a definite period. Thus, whenever the earth is at its nearest to the planet, namely, when this culminates at midnight, the inclination of the planet's orbit reaches its maximum; its minimum, in the opposite position; and its mean, halfway between. As a result, when the planet is at the limit of its greatest northern or southern latitude, its latitude appears much greater with the earth close than when it is at its greatest distance. The sole cause of this variation could be the earth's unequal distance, on the principle that things look bigger when nearer than when farther away. However, the latitudes of these planets increase and decrease with a greater variation [than would be produced solely by variations in the earth's distance]. This cannot happen unless the tilt of their orbits also oscillates. But, as I said above [III, 3], in motions which oscillate, a mean must be accepted between the extremes.

For the purpose of clarifying these remarks, in the plane of the ecliptic let $ABCD$ be the grand circle with its center at E. Let the planet's orbit be inclined to the grand circle. Let $FGKL$ be the orbit's mean and abiding declination, with F at the northern limit of its latitude, K at the southern limit, G at the descending node of the intersection, and L at the ascending node. Let the intersection [of the planet's orbit and the earth's grand circle] be BED. Extend BED along straight lines GB and DL. These four limits are not to shift except with the motion of the apsides. The planet's motion in longitude, however, is to be understood as occurring not in the plane of circle FG, but in another circle OP, concentric with FG and inclined to it. Let these circles intersect each other in that same straight line $GBDL$. Therefore, while the planet is carried on circle OP, that circle at times coincides with plane FK, and as a result of the motion in libration crosses over in both directions, and for that reason makes the latitude appear to vary.

Thus, first let the planet be at its greatest northern latitude at point O and at its closest to the earth, situated at A. At that time the planet's latitude will increase in accordance with angle OGF = the greatest inclination of orbit OGP. Its motion is an approach and withdrawal, because by hypothesis it is commensurable with the motion in parallax. Then if the earth is in B, O will coincide with F,



and the planet's latitude will appear smaller than before in the same place. It will even appear much smaller if the earth is at point *C*. For *O* will cross over to the outermost opposite part of its oscillation, and it will leave only as much latitude as exceeds the subtractive libration of the northern latitude, namely, the angle equal to *OGF*. Thereafter throughout *CDA*, the remaining semicircle, the northern latitude of the planet situated near *F* will increase until [the earth] returns to the first point *A*, from which it started out.

When the planet is located in the south near point *K*, its behavior and vicissitudes will be the same, when the earth's motion is taken as beginning at *C*. But suppose that the planet is in either node *G* or *L*, in opposition to or conjunction with the sun. Even though at that time circles *FK* and *OP* may be at their greatest inclination to each other, no latitude will be perceived in the planet since it occupies an intersection of the circles. From the foregoing remarks it is readily understood (I believe) how the planet's northern latitude decreases from *F* to *G*, and its southern latitude increases from *G* to *K*, while disappearing completely and crossing over to the north at *L*.

The three outer planets behave in the foregoing manner. Just as Venus and Mercury differ from them in longitude, so there is no little difference in latitude, because [the grand circle] is intersected by the orbits of the inner planets at their apogees and perigees. At their middle apsides, on the other hand, their greatest inclinations, like those of the outer planets, are varied by an oscillation. The inner planets, however, undergo an additional oscillation different from the former. Nevertheless, both are commensurable with the earth's revolutions, but not in the same way. For, the first oscillation has the property that while the earth returns once to the apsides of the inner planets, the motion in oscillation revolves twice, having as its axis the aforementioned fixed intersection through the apogees and perigees. As a result, whenever the line of the sun's mean motion is in their perigee or apogee, the angle of the intersection attains its maximum, whereas it is always at its minimum in the middle longitudes.

On the other hand, the second oscillation, which is superimposed on the first, differs from it in having a movable axis. As a result, when the earth is situated in a middle longitude of Venus or Mercury, the planet is always on the axis, that is, on this oscillation's intersection. By contrast, the planet is at its greatest divergence [from the axis of the second oscillation] when the earth is aligned with the planet's apogee or perigee, Venus inclining always to the north, as I said [VI, 1], and Mercury to the south. Yet at those times these planets would have had no latitude arising from the first and simple declination.

Thus, for example, suppose that the sun's mean motion is at Venus' apogee, and the planet is in the same place. Clearly, since at that time the planet is at the intersection of its orbit with the plane of the ecliptic, it would have no latitude due to the simple declination and first oscillation. But the second oscillation, which has its intersection or axis along a transverse diameter of the eccentric, superimposes its greatest divergence on the planet, because it cuts at right angles the diameter which passes through the higher and lower apsides. On the other hand, suppose that the planet is at either of the points at a quadrant's distance [from its apogee] and near the middle apsides of its orbit. At that time the axis of this [second] oscillation will coincide with the line of the sun's mean motion. To the northern divergence Venus will add the greatest deviation, which it will subtract from the southern divergence, leaving it diminished. In this way the deviation's oscillation is commensurate with the earth's motion.

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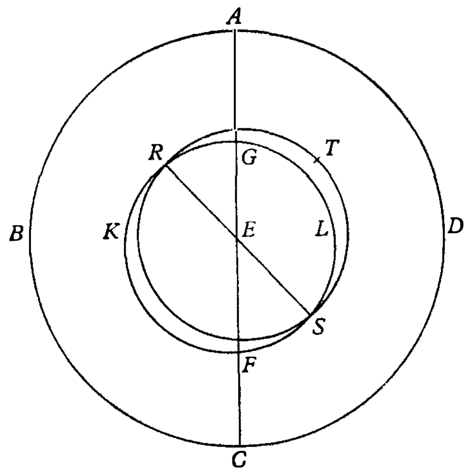
Thus, when the line of the sun's mean motion passes through the apogee or perigee of the planet, that body is then at its greatest deviation, in whatever part of its orbit it may be located, whereas it will have no deviation [when the line of the sun's mean motion] is near [the planet's] middle apsides].

To make the foregoing remarks likewise easier to understand, reproduce *ABCD*, the grand circle; *FGKL*, Venus' or Mercury's orbit, eccentric to circle *ABC* and inclined to it with a mean obliquity; and their intersection *FG*, through *F*, the orbit's apogee, and *G*, its perigee. For the sake of a more convenient demonstration, let us first take the inclination of *GKF*, the eccentric orbit, to be simple and constant or, if preferred, halfway between the minimum and maximum, except that intersection *FG* shifts with the motion of the perigee and apogee. When the earth is on the intersection, that is, at *A* or *C*, and the planet is on the same line, it would obviously have no latitude at that time. For its entire latitude lies at the sides in semicircles *GKF* and *FLG*. There the planet deviates to the



north or south, as was said [earlier in VI, 2], in accordance with the inclination of circle *FKG* to the plane of the ecliptic. This deviation of the planet is called the "obliquation" by some astronomers, and the "reflexion" by others. On the other hand, when the earth is in *B* or *D*, that is, at the planet's middle apsides, *FKG* and *GLF*, which are called the "declinations", will be the same latitudes, above and below. Thus they differ from the former in name rather than in fact, and at the middle places even the names are interchanged.

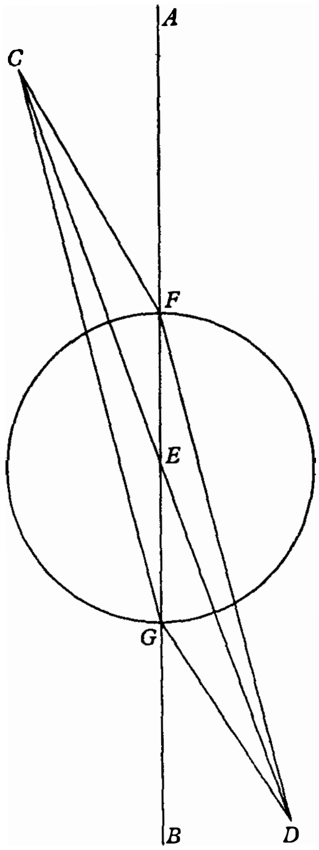
However, the angle of inclination of these circles is found to be greater in the obliquation than in the declination. Accordingly, this disparity was conceived to occur as a result of an oscillation, swinging around intersection *FG* as its axis, as was said earlier [in VI, 2]. Hence, when we know this angle of intersection on both sides, from the difference between them we would readily infer the amount of the oscillation from its minimum to its maximum.

Now conceive another circle of deviation, inclined to *GKFL*. Let it be concentric in the case of Venus; and in the case of Mercury, eccentreccentric, as will be indicated later [in VI, 2]. Let their intersection *RS* serve as this oscillation's axis, which moves in a circle according to the following rule. When the earth is in *A* or *B*, the planet is at the extreme limit of its deviation, wherever it may be, for instance, at point *T*. To the extent that the earth proceeds away from *A*, the planet is understood to move an equivalent distance away from *T*. Meanwhile, the obliquity of the circle of deviation diminishes. As a result, when the earth has traversed quadrant *AB*, the planet is understood to have arrived at this latitude's node, that is, *R*. At that time, however, the planes coincide at the oscillation's midpoint, and proceed in opposite directions. Therefore, the remaining semicircle of deviation, which previously was southern, jumps northward. As Venus advances into this semicircle, it leaves the south and proceeds northward, never to turn south as a result of this oscillation. In like manner Mercury pursues the opposite direction and remains southern. Mercury differs also in swinging not on a concentric of the eccentric, but on an eccentreccentric. I used an epicyclet in demonstrating the nonuniformity of its motion in longitude [V, 25]. There, however, its longitude is considered apart from its latitude; here, its latitude is considered apart from its longitude. These are comprised in one and the same revolution, and are equally completed thereby. Therefore, quite evidently, both variations could be produced by a single motion and the same oscillation, at once eccentric and oblique. There is no other arrangement than the one which I just described and which I shall discuss further below [VI, 5–8].

HOW MUCH ARE THE ORBITS OF SATURN, JUPITER, AND MARS INCLINED? *Chapter 3*

Having explained the theory of the latitudes of the five planets, I must now turn to the facts and analyze the details. First [I must determine] how much the individual circles are inclined. We compute these inclinations by means of the great circle which passes through the poles of the inclined circle at right angles to the ecliptic. On this great circle the deviations in latitude are determined. When these arrangements are understood, the road will be open to ascertaining the latitudes of each planet.

Once more let us begin with the three outer planets. At their farthest southern



limits of latitude, as shown in Ptolemy's Table [*Syntaxis*, XIII, 5], when they are in opposition, Saturn deviates 3° 5′, Jupiter 2° 7′, and Mars 7° 7′. On the other hand, in the opposite places, that is, when they are in conjunction with the sun, Saturn deviates 2° 2′, Jupiter 1° 5′, and Mars only 5′, so that it almost grazes the ecliptic. These values could be inferred from the latitudes observed by Ptolemy around the time of the planets' disappearances and first visibilities.

Now that the above assertions have been set forth, let a plane perpendicular to the ecliptic pass through its center and intersect the ecliptic in *AB*. But let its intersection with the eccentric of any of the three outer planets be *CD*, passing through the farthest southern and northern limits. Let the ecliptic's center be *E;* the diameter of the earth's grand circle, *FEG*; the southern latitude, *D*; and the northern, *C*. Join *CF, CG, DF,* and *DG*.

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Now as an example I shall use Mars because it exceeds all the other planets in latitude. Thus, when it is in opposition at point *D*, with the earth at *G* [corrected from *F*], angle *AFC* was known = 7° 7′. But *C* is given as Mars' position at apogee. From the previously established sizes of the circle, *CE* = 1ᵖ 22′ 20″, with *FG* [a slip for *FE*] = 1ᵖ. In triangle *CEF*, the ratio of the sides *CE* and *EF* is given, as well as angle *CFE*. Hence we shall also have as given *CEF* = the greatest angle of the eccentric's inclination = 5° 11′, according to the doctrine of plane triangles. However, when the earth is in the opposite place, that is, at *G* [should have been corrected to *F*], while the planet is still at *C*, *CGF* = the angle of the apparent latitude = 4′].

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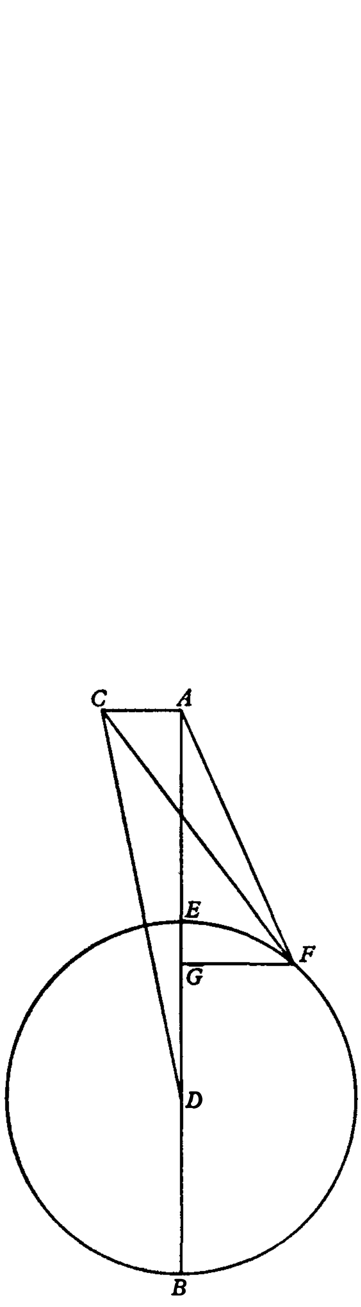
For each planet the ratio of *EG*, [the radius] of the earth's grand circle, to *ED*, [the radius] of the planet's eccentric, has already been shown above for any given places of earth and planet. But the places of the maximum latitudes are also given by observation. Therefore *BGD*, the angle of the greatest southern latitude, is given as an exterior angle of triangle *EGD*. In accordance with the theorems on Plane Triangles, the opposite interior angle *GED* will also be given as the angle of the eccentric's maximum southern inclination to the plane of the ecliptic. By means of the minimum southern latitude we shall likewise demonstrate the minimum inclination, for example, by means of angle *EFD*. In triangle *EFD*, the ratio of sides *EF* : *ED* is given as well as angle *EFD*. Therefore we shall have exterior angle *GED* given as the angle of the minimum southern inclination. Accordingly, from the difference between both inclinations we shall obtain the whole oscillation of the eccentric in relation to the ecliptic. Furthermore, by means of these angles of inclination we shall compute the opposite northern latitudes, such as *AFC* and *EGC*. If these agree with the observations, they will indicate that we have made no error.

However, as an example I shall use Mars, because it exceeds all the other planets in latitude. Its maximum southern latitude was noted by Ptolemy as about 7° when Mars was at perigee, and its maximum northern latitude at apogee as 4° 20′ [*Syntaxis*, XIII, 5]. However, having determined angle *BGD* = 6° 50′, I found the corresponding angle *AFC* ≅ 4° 30′. Given *EG* : *ED* = 1ᵖ : 1ᵖ 22′ 26″ [V, 19], from these sides and angle *BGD* we shall obtain angle *DEG* of the maximum southern inclination ≅ 1° 51′. Since *EF* : *CE* = 1ᵖ : 1ᵖ 39′ 57″ [V, 19] and angle *CEF* = *DEG* = 1° 51′, consequently the aforementioned exterior angle *CFA* = 4 ½° when the planet is in opposition.

Similarly, at the opposite place when it is in conjunction with the sun, suppose that we assume angle *DFE* = 5′. From the given sides *DE* and *EF* together with angle *EFD*, we shall obtain angle *EDF*, and exterior angle *DEG* of the minimum inclination ≅ 9′. This will furnish us also with angle *CGE* of the northern latitude ≅ 6′. Hence, if we subtract the minimum inclination from the maximum, that is, 1° 51′ − 9′, the remainder ≅ 1° 41′. This is the oscillation of this inclination, and 1/2 [of the oscillation] ≅ 50 1/2′.

In like manner the angles of inclination of the other two planets, Jupiter and Saturn, were determined together with their latitudes. Thus, Jupiter's maximum inclination = 1° 42′; its minimum inclination = 1° 18′; hence, its entire oscillation comprises not more than 24′. On the other hand, Saturn's maximum inclination = 2° 44′; its minimum inclination = 2° 16′; the intervening oscillation = 28′. Hence, through the smallest angles of inclination, which occur in the opposite place, when the planets are in conjunction with the sun, their deviations in latitude from the ecliptic will emerge as 2° 3′ for Saturn and 1° 6′ for Jupiter. These values had to be determined and retained for the construction of the Tables below [after VI, 8].

GENERAL EXPLANATION OF ANY OTHER LATITUDES OF THESE THREE PLANETS — *Chapter 4*



From what has been expounded above, the particular latitudes of these three planets will likewise be clear in general. As before, conceive the intersection *AB* of the plane perpendicular to the ecliptic and passing through the limits of their farthest deviations, with the northern limit at *A*. Also let straight line *CD* be the intersection of the planet's orbit [with the ecliptic], and let *CD* intersect *AB* in point *D*. With *D* as center, describe *EF* as the earth's grand circle. From *E*, where the earth is aligned with the planet in opposition, take any known arc *EF*. From *F* and from *C*, the place of the planet, drop *CA* and *FG* perpendicular to *AB*. Join *FA* and *FC*.

In this situation we first seek the size of *ADC*, the angle of the eccentric's inclination. It has been shown [VI, 3] to be at its maximum when the earth is in point *E*. Its entire oscillation, moreover, as is required by the oscillation's nature, was revealed to be commensurate with the earth's revolution on circle *EF*, as determined by diameter *BE*. Therefore, because arc *EF* is given, ratio *ED* : *EG* will be given, and this is the ratio of the entire oscillation to that which was just detached from angle *ADC*. Hence in the present situation angle *ADC* is given.

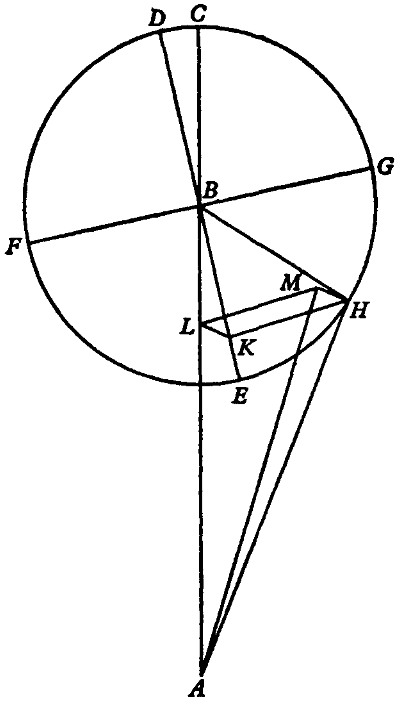
Consequently, in triangle *ADC*, the angles being given, all its sides are given. But ratio *CD* : *ED* is given by the foregoing. Also given, therefore, is [the ratio of *CD*] to *DG*, the remainder [when *EG* is subtracted from *ED*]. Consequently the ratios of both *CD* and *AD* to *GD* are known. Accordingly, *AG*, the remainder [when *GD* is subtracted from *AD*], is also given. From this information *FG* is likewise given, since it is half of the chord subtending twice *EF*. Therefore, in right triangle *AGF*, two sides [*AG* and *FG*] being given, hypotenuse *AF* is given, and so is ratio *AF* : *AC*. Thus, finally, in right triangle *ACF*, two sides [*AF* and *AC*] being given, angle *AFC* will be given, and this is the angle of the apparent latitude, which was sought.

Again I shall exemplify this analysis with Mars. Let its maximum limit of southern latitude, which occurs near its lower apse, be in the vicinity of *A*. However, let the place of the planet be C, where ADC, the angle of the inclination, was shown [VI, 3] to be at its maximum, namely, $1° 50'$, when the earth was at point E. Now let us put the earth at point F, and the motion in parallax along arc $EF = 45°$. Therefore, straight line FG is given $= 7071^p$ whereof $ED = 10{,}000^p$, and GE, the remainder [when $GD = FG = 7071^p$ is subtracted] from the radius [$= ED = 10{,}000^p$] $= 2929^p$. But half of ADC, the angle of the oscillation, has been shown $= 0° 50\,{}^1/_2{}'$ [VI, 3]. In this situation its ratio of increase and decrease $= DE : GE \cong 50\,{}^1/_2{}' : 15'$. When we subtract this latter quantity from $1° 50'$, the remainder $= 1° 35' = ADC$, the angle of the inclination in the present situation. Therefore, the angles and sides of triangle ADC will be given. CD was shown above to be $= 9040^p$ whereof $ED = 6580^p$ [V, 19]. Hence, in those same units $FG = 4653^p$; $AD = 9036^p$; AEG, the remainder [when $GD = FG = 4653^p$ is subtracted from $AD = 9036^p$] $= 4383^p$, and $AC = 249\,{}^1/_2{}^p$. Therefore, in right triangle AFG, perpendicular $AG = 4383^p$, and base $FG = 4653^p$; hence, hypotenuse $AF = 6392^p$. Thus, finally, triangle ACF has CAF as a right angle, together with given sides AC and AF [$= 249\,{}^1/_2{}^p, 6392^p$]. Hence, angle AFC is given $= 2° 15' =$ the apparent latitude when the earth is situated at F. We shall pursue the analysis in the same way for the other two planets, Saturn and Jupiter.

THE LATITUDES OF VENUS AND MERCURY *Chapter 5*

Venus and Mercury remain. Their deviations in latitude, as I said [VI, 1], will be demonstrated jointly by three interrelated latitudinal excursions. In order to be able to separate these from one another, I shall begin with the one called the "declination", since it is simpler to treat. It is the only one which sometimes happens to be separated from the others. This [separation occurs] near the middle longitudes and near the nodes when, as reckoned by the corrected motions in longitude, the earth is located a quadrant's distance from the planet's apogee and perigee. When the earth is near the planet, [the ancients] found $6° 22'$ of southern or northern latitude in Venus, and $4° 5'$ in Mercury; but with the earth at its greatest distance [from the planet], $1° 2'$ in Venus, and $1° 45'$ in Mercury [Ptolemy, *Syntaxis*, XIII, 5]. Under these circumstances the planets' angles of inclination are made known through the established tables of corrections [after VI, 8]. Therein, when Venus is at its greatest distance from the earth with its latitude $= 1° 2'$, and at its least distance [from the earth with its latitude =] $6° 22'$, an arc of approximately $2\,{}^1/_2{}°$ of orbital [inclination] fits both cases. When Mercury is most remote [from the earth], its latitude $= 1° 45'$, and when it is closest [to the earth, its latitude =] $4° 5'$ require an arc of $6\,{}^1/_4{}°$ [as the inclination] of its orbit. Hence, the orbits' angles of inclination $= 2° 30'$ for Venus, but for Mercury $6\,{}^1/_4{}°$, with $360° = 4$ right angles. Under these circumstances each of their particular latitudes in declination can be explained, as I shall presently demonstrate, and first for Venus.

Let the ecliptic be the plane of reference. Let a plane perpendicular to it and passing through its center intersect it in ABC. Let [the ecliptic's] intersection with Venus' orbital plane be DBE. Let the earth's center be A; the center of the planet's orbit, B; and the angle of the orbit's inclination to the ecliptic, ABE. With B as center, describe orbit $DFEG$. Draw diameter FBG perpendicular to



diameter *DE*. Let the orbit's plane be conceived to be so related to the assumed perpendicular plane that lines drawn therein perpendicular to *DE* are parallel to one another and to the plane of the ecliptic, in which *FBG* is the only [such perpendicular].

From the given straight lines *AB* and *BC*, together with *ABE*, the given angle of inclination, it is proposed to find how much the planet deviates in latitude. Thus, for example, let the planet be at a distance of 45° away from *E*, the point nearest to the earth. Following Ptolemy [*Syntaxis*, XIII, 4], I have chosen this point in order that it may be clear whether the inclination of the orbit produces any variation in the longitude of Venus or Mercury. For, such variations would have to be seen at their maximum about halfway between the cardinal points *D*, *F*, *E*, and *G*. The principal reason therefor is that when the planet is located at these four cardinal points, it experiences the same longitudes as it would have without any declination, as is self-evident.

Therefore, let us take arc $EH = 45°$, as was said. Drop *HK* perpendicular to *BE*. Draw *KL* and *HM* perpendicular to the ecliptic as the plane of reference. Join *HB*, *LM*, *AM*, and *AH*. We shall have *LKHM* as a parallelogram with 4 right angles, since *HK* is parallel to the plane of the ecliptic [*KL* and *HM* having been drawn perpendicular to the ecliptic]. The side [*LM* of the parallelogram] is enclosed by *LAM*, the angle of the longitudinal prosthaphaeresis. But angle *HAM* embraces the deviation in latitude, since *HM* also falls perpendicularly on the same plane of the ecliptic. Angle *HBE* is given $= 45°$. Therefore, $HK =$ half the chord subtending twice $HE = 7071^p$ whereof $EB = 10{,}000^p$.

Similarly, in triangle *BKL*, angle *KBL* is given $= 2\,{}^1\!/_2°$ [VI, 5, above], *BLK* is a right angle, and hypotenuse $BK = 7071^p$ whereof $BE = 10{,}000^p$. In the same units, the remaining sides $KL = 308^p$ and $BL = 7064^p$. But, as was shown above [V, 21], $AB : BE \cong 10{,}000^p : 7193^p$. In the same units, therefore, the remaining sides $HK = 5086^p$; $HM = KL = 221^p$; and $BL = 5081^p$. Hence *LA*, the remainder [when $BL = 5081^p$ is subtracted from $AB = 10{,}000^p$] $= 4919^p$. Now once more, in triangle *ALM*, sides *AL* and $LM =$ HK are given [$= 4919^p$, 5086^p], and *ALM* is a right angle. Hence we shall have hypotenuse $AM = 7075^p$, and angle $MAL = 45°57' =$ Venus' prosthaphaeresis or great parallax, as computed.

Similarly, in triangle [*MAH*], side *AM* is given $= 7075^p$, and side $MH = KL$ [$= 221^p$]. Hence, angle *MAH* is obtained $= 1°\,47' =$ the latitudinal declination. But if it is not boring to consider what variation in longitude is produced by this declination of Venus, let us take triangle *ALH*, understanding *LH* to be a diagonal of parallelogram $LKHM = 5091^p$ whereof $AL = 4919^p$. *ALH* is a right angle. From this information hypotenuse *AH* is obtained $= 7079^p$. Hence, the ratio of the sides being given, angle $HAL = 45°\,59'$. But *MAL* was shown $= 45°\,57'$. Therefore, the excess is only $2'$. Q. E. D.

Again, in like manner I shall demonstrate the latitudes of declination in Mercury by a construction similar to the foregoing. Therein assume arc $EH = 45°$, so that each of the straight lines *HK* and *KB* is taken, as before, $= 7071^p$ whereof hypotenuse $HB = 10{,}000^p$. In this situation, as can be inferred from the differences in longitude as shown above [V, 27], radius $BH = 3953^p$ and $AB = 9964^p$. In such units *BK* and *KH* will both be $= 2795^p$. *ABE*, the angle of inclination, was shown [VI, 5, above] $= 6°\,15'$, with $360° = 4$ right angles. Hence, in right triangle BKL the angles are given. Accordingly, in the same units base $KL = 304^p$, and the perpendicular $BL = 2778^p$. Therefore, AL, the remainder [when $BL = 2778^p$ is subtracted from $AB = 9964^p$] $= 7186^p$. But $LM = HK = 2795^p$. Hence, in triangle ALM, L is a right angle, and two sides, AL and LM, are given [$= 7186^p$, 2795^p]. Consequently, we shall have hypotenuse $AM = 7710^p$, and angle $LAM = 21° 16'$ = the computed prosthaphaeresis.

Similarly, in triangle AMH, two sides are given: AM [$= 7710^p$], and $MH = KL$ [$= 304^p$], forming right angle M. Hence, angle MAH is obtained $= 2° 16'$ = the latitude we were seeking. It may be asked how much [of the latitude] is owing to the true and apparent prosthaphaeresis. Take LH, the diagonal of the parallelogram. From the sides we obtain it $= 2811^p$, and $AL = 7186^p$. These show angle $LAH = 21° 23'$ = the apparent prosthaphaeresis. This exceeds the previous calculation [of angle $LAM = 21° 16'$] by about $7'$. Q. E. D.

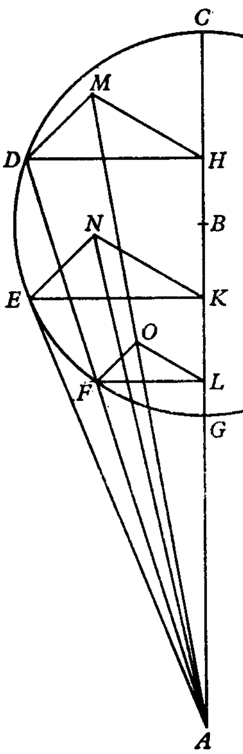
VENUS' AND MERCURY'S SECOND LATITUDINAL DIGRESSION, DEPENDING ON THE INCLINATION OF THEIR ORBITS AT APOGEE AND PERIGEE

Chapter 6

The foregoing remarks concerned that latitudinal digression of these planets which occurs near the middle longitudes of their orbits. These latitudes, as I said [VI, 1], are called the "declinations". Now I must discuss the latitudes which happen near the perigees and apogees. With these latitudes is mingled the deviation or third [latitudinal] digression. Such a deviation does not occur in the three outer planets, but [in Venus and Mercury] it can more easily be distinguished and separated out in thought, as follows.

Ptolemy observed [*Syntaxis*, XIII, 4] that these [perigeal and apogeal] latitudes appeared at their maximum when the planets were on the straight lines drawn from the center of the earth tangent to their orbits. This happens, as I said [V, 21, 27], when the planets are at their greatest distances from the sun in the morning and evening. Ptolemy also found [*Syntaxis*, XIII, 3] that Venus' northern latitudes were $^1/_3°$ greater than the southern, but Mercury's southern latitudes were about $1\ ^1/_2°$ greater than the northern. However, out of a desire to take into account the difficulty and labor of the computations, he accepted $2\ ^1/_2°$ as a sort of average quantity for the varying values of the latitude, mainly because he believed that no perceptible error would thereby arise, as I too shall soon show [VI, 7]. These degrees are subtended by the latitudes on the circle around the earth and at right angles to the ecliptic, the circle on which the latitudes are measured. If we now take $2\ ^1/_2°$ as the equal digression to either side of the ecliptic and for the time being exclude the deviation, our demonstrations will be simpler and easier until we have ascertained the latitudes of the obliquations.

Then we must first show that this latitude's digression reaches its maximum near the eccentric's point of tangency, where the longitudinal prosthaphaereses are also at their peak. Let the planes of the ecliptic and the eccentric, whether Venus' or Mercury's, intersect [in a line] through the [planet's] apogee and perigee. On the intersection take A as the place of the earth, and B as the center of the eccentric circle $CDEFG$, which is inclined to the ecliptic. Hence, [in the eccentric] any straight lines drawn perpendicular to CG form angles equal to the inclination



[of the eccentric to the ecliptic]. Draw *AE* tangent to the eccentric, and *AFD* as any secant. From points *D, E,* and *F,* furthermore, drop *DH, EK,* and *FL* perpendicular to *CG;* and also *DM, EN,* and *FO* perpendicular to the horizontal plane of the ecliptic. Join *MH, NK,* and *OL,* as well as *AN* and *AOM.* For, *AOM* is a straight line, since three of its points are in two planes, namely, the plane of the ecliptic, and the plane *ADM* perpendicular to the plane of the ecliptic. For the assumed inclination, then, angles *HAM* and *KAN* enclose the longitudinal prosthaphaereses of these planets, whereas their digressions in latitude are embraced by angles *DAM* and *EAN.*

I say, first, that the greatest of all the latitudinal angles is *EAN,* which is formed at the point of tangency, where the longitudinal prosthaphaeresis also is nearly at its maximum. For, angle *EAK* is the greatest of all [the longitudinal angles]. Therefore $KE : EA > HD : DA$ and $LF : FA$. But $EK : EN = HD : DM = LF : FO$, since the angles subtended [by the second members of these ratios] are equal, as I said. Moreover, *M, N,* and *O* are right angles. Consequently, $NE : EA > MD : DA$ and $OF : FA$. Once more, *DMA, ENA,* and *FOA* are right angles. Therefore, angle *EAN* is greater than *DAM* and all the [other] angles which are formed in this way.

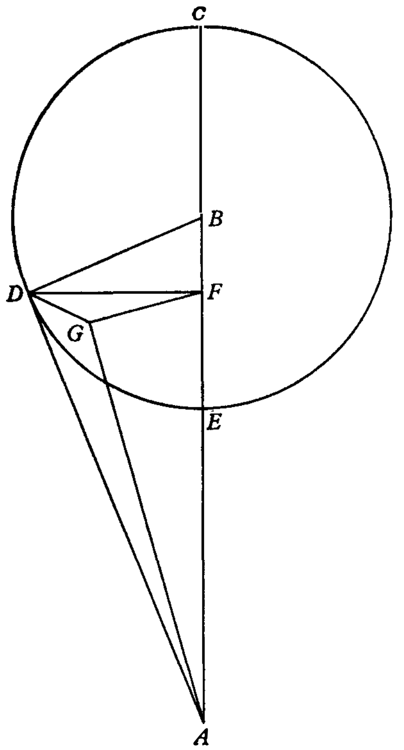
Of the difference in longitudinal prosthaphaeresis caused by this obliquation, consequently, clearly the maximum is also that which occurs at the greatest elongation near point *E.* For on account of the equality of the angles subtended [in the similar triangles], $HD : HM = KE : KN = LF : LO$. The same ratio holds good for their differences [$HD - HM, KE - KN, LF - LO$]. Consequently, the difference $EK–KN$ has a greater ratio to *EA* than the remaining differences have to sides like *AD.* Hence it is also clear that the ratio of the greatest longitudinal prosthaphaeresis to the maximum latitudinal digression will be the same as the ratio of the longitudinal prosthaphaereses of segments of the eccentric to the latitudinal digressions. For, the ratio of *KE* to *EN* is equal to the ratio of all the sides like *LF* and *HD* to the sides like *FO* and *DM.* Q. E. D.

THE SIZE OF THE OBLIQUATION ANGLES OF BOTH PLANETS, VENUS AND MERCURY — *Chapter 7*

Having made the foregoing preliminary remarks, let us see how great an angle is contained in the inclination of the planes of both these planets. Let us recall what was said above [VI, 5], that each of the planets, when [midway] between its greatest and least distances [from the sun], becomes farther north or south at the most by 5°, in opposite directions depending on its position in its orbit. For, at the eccentric's apogee and perigee Venus' digression makes a deviation imperceptibly greater or smaller than 5°, from which Mercury departs by $^1/_2$°, more or less.

As before, let *ABC* be the intersection of the ecliptic and the eccentric. Around *B* as center, describe the planet's orbit inclined to the plane of the ecliptic in the manner explained [previously]. From the center of the earth draw straight line *AD* tangent to the [planet's] orbit at point *D.* From *D* drop perpendiculars, *DF* on *CBE,* and *DG* on the horizontal plane of the ecliptic. Join *BD, FG,* and *AG.* Also assume that in the case of both planets angle *DAG,* comprising half of the aforementioned difference in latitude, $= 2\,^1/_2$°, with 4 right angles $= 360°$.

Let it be proposed to find, for both planets, the size of the angle of inclination of the planes, that is, angle *DFG*.



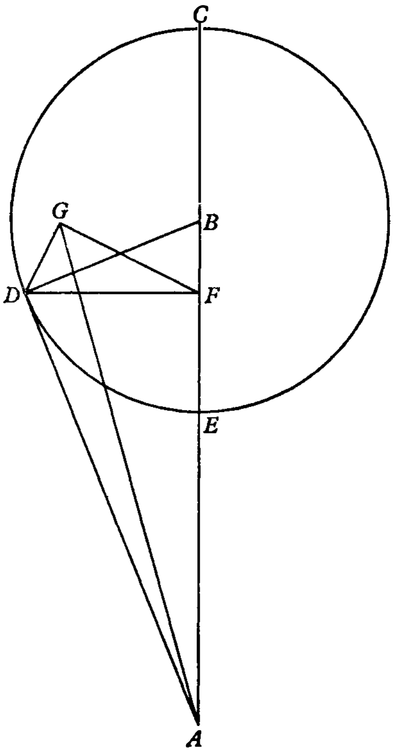
In the case of the planet Venus, in units whereof the orbit's radius = 7193^p, the planet's greatest distance [from the earth], which occurs at the apogee, has been shown = $10{,}208^p$, and its least distance, at perigee, = 9792^p [V, 21–22 : $10{,}000 \pm 208$]. The mean between these values = $10{,}000^p$, which I have adopted for the purposes of this demonstration. Ptolemy wanted to take laboriousness into account and, as far as possible, seek out short cuts [*Syntaxis*, XIII, 3, end]. For where the extreme values did not produce a manifest difference, it was better to accept the mean value.

Accordingly, $AB : BD = 10{,}000^p : 7193^p$, and *ADB* is a right angle. Then we shall have side $AD = 6947^p$ in length. Similarly, $BA : AD = BD : DF$, and we shall have $DF = 4997^p$ in length. Again, angle *DAG* is assumed = $2\,{}^1/_2{}^\circ$, and *AGD* is a right angle. In triangle [*ADG*, then], the angles being given, side $DG = 303^p$ whereof $AD = 6947^p$. Thus also [in triangle *DFG*] with two sides, *DF* and *DG*, being given [= 4997, 303], and *DGF* a right angle, *DFG*, the angle of inclination or obliquation, = $3^\circ 29'$. The excess of angle *DAF* over *FAG* comprises the difference in longitudinal parallax. Then the difference must be derived from the known sizes [of those angles].

It has already been shown that in units whereof $DG = 303^p$, hypotenuse $AD = 6947^p$, and $DF = 4997^p$, and also that $(AD)^2 - (DG)^2 = (AG)^2$, and $(FD)^2 - (DG)^2 = (GF)^2$. Then as a length *AG* is given = 6940^p, and $FG = 4988^p$. In units whereof $AG = 10{,}000^p$, $FG = 7187^p$, and angle $FAG = 45^\circ 57'$. In units whereof $AD = 10{,}000^p$, $DF = 7193^p$, and angle $DAF \cong 46^\circ$. In the greatest obliquation, therefore, the parallactic prosthaphaeresis is diminished by about $3'$ [$=46^\circ - 45^\circ 57'$]. At the middle apse, however, clearly the angle of the inclination between the circles was $2\,{}^1/_2{}^\circ$. Here, however, it has increased [to $3^\circ 29'$] by nearly a whole degree, which was added by that first libratory motion which I mentioned.

For Mercury the demonstration proceeds in the same way. In units whereof the orbit's radius = 3573^p, the orbit's greatest distance from the earth = $10{,}948^p$; its least distance = 9052^p; and between these values the mean = $10{,}000^p$ [V, 27]. $AB : BD = 10{,}000^p : 3573^p$. Then [in triangle *ABD*] we shall have the third side $AD = 9340^p$. $AB : AD = BD : DF$. Therefore $DF = 3337^p$ in length. *DAG* = the angle of the latitude, is assumed = $2\,{}^1/_2{}^\circ$. Hence $DG = 407^p$ whereof $DF = 3337^p$. Thus in triangle *DFG*, with the ratio of these two sides being given, and with *G* a right angle, we shall have angle $DFG \cong 7^\circ$. This is the angle at which Mercury's orbit is inclined or oblique to the plane of the ecliptic. Near the middle longitudes at a quadrant's [distance from apogee and perigee], however, the angle of inclination was shown = $6^\circ 15'$ [VI, 5]. Therefore, $45'$ [$= 7^\circ - 6^\circ 15'$] have now been added by the motion of the first libration.

Similarly, for the purpose of ascertaining the angles of prosthaphaeresis and their difference, it may be noticed that straight line *DG* has been shown = 407^p whereof $AD = 9340^p$ and $DF = 3337^p$. $(AD)^2 - (DG)^2 = (AG)^2$, and $(DF)^2 - (DG)^2 = (FG)^2$. Then we shall have as a length $AG = 9331^p$, and $FG = 3314^p$. From this information is obtained *GAF* = the angle of the prosthaphaeresis = $20^\circ 48'$, whereas $DAF = 20^\circ 56'$, than which *GAF*, which depends on the obliquation, is about $8'$ smaller.



It still remains for us to see whether these angles of obliquation and the latitudes connected with the orbit's maximum and minimum distance [from the earth] are found to conform with those obtained by observation. For this purpose in the same diagram again assume, in the first place, for the greatest distance of Venus' orbit [from the earth] that $AB : BD = 10{,}208^p : 7193^p$. Since ADB is a right angle, as a length $AD = 7238^p$ in the same units. $AB : AD = BD : DF$. Then in those units $DF = 5102^p$ in length. But DFG = the angle of the obliquity, was found $= 3° \, 29'$ [earlier in VI, 7]. The remaining side $DG = 309^p$ whereof $AD = 7238^p$. Then, in units whereof $AD = 10{,}000^p$, $DG = 427^p$. Hence, angle DAG is inferred $= 2° \, 27'$ at the [planet's] greatest distance from the earth. However, in units whereof BD = the orbit's radius $= 7193^p$, $AB = 9792^p$ [$= 10{,}000-208$] at the [planet's] least [distance from the earth]. AD, perpendicular to BD, $= 6644^p$. $AB : AD = BD : DF$. Similarly, as a length DF is given $= 4883^p$ in those units. But angle DFG has been put $= 3° \, 29'$. Therefore, DG is given $= 297^p$ whereof $AD = 6644^p$. Consequently in triangle [ADG], the sides being given, angle DAG is given $= 2° \, 34'$. However, neither $3'$ nor $4'$ [$2° \, 30' = 3' + 2° \, 27' = 2° \, 34' - 4'$] are large enough to be registered instrumentally with the aid of astrolabes. Hence, what was regarded as the maximum latitudinal digression in the planet Venus stands up well.

In like manner assume that the greatest distance of Mercury's orbit [from the earth is to the radius of Mercury's orbit], that is, $AB : BD = 10{,}948^p : 3573^p$ [V, 27]. Thus, by demonstrations like the foregoing, we obtain $AD = 9452^p$, and $DF = 3085^p$. But here again we have DFG, the angle of the inclination [between Mercury's orbit and the plane of the ecliptic] known $= 7°$, and for that reason straight line $DG = 376^p$ whereof $DF = 3085^p$ or $DA = 9452^p$. Hence in right triangle DAG, whose sides are given, we shall have angle $DAG \cong 2° \, 17'$ = the greatest digression in latitude.

At the [orbit's] least distance [from the earth], however, $AB : BD$ is put $= 9052^p : 3573^p$. Hence, in those units $AD = 8317^p$, and $DF = 3283^p$. However, on account of the same inclination [$= 7°$], $DF : DG$ is put $= 3283^p : 400^p$ whereof $AD = 8317^p$. Hence, angle $DAG = 2° \, 45'$.

The latitudinal digression associated with the mean value [of the distance of Mercury's orbit from the earth] is here too assumed $= 2\,^1/_2°$. From this quantity the latitudinal digression at apogee, where it reaches its minimum, differs by $13'$ [$= 2° \, 30' - 2° \, 17'$]. At perigee, however, where the latitudinal digression attains its maximum, it differs [from the mean value] by $15'$ [$= 2° \, 45' - 2° \, 30'$]. Instead of these [apogeal and perigeal differences], in computations based on the mean value, above it and below it I shall use $^1/_4°$, which does not differ perceptibly from the observations.

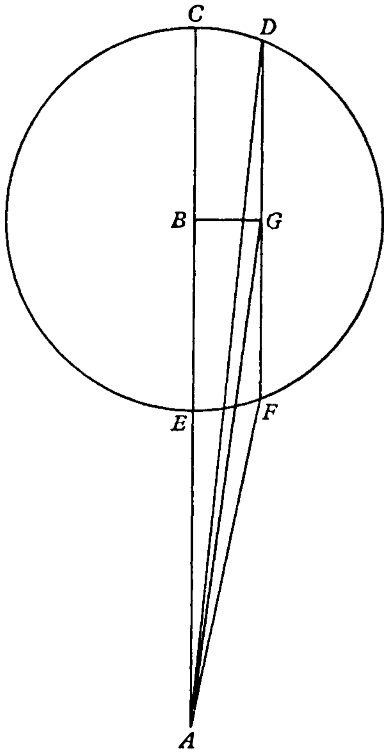
As a result of the foregoing demonstrations, and also because the greatest longitudinal prosthaphaereses have the same ratio to the greatest latitudinal digression as the partial prosthaphaereses in the remaining portions of the orbit have to the several latitudinal digressions, we shall obtain all the latitudinal quantities occurring on account of the inclination of the orbits of Venus and Mercury. But only the latitudes midway between apogee and perigee, as I said [VI, 5], are available. It has been shown that of these latitudes the maximum $= 2\,^1/_2°$ [VI, 6], while Venus' greatest prosthaphaeresis $= 46°$, and Mercury's $\cong 22°$ [VI, 5 : $45° \, 57'$, $21° \, 16'$]. And now in the tables of their nonuniform motions [after V, 33] we have the prosthaphaereses alongside the individual portions of the orbits. To the extent that each of the prosthaphaereses is smaller than the maximum, I shall take the corresponding part of those $2\,{}^{1}/_{2}°$ for each planet. I shall record that part numerically in the Table which is to be set out below [after VI, 8]. In this way we shall have in detail every individual latitude of obliquation which occurs when the earth is at the higher and lower apsides of these planets. In like manner I have recorded the latitudes of their declinations [when the earth is] at a quadrant's distance [midway between the planets' apogee and perigee], and [the planets are] at their middle longitudes. What occurs between these four critical points [higher, lower, and both middle apsides] can be derived by the subtlety of the mathematical art from the proposed system of circles, not without labor, however. Yet Ptolemy was everywhere as compact as possible. He recognized [*Syntaxis,* XIII, 4, end] that by themselves both of these kinds of latitude [declination, obliquation] as a whole and in all their parts increased and decreased proportionally like the moon's latitude. He therefore multiplied each of their parts by twelve, since their maximum latitude $= 5° = {}^{1}/_{12} \times 60°$. He made these [products] into proportional minutes, which he thought should be used not only in these two planets but also in the three outer planets, as will be explained below [VI, 9].

THE THIRD KIND OF LATITUDE, WHICH IS CALLED THE "DEVIATION", IN VENUS AND MERCURY — *Chapter 8*

Now that the foregoing topics in their turn have been thus expounded, something still remains to be said about the third motion in latitude, which is the deviation. The ancients, who station the earth in the middle of the universe, think that the deviation is produced by an oscillation of the eccentric, in phase with that of the epicycle, around the earth's center, the maximum occurring when the epicycle is located at the [eccentric's] apogee or perigee [Ptolemy, *Syntaxis,* XIII, 1]. In Venus the deviation is always ${}^{1}/_{6}°$ to the north, but in Mercury always ${}^{3}/_{4}°$ to the south, as I said above.

Yet it is not quite clear whether the ancients regarded this inclination of the circles as constant and always the same. For, this immutability is indicated by their numerical quantities when they ordain that ${}^{1}/_{6}$ of the proportional minutes always be taken as Venus' deviation, and ${}^{3}/_{4}$ as Mercury's [Ptolemy, *Syntaxis,* XIII, 6]. These fractions do not hold good unless the angle of inclination always remains the same, as is required by the scheme of those minutes which are based on that angle. Moreover, even if the angle does remain the same, it will be impossible to understand how this latitude of those planets suddenly rebounds from the intersection into the same latitude as that from which it previously came. You may say that this rebound happens like the reflection of light (as in optics). Here, however, we are discussing a motion which is not instantaneous, but by its very nature takes a determinable time.

It must be admitted, consequently, that these planets have a libration such as I have explained [VI, 2]. It makes the parts of the circle change [from one latitude] into the opposite. It is also a necessary consequence for their numerical quantities to vary, by ${}^{1}/_{5}°$ in the case of Mercury. Hence there should be no occasion for surprise if, according to my hypothesis, also this latitude varies and is not absolutely constant. Yet it does not produce a perceptible irregularity, distinguishable as such in all its variations.



Let the horizontal plane be perpendicular to the ecliptic. In the intersection [*AEBC* of these two planes] let A = the center of the earth; and at the greatest or least distance from the earth let B = the center of a circle *CDF*, which virtually passes through the poles of the oblique orbit. When the center of the orbit is at apogee and perigee, that is, on *AB*, the planet is at its greatest deviation, wherever it may be as determined by a circle parallel to the orbit. Of this circle parallel [to the orbit], the diameter *DF* is parallel to *CBE*, the diameter of the orbit. Of these [parallel circles], which are perpendicular to the plane of *CDF*, these diameters [*DF* and *CBE*] are taken to be the intersections [with *CDF*]. Bisect *DF* at *G*, which will be the center of the [circle] parallel [to the orbit]. Join *BG*, *AG*, *AD*, and *AF*. Put angle $BAG = \frac{1}{6}^\circ$, as at Venus' greatest deviation. Then in triangle *ABG*, with a right angle at *B*, we have the ratio of the sides $AB : BG = 10{,}000^p : 29^p$. But in those same units the whole of $ABC = 17{,}193^p$ [$CB = CA - BA = 17{,}193^p - 10{,}000^p = 7{,}193^p$; $CE = 2 \times 7193^p = 14{,}386^p$] and AE = the remainder [when $CE = 14{,}386^p$ is subtracted from $AC = 17{,}193^p$] $= 2807^p$. Half of the chords subtending twice *CD* and $EF = BG$. Therefore, angle $CAD = 6'$, and $EAF \cong 15'$. They differ from *BAG* [$= 10'$], in the former instance by only $4'$, and in the latter instance by $5'$, quantities which are generally ignored on account of their small size. Then Venus' apparent deviation, when the earth is located at its apogee and perigee, will be slightly greater or smaller than $10'$, in whatever part of its orbit the planet may be.

In the case of Mercury, however, we put angle $BAG = \frac{3}{4}^\circ$. $AB : BG = 10{,}000^p : 131^p$, $ABC = 13{,}573^p$, and the remainder $AE = 6427^p$ [$= AB - BE = 10{,}000^p - 3573^p$]. Then angle $CAD = 33'$, and $EAF \cong 70'$. In the former instance, therefore, $12'$ are lacking [$= 45' - 33'$], and in the latter instance there is an excess of $25'$ [$= 70' - 45'$]. Yet these differences are practically obliterated by the sun's rays before Mercury becomes visible to us. Hence the ancients investigated only its perceptible deviation, as though this were invariant.

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Nevertheless, if anybody wishes to scrutinize also those divagations of Mercury which are hidden by the sun, he will expend more labor [on them] than on any of the aforementioned latitudes. Let me therefore disregard this topic, and use the space for the calculations of the ancients, which do not depart much from the truth, lest in such a minor matter I seem to have struggled over the shadow of an ass (as the saying goes). And let the foregoing statements suffice for the deviations of the five planets in latitude, concerning which I have drawn up another Table in 30 lines, like the previous Tables [after V, 33].

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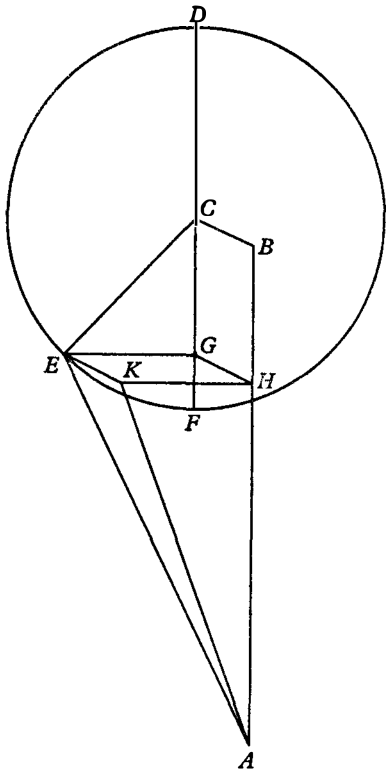
Nevertheless, if anybody is not wearied by the labor and wishes to obtain an exact knowledge also of those divagations which are hidden by the sun, I shall explain how to do so in the following way.

As the example I shall use Mercury, because its deviation is more notable than Venus'. Let the straight line *AB* be in the intersection of the planet's orbit and the ecliptic. Let the earth at *A* be at the apogee or perigee of the planet's orbit. Put line $AB = 10{,}000^p$ without any variation as the length midway between the maximum and minimum, as I did with regard to the obliquation [VI, 7].



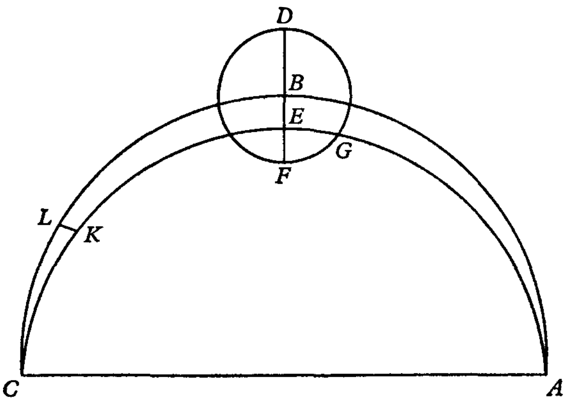
With C as center, describe circle DEF, parallel to the eccentric orbit at distance CB. Conceive the planet as undergoing its maximum deviation at that time on this parallel circle. Let this circle's diameter be DCF, which must likewise be parallel to AB, while both lines are in the same plane, perpendicular to the planet's orbit. Assume $EF = 45°$, for example, the arc at which we investigate the planet's deviation. Drop EG perpendicular to CF, as well as EK and GH perpendicular to the horizontal plane of the orbit. By joining HK, complete the rectangle. Also join AE, AK, and EC.

On the basis of the maximum deviation in Mercury, $BC = 131^p$ whereof $AB = 10{,}000^p$ and $CE = 3573^p$. In right triangle [CEG], the angles being given, side $EG = KH = 2526^p$. When $BH = EG = CG$ [$= 2526^p$] is subtracted [from $AB = 10{,}000^p$], the remainder $AH = 7474^p$. In triangle AHK, therefore, the sides forming right angle H being given [$= 7474^p$, 2526^p], hypotenuse $AK = 7889^p$. But [KE] $= CB = GH$ has been taken $= 131^p$. Hence in triangle AKE two given sides, AK and KE, form right angle K, and angle KAE is given. This corresponds to the deviation which we were seeking for the assumed arc EF, and it differs little from the observations. Proceeding similarly in the other [deviations of Mercury] and in Venus, I shall enter the results in the subjoined Table.

Having made the foregoing explanation, for the deviations between these limits I shall adjust the sixtieths or proportional minutes in both Venus and Mercury. Let circle ABC be the eccentric orbit of Venus or Mercury. Let A and C be the nodes of this latitude. Let B be the limit of the maximum deviation. With B as center, describe a circlet DFG, whose transverse diameter is DBF. Let the libration of the motion in deviation occur along DBF. It is assumed that when the earth is in the apogee or perigee of the planet's eccentric orbit, the planet executes its greatest deviation at point F, where the planet's deferent is tangent to the circlet.

Now let the earth be at any distance whatever from the apogee or perigee of the planet's eccentric. In accordance with this motion take FG as a similar arc on the circlet. Describe AGC as the planet's deferent. AGC will intersect the circlet and [cut its] diameter DF in point E. On AGC put the planet at K, with arc EK similar to FG by hypothesis. Drop KL perpendicular to circle ABC.

From FG, EK, and BE it is proposed to find magnitude KL = the planet's distance from circle ABC. From arc FG, EG is known as though it were a straight



line barely different from a circular or convex line. Likewise, *EF* will be given in the same units as the whole of *BF* and *BE*, the remainder [when *EF* is subtracted from *BF*]. *BF* : *BE* = chord subtending twice the quadrant *CE* : chord subtending twice *CK* = *BE* : *KL*. Therefore, if we compare both *BF* and the radius of *CE* to the same number 60, from them we shall obtain the value of *BE*. When this is squared, and the product is divided by 60, we shall obtain *KL* = the desired proportional minutes of arc *EK*. In like manner I have entered these minutes in the fifth and last column of the Table which follows.

LATITUDES OF SATURN, JUPITER, AND MARS															
Common Numbers		Saturn's Latitude				Jupiter's Latitude				Mars' Latitude				Proportional Minutes	
		Northern		Southern		Northern		Southern		Northern		Southern			
°	°	°	′	°	′	°	′	°	′	0	′	0	′	Min.	Sec.
3	357	2	3	2	2	1	6	1	5	0	6	0	5	59	48
6	354	2	4	2	2	1	7	1	5	0	7	0	5	59	36
9	351	2	4	2	3	1	7	1	5	0	9	0	6	59	6
12	348	2	5	2	3	1	8	1	6	0	9	0	6	58	36
15	345	2	5	2	3	1	8	1	6	0	10	0	8	57	48
18	342	2	6	2	3	1	8	1	6	0	11	0	8	57	0
21	339	2	6	2	4	1	9	1	7	0	12	0	9	55	48
24	336	2	7	2	4	1	9	1	7	0	13	0	9	54	36
27	333	2	8	2	5	1	10	1	8	0	14	0	10	53	18
30	330	2	8	2	5	1	10	1	8	0	14	0	11	52	0
33	327	2	9	2	6	1	11	1	9	0	15	0	11	50	12
36	324	2	10	2	7	1	11	1	9	0	16	0	12	48	24
39	321	2	10	2	7	1	12	1	10	0	17	0	12	46	24
42	318	2	11	2	8	1	12	1	10	0	18	0	13	44	24
45	315	2	11	2	9	1	13	1	11	0	19	0	15	42	12
48	312	2	12	2	10	1	13	1	11	0	20	0	16	40	0
51	309	2	13	2	11	1	14	1	12	0	22	0	18	37	36
54	306	2	14	2	12	1	14	1	13	0	23	0	20	35	12
57	303	2	15	2	13	1	15	1	14	0	25	0	22	32	36
60	300	2	16	2	15	1	16	1	16	0	27	0	24	30	0
63	297	2	17	2	16	1	17	1	17	0	29	0	25	27	12
66	294	2	18	2	18	1	18	1	18	0	31	0	27	24	24
69	291	2	20	2	19	1	19	1	19	0	33	0	29	21	21
72	288	2	21	2	21	1	21	1	21	0	35	0	31	18	18
75	285	2	22	2	22	1	22	1	22	0	37	0	34	15	15
78	282	2	24	2	24	1	24	1	24	0	40	0	37	12	12
81	279	2	25	2	26	1	25	1	25	0	42	0	39	9	9
84	276	2	27	2	27	1	27	1	27	0	45	0	41	6	24
87	273	2	28	2	28	1	28	1	28	0	48	0	45	3	12
90	270	2	30	2	30	1	30	1	30	0	51	0	49	0	0

LATITUDES OF SATURN, JUPITER, AND MARS

Common Numbers		Saturn's Latitude				Jupiter's Latitude				Mars' Latitude				Proportional Minutes	
		Northern		Southern		Northern		Southern		Northern		Southern			
°	°	°	′	°	′	°	′	°	′	°	′	°	′	Min.	Sec.
93	267	2	31	2	31	1	31	1	31	0	55	0	52	3	12
96	264	2	33	2	33	1	33	1	33	0	59	0	56	6	24
99	261	2	34	2	34	1	34	1	34	1	2	1	0	9	9
102	258	2	36	2	36	1	36	1	36	1	6	1	4	12	12
105	255	2	37	2	37	1	37	1	37	1	11	1	8	15	15
108	252	2	39	2	39	1	39	1	39	1	15	1	12	18	18
111	249	2	40	2	40	1	40	1	40	1	19	1	17	21	21
114	246	2	42	2	42	1	42	1	42	1	25	1	22	24	24
117	243	2	43	2	43	1	43	1	43	1	31	1	28	27	12
120	240	2	45	2	45	1	45	1	44	1	36	1	34	30	0
123	237	2	46	2	46	1	46	1	46	1	41	1	40	32	36
126	234	2	47	2	48	1	47	1	47	1	47	1	47	35	12
129	231	2	49	2	49	1	49	1	49	1	54	1	55	37	36
132	228	2	50	2	51	1	50	1	51	2	2	2	5	40	0
135	225	2	52	2	53	1	51	1	53	2	10	2	15	42	12
138	222	2	53	2	54	1	52	1	54	2	19	2	26	44	24
141	219	2	54	2	55	1	53	1	55	2	29	2	38	46	24
144	216	2	55	2	56	1	55	1	57	2	37	2	48	48	24
147	213	2	56	2	57	1	56	1	58	2	47	3	4	50	12
150	210	2	57	2	58	1	58	1	59	2	51	3	20	52	0
153	207	2	58	2	59	1	59	2	1	3	12	3	32	53	18
156	204	2	59	3	0	2	0	2	2	3	23	3	52	54	36
159	201	2	59	3	1	2	1	2	3	3	34	4	13	55	48
162	198	3	0	3	2	2	2	2	4	3	46	4	36	57	0
165	195	3	0	3	2	2	2	2	5	3	57	5	0	57	48
168	192	3	1	3	3	2	3	2	5	4	9	5	23	58	36
171	189	3	1	3	3	2	3	2	6	4	17	5	48	59	6
174	186	3	2	3	4	2	4	2	6	4	23	6	15	59	36
177	183	3	2	3	4	2	4	2	7	4	27	6	35	59	48
180	180	3	2	3	5	2	4	2	7	4	30	6	50	60	0

LATITUDES OF VENUS AND MERCURY

Common Numbers		Venus				Mercury				Venus		Mercury		Proportional Minutes of the Deviation	
		Declination		Obliquation		Declination		Obliquation		Deviation		Deviation			
°	°	°	′	°	′	°	′	°	′	°	′	°	′	Min.	Sec.
3	357	1	2	0	4	1	45	0	5	0	7	0	33	59	36
6	354	1	2	0	8	1	45	0	11	0	7	0	33	59	12
9	351	1	1	0	12	1	45	0	16	0	7	0	33	58	25
12	348	1	1	0	16	1	44	0	22	0	7	0	33	57	14
15	345	1	0	0	21	1	44	0	27	0	7	0	33	55	41
18	342	1	0	0	25	1	43	0	33	0	7	0	33	54	9
21	339	0	59	0	29	1	42	0	38	0	7	0	33	52	12
24	336	0	59	0	33	1	40	0	44	0	7	0	34	49	43
27	333	0	58	0	37	1	38	0	49	0	7	0	34	47	21
30	330	0	57	0	41	1	36	0	55	0	8	0	34	45	4
33	327	0	56	0	45	1	34	1	0	0	8	0	34	42	0
36	324	0	55	0	49	1	30	1	6	0	8	0	34	39	15
39	321	0	53	0	53	1	27	1	11	0	8	0	35	35	53
42	318	0	51	0	57	1	23	1	16	0	8	0	35	32	51
45	315	0	49	1	1	1	19	1	21	0	8	0	35	29	41
48	312	0	46	1	5	1	15	1	26	0	8	0	36	26	40
51	309	0	44	1	9	1	11	1	31	0	8	0	36	23	34
54	306	0	41	1	13	1	8	1	35	0	8	0	36	20	39
57	303	0	38	1	17	1	4	1	40	0	8	0	37	17	40
60	300	0	35	1	20	0	59	1	44	0	8	0	38	15	0
63	297	0	32	1	24	0	54	1	48	0	8	0	38	12	20
66	294	0	29	1	28	0	49	1	52	0	9	0	39	9	55
69	291	0	26	1	32	0	44	1	56	0	9	0	39	7	38
72	288	0	23	1	35	0	38	2	0	0	9	0	40	5	39
75	285	0	20	1	38	0	32	2	3	0	9	0	41	3	57
78	282	0	16	1	42	0	26	2	7	0	9	0	42	2	34
81	279	0	12	1	46	0	21	2	10	0	9	0	42	1	28
84	276	0	8	1	50	0	16	2	14	0	10	0	43	0	40
87	273	0	4	1	54	0	8	2	17	0	10	0	44	0	10
90	270	0	0	1	57	0	0	2	20	0	10	0	45	0	0

LATITUDES OF VENUS AND MERCURY

Common Numbers		Venus Declination		Venus Obliquation		Mercury Declination		Mercury Obliquation		Venus Deviation		Mercury Deviation		Proportional Minutes of the Deviation	
°	°	°	′	°	′	°	′	°	′	°	′	°	′	Min.	Sec.
93	267	0	5	2	0	0	8	2	23	0	10	0	45	0	10
96	264	0	10	2	3	0	15	2	25	0	10	0	46	0	40
99	261	0	15	2	6	0	23	2	27	0	10	0	47	1	28
102	258	0	20	2	9	0	31	2	28	0	11	0	48	2	34
105	255	0	26	2	12	0	40	2	29	0	11	0	48	3	57
108	252	0	32	2	15	0	48	2	29	0	11	0	49	5	39
111	249	0	38	2	17	0	57	2	30	0	11	0	50	7	38
114	246	0	44	2	20	1	6	2	30	0	11	0	51	9	55
117	243	0	50	2	22	1	16	2	30	0	11	0	52	12	20
120	240	0	59	2	24	1	25	2	29	0	12	0	52	15	0
123	237	1	8	2	26	1	35	2	28	0	12	0	53	17	40
126	234	1	18	2	27	1	45	2	26	0	12	0	54	20	39
129	231	1	28	2	29	1	55	2	23	0	12	0	55	23	34
132	228	1	38	2	30	2	6	2	20	0	12	0	56	26	40
135	225	1	48	2	30	2	16	2	16	0	13	0	57	29	41
138	222	1	59	2	30	2	27	2	11	0	13	0	57	32	51
141	219	2	11	2	29	2	37	2	6	0	13	0	58	35	53
144	216	2	25	2	28	2	47	2	0	0	13	0	59	39	15
147	213	2	43	2	26	2	57	1	53	0	13	1	0	42	0
150	210	3	3	2	22	3	7	1	46	0	13	1	1	45	4
153	207	3	23	2	18	3	17	1	38	0	13	1	2	47	21
156	204	3	44	2	12	3	26	1	29	0	14	1	3	49	43
159	201	4	5	2	4	3	34	1	20	0	14	1	4	52	12
162	198	4	26	1	55	3	42	1	10	0	14	1	5	54	9
165	195	4	49	1	42	3	48	0	59	0	14	1	6	55	41
168	192	5	13	1	27	3	54	0	48	0	14	1	7	57	14
171	189	5	36	1	9	3	58	0	36	0	14	1	7	58	25
174	186	5	52	0	48	4	2	0	24	0	14	1	8	59	12
177	183	6	7	0	25	4	4	0	12	0	14	1	9	59	36
180	180	6	22	0	0	4	5	0	0	0	14	1	10	60	0

COMPUTING THE LATITUDES OF THE FIVE PLANETS

Chapter 9

The method of computing the latitudes of the five planets by means of the foregoing Tables is as follows. In Saturn, Jupiter, and Mars we obtain the common numbers from the adjusted or normalized anomaly of the eccentric. In Mars we keep the anomaly as it is; in Jupiter we first subtract 20°; but in Saturn we add 50°. Then we record the results in the last column under the sixtieths or proportional minutes.

Similarly, from the adjusted parallactic anomaly we take each planet's number as its associated latitude. We take the first and northern latitude if the proportional minutes [descend from] higher [to lower]. This happens when the eccentric's anomaly falls below 90° or exceeds 270°. But we take the second and southern latitude if the proportional minutes [rise from] lower [to higher], that is, if the eccentric's anomaly (with which we enter the Table) is more than 90° or less than 270°. If we then multiply either of these latitudes by its sixtieths, the product will be the distance north or south of the ecliptic, depending on the classification of the assumed numbers.

In Venus and Mercury, on the other hand, from the adjusted parallactic anomaly we must first take the three latitudes which occur: declination, obliquation, and deviation. These are recorded separately. By an exception, in Mercury $^1/_{16}$ of the obliquation is subtracted if the eccentric's anomaly and its number are found in the upper part of the Table, or an equal fraction is added if [the eccentric's anomaly and its number are found] in the lower [part of the Table]. The remainder or sum resulting from these operations is retained.

However, the classification of these latitudes as northern or southern must be ascertained. Suppose that the adjusted parallactic anomaly lies in the apogeal semicircle, that is, is less than 90° or more than 270°, and also that the eccentric's anomaly is less than a semicircle. Or again, suppose that the parallactic anomaly lies in the perigeal arc, namely, is more than 90° and less than 270°, and the eccentric's anomaly is larger than a semicircle. Then, Venus' declination will be northern, and Mercury's southern. On the other hand, suppose that the parallactic anomaly lies in the perigeal arc while the eccentric's anomaly is less than a semicircle, or that the parallactic anomaly lies in the apogeal region, while the eccentric's anomaly is greater than a semicircle. Then, conversely, Venus' declination will be southern, and Mercury's northern. In the obliquation, however, if the parallactic anomaly is less than a semicircle and the eccentric's anomaly is apogeal, or if the parallactic anomaly is greater than a semicircle and the eccentric's anomaly is perigeal, Venus' obliquation will be northern and Mercury's southern; here too the converse holds true. However, the deviations always remain northern for Venus and southern for Mercury.

Then, with the adjusted anomaly of the eccentric take the proportional minutes common to all five planets. Those proportional minutes which are ascribed to the three outer planets, even though they are so ascribed, are to be assigned to the obliquation, and the remainder to the deviation. Thereafter add 90° to that same anomaly of the eccentric. The common proportional minutes which are connected with this sum are again to be applied to the latitude of declination.

When all these quantities have been so arranged in order, multiply by its own proportional minutes each of the three separate latitudes that have been set down. They will all emerge corrected for time and place, so that finally we have the complete account of the three latitudes in these two planets. If all the latitudes are of the same classification, add them together. But if they are not, combine only those two which are of the same classification. According as these two amount to more or less than the third latitude of the opposite classification, it is subtracted from them, or they are subtracted from it, and the preponderant remainder will be the latitude which we were seeking.

End of the Sixth and Last Book of the Revolutions

P. 41. Second diagram to face paragraph II D (lines 10-19).

P. 270,27. For (of Smyrna?) read [of Smyrna?].

TO THE REVEREND BERNARD WAPOWSKI, Cantor and Canon of the Church of Cracow, and Secretary to His Majesty the King of Poland, from Nicholas Copernicus.

Some time ago, my dear Bernard, you sent me a little treatise on *The Motion of the Eighth Sphere* written by John Werner of Nuremberg. Your Reverence stated that the work was widely praised and asked me to give you my opinion of it. Had it been really possible for me to praise it with any degree of sincerity, I should have replied with a corresponding degree of pleasure. But I may commend the author's zeal and effort. It was Aristotle's advice that "we should be grateful not only to the philosophers who have spoken well, but also to those who have spoken incorrectly, because to men who desire to follow the right road, it is frequently no small advantage to know the blind alleys."[1] Faultfinding is of little use and scant profit, for it is the mark of a shameless mind to prefer the role of the censorious critic to that of the creative poet. Hence I fear that I may arouse anger if I reprove another while I myself produce nothing better. Accordingly I wished to leave these matters, just as they are, to the attention of others; and I intended to reply to your Reverence substantially along these lines, with a view to a favorable reception of my work. However, I know that it is one thing to snap at a man and attack him, but another thing to set him right and redirect him when he strays, just as it is one thing to praise, and another to flatter and play the fawner. Hence I see no reason why I should not comply with your request or why I should appear to hamper the pursuit and cultivation of these studies, in which you have a conspicuous place. Consequently, lest I seem to condemn the man gratuitously, I shall attempt to show as clearly as possible in what respects he errs regarding the motion of the sphere of the fixed stars and maintains an unsound position. Perhaps my criticism

[1] *Metaphysics* i minor.1 993b11-14; Copernicus departs considerably from the original and appears to be quoting from memory.

may even contribute not a little to the formation of a better understanding of this subject.

In the first place, then, he went wrong in his calculation of time. He thought that the second year of Antoninus Pius Augustus, in which Claudius Ptolemy drew up the catalogue of the fixed stars as observed by himself,[2] was C.E. 150,[3] when in fact it was C.E. 139. For in the *Great Syntaxis*, Book III, chapter i,[4] Ptolemy says that the autumnal equinox observed 463 years after the death of Alexander the Great fell in the third year of Antoninus.[5] But from the death of Alexander to the birth of Christ there are 323 uniform Egyptian years[6] and 130 days, because the interval between the beginning of the reign of Nabonassar and the birth of Christ is computed as 747 uniform years and 130 days.[7] This computation, I observe, is not questioned, certainly not by our author, as can be seen in his Proposition 22.[8] It is true that according to the Alfonsine

[2] The view that Ptolemy copied his star catalogue from that of Hipparchus corrected for precession was critically examined by J. L. E. Dreyer and rejected (*Monthly Notices of the Royal Astronomical Society*, LXXVII [1917], 528-39; LXXVIII [1918], 343-49); J. K. Fotheringham took the same position (LXXVIII [1918], 419-22). See also Armitage, *Copernicus*, p. 106.

[3] In Proposition 4 (not 3 as in PII, 173 n) of his *De motu octavae sphaerae tractatus primus* Werner dates an observation of the second year of Antoninus *anno dominicae incarnationis 150 incompleto*, "in the 150th year of the incarnation of our Lord." Copernicus does not call attention to a related error committed by Werner. The latter regarded February 22, 150, as the epoch of Ptolemy's catalogue of the fixed stars: "Therefore it is clear that Ptolemy established the true places of the fixed stars in the zodiac for the 22d day of February, according to the Roman calendar, C.E. 150." (Prop. 4); ". . . the era of Ptolemy, that is, 149 years, 53 days from the incarnation of the Lord . . ." (Prop. 10). But Ptolemy gives his epoch as the beginning of the reign of Antoninus (HII, 36.13-16).

[4] HI, 204.7-11.

[5] Siegmund Günther erred when he stated (MCV, II[1880], 5.15-19) that, according to Copernicus, Ptolemy equated the year 463 of Alexander with the second year of Antoninus.

[6] Of exactly 365 days, with 12 months of 30 days each and 5 additional days (cf. Th 172.29-173.12).

[7] Günther's "747 years and 140 days" (MCV, II, 6.3) is evidently a misprint, for ten lines below (*ibid.*, 6.13) he gives the correct number.

[8] Werner there states that "the interval between the years of Christ and Nebuchadnezzar is, according to the Alfonsine Tables, 747 uniform years, 131 days."

Tables there is one additional day. The reason for this discrepancy is that Ptolemy takes noon of the first day of the first Egyptian month Thoth as the starting point of the years reckoned from Nabonassar and Alexander the Great,[9] while Alfonso starts from noon of the last day of the preceding year,[10] just as we compute the years of Christ from noon of the last day of the month December.[11] Now the interval from Nabonassar to the death of Alexander the Great is given by Ptolemy, Book III, chap. viii,[12] as 424 uniform years; and Censorinus, relying on Marcus Varro, agrees with this estimate in his *De die natali*,[13] addressed to Quintus Caerellius.[14] This interval, subtracted from 747 years and 130 days, leaves a remainder of 323 years and 130 days as the period from the death of Alexander to the birth of Christ. Then from the birth of Christ to the aforementioned observation of Ptolemy there are 139 uniform years and 303 days.[15] Therefore it is clear that the

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In *De rev.* Copernicus calls attention to the error, frequently made, of identifying Nebuchadnezzar with Nabonassar (Th 186.20-24).

[9] HI, 256.13-16.

[10] *Libros del saber de astronomía*, ed. Rico y Sinobas, IV, 120.

[11] Cf. Werner, Prop. 16, corollary 3: "Roman years . . . that is, years computed from the birth of the Savior and from noon of the last day of December."

[12] HI, 256.10-12. In H these lines are in chap. vii. Copernicus cites them from chap. viii, because he is using the translation of 1515, which divides Book III into ten chapters instead of nine. It makes a separate chapter of the penultimate paragraph of Heiberg's chap. v (HI, 251.10–252.8). L. A. Birkenmajer discovered Copernicus's copy of the 1515 translation (see his *Mikołaj Kopernik*, ch. x, and *Bulletin international de l'académie des sciences de Cracovie*, classe des sciences math. et naturelles [1909[2]], p. 24, n. 2).

[13] 21.9; Censorinus equates the year 986 of Nabonassar with the year 562 of Alexander.

[14] On the form of this name Curtze wrote (MCV, I, 25) a perplexing note, which Prowe repeats (PII, 174) with all its blunders. Hultsch's edition of Censorinus (Leipzig, 1867) reads Caerellius, not Caerellus (1.1, 15.1). The incunabular editions (GW 6,471–72) read Cerelius and Cerellius, not Cerillius. The praenomen is Quintus, not Gaius (PW s. v. Caerellius, No. 4; *Prosopographia imperii Romani* [2d ed.; ed. Groag and Stein, Berlin and Leipzig, 1933–], II, 30, No. 156).

[15] Günther's "139 years and 333 days" (MCV, II, 6.16) is a misprint. The observation was made in the early morning of the 9th day of Athyr (the third month in the Egyptian year), and is therefore assigned to the 68th astronomical day of the year. Then Copernicus reckons the interval from the death of Alex-

autumnal equinox observed by Ptolemy occurred 140[16] uniform years after the birth of our Lord, on the ninth day of the month Athyr; or 139 Roman years, September 25, the third year of Antoninus.[17]

Again, in the *Great Syntaxis*, Book V, chap. iii, Ptolemy counts 885 years and 203 days from Nabonassar to his observation of the sun and moon in the second year of Antoninus.[18] Therefore 138 uniform years and 73 days must have elapsed since the birth of Christ.[19] Hence the fourteenth day thereafter, that is, the ninth of Pharmuthi, on which Ptolemy observed

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ander to the observation as $463^y\ 68^d$;

subtract $323^y\ 130^d$ from the death of Alexander to the birth of Christ

$139^y\ 303^d$

[16] Since the first day of the Roman year fell on the 130th day of the Egyptian year, and $130 + 303 > 365$, the observation took place in the following Egyptian year.

[17] Since $\frac{1}{4} \times 139 = 34\frac{3}{4}$, subtract 35 days on account of leap-years: $303 - 35 = 268$. Hence the observation took place on the 268th day of the Roman year, or September 25. Copernicus assigns the observation to C.E. 140, although C.E. 139 is clearly correct (see emendation No. 12 on p. 254 in L. A. Birkenmajer, *Mikolaj Kopernik*). It was pointed out in n. 15, above, that he counted 463 years, 68 days, from Alexander to the observation. Yet the 1515 edition of the *Syntaxis* reads . . . *quod quidem fuit post mortem Alexandri in .463. anno* . . . (p. 28r), ". . . which was in the 463d year after the death of Alexander . . ." making the interval 462 years, 68 days. We are reduced to choosing between two alternatives: either Homer nodded, or the MSS of the *Letter against Werner* should read 138 years, 303 days, instead of 139 years, 303 days. Werner makes a commendable effort to avoid such ambiguity by attaching to a total number of years the adjective "complete" or "incomplete," so that $n + 1$ incomplete years $= n$ complete years. He writes: "Therefore it is clear that Ptolemy established the true places of the fixed stars in the zodiac for the 22d day of February, according to the Roman calendar, while the 150th year of our Lord was still incomplete" (Prop. 4); "It is clear, however, from Proposition 4 that Claudius Ptolemy established the true places of the fixed stars for 149 complete Roman years and 53 days from the beginning of the years of Christ" (Prop. 20).

Günther's statement that Copernicus assigns the observation under discussion to September 25, 138 (MCV, II, 6.18-19) is clearly erroneous.

[18] HI, 362.9-10, 19-21; the observation was made on the twenty-fifth of Phamenoth.

[19] $885^y\ 203^d$ from Nabonassar to observation

$747\ \ 130$ from Nabonassar to Christ

$138^y\ \ 73^d$ from the birth of Christ to the observation.

Basiliscus in Leo,[20] was the 22d day of February in the 139th Roman year after the birth of Christ.[21] And this was the second year of Antoninus, which our author thinks was C.E. 150. Consequently his error consists of an excess of eleven years.

If anyone is still in doubt and, not satisfied by our previous criticism, desires a further test of this treatise, he should remember that time is the number or measure of the motion of heaven considered as "before" and "after." From this motion we derive the year, month, day, and hour. But the measure and the measured, being related, are mutually interchangeable.[22] Now since Ptolemy based his tables on fresh observations of his own, it is incredible that the tables should contain any sensible error or any departure from the observations that would make the tables inconsistent with the principles on which they rest. Consequently if anyone will take the positions of the sun and moon, which Ptolemy determined by the astrolabe in his examination of Basiliscus, in the second year of Antoninus, on the ninth day of the month Pharmuthi, 5½ hours after noon, and if he will consult Ptolemy's tables for these positions, he will find them, not 149 years after Christ, but 138 years, 88 days, 5½ hours, equal to 885 years after Nabonassar, 218 days, 5½ hours.[23] Thus is laid bare the error which frequently

[20] HII, 14.1-14; for Basiliscus see above, p. 76, n. 56.

[21] 138 years and 73 days + 14 days — 34 days (on account of leap-years) = 138 years and 53 days or February 22, 139. In *De rev.*, where Copernicus considers this observation without reference to any other, he gives the day as February 24 (Th 114.20-22). His computation may be reconstructed as follows. Pharmuthi 9 is the 219th day of the Egyptian year. Subtract 34 days on account of leap-years, and 130 days to get the equivalent day in the Roman year: 219 — (34 + 130) = 55th day of the Roman year, or February 24. Robert Schram's tables, *Kalendariographische und chronologische Tafeln* (Leipzig, 1908), give February 23.

[22] These reflections on the nature of time are an echo of Aristotle's views; see *Physics* iv.11 219b1-2, iv.12 220b14-16, iv.14 223b21-23, and vi.4 235a10-24.

[23]

747^y	130^d	0^h	from Nabonassar to Christ
+138	88	5½	from Christ to observation
885^y	218^d	$5½^h$	

We established in n. 12 (p. 95, above) that when Copernicus wrote the *Letter against Werner* he was using for his references to Ptolemy the translation of 1515. Consulting the tables for the sun in that work (pp. 29r, 33v), we get the following result:

vitiates our author's examination of the motion of the eighth sphere when he mentions time.

The hypothesis in which he expresses his belief that during the four hundred years before Ptolemy the fixed stars moved with equal motion only[24] involves a second error no less important than the first. To clarify this matter and make it more intelligible, attention should be directed, I think, to the propositions stated below. The science of the stars is one of those subjects which we learn in the order opposite to the natural order. For example, in the natural order it is first known that

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810^y	163° 4′ 13″	
72^y	342° 29′ 42″	
3^y	359° 16′ 14″	
210^d	206° 59′ 0″	
8^d	7° 53′ 6″	
5^h	0° 12′ 19″	
$\frac{1}{2}^h$	0° 1′ 13″	
	1,079° 55′ 47″	
initial position	+ 265° 15′	
	1,345° 10′	
	− 1,080°	
	265° 10′	
Gemini	5° 30′	
mean place of sun	270° 40′ or Pisces	0° 40′
anomaly		+2° 23′
true place of sun	Pisces	3° 3′

And Ptolemy states that the true place of the sun was Pisces 3° 3′ (HII, 14.14-16). There is no need to set down here the longer calculations required for the moon, inasmuch as Manitius gives them (*Ptolemäus Handbuch*, II, 397-98). For the method of using Ptolemy's solar tables, cf. A. Rome, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste*, I (Rome, 1931), xxxv-xxxvii.

[24] Werner, Prop. 6: "To prove that the motion of the fixed stars in the zodiac for approximately four hundred years before the era of Ptolemy was nearly uniform and equal. In many passages of his *Great Syntaxis* Ptolemy shows with reference to the motion of the stars that previous to him and to his observation of the fixed stars, they moved for about four hundred years only one degree in each century. Therefore if for four hundred years the motion of the fixed stars completed one degree in each century (*centenariis*, not *centenarios*, as Curtze [MCV, I, 26 n] and PII, 175-76 n), the consequence is that the motion of the fixed stars for four hundred years previous to Ptolemy was nearly uniform and equal." Prop. 8: "Therefore it is clear that the fixed stars moved only with equal motion, and lacked unequal motion; or if they had any unequal motion, it was very small and almost imperceptible."

the planets are nearer than the fixed stars to the earth, and then as a consequence that the planets do not twinkle. We, on the contrary, first see that they do not twinkle, and then we know that they are nearer to the earth.[25] In like manner, first we learn that the apparent motions of the planets are unequal, and subsequently we conclude that there are epicycles, eccentrics, or other circles by which the planets are carried unequally. I should therefore like to state that it was necessary for the ancient philosophers, first to mark with the aid of instruments the positions of the planets and the intervals of time, and then with this information as their guide, lest the inquiry into the motion of heaven remain interminable, to work out some definite planetary theory, which they seem to have found when the theory agreed in some harmonious manner with all[26] the observed and noted positions of the planets. The situation is the same with respect to the motion of the eighth sphere. However, by reason of the extreme slowness of this motion, the ancient mathematicians were unable to pass on to us a complete account of it. But if we desire to examine it,[27] we must follow in their footsteps and hold fast to their observations,[28] bequeathed to us like an inheritance. And if anyone on the contrary thinks that the ancients are untrustworthy in this regard, surely the gates of this art are closed to him. Lying before the entrance, he will dream the dreams of the disordered about the motion of the eighth sphere and will receive his deserts for supposing that he must support his own hallucination by defaming the ancients. It is well known that they observed all

[25] An echo of Aristotle's *De caelo* ii.8 290a17-24 and *Posterior Analytics* i.13 78a30-78b4.

[26] I take it that *omnibus* appears twice by dittography in the texts of Curtze (MCV, I, 27.7-8) and Prowe (PII, 176.15-16).

[27] Prowe reads incorrectly *cum* instead of *eum* (PII, 176.19).

[28] Curtze preferred *observationibus*, the reading of the Vienna MS, to *considerationibus*, the reading of the Berlin MS (MCV, I, 27 n). In his note, which Prowe follows (PII, 176 n), Curtze equates *consideratio* with "Betrachtung" (contemplation), and seems unaware of the use of *consideratio* in the technical sense of "observation" (e.g., Th 192.11, 28; 259.26-27; 261.8; 276.3; 337.25; 338.20-21, 29; 351.32-352.1; 357.19; 365.6-7; 366.2, 6; 367.17; 379.13; 385.6; p. 104, n. 45, below; for an example in the *Epitome* see p. 124, n. 62, below).

these phenomena with great care and expert skill, and bequeathed to us many famous and praiseworthy discoveries. Consequently I cannot be persuaded that in noting star-places they erred by $\frac{1}{4}°$ or $\frac{1}{5}°$ or even $\frac{1}{6}°$, as our author believes. But of this I shall say more below.

Another point must not be overlooked. In every celestial motion that involves an inequality, what we want above all is the entire period in which the apparent motion passes through all its variations. For an apparent inequality in a motion is what prevents the whole revolution and the mean motion from being measured by their parts. As Ptolemy and before him Hipparchus of Rhodes, in their investigation of the moon's path, divined with keen insight, in the revolution of an inequality there must be four diametrically opposite points, the points of extreme swiftness and slowness, and, at each end of the perpendicular, the two points of mean uniform motion. These points divide the circle into four parts, so that in the first quadrant the swiftest motion diminishes, in the second the mean diminishes, in the third the slowest increases, and in the fourth the mean increases.[29] By this device they could infer from the observed and examined motions of the moon in what portion of its circle it was at any specified time; and hence, when a similar motion recurred, they knew that a revolution of the inequality had been completed. Ptolemy explained this procedure more fully in the fourth book[30] of the *Great Syntaxis*.

This method should have been adopted also in studying the motion of the eighth sphere. But because it is extremely slow, as I have said, in thousands of years the unequal motion quite clearly has not yet returned upon itself; and we are not permitted to give a final statement forthwith in dealing with a motion that extends beyond many generations of men. Nevertheless it is possible to attain our goal by a reasonable conjecture; and we now have the assistance of some observations, added since Ptolemy, which agree with this explanation. For what has been determined cannot have innumerable explanations; just as, if a circumference is drawn through three given

[29] Cf. p. 112, n. 13, below.

[30] Chap. ii.

points not on a straight line, we cannot draw another circumference greater or smaller than the one first drawn.[31] But let me postpone this discussion to another occasion in order that I may return to the point where I digressed.

We must now see whether during the four hundred years before Ptolemy the fixed stars indeed moved, as our author says, with equal motion only. But let us not be mistaken in the meaning of terms. I understand by "equal motion," usually called also "mean motion," the motion that is half way between the slowest and the swiftest. We must not be deceived by the first corollary to the seventh proposition. There he says[32] that the motion of the fixed stars is slower when on his hypothesis the equal motion occurs, while the rest of the motion is more rapid and hence would at no time be slower than the equal motion. I do not know whether he is consistent in this regard when later on he uses the expression "much slower."[33] He derives his measure of the equal motion from the following uniformity: in the period from the earliest observers of the fixed stars, Aristarchus and Timocharis, to Ptolemy, and in equal periods of time, the fixed stars moved equal distances, namely, approximately 1° in a century. This rate is given quite clearly by Ptolemy,[34] and is repeated by our author in his seventh proposition.[35]

But being a great mathematician, he is not aware that at the points of equality, that is, the intersections of the ecliptic of the

[31] The foregoing passage, in a slightly altered form, was quoted by Tycho Brahe, as I pointed out in n. 15 on pp. 8-9, above. Then Brahe added: "From these remarks it is clear that Copernicus, who came to the science of astronomy with talents certainly equal to Ptolemy's, thought that it was not utterly useless to construct, from some carefully examined part of its motion, a probable conception of the entire motion of the eighth sphere" (*Tychonis Brahe opera omnia*, ed. Dreyer, IV, 292.20-23).

[32] . . . *motum fixorum siderum tardiorem existere* . . .

[33] Copernicus has placed his finger on a logical difficulty. For Werner the mean rate (Prop. 8: "Hence it can without difficulty be inferred that the fixed stars in their equal motion move only one degree in each century of uniform years. Corollary. Hence it is clear that the fixed stars in their equal motion complete one revolution in 36,000 uniform years.") is also the slowest (Prop. 13: "Therefore the motion of the fixed stars in Ptolemy's time was slower or slowest.").

[34] HII, 36.21-37.2.

[35] The reference should be to the sixth proposition.

tenth sphere with the circles of trepidation, as he calls them,[36] the motion of the stars cannot possibly appear more uniform than elsewhere.[37] The contrary is necessarily true: at those times the motion appears to change most, and least when the apparent motion is swiftest or slowest. He should have seen this from his own hypothesis and system and from the tables based on them, especially the last table which he drew up for the revolution of the entire equality or trepidation.[38]

In this table the apparent motion is found to be, according to the preceding calculation, only 49′ for the century following 200 B.C., and 57′ for the next century. During the first century C.E. the stars must have moved about 1°6′, and during the second about 1°15′. Thus in equal periods of time the motions were successively greater by a little less than ⅙°.[39] If you add the motion of the two centuries in either era, the total for the first interval will fall short of 2° by more than ⅙°, while the total for the second will exceed 2° by about ¼°.[40] Thus again in equal times the later motion will exceed the earlier by about 34′,[41] whereas our author had previously reported, trusting in Ptolemy, that the fixed stars moved 1° in a century. On the other hand, by the same law of the circles which he assumed, in the swiftest motion of the eighth sphere it happens that during 400 years a variation of scarcely 1′ is found in the apparent motion, as can be seen in the same table for the years

[36] Prop. 11: "... the apparent or unequal motion of the sphere of the fixed stars or of the eighth sphere is caused by the circumstance that the first points of Cancer and Capricornus of the ecliptic of the ninth sphere revolve on small circles. This revolution is called by Thābit and the Alfonsine Tables the forward and backward motion or trepidation of the eighth sphere. This trepidation pro-ceeds sometimes in the order of the signs, sometimes in the contrary order. Hence the motion of the fixed stars is sometimes slow and sometimes rapid. It is clear, moreover, that the motion of the fixed stars is composed of the equal motion of the eighth sphere, and the trepidation or forward and backward motion of the ninth sphere on the small circles."

[37] In Prop. 13 Werner states: "Therefore it is clear that the first points of Cancer and Capricornus of the ninth sphere were, about Ptolemy's time, near the aforesaid intersections of the small circles with the ecliptic of the tenth sphere."

[38] This table is placed at the end of Prop. 30.

[39] The successive increases are 8′, 9′, 9′.

[40] 49′ + 57′ = 1° 46′; 1° 6′ + 1° 15′ = 2° 21′.

[41] 2° 21′ — 1° 46′ = 35′.

600–1000 C.E.; and similarly in the slowest motion, from 2060 B.C. for 400 years thereafter. Now the law governing an inequality is that, as was stated above,[42] in one semicircle of trepidation, the one that extends from extreme slowness to extreme swiftness, the apparent motion constantly increases; and in the other semicircle, the one that extends from extreme swiftness to extreme slowness, the motion, previously on the increase, constantly diminishes. The greatest increase and decrease occur at the points of equality, diametrically opposite to each other. Hence in the apparent motion for two continuous equal periods of time equal motions cannot be found, but one is greater or smaller than the other. An exception occurs only at the extremes of swiftness and slowness, where the motions to either side pass through equal arcs in equal times; beginning or ceasing to increase or decrease, they equal each other at those times by undergoing opposite changes.

Therefore it is clear that the motion during the four hundred years before Ptolemy was not at all mean, but rather the slowest. I see no reason why we should suppose any slower motion, for which we have not been able thus far to get any evidence. No observation of the fixed stars made before Timocharis has come down to us, and Ptolemy had none. Since the swiftest motion has already occurred, we are now as a consequence not in the same semicircle with Ptolemy. In our semicircle the motion diminishes, and no small part of it has already occurred.

Hence it should not be surprising that with these assumptions our author could not more nearly approach the recorded observations of the ancients; and that in his opinion they erred by $\frac{1}{4}°$ or $\frac{1}{5}°$, or even $\frac{1}{2}°$ and more. Yet nowhere does Ptolemy seem to have exercised greater care than in his effort to hand down to us a flawless treatment of the motion of the fixed stars. He could be successful only in that small portion of it from which he had to reconstruct the entire revolution. If an error, however imperceptible, entered that whole vast realm, it might have prodigious effects on the outcome. Therefore he seems to have joined Aristarchus to Timocharis of Alexandria, his contemporary, and Agrippa of Bithynia to Menelaus of

[42] Page 100.

Rome; in this way he would have most certain and unquestionable evidence when they agreed with each other, although separated by great distances. It is incredible that such great errors were made by these men or Ptolemy, who could deal with many other more difficult matters and, as the saying goes, put the finishing touches to them.

Finally, our author is nowhere more foolish than in his twenty-second proposition, especially in the corollary thereto. Wishing to praise his own work, he censures Timocharis with regard to two stars, namely, Arista Virginis,[43] and the star which is the most northerly of the three in the brow of Scorpio,[44] on the ground that for the former star Timocharis's calculation fell short, and for the latter was excessive.[45] But here our author commits a childish blunder. For both stars the difference in the distance, as determined by Timocharis and Ptolemy, is the same, namely, 4° 20′ in approximately equal intervals of time; and hence the result of the calculation is practically the same. Yet our author disregards the fact that the addition of 4°7′ to the place of the star which Timocharis found in 2° of Scorpio[46] cannot possibly produce 6°20′ of Scorpio, the place where Ptolemy found the star.[47] Conversely when the same number is subtracted from 26°40′, the place of Arista according to Ptolemy,[48] it cannot yield 22°20′, as it

[43] Commonly called Spica; cf. p. 67, above.

[44] Scorpio 1 (HII, 108.18; Th 137.31), β Scorpii.

[45] Corollary to Prop. 22 (not 27, as in Curtze, MCV, I, 31 n. and Prowe, PII, 181 n): "This is clear from the observations [*considerationibus*] of Timocharis. In the case of the fixed star called Arista, they fall short of my calculation, but in the case of the star which is the most northerly of the three bright stars in the brow of Scorpio, they exceed my computation. However, if these observations [*considerationes*] made by Timocharis had both been true, they should equally fall short of my calculation, or equally exceed it. Therefore the trustworthiness of my tables is not less than that of the observations and discoveries of the ancients."

[46] HII, 32.20-33.1.

[47] HII, 109.18. Werner's remarks are: "The true motion of the fixed stars, in the interval between Timocharis and Ptolemy, will turn out to be 4° 7′ 3″ 28‴. If we add this difference to the true place of the fixed star which is the most northerly in the brow of Scorpio, to the place, that is, which Timocharis found in his observation, the result will be 6° 7′ 3″. But Ptolemy's tables place this star in 6° 20′ of Scorpio" (Prop. 22).

[48] HII, 103.16. This statement enables us to correct the slip in Th 160.1; see n. 14 on p. 112, below.

should,[49] but it gives 22° 32′.[50] Thus our author thought that in the one case the computation was deficient by the amount by which in the other case it was excessive, as though this irregularity were inherent in the observations, or as though the road from Athens to Thebes were not the same as the road from Thebes to Athens. Besides, if he had either added or subtracted the number in both cases, as parity of reasoning required, he would have found the two cases identical.

Moreover, between Timocharis and Ptolemy there were in reality not 443 years,[51] but only 432, as I indicated in the beginning.[52] Since the interval is shorter, the difference should be smaller; hence he departs from the observed motion of the stars not merely by 13′[53] but by ⅓°. Thus he imputed his own error to Timocharis, while Ptolemy barely escaped. And while he thinks that their reports are unreliable,[54] what else is left but to distrust his observations?

So much for the motion in longitude of the eighth sphere. From the foregoing remarks it can easily be inferred what we

[49] HII, 29.9-11.

[50] Not 22° 33′, because Werner is subtracting 4° 7′ 57″: "The difference will be 4° 7′ 57″, the true motion of the fixed stars for the 442 complete Roman years and 350 days between the observations of Ptolemy and Timocharis. If, finally, this value of 4° 7′ 57″ is subtracted from the true place of Arista as observed or calculated by Ptolemy, the remainder is 22° 32′ 3″ of Virgo, the true place of Arista in the zodiac, near the place found by Timocharis in his observation" (Prop. 22).

[51] Prop. 22: "Finally, between this observation of Timocharis and Ptolemy's investigation of the fixed stars there intervened 443 Roman years and 64 days."

[52] In his treatment of Werner's "first error," Copernicus established that Werner postdated Ptolemy by eleven years; cf. pp. 94-97, above.

[53] Prop. 22: "Therefore my tables would diminish the position of this star by 13′." Again, "However, my computation exceeds Timocharis's observation by 12′."

[54] Prop. 22, Corollary: "For this weakens not a little the reliability of the ancient observations of the fixed stars, since some of these observations exceed the computation based on the foregoing canons and tables, while certain of them fall short of this computation. Now if all the results of the ancient observations of the fixed stars coincided exactly with the truth, they should, with perfect propriety, all together fall short of the calculation based on the aforesaid tables, or they should all equally exceed it. But it has been shown above that the ancient observations partly fall short of, and partly exceed the calculation based on my tables." Yet with regard to the length of the year Werner is less confident: "For I do not venture to charge the ancient observers of the stars with any error" (Prop. 33).

must think about the motion in declination, which our author has complicated with two trepidations, as he calls them, piling a second one upon the first.[55] But since the foundation has now been destroyed, of necessity the superstructure collapses, being weak and incohesive. What finally is my own opinion concerning the motion of the sphere of the fixed stars? Since I intend to set forth my views elsewhere, I have thought it unnecessary and improper to extend this communication further. For it is enough if I satisfy your desire to have my judgment of this work, as you requested.[56] May your Reverence be of sound health and good fortune.

NICHOLAS COPERNICUS
Frauenburg, June 3, 1524

To the Reverend Bernard Wapowski,
Cantor and Canon of the Church of Cracow,
Secretary to His Majesty the King of Poland,
my highly esteemed lord and patron, etc.

[55] Prop. 18: "The first trepidation or forward and backward motion is a property of the ninth sphere and its small circles. This trepidation of the ninth sphere is called the first trepidation because, by reason of the variation in the maximum declination of the sun, a revolution or upward and downward movement on small circles must be assigned to the ecliptic of the tenth sphere also. This movement will, then, be named the second trepidation."

[56] We have already seen (cf. pp. 7-8, n. 14, above) how difficult it was for Tycho Brahe to obtain a copy of Werner's treatise on *The Motion of the Eighth Sphere*. His critical comment on it follows: "I have examined it, studied it thoroughly, and set it aside for a reason which I may briefly explain. Werner uses three stars as a basis for dealing with the rest, and from these three he attempts to construct complicated movements of the eighth sphere. He did not carefully observe the three stars in the heavens, as he should have, although he pretends to have done so (I wish, however, to say this with due respect to the memory of a man who was otherwise very learned, and who served the cause of mathematics admirably). Rather, he represented them as he pleased and adjusted them to fit his purpose. This is quite clear from the fact that he retains everywhere the ancient values for their latitudes and nevertheless, assuming his own motions in declination, works out changes in their longitude equal to the accepted account. These views cannot possibly be consistent. For the ancient determinations of the latitudes of these stars do not accord with what is in the heavens, except in the case of Spica alone, where only a single minute is lacking; and the accepted shifts in their longitude do not agree with the appearances. Hence it is clear how the rest of his argument, which he strives to erect not without keenness and subtlety of mind, turns out to be feeble and broken. I make no mention at present of the fact that neither Werner nor the great Copernicus noticed that the latitude of the stars changes in accordance with the shift in the obliquity of the ecliptic (as has been clearly established by me); nor did they explain the displacement in latitude by any hypothesis" (*Tychonis Brahe opera omnia*, ed. Dreyer, VII, 295.23-42; cf. II, 223.29-226.11).